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Information fusion for natural hazards in mountains

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Abstract: *From a real case application based on snow-avalanche risk management, an integrated framework mixing evidential reasoning and multi-criteria decision analysis (ER-MCDA) is proposed. This methodology considers a simplified decision sorting problem based on qualitative and quantitative criteria on which more or less reliable sources provide uncertain and imprecise evaluations. The Analytical Hierarchy Process (AHP) is used both to model the problem in a conceptual way and to elicit preferences between key criteria. Fuzzy Sets and Possibilities theories are used to transform quantitative and qualitative criteria into a common frame for Dempster-Shafer Theory (DST) and Dezert-Smarandache Theory (DSmT). It is shown that DSmT offers an interesting framework to take incomplete information into account and we use it for decision-making. Evidential reasoning allows merging different uncertain and incomplete pieces of information to identify the sensitivity of an avalanche prone area and to determine an avalanche hazard map. This approach emphasizes some implementation guidelines based on a Unified Modeling Language (UML) of the problem. We point out also some important issues of information fusion such as basic belief assignment elicitation, conflict identification, fusion rules choice and results validation.*

23.1 Introduction

23.1.1 Natural hazards in mountains

How and why expertise is needed in the risk management process?

Natural hazards in mountains such as snow avalanches or floods threaten human or material stakes with sometimes dramatic consequences including damages for people and material assets (see FIG. 23.1).

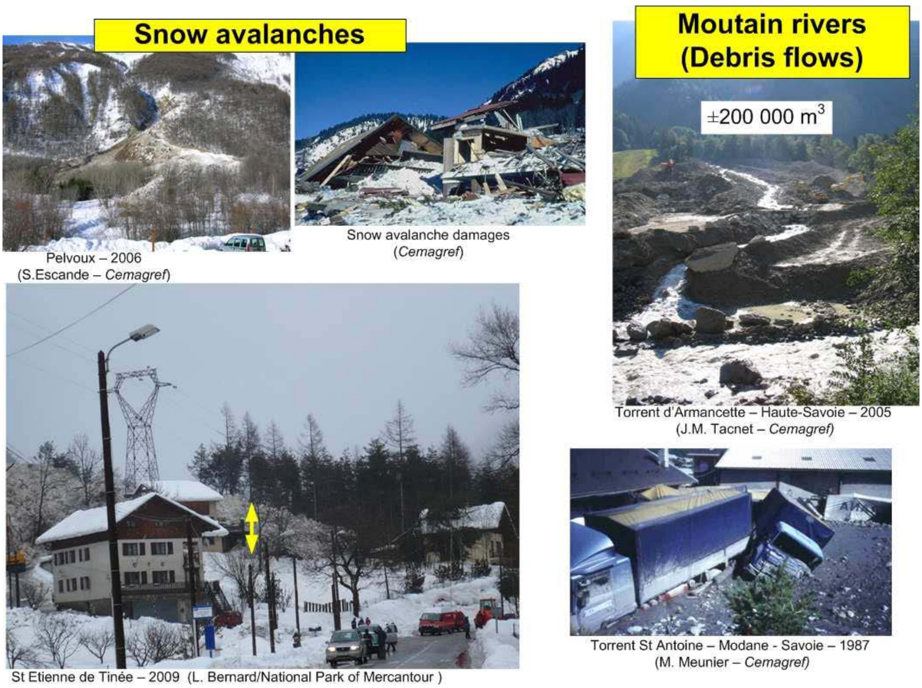


Figure 23.1: Examples of natural hazards in mountains.

The effects of physical phenomenon on existing stakes such buildings, persons, infrastructures are cross-analyzed with their temporal occurrence. In a classical way, risk can be considered as a combination of *Hazard level* and *Vulnerability* (see FIG. 23.2):

- **Hazard level** represents the physical effects of a natural phenomenon described through its intensity and frequency. This produces a hazard level factor mixing frequency and intensity. For a snow avalanche, the effects can be snow deposition, impacts of avalanche and/or blocks, trees carried by the flow, etc. For debris flows, the effects can be the static and/or dynamic pressure due to the height of fluid, the impacts of blocks, etc. The more intense and frequent is the phenomenon, the higher will be the hazard level. A same hazard level can be due either to a very frequent phenomenon with low-medium intensity or to a rare event with high intensity (potential effects);

- **Vulnerability** represents the consequences due to the direct physical or indirect effects of the phenomenon on people, material assets, organization. These consequences correspond to losses or damages which are first described in a physical way and then valuated according to their economic value for material assets and evaluate a risk level.

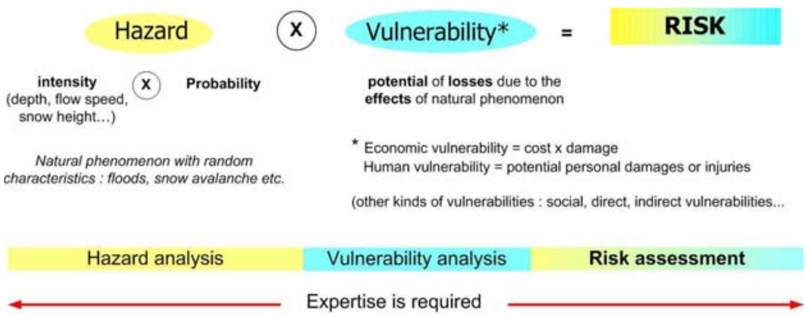


Figure 23.2: Risk is a combination of hazard and vulnerability.

Risk management can be also viewed as a decision process : in a given situation, several strategies do exist to reduce the level of risk [Tacnet and Richard 2008]. Prioritization and choice have always to be done by the decision-makers (ministries, local authorities, private companies or technical staff involved in risk management). The *risk management process* can be considered as a combination of decisions related both to the temporal steps of the physical process and to the functional steps of the risk management framework in itself. Therefore, decision support systems are helpful to propose synthesis of the different criteria involved in the decision. To a certain extent, the decision process when it dysfunctions can also induce disasters [Weichselgartner and Bertens 2000].

23.1.2 Experts are expected to manage and integrate the overall uncertainty

In a natural hazard context, the practical implementation of these principles will concern approaches going from physical phenomenon description (the risk analysis) to risk evaluation and management. Risk analysis begins with the hazard assessment. It requires first to identify the phenomena and the physical processes such as triggering, propagation and deposition. These processes correspond to the successive temporal steps of phenomena. This begins with a qualitative description of the different phenomenon that have already occurred or that may occur in the risk basin. For each step, different characteristics related to the possible effects are analyzed by the experts. For avalanche risk analysis, experts collect and choose parameters that are used to define the intensity and characteristics of the reference phenomena: the expertise process can be seen as a serial of decisions related to the different parameters (see Fig. 23.3). In a second step, frequency of the phenomena is evaluated. Data sources are historical information, pictures, hydrological chronicles, topographic information. Risk analysis consists afterwards in the estimation of consequences on exposed people and assets.

All over this expertise process, uncertainty arises both from expert basic knowledge of the different phenomena, the intermediate tools such as models, the expert evaluations for data collection and finally from the decision step. In most of cases, choosing limits on continuous physical values does not make much sense: if a natural slope is supposed to highly contribute to the sensitivity level of an exposed site and if its inclination is over 30%, what should we think of a 28% slope? Reasoning on classes with artificial thresholds does not correspond to the reality. In the natural hazards risk management context, there is a great need for tools and methodologies that allow considering both uncertain and imprecise information. Specific needs and developments about uncertainty in natural hazards risk management processes concern floods [Apel et al. 2004, Van Der Most and Wehrung 2005], rock-falls in relation with spatial data accuracy [Dorren and Heuvelink 2004], debris-flows [Lin et al., 2004] or snow-avalanches [Barbolini and Savi 2001].

This justifies the development of general framework to consider decision and uncertainty.

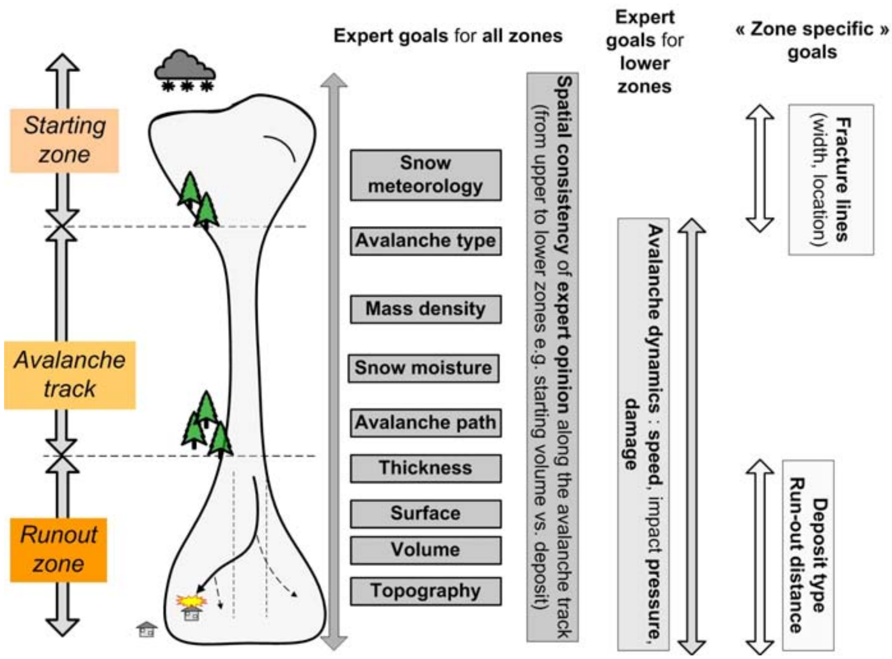


Figure 23.3: Expertise required during the hazard analysis step.

The problem complexity requires using different approaches to analyze the risk situation: descriptive and qualitative approaches are used as well as numerical modeling. In many cases, they must be considered as complementary [Ancey 2006]. Involving experts whose backgrounds, methods are different is as useful as necessary to capture all the complexity of the studied phenomena [Lacroix 2006]. Natural hazards expertise consists in a complex framework involving several decision levels based on incomplete and uncertain information (see Fig. 23.4). Expertise is required to fill the gap between the needs and the available knowledge. This lack of knowledge can exist at different stages of the *risk management process* and can be due to incomplete historical information describing the extension area [Tacnet et al. 2006], lack of scientific knowledge, unknown phenomenon scenarios but also to insufficient means (time,money) for risk analysis and evaluation, etc.

