-A New Law of Kinematics-

JOSEPH PALAZZO

Three laws of kinematics are identified. The first two were already known, and are presented with a slightly different perspective. However, the third law of kinematics is a new law of nature. And it states: for every elastic collisions, a free body with the higher kinetic energy can only lose energy from another body with lower kinetic energy, and the lower kinetic energy body will always gain energy from the higher kinetic energy body. Moreover, under general considerations from the theory of Special Relativity, a particle can decay into other particles only if there is a source of positive energy necessary to insure that the new particles have positive kinetic energy, which is why E=mc2 provides the source of this positive energy in the form of mass conversion into energy. Also this new kinematics leads to the reformulation of the 2nd law of thermodynamics which can now explain why heat only flows from a hot body to a cold body, and never the other way around, that a system left on its own will see its disorder increase in time, and that the entropy as defined according to Boltzmann tends to increase. The insight that we gain from this deeper concept which underlies the new law of kinematics and the reformulation of the second law of thermodynamics is that vacuum energy is a necessary condition for particles to pop out of the vacuum. However, this spells trouble for Hawking radiation as it has been formulated - it would lead to a violation of this new kinematics law. Surprisingly this new law of kinematics also sheds new insights into Planck's ad hoc hypothesis, E = ħω, and why energy must be quantized. As a general conclusion, every dynamical theory must reconcile with this new kinematics.

Preliminary

We are considering the case in which no forces of any kind are involved, that is, no work is done on the system under considerations, and no work is done by our system to another system. We just have free particles with kinetic energy. From quantum mechanical considerations, we are not interested in measuring the position of these particles, and so we can know their momentum, and consequently their kinetic energy to any degree of precision we wish. We also assume that there is no exchange of angular momentum (no spin involved).

First Law of Thermodynamics

The 1st law of thermodynamics states that the total energy of a system is always conserved. But it does not specify how the energy is redistributed after an elastic collision. Denote the higher kinetic energy particle as B-particle, B ≡ "Big" and the lower kinetic energy particle as L-particle, L≡ "Little". The 1st law of thermodynamics states after an elastic collision,

$$KE_B + KE_L = KE'_B + KE'_L \qquad (1)$$

Where KE ≡ Kinetic Energy before collision, KE′ ≡ Kinetic Energy after collision.

The Second Law of Thermodynamics

The new law of kinematics states that,

$$KE_B > KE'_B \qquad (2)$$

That is, the B-particle always loses energy after the elastic collision. Rewriting (1) as,

$$KE_B = KE'_B + KE'_L - KE_L \quad (3)$$

Substitute (3) into (2),

$$KE'_B + KE'_L - KE_L > KE'_B$$

$$KE'_L - KE_L > 0$$

$$KE'_L > KE_L \quad (4)$$

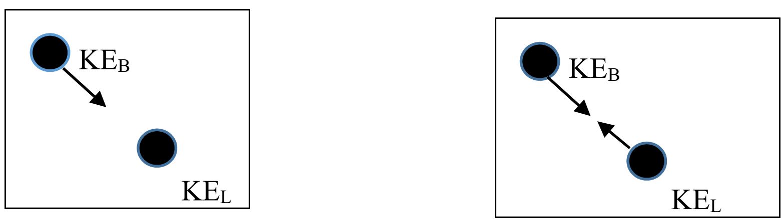
Consequently, after collision the corresponding L-particle always gains kinetic energy.

Special Relativity

At the core of the theory of Special Relativity is that the laws of physics are the same in all inertial frames.

<u>Case A</u>

Before Collision



(a) $KE_L = 0$ (b) $KE_B > KE_L$

Fig. 1

In Fig. 1b, we have a case that could arise in the lab frame. We can always choose a different frame such that the L-particle is at rest, $KE_L = 0$, (Fig. 1a).

After Collision

(a) $KE'_L > 0$ (b) $KE_B > KE'_B$

Fig. 2

We make the claim that a particle can never have negative kinetic energy. From (4), after an elastic collision the lower kinetic energy will then have,

$$KE'_L > 0 \quad (4)$$

And therefore the L-particle must gain energy.