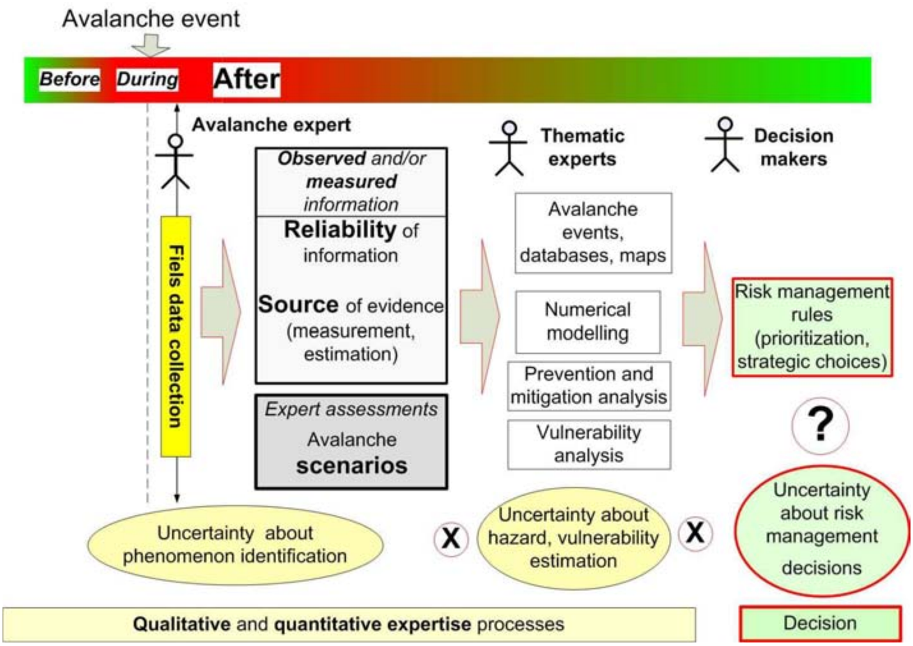


Figure 23.4: The uncertainty comes from the different steps of the expertise.

Expertise is therefore the result of multiple thematic evaluations based on more or less reliable and conflicting sources. All the different steps of the expertise are based on uncertainties that will influence the final result. At the end, the whole expertise process appears as a sequential process ranging from primary and more or less uncertain data to the processed data (or decision) is quite difficult to trace in a detailed way. It is therefore possible to settle decision on very uncertain hypothesis without being really able to know it precisely (see FIG. 23.5). Even when advanced tools such as numerical modeling are used for hazard and risk assessment, the experts always never consider the results directly as decisions but always interpret to provide an operational result [Tacnet et al. 2005a]. This reality corresponds to the difference between decision-aid and decision-making [Roy 1990].

23.1.3 A more realistic description of the expertise process

Expertise is expected to help for decision-making in poor available knowledge conditions but appears as a very paradoxical and difficult exercise. Uncertainty and imprecision do exist on their main steps because of lack of data and knowledge without being clearly elicited. Final results often come from various sources whose reliability and mutual conflict are not easily traced all along the technical decision process.

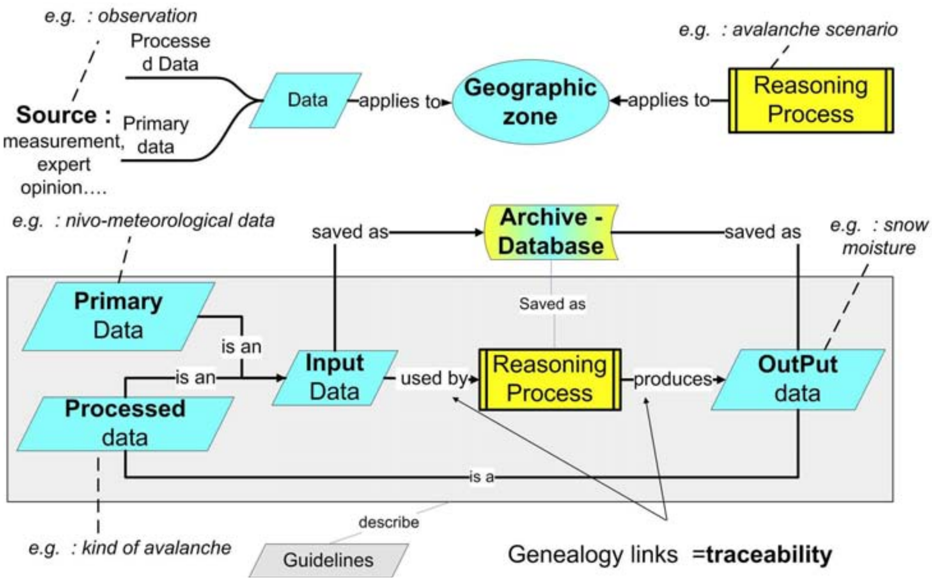


Figure 23.5: From primary data to processed data in the expertise process.

Uncertainty does not affect equally all the decision parameters which are themselves known to be more or less important.

Three main questions can therefore be pointed out:

- Can we find theoretical frameworks that would help decision-making and would be able to represent in a more realistic way the available knowledge level, the reliability of sources and the uncertainty of their evaluations?
- How far can we be confident in the expertise results? How can we make a link between a decision and the way it was obtained: what is the global confidence in the result? do all the sources agree with this result (in particular when results come from contradicting positions and criteria)?
- Assuming that we are able to describe and evaluate the uncertainty sources, how can we make a decision that would be considered?

This chapter proposes an alternative methodology to the classical risk evaluation method used in the natural hazards mountains management. It is based on a combination of a multi-criteria hierarchical method and Evidence Theory based approaches. We present a mixed framework involving both information fusion and multi-criteria decision analysis (MCDA) in the context of natural hazards in mountains. In the next sections, we focus on the different ways to introduce evidential reasoning in a multi-criteria decision analysis model. In Section 23.2, we briefly remind of formal theories to manage uncertainty insisting on the advantages of the DST and DSmT. We also present multi-criteria methods and the way they can use or consider uncertainty in the DST context. Section 23.3 focuses on methodology used to mix multi-criteria approach and information fusion. Section 23.4 deals with two applications. The first one is a simplified version of a global framework to analyze the exposure level of a snow-avalanche prone area. The second one relates to the risk zoning methodology and focuses on specific points related to spatial applications. Section 23.5 is a general discussion and section 23.6 is the conclusion.

23.2 Backgrounds on MCDA and evidential reasoning

Managing uncertainty requires being able to analyze its sources, to evaluate it and to propagate it through the evaluation process. This section briefly presents existing approaches for decision based either on multi-criteria decision analysis (MCDA) and evidential reasoning (ER).

23.2.1 Multi-criteria decision analysis

23.2.1.1 MCDA methods

MCDA is usually used in cases where optimization is not efficient.

In the decision theory, the first theory developed in Economics [Von Neumann and Morgenstern 1967], the concept of *decision under risk* corresponds to situations where objective probabilities of events can be calculated. in that context, the decision relies on the maximum of expected utility. Due to the complexity of real-life problems and the limited rationality of human decision, the concept of utility and optimum for decision have been criticized [Scharlig 1985, Roy 1989, Climaco 2004] leading to the development of alternative methods for decision-making known as multi-criteria methods. Decisions support systems based on multi-criteria paradigm try to reach a compromise through various aggregation methods. Several methods are available to produce an evaluation of solutions or alternatives but none of the numerous existing multi-criteria decision aid methods can be considered as a perfect and universal method that would be appropriate for any decision problem. A comparative analysis has been handled by [Guitouni and Martel 1998] to propose some guidelines for choosing the ad-hoc method. Another review is proposed by [Linkov et al. 2006] in the context of environmental comparative risk assessment.

Multi-criteria decision analysis mainly focuses on reaching a compromise between those sources. Most of existing methods have been initially developed to consider only one decision maker [Jabeur and Martel 2005]. Others approaches related to group-decision making consider the case of several decision-makers. In such cases, compromises are searched between at the valuation level.

A complete review of all the MCDA methods would be difficult. Two main classes of methods can be distinguished: those whose final evaluation is the result of a complete aggregation process as *Analytic Hierarchy Process* (AHP), *Multi Attribute Utility Theory* (*M.A.U.T.*) and those based on an incomplete aggregation process or outranking methods such as *ELECTRE* or *PROMETHEE*. The first category of methods is widely used in Anglo-Saxon community which is sometimes described as the "MCDA [1] American school" (MCDA). The second class corresponds to the so-called "MCDA European school". The complete aggregation methods have been criticized notably because they do not consider un-comparability and preferences un-transitivity. [Guitouni and Martel 1998] proposes some guidelines to choose a MCDA framework between all the existing methods. We only cite here elements of comparison between three advanced MCDA methods [Linkov et al. 2006, Guitouni and Martel 1998]:

- **MAUT or MAVT:** The Multi-Attribute Utility Theory (MAUT) [Keeney and Raiffa 1976] or Multi-Attribute Value Theory (MAVT) is certainly the MCDA method which looks like the classical decision theory in a closer way. MAUT relies on the hypothesis that decision-maker is rational (he prefers more an higher utility level than a lower one), that he has perfect knowledge and that he is consistent in his judgments. For each attribute, the decision maker must be able to propose a utility function (using as example indirect methods such as UTA);

- **AHP:** The Analytic Hierarchy Process (AHP) [Saaty 1980] is a single synthesizing criterion approach. It uses pairwise comparisons with a semantic and ratio scale to assess the decision maker preferences. The axiomatic foundations suppose that there must be outer and inner independence between the different hierarchical levels.

- **ELECTRE:** This outranking synthesizing method [Roy 1968] is based on the principle that one alternative may have a degree of dominance over another. Dominance occurs when one option performs better than another on at least one criterion and not worse than the other on all criteria. These methods accept and manage potential un-comparability between different criteria through as an example, the principle of discordance in ELECTRE methods.

[1] multi-criteria decision analysis

Three main problematics are identified to describe the MCDA methods which are presented in Fig. 23.6 below.

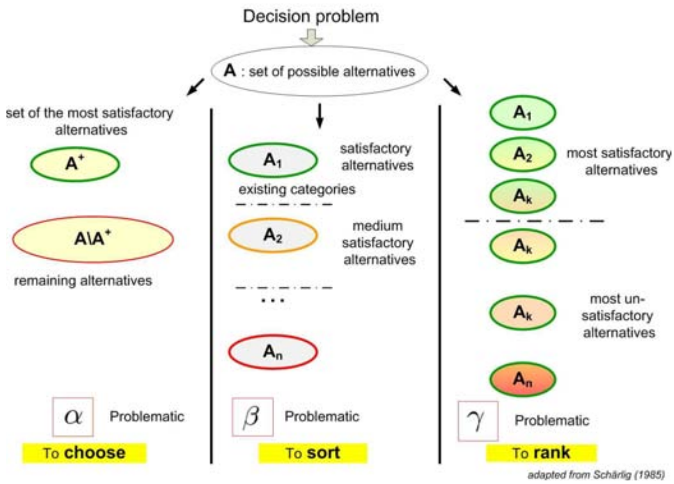


Figure 23.6: Main problematics addressed by MCDA methods [Scharlig 1985].

The main steps of a multi-criteria analysis can be summarized as follows:

1. decision purpose identification;
2. criteria identification;
3. preferences between criteria;
4. evaluation;
5. sensitivity analysis with regard to weights, thresholds, . . .

MCDA: an useful tool to aid decision and elicit the natural hazard expert reasoning process.

From a conceptual point of view, Risk evaluation is based on a combination of hazard and vulnerability. In most cases, this combination appears more as an expert choice than a real deterministic process based on a precise quantification. This is due both to the uncertainty attached to the two parts of the global risk equation. In Risk Prevention Plans, expert choices are often the main sources for risk zoning. A so-called risk equation is supposed to be used but in fact its terms are not evaluated on the same scale. Some recent progress does exist with the use of deterministic modeling in connection with protection works. A risk level can be calculated and optimized using Bayesian probabilistic framework. The risk level is optimized on the basis of a utility economic function.

23.2.1.2 The original Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP) method is world-wide used in almost all applications related with decision-making [Vaidya and Kumar 2006]. AHP is a special case of complete aggregation method and can be considered as an approximation of multi-attribute preference models [Dyer 2005]. Its principle is to arrange the factors considered as important for a decision in a hierarchic structure descending from an overall goal to criteria, sub-criteria and finally alternatives in successive levels (see Fig. 23.7). It is therefore based on three basic principles: decomposition of the problem, comparative judgments and hierarchic composition or synthesis of priorities.

At each level, a preference matrix is built up with pairwise comparison between the criteria of each level [Saaty 1982, Saaty 1990]. Through the AHP pairwise comparison process, weights and priorities are derived from a set of judgments that can be expressed either verbally, numerically or graphically [Forman and Selly 2002][2]. It can be considered as a kind of conjunctive consensus between different criteria evaluation. The original AHP method uses an additive preference aggregation.

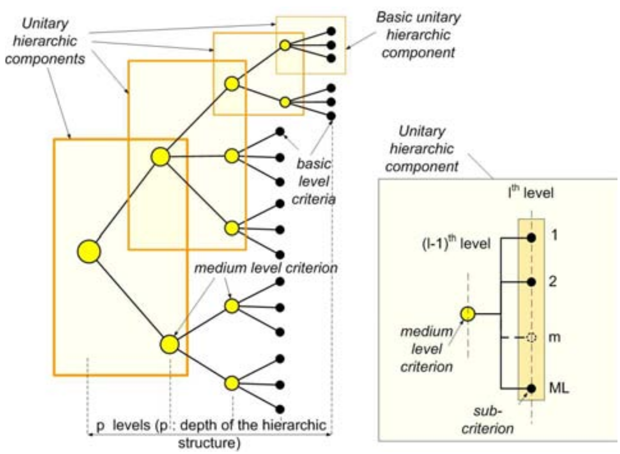


Figure 23.7: A multi-criteria hierarchical structure is broken down into unitary hierarchic components.

[2] p.45

The final evaluation index is the result of a sum of products of weights from the tree root to the leaves (see Fig. 23.8). At the leave level, the evaluation expert has to choose in an exclusive way between several classes.

To implement the AHP method, two different strategies can be used to provide valuations of alternatives on which we want to make a decision. The original AHP process consists in comparing the solutions from one to each other in a so-called "Criterion-alternative approach". This implies to make pairwise comparisons between all the solutions or alternatives in order to obtain preferences levels between these alternatives. A methodology based on a relative verbal scale is provided to calibrate the numeric scale for measurement of quantitative as well as qualitative performances (see Fig. 23.9). When dealing with great amount of data, this becomes quickly quite difficult. The preferences are here the result of a comparative approach of solutions according to criteria. It is impossible to calculate an index or a rating value for a unique solution.

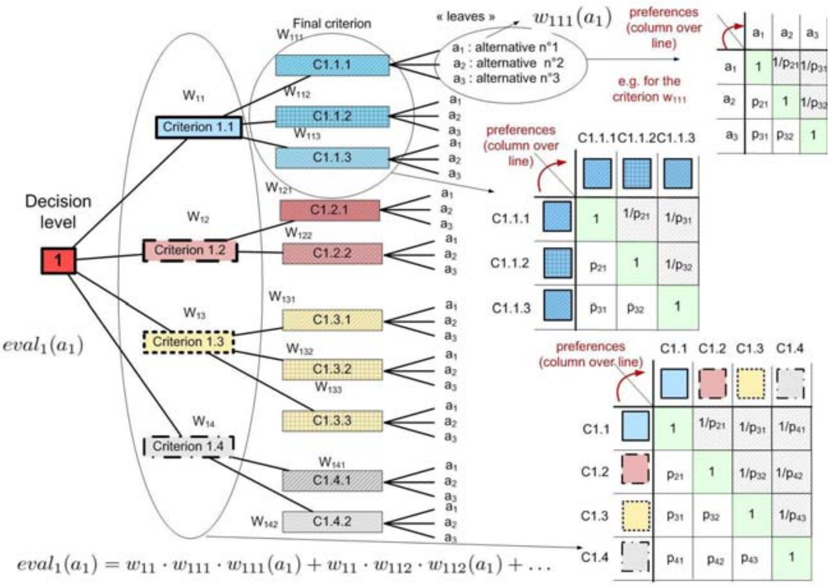


Figure 23.8: Principle of the Analytic Hierarchy Process (AHP).

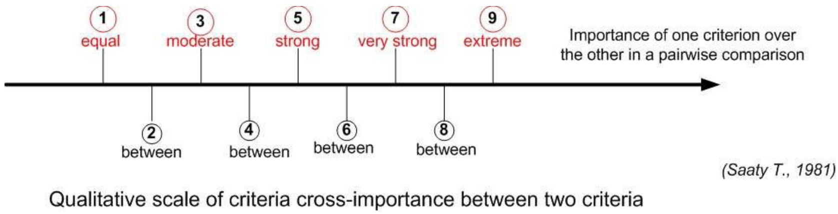


Figure 23.9: Saaty's verbal scale for pairwise comparison.

A second approach so-called "Criterion-index (or estimator)-alternative" can be imagined (see FIG. 23.10). Instead of comparing all the alternatives, the decision analyst proposes classes for each criterion. To a certain extent, theses classes correspond to an increasing or decreasing level of satisfaction of a given criterion. These classes code some kind of ordinal levels corresponding to a low, medium or strong contribution (or satisfaction) to (or of) the criterion. For example, the criterion *human vulnerability* exposed to natural hazards can be assessed according to three classes based on a number of existing and exposed buildings. This approach prevents from the well-known "Rank reversal" problem of the AHP method [Wang and Elhag 2006]: introducing twice the same alternative modify its relative rank compared to all the others unchanged alternatives. In that way, the AHP method, despite the known issues of complete aggregation methods, fits quite well to decision ranking problems where the alternatives are not all known.

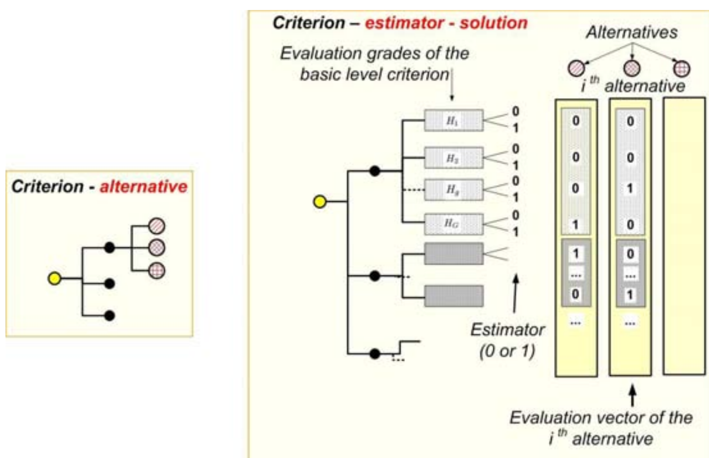


Figure 23.10: Criterion-Alternative and Criterion-Estimator-Solution approaches.

23.2.1.3 Uncertainty and MCDA methods

Uncertainty and imprecision in multi-criteria decision models has been early considered [Roy 1989]. Different kinds of uncertainty can be considered: on the one hand the internal uncertainty is linked to the structure of the model and the judgmental inputs required by the model, on the other hand the external uncertainty refers to lack of knowledge about the consequences about a particular choice. Forma modeling of uncertainties is necessary when risk and uncertainties are as critical as the issue of conflicting management goals [Stewart 2005].

Several different techniques have been used to manage uncertainty in the MCDA process. Fuzzy approaches have been introduced either in the *analytic hierarchy process* (AHP) context [Salo and Hamalainen 1995, Salo 1996], in the multi attribute value theory (MAVT or MAUT) preferences ratios based methods [Salo and Haimailaiinen 2001]. Interval judgments are introduced as a an easy way to handle imprecise information [Mustajoki et al. 2005].

Fuzzy sets theory is used to consider, according to [Fenton and Wang 2006], risk and confidence of a decision maker in a multi-criteria decision making problem. Fuzzy number are then used to valuate the performance index (weights) of the criteria ("risk attitude" depending on the decision attitude of the decision maker ranging from an optimistic to pessimistic) and to valuate the alternatives denoted as a "confidence" level. Fuzzy approaches have been introduced into the AHP to valuate the alternatives [Kuo et al. 2006, Pan 2008, Dweiri 1999]. This method can be (has already been) criticized [Linkov et al. 2006][3] notably on the basis of the aggregation issues and its ability to deal with uncertainty [Forman 1993]. [Saaty and Vargas 1987] has studied the way to consider uncertainty in the AHP process but considers that such an approach of fuzzifying the numerical judgments used in AHP has no interest since the numerical values used for pairwise comparisons already correspond to some fuzzy evaluation by the decision-maker [Saaty and Tran 2007]. Taking perturbations or catastrophes into account in the decision process was an earlier issue recognized by [Saaty 1990][4]. He suggested to always including a criterion that would gather all what is unknown and represent a cluster of unforeseen threats in the decision model. He also considers that the AHP is able to manage uncertainty through its ability to elicit the subjective probabilities [Ozdemir and Saaty 2006].

More recently, the question of decision under risk has been addressed by Matos in [Matos 2007]. He suggests a two-step decision method. The first step consists in the evaluation of the alternatives according to their uncertainty level using different theories such as classical probabilities, fuzzy sets theory. The second step uses multi-criteria methods to interpret the result. He advocates that the *"transformation of a decision problem under uncertainty into a deterministic multi-criteria problem provides more meaningful information to the Decision maker"*.

[3] p. 1076
[4] p.23, §10

The integration of multiple criteria decision analysis and scenario planning is presented as a future way of development. Scenario planning is a *"technique for facilitating the process of identifying uncertain and uncontrollable factors that may impact the consequence of decision in a strategic management context"*. Integration of external uncertainties in a mixed approach using MCDA and scenario planning is still a research challenge [Stewart 2005]. On this basis, the methodology proposed in the following sections tries to follow the main previous guidelines and principles:

- to evaluate the uncertainty about input evaluation (using the different theories for uncertainty and imprecision) and inject those results into a decision-aid method;

- to propose a scenario-based approach that would remain understandable for decision-makers. This scenario planning approach fits perfectly to the context of natural hazards where knowledge and objective probabilities are often lacking.