Case B

We can also choose a frame of reference in which the B-particle is at rest. In terms of case A, the B-particle is now the L-particle, with zero kinetic energy. And so equation (4) still stands. That is,

$$KE'_L > 0 \quad \text{with } B \rightarrow L \quad (5)$$

And so, there are no frames of reference in which the L-particle can have negative kinetic energy. And the L-particle must always gain energy. This is the new law of kinematics: in every collision, it will always be the case that the higher kinetic energy body will lose kinetic energy and the lower kinetic energy body will gain kinetic energy. Stated differently: in every frame of reference, the B-particle loses kinetic energy, and the L-particle gains kinetic energy. There are no exceptions.

In what ways this leads to a reformulation of the 2nd law of Thermodynamics?

Heat Transfer from Hot to Cold

A hot body is a system in which the average kinetic energy of its constituents is high, whereas a body is considered cold with respect to the hot body if it has a lower average kinetic energy. When these two systems are placed in contact with each other, the new law of kinematics says that in every collision, B-particles always lose energy, while L-particles gain energy. Given sufficient time, the average kinetic energy of all the B-particles will decrease, while the average kinetic of all the L-particles will increase. This is a manifestation of heat flowing from a hot body to a cold body.

Let us demonstrate why this is so.

Consider that for the hot body, the system is made up of B-particles. For every collision taking place when the two system are in contact, we get the following,

$$KE_{B1} > KE'_{B1}$$

$$KE_{B2} > KE'_{B2}$$

$$KE_{B3} > KE'_{B3}$$

….

$$KE_{Bi} > KE'_{Bi}$$

….

We get,

$$\Sigma\, KE_{Bi} > \Sigma\, KE'_{Bi} \quad (6)$$

The last step was obtained by summing up over all the B-particles. But we're missing one element to work out the average kinetic energy. The temperature is a measure of the average kinetic energy, but we need to divide the sum by the number of particles. Once the two system are in contact, the question is, will the number of B-particles remains constant, or will it change? To answer this, we will use the following illustration.

We will consider a system of three particles where $KE_1 > KE_2 > KE_3$

Case	Collision	KE_1	KE_2	KE_3	N_B	N_L	Total Energy	Ave. Total Energy
1	none	30	20	10	1	1	60	20
2	1↔2	29	21	10	1	1	60	20
3	2↔3	29	20	11	1	1	60	20
4	1↔3	27	20	13	1	1	60	20
5	1↔2	26	21	13	1	1	60	20
6	2↔3	26	19	15	1	1	60	20
7	1↔3	24	19	17	1	1	60	20
8	1↔2	23	20	17	1	1	60	20
9	2↔3	23	19	18	1	1	60	20
10	1↔3	21	19	20	1	1	60	20

Table 1

(a) In case 1, we have no collision. This is our starting point. The energy assigned to the particles are arbitrary and will be sufficient to illustrate what is happening just by applying the new law of kinematics. So we have one B-particle, particle 1 (N_B= 1), and one L-particle, particle 3 (N_L= 1). We ignore particle 2 for reasons that will be obvious.
(b) We notice that particle 2 with a kinetic energy close to the average kinetic energy (Total energy =60, Ave. KE= 20) fluctuates around the average. When it interacts with particle 1, it is the L-particle, and so it gains energy (cases 2, 5, 8). When it interacts with particle 3, it is the B-particle, so it loses energy (cases 3, 6, 9). So we don't count it in the N_B column nor in the N_L column.
(c) Particle 1 is always the B-particle, so it loses energy in every collision.
(d) Particle 3 is always the L-particle, so it gains energy (except for case 10, which we will discuss later). The net result is that for all particles, their kinetic energy tends towards the average kinetic energy.
(e) In case 10, the last entry, particle 2 has less energy than particle 3. We just need to relabel, 2 → 3, and 3 → 2. And so N_B= 1 and N_L= 1.