23.2.2 Evidential reasoning (ER)

Several theoretical frameworks exist to handle uncertainty in human reasoning and decision processes.

Three main theories are mainly used to handle uncertain and incomplete information in a decision process: probabilities, possibilities and evidence theories. Classical probabilities are the traditional tool for situations of incomplete information. Most of time, the decision processes used for risk evaluation supposes that objective probabilities are available for each component of the risk. This principle is considered as imperfect since probabilities and data used for numerical modeling often result from expert assessments. Moreover, these expert opinions in an uncertain context are known to be influenced by cognitive biases leading to different types of risk aversion [Ellsberg 1961]. For environmental or sustainable development related problems, other decision models are required: they should consider, from one hand, the risk evaluation step and from the other hand, the decision process itself [Magne and Vasseur 2006][5].

Probabilities are criticized especially when they are known to be highly subjective. Recent developments have studied the use of probability-possibility to improve decision making under uncertainty in the classical decision theory framework [Gajdos et al. 2008]. Subjective approaches of probability have been recently proposed according to Bayesian approaches (probability on probability law parameters). This Bayesian framework can take this subjectivity into account in a rigorous and axiomatically based framework. Soundappan et al. [Soundappan et al. 2004] states that Bayesian framework and evidential reasoning can be used to model uncertainty

[5] Chapter 12, p.397

and safety of a model when the available evidence consists of intervals bounding the values of input variables. Bayesian approach has recently been applied to snow avalanches context using large available data bases about avalanches extension to optimize the size of a passive avalanche defense structure [Eckert et al. 2008, Eckert et al. 2008a]. The decision application is based on (mostly economic) optimization principle. This "optimum" resulting from a complex calculation process is proposed as a unique result to the decision maker.

When data are not available, when experts judgments are essential part of the expertise process or to capture the reasoning hypotheses, this powerful probabilistic framework is not fully adapted. Alternative methods may be useful to complete this approach. Though probabilities remain the traditional tool and powerful tool for uncertainty management (as long as data are certain and available), others theories for uncertainty fit quite well to the context of expertise for natural hazards in mountains. *Fuzzy Sets*, *Possibilities* and *Belief Functions* theories can be used in the natural hazards management context to consider information at its effective level including uncertainty, imprecision, heterogeneity and reliability of sources. Nevertheless, evidential reasoning which has already been widely used in domains such as classification, cartography, expert systems, decision-making, . . ., as reviewed by [Sentz and Ferson 2002], has quite few applications in the natural hazards context [Binaghi et al. 1998].

Our methodology explores a way to introduce evidential reasoning and its more recent developments such as DSmT theory in decision processes related to natural hazards management. Main goals are to make decision but also to trace the reasoning process used by the experts to build their judgments in the complex and uncertain context of natural hazards in mountains. The following section presents some notions about evidential reasoning. Basic principles of the belief function theory, and specifically *Dempster-Shafer* (DST) and *Dezert-Smarandache* (DSmT) theories are widely and extensively described in the others chapters of this volume, and will not be described again. We will only focus on some interesting features and specificities of these theories in relations with our application context. Secondly, the fusion rules still work even in a high level of conflict between sources. The DSmT theoretical framework appears as quite versatile but in fact it is quite difficult to find applications based on a non-exhaustive frame of discernment.

What is fusion?

Information fusion consists of merging or exploiting conjointly, several sour-ces of information so as to answer questions of interest and make proper decisions [Dubois and Prade 2004]. The following definition was proposed: *"Fusion consists in conjoining or merging information that stems from several sources and exploited that conjoined or merged information in various tasks such as answering questions, making decisions, numerical estimation, ... "* from European working group *FUSION* cited in [Bloch et al. 2001].

In practice, fusion is operated through fusion rules that allow aggregating the more or less uncertain information issued from the different sources. The DST framework is based on exhaustive and exclusive hypotheses while DSmT framework does not require such constraints (e.g. in Fig. 23.33). In comparison with other theories, DST and DSmT offer a wide and powerful range of fusion methods to aggregate the different basic belief assignments (bba). An exhaustive review of the fusion rules has been proposed by [Sentz and Ferson 2002]. Their analysis also provide a valuable summary of the elements under consideration in a combination problem in DST context (see Fig. 23.11).

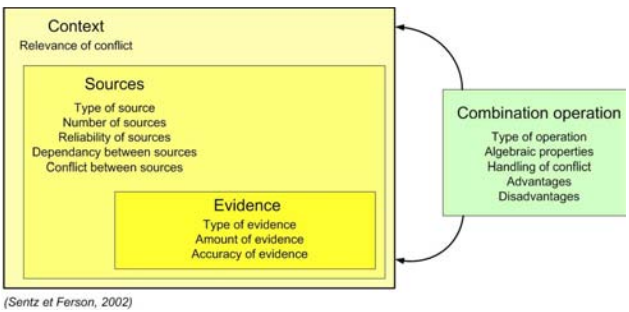


Figure 23.11: Elements under consideration for the fusion with DST.

For [Haenni 2002], there is no need for alternative fusion rules to classical Dempster's fusion rule, refining the model is sufficient. Such argumentation doesn't hold because the refinement becomes very hard to do when the cardinality of the frame of discernment and the number on non-empty intersections increases (the model's complexity increases), and the elements of the refined space can have no physical sense/meaning/existence at all and finally they cannot truly be considered as useful finer exclusive information granules. Moreover, several authors such as [Smarandache and Dezert 2006b] and [Martin and Osswald 2006] show that alternative fusion rules perform better than the classical Dempster's fusion rule in high conflict situation. For this reason, we will compare in our applications the classical normalized Dempster fusion rule with proportional conflict redistribution rule such as PCR6 rule. To illustrate the conflict level, we will also use in our application Smets' rule which transfers conflict on the empty set. From a general point of view, the fusion process depends on a great number of elements (see Fig. 23.11). A fusion approach used in a decision context implies four steps: modeling (often considered as the most difficult step), parameters estimation (depending on the model), combination and decision [Martin and Osswald 2006]. In the following section, we briefly analyze the existing approaches that use both MCDA and evidential reasoning.

23.2.3 Mixing MCDA and evidential reasoning

Trying to mix multi-criteria decision analysis (MCDA) and evidential reasoning (ER) quickly leads the question of the difference between aggregation of preferences and information fusion and therefore to the validity of an analogy between aggregation of preferences and data fusion. Data fusion is considered as a way to extract the truth between a set of hypothesis evaluated by different sources. Those two problems are considered as different: aggregation problem consists in deriving a global preference profile corresponding to a consensus between the preference profiles induced by the various sources [Dubois and Prade 2004][6]. Fusion and aggregation should be considered as mainly different problems [Dubois and Prade 2006] while some applications do not make such a difference between the two application domains [Dubois et al. 2001]. Despite of these analysis, many authors have already introduced evidential reasoning (fusion) in MCDA frameworks (based on aggregation of preferences).

23.2.3.1 Existing approaches

Evidential reasoning and multi-criteria decision analysis have already been used in a common framework. In these approaches, data fusion is mainly applied either to the criteria considered as sources of a fusion process. Our analysis briefly focuses here on four main points: How do these models consider the complexity and the implicit hierarchy between criteria? How does the analyst extract the basic belief assignment elicitation? How is considered the difference between the importance and the uncertainty level linked to each criterion? Which fusion rules are used? Do they consider conflict? ER has been already combined with multiple attribute decision analysis (MADA) problems of both qualitative and quantitative nature [Yang 2001, Yang and Xu 2002, Yang et al. 2006]. Basic belief assignments (bba's) are derived directly from utility functions. A specific process, based on Dempster's rule of combination is used to mix criteria without specific consideration of conflict between sources (criteria). This methodology is applied to environmental problems [Wang et al. 2006].

Using Belief function theory and multi-criteria decision analysis requires evaluating all the criteria on the basis of the same frame of discernment. Including DST

[6] §5.2

or DSmT in a multi-criteria approach requires adopting a common frame of discernment. Total aggregation methods such as MAUT [7] or AHP[8] gather in a unique index the result of the evaluation of alternative. For the partial aggregation methods, such as *outranking methods*, un-comparability and intransitivity of preferences are basic paradigms: alternatives are compared to each other without aggregating all the criteria into one and only value. If we consider total aggregation methods, the attempt to mix multi-criteria approaches and evidence theory using a unique common frame of discernment for all the criteria does not sound initially so illegitimate.

Such an approach has been proposed by [Beynon et al. 2000] in a mixed framework called DS-AHP. His decision problem consists in choosing the best alternative between a complete set of alternatives according several criteria. Belief function theory is used essentially to reduce the high number of pairwise comparisons required when the number of alternatives increase. DS-AHP was first proposed as a way to compare sets of alternatives instead of unique alternatives [Beynon et al. 2000]. Preferences of each criterion are calculated according the classical pairwise comparison method. For each criterion, basic belief assignments are calculated on singletons or sets of alternatives on the basis of the perceived amount of favorable information in comparison with a total ignorance (i.e. the whole set of alternatives). The basic belief assignment (bba) on each evaluation grade or alternative is assessed through an indirect analysis of the "*favorability*" of knowledge. Each criterion is always compared to the whole set of hypothesis using a very specific pairwise comparison matrix. Some issues can be identified in its process:

- The mass elicitation mixes to different kinds of concepts: in the Belief function Theory, putting bba on a group of alternatives does not mean that all the included alternatives have the same level of information. On the contrary, it implies that knowledge is shared between all the groups without being able to put some more precise probability (of satisfying the criterion) on each of them. Reasoning on sets in the *Evidential reasoning* framework is not a faster way to put masses on singletons;

- As this principle mixes preferences and uncertainty in a unique bba, classical preference weights are then applied to reduce this bba without assigning any additional bba;

- Despite of the presumed ability to consider high number of alternatives, exposed examples only deal with rather small numbers of alternatives. With a high number of alternatives, reasoning with sets do not facilitate the decision since assigning basic belief assignments on sets mean that we are not able to share the knowledge between all the elements included inside. Taking a decision resulting from a fusion process is not that easy to interpret;

[7] Multi Attribute Utility Theory
[8] Analytic Hierarchy Process

- Decision rules used only use minimum or maximum of credibility and plausibility. In fact, the decision is possible only if some focal elements exist;
- Because of the use of the (grouped) pairwise comparison matrix, mass elicitation is sensitive to the number of levels in the evaluation scale [Beynon 2002]: a residual mass is always put on the total ignorance even whatever the choices of the decision analyst;
- Only a basic one level hierarchic model is considered. Criteria are considered as the only sources to be fused while several experts may proceed to evaluation;
- The fusion process is only based on the classical *Dempster's rule* known as a failure cause when the level of conflict increases.