In light of the reformulation of the 2nd law, we can look at our B-particles, in the system we've designated as the hot body, as particles wearing a tag which reads "B". Similarly, for the cold system, the particles are wearing tags with the label, "L". So now we let them loose in the same room. They start bumping into each other. Suppose in that process that one of the L-particles has gained enough energy so that it has earned to be part of the B-team. Call it the "lucky" particle. All we need to do is switch tags: there is at least one B-particle that has less energy than lucky particle, otherwise our lucky particle isn't "lucky". After the switch, the number of B-particles and L-particles, both remain the same. We can do that for every single collision: either we switch tags or we don't.

We can safely say that the number of B-particles (N_B) remains constant, as well as for the L-particles (N_L). So to calculate the average kinetic energy, we add the energy of each particle divided by the number of particles. Equation (6) becomes,

$$(\Sigma\ KE_{Bi})/\ N_B > \Sigma\ KE'_{Bi}\ /\ N_B \qquad (7)$$

Or

$$\text{Ave. } KE_B > \text{Ave. } KE_B \qquad (8)$$

And this can only hold if N_B is the same throughout the process when the two systems are placed in contact, which our table demonstrates. The average kinetic energy is also a measure of the temperature.

Thermal Equilibrium

In standard interpretation of thermodynamics, equilibrium is defined in terms of an isolated system which is found with equal probability in each one of its accessible states [1]. We redefine thermal equilibrium as a limiting process. We need to examine table 1 and compare each particle's kinetic energy with the average kinetic energy of the combined system. We notice that the difference of the B-particle's kinetic energy with the total system's average energy tends to decrease towards the average kinetic energy of the combined system. We can express that as,

$$\text{Ave. } KE_B \rightarrow \text{Ave. KE of the combined system,} \qquad (9)$$

And also, the difference of the L-particle's kinetic energy with the total system's average energy tends to increase to the same average.

$$\text{Ave. } KE_L \rightarrow \text{Ave. KE of the combined system.} \qquad (10)$$

Since temperature is a measure of the average kinetic energy for any given system, for the combined system in our case, this temperature is the equilibrium towards which the kinetic energy of both the B-particles and the L-particles are moving. The result is that the number of B-particles (and the L-particles) remains constant. When the combined system has finally reached thermal equilibrium, the B-particle (the L-particles) being near the equilibrium will gain (lose) kinetic energy sufficiently, requiring a frequent change of tags, nevertheless, the collisions even as tiny these differences in kinetic energy can get will continue relentlessly, which can be observed as fluctuations. Thus, a thermometer sensitive enough will be able to show those fluctuations.

The combination of the 1st and 2nd laws makes this process necessary, and not just probable. This is a manifestation that heat flows from a hot body to a cold body such that the combined system will eventually reach thermal equilibrium.

Positive Definite

So the question arises, how does this reformulation fit in the scheme of Boltzmann's microscopic states? Boltzmann had to assume that all those states were equally probable, and so it necessitated the use of probability theory in which each outcome is a number between 0 and 1, and the sum of all outcomes is 1. What this means is that Probability theory uses the concept of positive definite – you can't have negative probabilities. It turns out that kinetic energy is also positive definite, [see equation (4)]. However he assumed that the energy distribution could take values that we see would be wrong according to the new law of kinematics. If we look again at table 1, Boltzmann assumed that the B-particles could equally take any value between 0 and 60; similarly with the L-particle, any value with equal probability between 0 and 60 as long as the law of conservation of energy is obeyed. What the new law of kinematics say is that the B-particle in that particular frame of reference can take values with equal probability but only between 30 and 20, while the L-particles can only take those values with equal probability only between 10 and 20.

Needless to say that in a different frame of reference these ranges of values would change. In case A above, the L-particle being at rest would take values of energy ranging between 0 and 20 after collision, while the B-particle would be in the range 20 to 40 in the aftermath of the collision. In case 2, those numbers would be reversed. So we can see that the number of microscopic states available to each particles is restricted and depends on the frame of reference.

Though Boltzmann's work was a step in the right direction, his analysis of ensemble of particles obscured what was happening at the individual level, where one particle collides with another particle. And only by taking into account this new law of kinematics and a reformulation of the 2nd law can bring to the fore why heat always flow from hot to cold.

We can now claim that we've inherited all the mechanism of what has been established in thermodynamics.

We are all familiar with an ice cube left in the open, and finding out later on it has evaporated. The standard description is that the atoms in the cube gain energy from the air molecules sufficient enough to escape their bondage in the cubic crystal. This is often interpreted as the entropy increases, that is, the atoms arranged in the cube in a certain order but later on, as free particles in the air, they have less order, or greater disorder. Often, the belief is that some molecules in the air could join the water molecules in the cube, but that the probability for that to happen is so small, so insignificant that we can ignore that. We are also familiar with the scenario often repeated that the air in the room could all gather in one corner, leaving everyone in the room gasping for air. But have no fear we are told, not to worry that for this to happen, it is very unlikely.

With the reformulation of the 2nd law we can say that the air molecules are the "Big" particles in the room, those in the ice cube are the "Little" particles. The average kinetic energy of the "Big" particle tends to decrease to some equilibrium. Unless the outside temperature is well below zero centigrade, which is the temperature at equilibrium for this case, the air molecules in their tendency to lose kinetic energy up to thermal equilibrium will still have too much energy to be solidified as the water molecules are. So air molecules freezing to join the ice cube is not going to happen.

In the case of the air molecules all moving towards the corner of the room, we would observe an increase in temperature in that corner, and for that to happen, the "Big" particles which have kinetic energies slightly above thermal equilibrium would have to gain energy, and we know this is forbidden. Unless there is a force shooing in these particles into one single corner, no such phenomenon is going to happen.

$E = mc^2$ Revisited

Consider a particle at rest that decays into two particles,

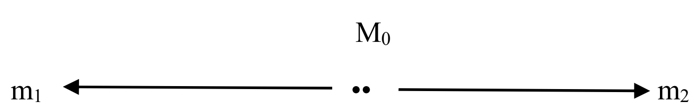


Fig. 3

Initially, the total energy is zero. After it decays into two particles, they will fly away from each other with equal and opposite momentum (conservation of momentum) and with equal and opposite spin (conservation of angular momentum). But they both have kinetic energy, and by the conservation of energy,

$$0 = KE_1 + KE_2 \quad (11)$$

This necessitates that one of the particles would carry NEGATIVE kinetic energy?! However according to the new law of kinematics, this is forbidden. The only way this can happen is that some positive energy is given to these two particles, and Einstein provided the answer in $E= mc^2$, that is, there is a mechanism by which this process can take place. I will make this statement stronger: It would be impossible for decay to occur in nature if mass could not be converted to energy. What Einstein discovered, although admittedly by other means, is that the existing laws of kinematics had to be amended in view of the constancy not only of the speed of light but all laws of physics in all inertial frames by making everyone aware that the Galilean transformations must be replaced by the Lorentz transformations – an idea that was successfully incorporated in Quantum Mechanics to yield Quantum Field Theory. Einstein revised the above equation to read as,

$$(\text{Mass} \rightarrow \text{energy}) = KE_1 + KE_2 \quad (12)$$

More specifically, the above equation is worked out to give for one particle (c=1),

$$E^2 = p^2 + m^2 \quad (13)$$

In the case of a particle of mass M_0 decaying into two particles of mass m_1 and m_2, we get

$$M_0^2 = p_1^2 + m_1^2 + p_2^2 + m_2^2 \quad (14)$$

Rearranging,

$$M_0^2 – (m_1^2 + m_2^2) = p_1^2 + p_2^2 \quad (15)$$

The condition for decay to take place and making sure it obeys the new kinematics, that is, no particle can have negative energy in any frame of reference is,

$$M_0^2 > (m_1^2 + m_2^2) \quad (16)$$

The mass can change as long as it obeys this restriction. The implication is a mass can decay through different channels, an observation that has been confirmed multiple times in high energy physics. However a re-interpretation is necessary in light of the new kinematics: whenever new particles appear, there must be a source of energy such that no new particle can have negative kinetic energy.