This approach is presented as extended by [Hua et al. 2008] for the case where information is incomplete. However, we can consider here that *Beynon*'s approach had only not emphasized this intrinsic ability of the belief function theory. Another extension of this method was proposed to consider a multi-expert environment [Beynon 2005].

23.2.3.2 Requirements for an ER-MCDA methodology

If the belief function theory appears as a powerful framework to consider both uncertainty and imprecision, one of its main drawback consists of choosing bba's to be used in the fusion process, especially when information only comes from expert judgments. Many different methods have been proposed to elicit those bba [Bryson et al. 1994, Wong and Lingras 1994]. Using a common scale in order to describe a reasoning process can consist of some kinds of correspondence tables between a common numerical or ordinal scale and evaluation made by the experts as used as an example to evaluate the damage level of dams: each failure piece of evidence is rated in a numerical scale corresponding to an increasing level of gravity [Curt 2008]. This difficulty does exist in our framework: at least, our proposition introduces a way to fully describe the decision process (from its design to the evaluation steps) in a less ambiguous and more complete manner.

Face to the problematic of expertise of natural hazards in mountains, our goal is a methodology that would allow to make decision such as determining the most dangerous areas, the best prevention strategy, ... according to the following requirements: on the one hand, the decision framework should allow and trace a multi-expert analysis of the criteria importances, on the other hand, the model should allow to gather the more or less uncertain, imprecise evaluations provided by more or less reliable different sources (data sensors, data-sets, expert judgments). Many combinations related to the nature and quality of information can be observed. A secondary criterion can be assessed in a very precise and certain way by a fully reliable source. On the contrary, an important criterion can be evaluated precisely, in a certain way by a not reliable source. Our model aims to support decision in an uncertain context where several sources provide information about the problem. Its initial main purposes are not only to provide a help for decision but also to consider the way the decision is reached considering the reliability of sources and the uncertainty of information. The system must be able to model a complex hierarchical decision framework (some criteria are more important than others), different kinds of criteria (either qualitative or quantitative criteria). This versatile system (see Fig. 23.12) should therefore consider importance, uncertainty, imprecision and reliability in a multi-sources environment.

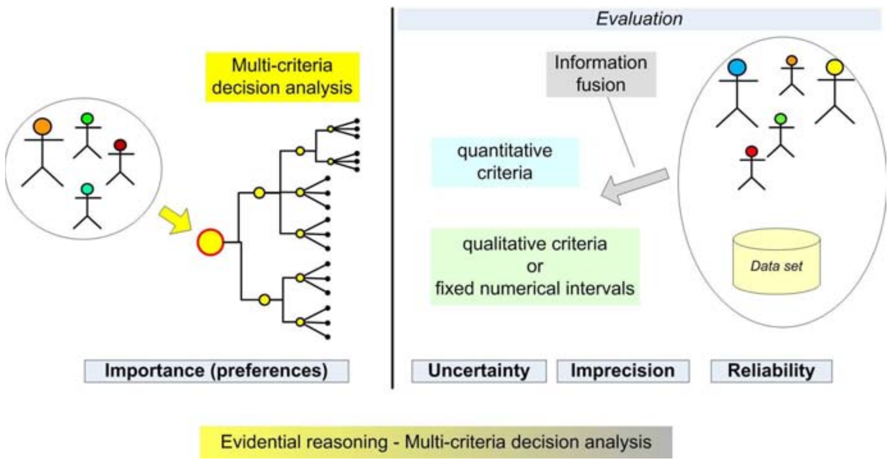


Figure 23.12: Principle of a versatile ER-MCDA.

We will use one of the most simple multi-criteria decision analyses (MCDA) method denoted as AHP recognized as a powerful and easy framework to help decision and reduce complexity in real-case decision situations. Our goal is to help decision but also to trace reasoning process. Evidential reasoning through *Dempster-Shafer* and *Dezert-Smarandache* theories is used to consider uncertainty and imprecision. The global methodology is presented in section 23.3 and a simplified application case in section 23.4.

23.3 ER-MCDA methodology

This section describes the global ER-MCDA framework and its two main parts: the multi-criteria model and the evaluation and fusion step. The proposed global methodology mixes those theories with an evidential reasoning (ER) process, known as a more general, versatile and integrating framework. These approaches are used in a multi-criteria decision framework. Many solutions do exist and we discuss in following sections some issues related to class level belonging, fusion order, etc.

What and how should we fuse?

Several *alternatives* or *actions* are evaluated according to different criteria through a preference relation. In addition to this, the evidential reasoning theories and their associated fusion rules are used to evaluate and propagate an uncertainty level in the decision process. This methodology considers both experts and criteria used in the hierarchic approach as sources in the fusion process. Different strategies to aggregate or fuse information are analyzed according to fusion rules, fusion order.

The methodology can be summarized as following:

- Identification and prioritization of criteria in a hierarchic MCDA framework;
- Definition of the frame of discernment considering either exclusive hypotheses (*Dempster-Shafer model*) or non exclusive hypotheses (*DSmT* framework).

In our problem, both quantitative (mainly related to physical data) and qualitative criteria are used in the model. Quantitative criteria are evaluated with some imprecision (corresponding to intervals values) and uncertainty (corresponding to a confidence level linked to these evaluations). The number of sources can also be different from one source to another. In that way, this model offers a versatile framework where several criteria are valuated by several sources whose reliability and kinds of valuation (precise or imprecise way) may also change.

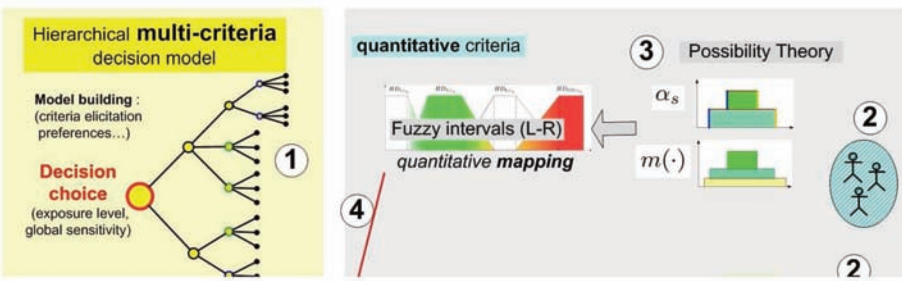
23.3.1 Possibility and Evidence Theory: why and what for?

In our context, expert evaluations often deal with continuous factors such as slope, surface, etc. These quantitative values are then linked to class levels according to a common qualitative scale enabling the fusion and/or aggregation process. A numerical value becomes correspond to a linguistic such as *high sensitive*, *sensitive*, *low sensitive*. This means that, for a given alternative (avalanche site), a numerical value would respectively, accordingly to their relative importance in the whole process, induce a global evaluation at the levels *high sensitive*, *sensitive* and *low sensitive*. Change of sensitivity level correspond to a fuzzy relation: let us suppose that the expert evaluation is 6% and that the two different classes of slopes are *between* 5% *and* 10% (low sensitive) and *between* 10% *and* 15% (sensitive). From a strict point of view, a 6% value belongs to the class *sensitive* but everybody has got the feeling that it is not so far from the *low sensitive* level. This *relative belonging strength* must be valuated. Expert evaluations can also result from an imprecise evaluation. An expert may be unable to fix a unique value for a slope inclination. In many cases, the only result that expert can provide is an interval with some confidence values: as an example, the expert would be able to say that the slope inclination is between 4% and 7%. For those reasons, it appears that the mixed use of fuzzy set and possibilities theories is useful to take into account the real knowledge that the expert is able to put inside the decision process.

23.3.2 AHP and ER within uncertain and complex context

23.3.2.1 Description of AHP-ER framework

The global framework is based on a combination of multi-criteria decision analysis (MCDA) techniques and evidential reasoning (ER) through the use of the theory of belief functions which is implemented in a classical way through DST framework and also in the new DSmT framework. The global framework considers both importance, uncertainty and imprecision in criteria assessment. Uncertainty and imprecision are considered through Belief Functions, Fuzzy Sets [Zadeh 1978] and Possibility theories [Dubois et al. 2000]. Importance is assessed according to the multi-criteria framework and especially through the classical pairwise comparison matrix using Saaty's scale. As recommended in literature [Saaty and Tran 2007], we do not introduce some additional fuzziness on the comparison rates in this matrix. Such attempts to mix different approaches related to uncertainty management already exist. As an example Omnari et al. [Omrani et al. 2007] have proposed a model for transportation strategies evaluation.



This framework implies the following steps (see Fig. 23.13):

- Hierarchical model implementation with the experts of the domain (criteria elicitation, qualitative and quantitative criteria identification);
- Choice of decision model (criteria-solution) or (criteria-estimator-solution);
- Choice of the common decision frame of discernment (*criteria-solution framework* implies that the frame consists of solutions while *criteria-estimator-solution* implies that the frame consists of a common scale for every criterion;
- Mapping process to transform the evaluations of the basic level criteria (handled according the hierarchical decision framework) into a common frame of discernment allowing a fusion process;
- Choice of fusion strategy (fusion of the different experts choices at the criterion level or at the evaluation stage);
- Choice of decision rule.

The fuzzy mapping for qualitative and quantitative criteria.

The second step of the ER-MCDA framework (see Fig. 23.13) consists of setting up, for each criterion c_j, a fuzzy mapping process that enables to transform uncertain evaluation of the criteria into bba's according to the common frame of discernment. This mapping process proposes a correspondence between the evaluation of the criteria and the elements of common frame of discernment used for the fusion process and the decision. A mapping model is a set of fuzzy numbers (see Fig. 23.14) or fuzzy intervals (see Fig. 23.15).

Since the evaluation of criteria can be uncertain and imprecise, the fuzzy intervals used for this mapping process may differ from one source to another. Therefore, $nbModels$ mapping models $mapModel_{x,c_j}$ (for $x = 1$ to $nbModels$) can exist depending from one hand on the experts involved in the model building and from the other hand on the theory used to represent the decision (DST or DSmT mapping). Two different mapping rules are used depending from one hand on the qualitative or quantitative nature of the criteria and on the other hand from the nature of the evaluation (numerical or membership assessment). For quantitative criteria, the mapping process transforms a possibility distribution, derived from necessity inputs, into bba's. For a given quantitative criterion c_j, each source s provide $nbInt_s$ numerical evaluation intervals described by a minimum, a maximum and a necessity value. This necessity value represents the minimum confidence of the source in the proposition *"the value of the criterion c_j belongs to the interval"*. For qualitative criterion, the fuzzy number are defined according to credibility values defined on each class.