Vacuum Energy

It was Schrödinger who came up with the Klein-Gordon equation in his search to describe de Broglie waves. But it gave out negative frequencies and negative probabilities. Hence he abandoned it and used a non-relativistic approximation in his work. Dirac was able to give an explanation in terms of a negative sea with negative particles and the condition that it was all filled up. Occasionally a hole would respond to electric fields as though it were a positively charged particle and predicted the existence of anti-matter, which was discovered subsequently a few years later. Today we treat the positron as a "real" particle rather than a hole or the absence of a particle, and the vacuum is thought as the state in which no particles exist instead of an infinite sea of particles. On the other hand quantum field theory postulates that at every point in space, there is a field made of an infinite number of harmonic oscillators, which are expressed as Fourier series and annihilation and creation operators. With what has been established in this paper, we can say that it is only through this device of a zero-point energy that quantum fluctuations are a possibility. So even though some are uncomfortable with this infinite zero-point energy, it is a necessity as it is a testimony that the new law of kinematics must prevail: no particles can pop out of the vacuum without a source to supply it with energy sufficient enough so that these particles can escape with positive kinetic energy. What remains to be determined is how big this ground zero energy is as it is at variance with Dark Energy by the most mismatched ratio in science history. Hopefully the new law of kinematics and this reformulation of the 2^{nd} law of thermodynamics are steps in that direction.

Hawking Radiation

Hawking assumed that particles can pop out of the vacuum. As it was pointed out above, this process is permissible as long as we take the vacuum energy as necessary to allow this process. And this is further emphasized in the Casimir force and the Lamb shift, to name two instances. In the case of Hawking radiation, one can say that the Black Hole loses energy by absorbing one of the free particle with negative kinetic energy, and the other free particle just outside the Horizon escapes to infinity [2]. In the process, the Black Hole's mass would decrease, and Hawking formulated that its mass would be inversely proportional to its temperature. This is all well within the 1^{st} law of thermodynamics. What is no longer acceptable in the context of this new law of kinematics is that one of the particles has negative kinetic energy. So unless there is an unknown mechanism to provide positive kinetic energy, the process cannot take place. And if there is such a mechanism, that particle would have positive kinetic energy making the argument that the Black Hole loses mass untenable.

The Three Laws of Kinematics

First law

The first law of kinematics is known as Galileo's inertial law of motion, which states: if a body is at rest or in motion, it will continue to do so unless there is a force acting on it. We will represent this as one body viewed in two different frames of reference.

Fig. 4

We have two observers: one is at rest with the body, Frame 1; the other observer is in uniform motion with respect to the same body in question, Frame 2.

Second Law

The second law of kinematics is concerned with how in different inertial frames of reference such as Frame 1 and 2 the laws of nature are transformed. Initially, we had the Galilean transformation equations.

$$x_2 \rightarrow x_1 - vt \quad (22)$$

In the theory of Special Relativity (SR) Einstein made us all aware that in order to have the constancy of the speed of light in every inertial frame of reference, the Galilean transformation equations had to be replaced by the Lorentz transformation equations.

$$x_2 \rightarrow \gamma (x_1 - vt_1) \quad (22)$$

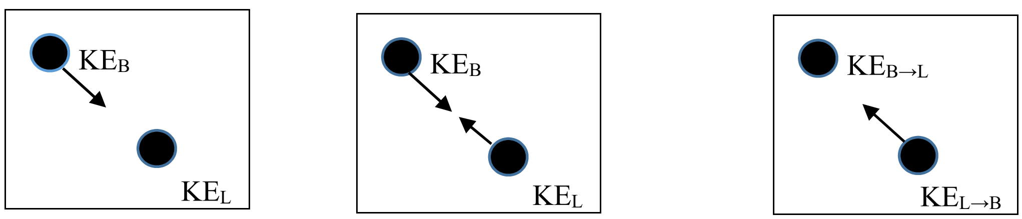
$$t_2 \rightarrow \gamma (t_1 - vx_1/c^2) \quad (23)$$

Where $\gamma = (1 - v^2/c^2)^{-½}$

The surprising result was that time is no longer an absolute quantity but when measured in a given frame it will yield a different time measure compared to another frame, which is generally known as time dilation. What was demonstrated in this paper is that in decay processes. Mass conversion has a restriction expressed as (equation 16),

$$M_0^2 > (m_1^2 + m_2^2) \quad (24)$$

<u>Third Law</u>



(a) $KE_L = 0$ (b) $KE_B > KE_L$ (c) $KE_{B\rightarrow L} = 0$

Fig. 5

This paper is about the new law of kinematics, which now becomes the third law of kinematics, and this can be stated as: after an elastic collision, in every inertial frame of reference, the L-particle always gains energy, and the B-particle always loses energy. This can be stated as:

$$KE_B > KE'_B \quad (25)$$

$$KE'_L > KE_L \quad (26)$$

Where the unprimed quantities denotes before collision; and the primed quantities, after collision. These two equations demonstrates that they are restrictions imposed on the distribution of kinetic energy after an elastic collision.

Center of Kinetic Energy

<u>Definition</u>: The center of kinetic energy is that frame of reference in which both particles, the B-particle and the L-particle, have their kinetic energy equal to the average kinetic energy.

We have established that a system tends to move towards the equilibrium temperature, that is, towards the average kinetic energy of the combined system. There is one frame of reference in which each particle has its KE = Ave. KE. And that is the center of kinetic energy (Fig 6a, below).

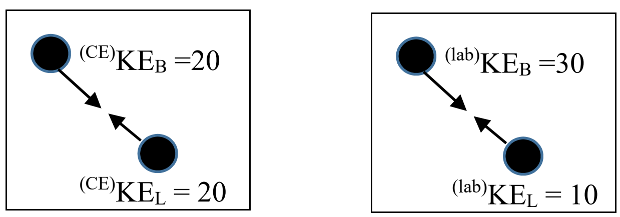
Note: The center of kinetic energy is an idealized frame of reference. It cannot be realized in the real world as one would have to measure the kinetic energy to an infinite precision.

Definition: The deviation is the energy difference between the body's kinetic energy in a given frame and its kinetic energy in the frame of the center of kinetic energy.

Irrespective of how we define our energy scales according to one frame of reference, what matters is that we measure energy differences. And this difference, or the deviation, is frame invariant [3].

So the questions arises: consider the B-particle, which we claim that it will always lose energy in a given collision, could it loses all of its energy to the L-particle?

BEFORE COLLISION



(a) $KE_B = KE_L =$ Ave. KE (b) $KE_B > KE_L$

Fig. 6

Going back to table 1, where case 1 is the starting point with $^{(lab)}KE_B = 30$ and $^{(lab)}KE_L = 10$, (Fig. 6b); while Fig. 6a depicts the situation for an observer in the frame of reference which is the center of kinetic energy. For this case, we have $^{(CE)}KE_B = 20$ and $^{(CE)}KE_L = 20$. In terms of the deviation we write,

$$^{(lab)}KE_B - {}^{(CE)}KE_B = \text{Dev. KE} \quad (27)$$

$$^{(lab)}KE_L - {}^{(CE)}KE_L = -\,\text{Dev. KE} \quad (28)$$

Using the values from fig 4,

$$\text{Dev. KE} = 10 \quad (29)$$

Suppose that the B-particle loses all of its energy to the L-particle. Then what we would see after collision, (after applying equations 27, 28 and 29) is,

AFTER COLLISION (hypothetically)

(a) (b)

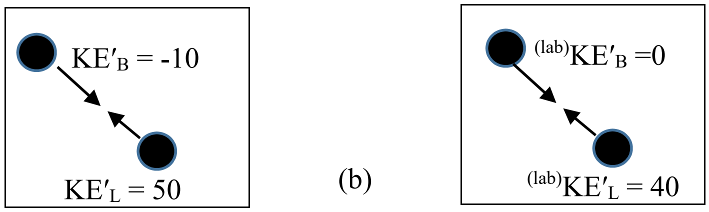
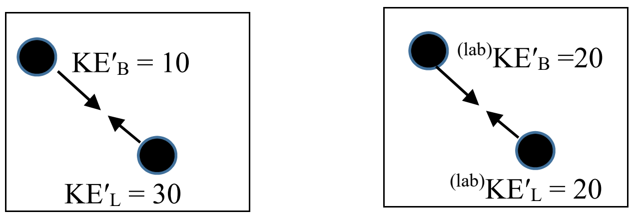


Fig. 7

Note that the average kinetic energy hasn't changed, but Fig. 7a no longer depicts the center of energy, and KE'_B has now negative kinetic energy in that frame, and this is forbidden. And so this is an unphysical situation.