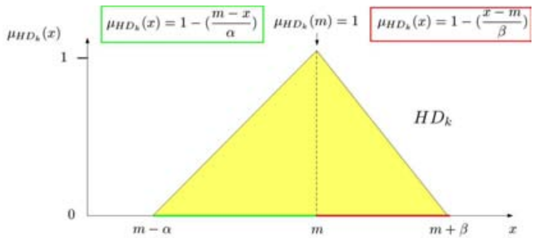


Figure 23.14: Fuzzy number $L - R$.

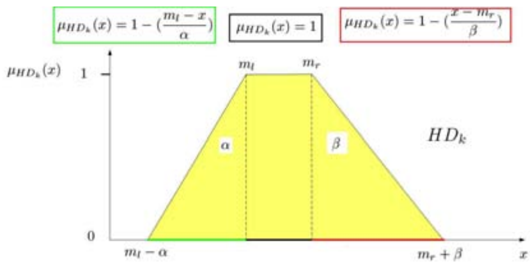


Figure 23.15: Fuzzy interval $L - R$.

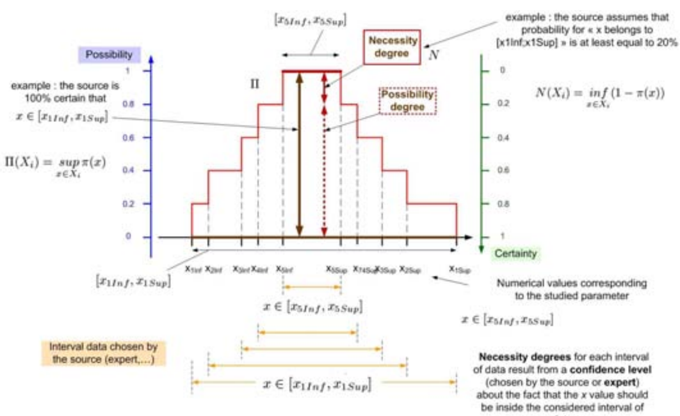


Figure 23.16: Possibility and necessity distributions.

Valuations of the criteria: from possibility to bba.

In [Dubois and Prade 2006][9], the authors describe relations between Possibility theory and Belief function theory. Given a bba m defined on a finite set S, the possibility distribution π resulting from m is defined by $\pi(s) = Pl(\{s\})$ (singletons plausibility). For different consonant focal elements E_i such as $E_1 \subset E_2 \subset \ldots E_n$, with $E_i = \{s_1, \ldots, s_i\}$, the possibility measure Π and the necessity measure N correspond to plausibility and credibility functions (see Fig. 23.16).

Example [Baudrit et al. 2005a, Baudrit et al. 2005b, Baudrit et al. 2007]: An expert provides n evaluation intervals of a quantitative criterion and assigns a confidence level λ_i to each of them. E_i corresponds to the i^{th} interval chosen by the expert (considered as a source) with $i \in \{1, 2, \ldots, n\}$. λ_i is the confidence degree associated to the interval E_i with $\lambda_i = N(E_i)$ (see Fig. 23.17).

$$\forall x \in \mathbb{R}, \pi(x) = \min_{i \in \{1,2,\ldots,n\}} (\max(1 - \lambda_i), \mathfrak{X}_{E_i}(x))) \tag{23.1}$$

with

$$\mathfrak{X}_{E_i(x)} = \begin{cases} 1 & \text{if } x \in E_i \\ 0 & \text{if } x \ni E_i \end{cases} \tag{23.2}$$

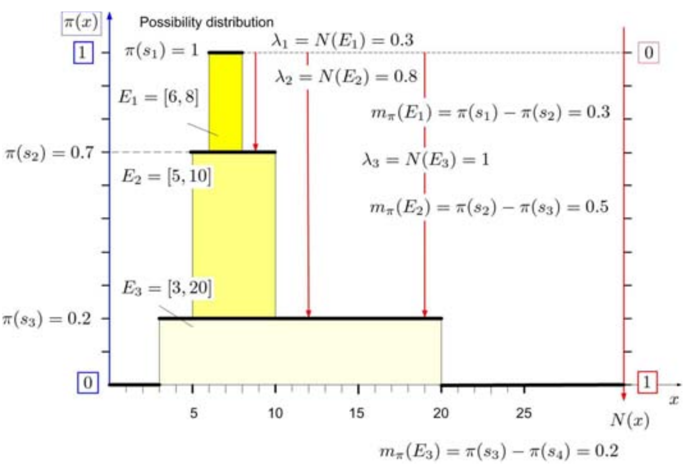


Figure 23.17: From expert necessity values to bba: numerical example.

[9] vol. 1, p. 140.

23.3.3 Step 1: Problem modeling

Risk management process is a complex framework in itself. Faced to any decision problem, the decision analyst always begins with a modeling phase which is an essential part and a main difficulty of the methodology. In our context, the modeling both concerns and cumulates known difficulties related both to the multi-criteria approach [Roy 1985] and to the evidential reasoning process [Martin and Osswald 2006]. To a certain extent, one of the more natural and intuitive way to cope with the complexity of the problem is very often to break its components down into several smaller ones. This approach is used in the hierarchical analysis proposed in AHP [Saaty 1982] but also in reliability and safety models through failure trees [Wang et al. 1996] or any systemic-like models. In [Forman and Selly 2002], Forman considers that hierarchical analysis is equivalent to the well-known cause tree or *Ishikawa* diagram. As a standard language, the *Unified Modeling Language (UML)* [Rumbaugh et al. 1999, Fowler 2000] is used to model the problem. This language is widely used in Computer Sciences and Information systems design to elicit the initial requirements, to represent the data model. In comparison with any other graphical flowcharts or diagrams, it represents a normalized framework that can be understood in the same way by all of its possible users: every graphical software is able to provide flow-charts diagrams that are not always interpreted in the same way. In our context, building conceptual models is one of the first and essential step to describe to consider the different types of sources, including both experts, databases or criteria evaluation involved in the fusion process. This approach allows building a link with calculation tools such as PCR5, PCR6 or DSmH routines.

The modeling step concerns on the one hand the decision problem description (through a hierarchical decision structure) and, on the other hand, the fusion problem modeling.

In a criterion-estimator-solution framework, the decision consists of choosing an evaluation grade for a given alternative. The common Decision Frame of discernment used in our *ER-MCDA* framework consists of a set of evaluation grades denoted $\Theta_{Decision} = \{HD_1, HD_2, \ldots, HD_k, \ldots, HD_{GD}\}$ with $k \in \{1, 2, \ldots, GD\}$. The decision is broken down into qualitative and/or quantitative criteria (see FIG. 23.18 and FIG. 23.19).

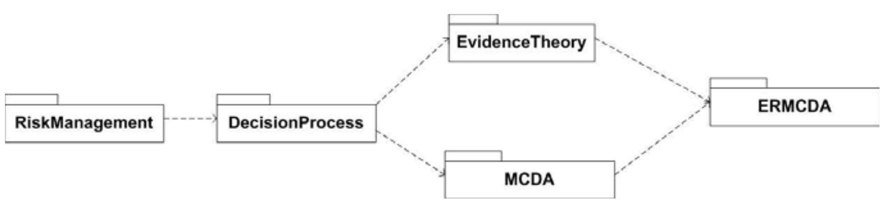


Figure 23.18: ER-MCDA framework - UML modeling - Main packages.

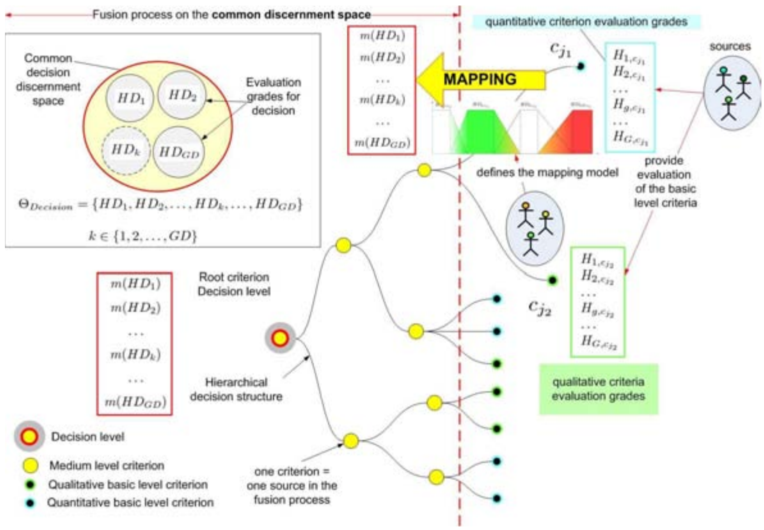


Figure 23.19: Principle of the AHP based ER-MCDA framework.

23.3.3.1 The multi-criteria analytical hierarchical model

The real problem is first analyzed through the hierarchical decision framework used a conceptual support for criteria and preferences identification. Criteria are ranked and weighted according to their importance in the decision process. The basic level criteria are assessed according either quantitative (numerical) evaluation grades. The criterion *Human vulnerability*, as an example of quantitative criterion, is assessed through the number of winter occupants. This number can be a single integer or an interval with a minimum and a maximum value such as $[1, 5]$. The *Living places/infrastructures*, as an example of qualitative classes, is assessed through a membership level for each class.

Unitary Hierarchic Component.

The Hierarchic Structure is composed of *Unitary Hierarchic Components* such as described in Fig. 23.7

$$SubC_j = SubC_{[r_1,r_2,\ldots,r_l]} = \{C_{[r_1,r_2,\ldots,r_l,1]}, C_{[r_1,r_2,\ldots,r_l,2]}, \ldots, C_{[r_1,r_2,\ldots,r_l,k]}, \ldots, C_{[r_1,r_2,\ldots,r_n]}\} \quad (23.3)$$

For a given medium level criterion C_j or for the general attribute of the hierarchic structure , $SubC_j$ is the set of its ML sub-criteria.

Criterion Identification Vector (CIV).

A practical and simple codification is used to identify any criterion in the hierarchical structure and to implement the software application. For the criterion number j, at the level l of the hierarchy, the *Criterion Identification Vector* denoted as CIV_j is defined as $CIV_j = \underbrace{[r_1, r_2, \ldots, r_k, \ldots, r_{l-1}, r_l]}_{l \text{ terms}}$ where $CIV_j(l) = r_l$ is the l^{th} term of CIV_j.

By definition, the general attribute of the hierarchic structure is the first criterion of any *Criterion Identification Vector*: $\forall j \in \{1, 2, \ldots, M\}, CIV_1 = [1]$. It is also called the root of the hierarchic structure. $CIV_j(k)$ is the k^{th} term of CIV. $CIV_j(k) = r_k$ means that the rank of the given criterion is r_k relatively to its parent-criterion in the unitary hierarchic component whose root criterion is the criterion denoted as $CIV = [r_1, r_2, \ldots, r_{l-1}]$ which has $l-1$ terms.