<u>After Collision</u> (considering thermal equilibrium as the limiting process)



(a) (b)

Fig. 8

Note that the lab frame is now the center of energy as the B-party (L-particle) has lost (gained) the maximum to bring it to the average kinetic of the system, and no particle has negative kinetic energy.

Note that the center of energy serves as a litmus check if the 3^{rd} law of kinematics was correctly applied.

And so, we can conclude that the B-particle will lose energy in the lab frame as depicted in table 1, but will never lose all of its energy. The maximum it can lose is up to it reaches the average kinetic energy at the thermal equilibrium.

Quantized Energy

The third law of kinematics makes it imperative that energy at sub-microscopic scales is quantized.

In the exchange of kinetic energy between the B-particles and the L-particles, as it was mentioned before, this leads to thermal equilibrium and produces fluctuations. But why should this process stop there? Why not go all the way until all the B-particles (L-particles) have lost (gained) all of their energy permissible, in which their kinetic energy would then equal exactly the average kinetic energy as depicted in Fig 8b? If that would happen, then all of the B-particles and all the L-particles are identical, in that, they are all have equal in kinetic energy, the average kinetic energy of the combined system, and no exchange of energy is ever possible. In this universe, there are only bouncing balls, in which all the B-particles and all the L-particles are the same. Call that the heat catastrophe. But that's not the real world. The only way out is that there exists a minimum non-zero energy in which the B-particles can still exchange with the L-particles, and in every single collision we either exchange tags or we don't. So we can say that,

$$E = k(?) \quad (30)$$

Where the k is a constant that represents some minimum value, and the ? represents a quantity of which the energy can be a function.

And so what is/are the candidate(s) possible suitable for that unknown quantity?

The Harmonic Oscillator

Quantum Field Theory (QFT) began with the works of Dirac and others, and the key idea was to unify Quantum Mechanics (QM) with SR. By demanding that the Lagrangian is Lorentz invariant (the 2^{nd} law of kinematics), it guarantees that SR is incorporated into the theory. From QM, what was used essentially is the quantized harmonic oscillator – the only problem with an exact solution to the Schrödinger's equation – which contains everything you need in QM.

In our previous assumptions, it was stated that no energy enters our system, no energy leaves our system. It was also stated that if a particle breaks up into two or more particles, a source of energy had to be available to give each particle positive kinetic energy, and that was the mass in the form of $E = mc^2$. In an elastic collision what we're dealing with is not a break-up, but just an exchange of energy. So mass as a possible candidate for our unknown quantity is ruled out. Consider now that our particles are really harmonic oscillators, the only remaining candidate would be their frequencies, and that's what oscillators do – they oscillate at a certain frequency. So we can write equation 30 as,

$$E_{min} = k\omega \quad (31)$$

This would be the minimum quantity of energy that must be exchanged in order to avoid the heat catastrophe. But what still remains to be shown is that the energy is quantized.

Let us assume that in any elastic collision, the energy exchanged can be expressed as,

$$E_{exchanged} = nE_{min} = nk\omega \quad (31)$$

All we need to do is to show that n is an integer.

Normal Modes

And now we go back to Planck and the experiment needed to explain. What was used in those days is that radiation inside a black box was made up of frequencies of all modes. That was permissible as Fourier had successfully demonstrated that such frequencies can expressed as a series of fundamental modes, also called normal modes. More specifically, we will be concerned with the wavelength,

$$c = \lambda\nu \quad (32)$$

Multiply both sides by 2π, and $\omega = 2\pi\nu$,

$$2\pi c = \lambda\omega$$

Rearranging,

$$\omega = 2\pi c/\lambda \quad (33)$$

But these normal modes obey boundary conditions in the box, such that the 1st mode is L = ½ λ, the 2nd mode is, L = λ, the third, is L = 1½ λ… L = (n +½) λ, where n is an integer. Substitute that into equation 33,

$$\omega = (2\pi c/L)\,(n + \tfrac{1}{2}) \quad (33)$$

With this result, we can repackage equation 31 as,

$$E = k\omega = (n + \tfrac{1}{2})\hbar\omega$$

Where ħ is a universal constant known as the Planck constant (in some text, the reduced Planck constant), and the energy is quantized.

Conclusion

There is a new law of kinematics. Stated again: it will always be the case that the higher kinetic energy particle will lose kinetic energy in an elastic collision, and the lower kinetic energy particle will gain energy. As a corollary, a free particle can never have negative kinetic energy. What was explored in this paper is how it gives new perspectives to the 2nd law of thermodynamics which is reformulated as: when two systems at different temperatures are place in contact with each other, the average kinetic energy of the higher kinetic energetic particles will decrease to the limiting point called the thermal equilibrium, while the average kinetic energy of the lower kinetic energetic particles will increase to the thermal equilibrium. This new law of kinematics also explains that decay of particles are possible only because a source is available through Einstein's $E = mc^2$, to give each particle after decay positive kinetic energy. It also provides an explanation why the zero-point energy is a necessary condition for the possibility of particles to pop out of the vacuum. As a general consequence, every dynamical theory must take into account this new revised kinematics.

APPENDIX

Quantum Field Theory

This part is more of a suggestive nature. We can point to where the new kinematics could be playing a role unsuspectedly in QFT. The φ^4-theory was particularly chosen for illustration purposes.

One of the major problems encountered in the development of QFT that had to be overcome was the many infinities that plagued the theory from its onset. This necessitated several methods to remove these infinities that resulted from different considerations. One of these methods is known under the name of adding counter terms. So for a scalar field, you start with a Lagrangian that comes from the Klein-Gordon equation – an equation which is the quantized version of equation (13) and to that, there is an interaction that is added [4],

$$\mathbf{L} = \tfrac{1}{2}(\partial_\mu\varphi)^2 - \tfrac{1}{2}m^2\varphi^2 - \tfrac{1}{4!}\lambda\,\varphi^4 \quad (17)$$

Under the scheme of renormalization, which we will not be exploring in this paper, the counter terms are added in order to subtract the infinities, and so the above equation is modified as,

$$\mathbf{L} = \tfrac{1}{2}(\partial_\mu \varphi_r)^2 - \tfrac{1}{2}m^2\varphi_r^2 - \tfrac{1}{4!}\lambda\, \varphi_r^4$$

$$- \tfrac{1}{2}B(\partial_\mu \varphi_r)^2 + \tfrac{1}{2}A\, \varphi_r^2 + \tfrac{1}{4!}C\, \varphi_r^4 \quad (18)$$

We see that the kinetic term is renormalized as,

$$\tfrac{1}{2}(\partial_\mu \varphi)^2 \rightarrow \tfrac{1}{2}(\partial_\mu \varphi_r)^2 - \tfrac{1}{2}B(\partial_\mu \varphi_r)^2 \quad (19)$$

Similarly for the mass term,

$$\tfrac{1}{2}m^2\varphi^2 \rightarrow \tfrac{1}{2}m^2\varphi_r^2 - \tfrac{1}{2}A\, \varphi_r^2 \quad (20)$$

The interaction term is also modified

$$\tfrac{1}{4!}\lambda\, \varphi^4 \rightarrow \tfrac{1}{4!}\lambda\, \varphi_r^4 - \tfrac{1}{4!}C\, \varphi_r^4 \quad (21)$$

In light of what has been established in this paper, we can see that the K-G equation in its original form was not taking into consideration the new kinematics and had to be modified, in particularly (a) the kinetic term – the distribution of kinetic energy among new particles have restrictions as in equation (2); (b) the mass term – as indicted with equation (16) that mass though it must obey this restriction can assume different values when it converts to energy ; and consequently (3) this new kinematics also affects the interaction (the dynamics) of the theory. The 3rd law of kinematics has the potential of shedding new light on the origin of these infinities.

Reference

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