Example: Let's consider a criterion defined by its identification vector $CIV = [1, 3, 2, 2]$. Its vector length is 4. This criterion is the 2^{nd} sub-criterion of the criterion whose CIV is $[1, 2, 3]$. The criterion c_j is described by CIV_j. $ML(CIV_j) = ML([(r_1, \ldots, r_k, \ldots, r_l]$ is the number of sub-criteria of the criterion described by its identification vector CIV_j. The sub-criteria of c_j are referenced by *Criterion Identification Vector* such as $[(r_1, r_2, \ldots, r_k, \ldots, r_l, r_{l+1}]$ with $r_{l+1} \in \{1, 2, \ldots, ML(CIV_j)\}$ (see FIG. 23.20). For any criterion, ML is a function of a vector whose length ranges from 1 to D (maximal depth of the hierarchic structure) defined by $ML : C_j \longrightarrow \mathbb{N}$ and $CIV_j \longrightarrow ML(CIV_j)$.

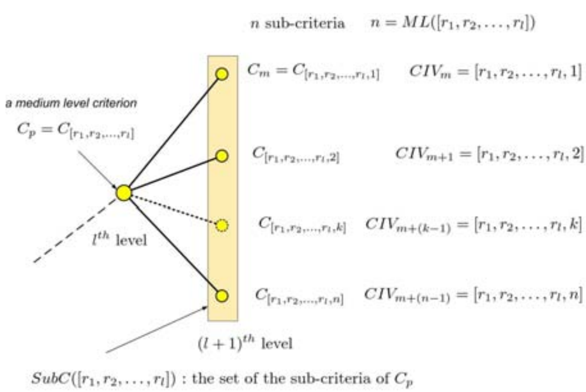


Figure 23.20: Criterion and sub-criteria codification in the hierarchical structure.

An UML class diagram using a composite pattern diagram [Gamma et al. 1995] can represent this hierarchical structure as described in (see Fig. 23.21).

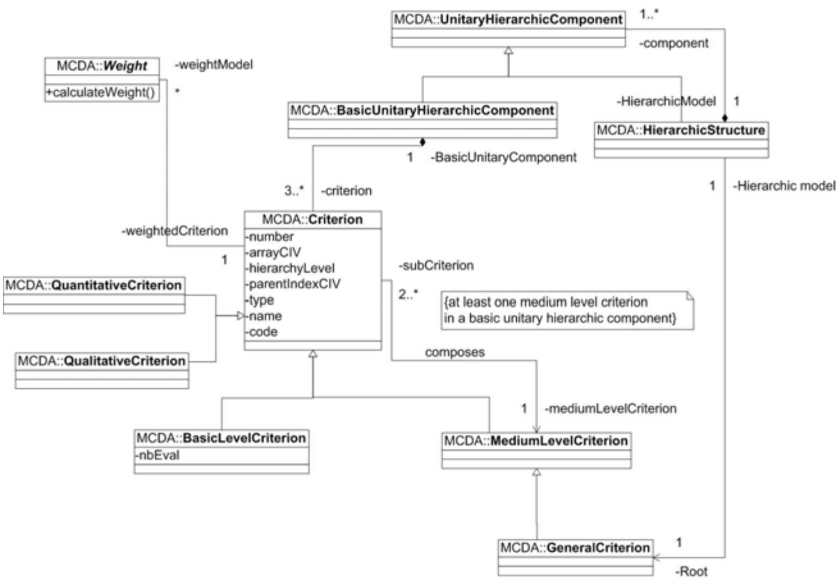


Figure 23.21: ER-MCDA framework - UML Class diagram - Decision hierarchical structure.

Evaluation of the basic level criteria.

For each basic level criterion (or attribute), S sources provide an evaluation of the criterion based on a common evaluation scale $H = \{H_1, H_2, \ldots, H_G\}$ with G corresponding to the number of levels of the scale. H is the frame of discernment on which the evaluation is done.

23.3.3.2 A sample decision model

We introduce here a simplified model to illustrate the coupled use of fusion process and MCDA approaches. This model is derived from a real decision-aid model that calculates the sensitivity level of a natural site exposed to avalanches.

A common frame of discernment is required for decision.

Any fusion problem requires defining a common frame of discernment. Its definition is closely linked to the nature of decision such as choosing a sensitivity or exposure level for a site in a natural hazards prone area, choosing the more important areas to protect, choosing the level of confidence for an expertise. The fusion process will provide basic belief assignments on each or combination of the elements of the frame of discernment. We can obviously question ourselves about the interest of using the DSmT framework (allowing non-empty intersections) instead of the classical DST framework based on exhaustive and exclusive hypotheses.

Two frames of discernment Θ are considered in this work:

- in the DST framework (see FIG. 23.22), the frame Θ is composed of 4 exclusive elements defined by HD_1 = 'No sensitivity', HD_2 = 'Low sensitivity', HD_3 = 'Medium sensitivity' and HD_4 = 'High sensitivity';

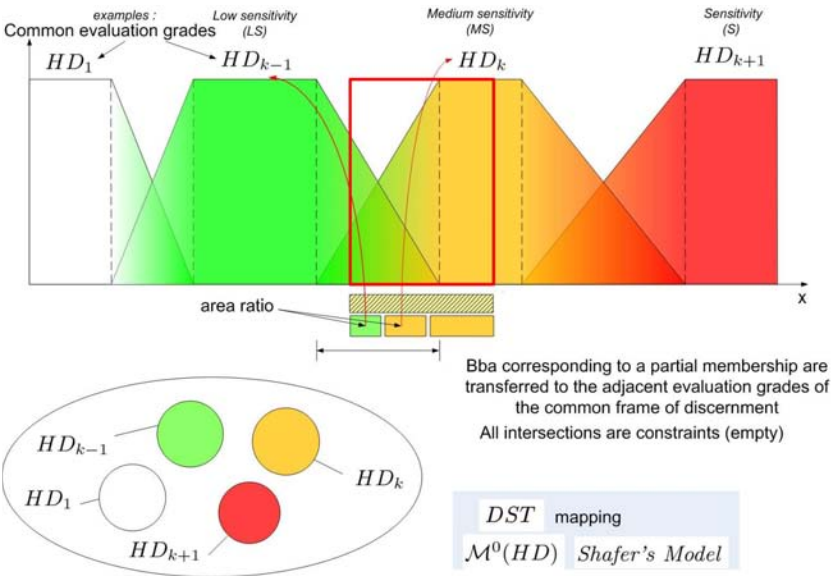


Figure 23.22: Modeling the common evaluation grades in a DST framework.

- in the DSmT framework (see FIG. 23.23), the frame Θ is composed of 3 elements defined by HD_1 = 'No sensitivity', HD_2 = 'Low sensitivity' and HD_3 = 'High sensitivity';

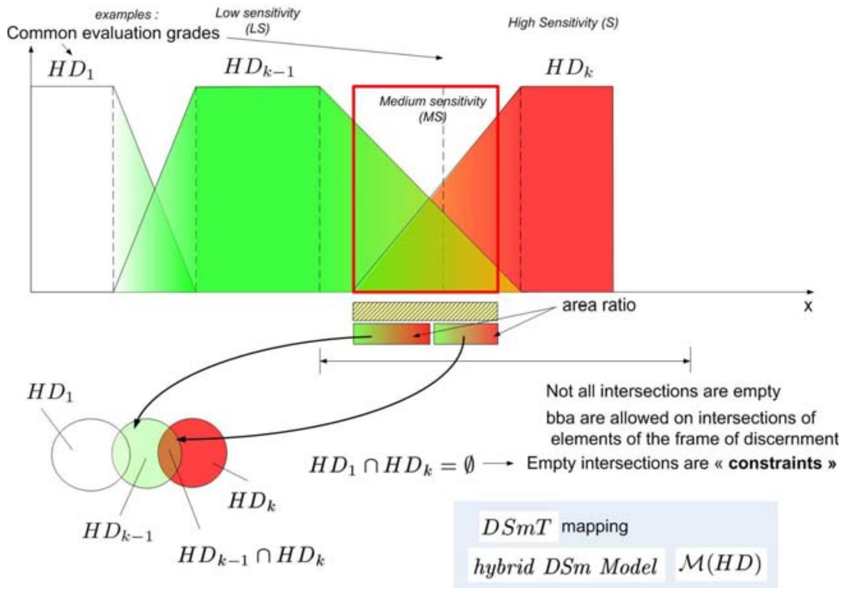


Figure 23.23: Modeling the common evaluation grades in a DSmT framework: simplified version.

23.3.4 Step 2: Mapping quantitative criterion into a common frame

This section describes the transformation of the evaluations provided by the different sources on quantitative criteria into the common frame of discernment.

23.3.4.1 Mapping quantitative criteria

For a given quantitative criterion c_j, the mapping process $mapModel_x$ transforms a quantitative evaluation into bba defined in the common frame of discernment (see FIG. 23.24):

$$\begin{cases} mapModel_{(x,c_j)} : [0,1] \rightarrow [0,1] \\ mapModel_{(x,c_j)}(I_{(s,int_j)}) = \{m_{s,I_{(s,int_j)}}(HD_1), \ldots, m_{s,I_{(s,int_j)}}(HD_{GD})\} \end{cases}$$

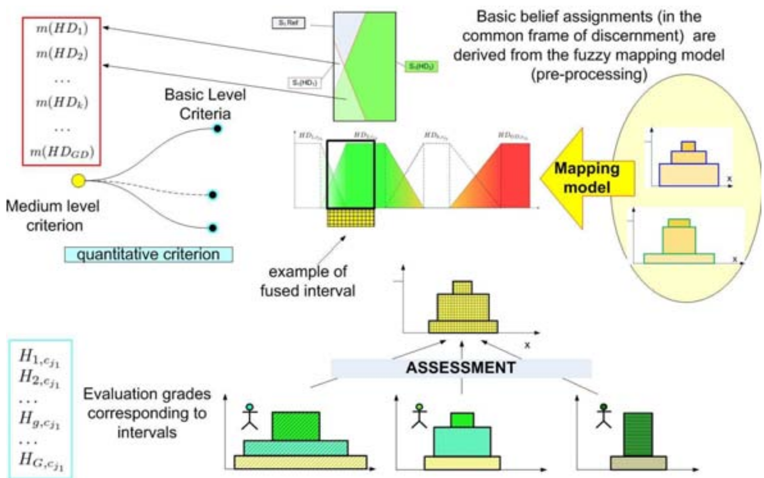


Figure 23.24: Mapping from quantitative criterion assessment to the common frame of discernment.

For a quantitative criterion c_j, the evaluation by a source s (an expert) can be either a single value (e.g. x_1) or numerical intervals. Different consonant intervals, corresponding to different levels of confidence can be proposed by the source. The evaluation of the source s is therefore a possibility distribution whom we are able to extract intervals denoted $I_{(s,int_j)}$ and corresponding basic belief assignments denoted $m_s(I_{(s,int_j)})$ [Dubois and Prade 2006]. For each interval $I_{(s,int_j)}$, the fuzzy mapping function of the x^{th} mapping model, for the criterion c_j distributes $m_s(I_{(s,int_j)})$ on the elements of the common frame of discernment $\Theta_{Decision} = \{HD_1, HD_2, \ldots, HD_k, \ldots, HD_{GD}\}$ on which the global decision is taken. The distribution of $m_s(I_{(s,int_j)})$ on HD_k ($k \in \{1, 2, \ldots, GD\}$) is proportional to the intersection of the following areas (see Fig. 23.25):

- a rectangle whose width is equal to the length of the interval $m_s(I_{(s,int_j)})$ and height is equal to 1;
- intersection of the previous rectangle with the areas of fuzzy intervals defined in the mapping model $mapModel_{x,c_j}$, denoted $A_{mapModel_{x,c_j}}(HD_k)$.

The evaluation source is described through:

- its confidence, resulting from its own assessment and valuated by a necessity value attached to each interval;
- its reliability, resulting from an external assessment, and valuated through a discounting factor.

Different cases are considered depending on the nature and number of evaluations provided by one source for a given criterion (numerical intervals or single discrete values).

23.3.4.2 Case of one source with one evaluation

- **Case of one totally reliable source with one imprecise evaluation** This case corresponds to a source s which evaluates the quantitative criterion with a unique interval ($nbInt_s = 1$) whose necessity value equals to 1. The bba of the interval ($m_s(I_{s,1}) = 1$) is transferred to the elements of the common frame of discernment (see FIG. 23.25). The interval $I_{(s,1)} = [x_{Inf_{(s,1)}}, x_{Sup_{(s,1)}}]$ corresponds to a total area of

$$A_{I_{(s,1)}} = length(I_{(s,1)}) = x_{Sup_{(s,1)}} - x_{Inf_{(s,1)}}.$$

$A_{I_{(s,1)}}$ represents the total membership area of the interval with $A_{I_{(s,1)}} = A_{I_{(s,1)}}(HD_{k-1}) + A_{I_{(s,1)}}(HD_k)$.The bba transferred on HD_{k-1} is

$$m_{s,I_{(s,1)}}(HD_{k-1}) = \frac{A_{I_{(s,1)}}(HD_{k-1})}{A_{I_{(s,1)}}} \cdot m_s(I_{(s,1)}).$$

The bba transferred on HD_k is

$$m_{s,I_{(s,1)}}(HD_k) = \frac{A_{I_{(s,1)}}(HD_k)}{A_{I_{(s,1)}}} \cdot m_s(I_{(s,1)}).$$

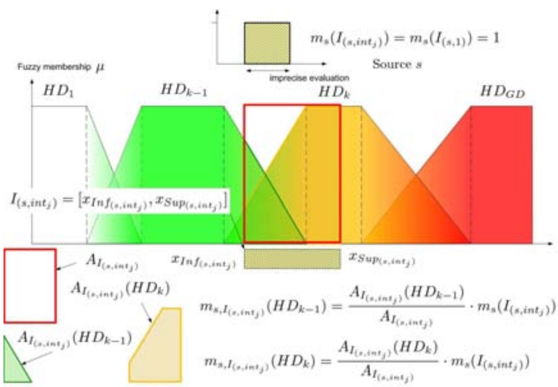


Figure 23.25: Quantitative criterion mapping: One totally reliable source with imprecise evaluation.

- **Case of one partially reliable source with imprecise evaluation**

  The source s is assumed partially reliable. A discounting factor is applied to the bba corresponding to the evaluated intervals (see FIG. 23.26).

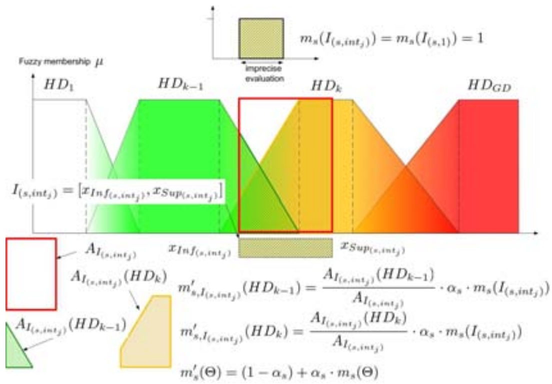


Figure 23.26: Quantitative criterion mapping: One partially reliable source with imprecise evaluation.

- **Case of a partially reliable source with precise evaluation**

  The source s provides a single discrete evaluation x_1 of the quantitative criterion c_j. $m(x_1)$ is derived from the fuzzy mapping intervals by the intersection of a vertical line with these fuzzy intervals (see FIG. 23.27). The reliability of the source is taken into account by a discounting factor $\alpha_s \in [0, 1]$.

23.3.4.3 Case of one source with two evaluation intervals

Based on the necessity-possibility functions inputs, one transfers the initial bba to a bba related to the common frame of discernment chosen for decision. This transfer uses the proportion of intersected areas of the whole area of the interval with each fuzzy $L - R$ interval of the mapping model (see FIG. 23.28).

We consider here a source s that provides two evaluation intervals ($nbInt_s = 2$). The first evaluation of the source s is interval $I_{(s,1)} = [x_{Inf_{(s,1)}}, x_{Sup_{(s,1)}}]$. The membership area (see FIG. 23.29) of this interval equals to

$$A_{I_{(s,1)}} = A_{I_{(s,1)}} = A_{I_{(s,1)}}(HD_{k-1}) + A_{I_{(s,1)}}(HD_k).$$

The bba's transferred respectively on HD_{k-1} and on HD_k are: $m_{s,I_{(s,1)}}(HD_{k-1}) = (A_{I_{(s,1)}}(HD_{k-1})/A_{I_{(s,1)}}) \cdot m_s(I_{(s,1)})$ and $m_{s,I_{(s,1)}}(HD_k) = (A_{I_{(s,1)}}(HD_k)/A_{I_{(s,1)}}) \cdot m_s(I_{(s,1)})$.

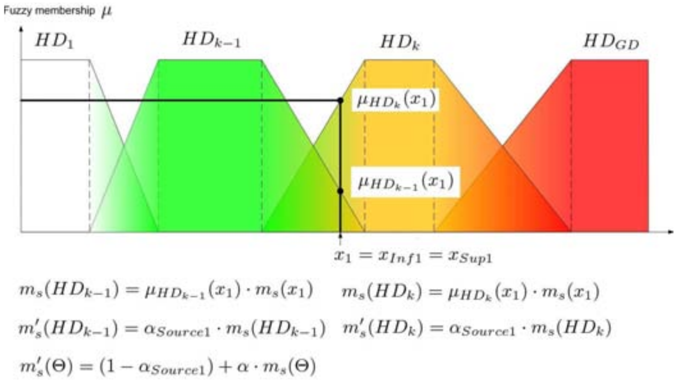


Figure 23.27: Quantitative criterion mapping: One partially reliable source with precise evaluation.

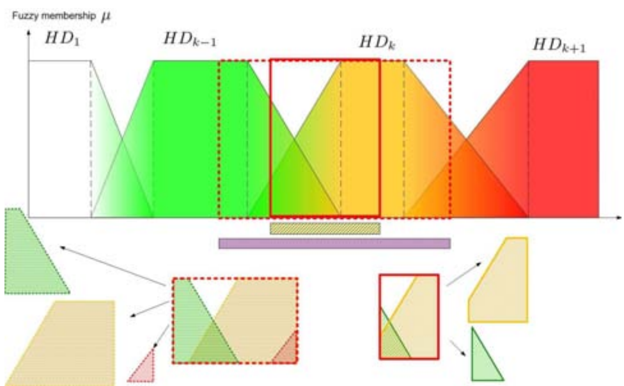


Figure 23.28: Quantitative criterion mapping: Two imprecise evaluations - Principle of area mapping calculation.

The second evaluation of the source s is $I_{(s,2)} = [x_{Inf_{(s,2)}}, x_{Sup_{(s,2)}}]$. The membership area (see Fig. 23.30) of the interval equals to

$$A_{I_{(s,2)}} = A_{I_{(s,2)}} = A_{I_{(s,2)}}(HD_{k-1}) + A_{I_{(s,2)}}(HD_k) + A_{I_{(s,2)}}(HD_{k+1}).$$

The bba's transferred on HD_{k-1}, on HD_k and on HD_{k+1} are respectively given by:

$$m_{s,I_{(s,2)}}(HD_{k-1}) = \frac{A_{I_{(s,2)}}(HD_{k-1})}{A_{I_{(s,2)}}} \cdot m_s(I_{(s,2)})$$

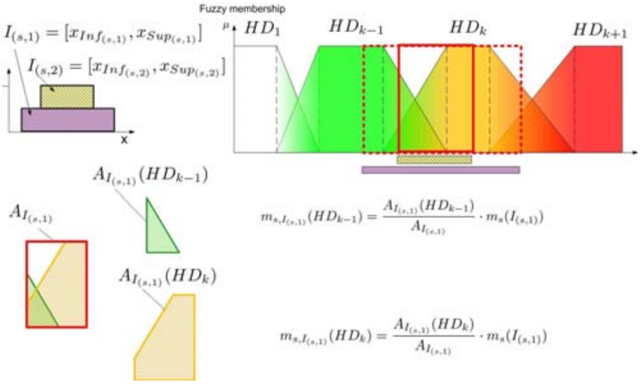


Figure 23.29: Quantitative criterion mapping: One partially confident source and a totally reliable source - interval 1.

$$m_{s,I_{(s,2)}}(HD_k) = \frac{A_{I_{(s,2)}}(HD_k)}{A_{I_{(s,2)}}} \cdot m_s(I_{(s,2)})$$

$$m_{s,I_{(s,2)}}(HD_{k+1}) = \frac{A_{I_{(s,2)}}(HD_{k+1})}{A_{I_{(s,2)}}} \cdot m_s(I_{(s,2)}).$$

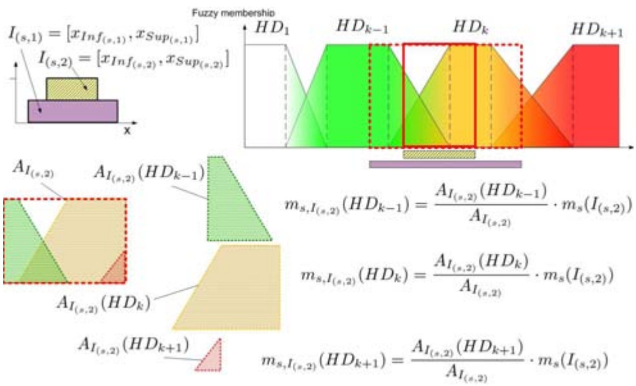


Figure 23.30: Quantitative criterion mapping: One partially confident source and a partially reliable source - interval 2.

23.3.4.4 Generalization: evaluation of one source s with $nbInt$ numerical intervals

- **One totally reliable source with $nbInt$ imprecise evaluations**

  We consider here the source s that provides $nbInt$ evaluation intervals. The j^{th} interval is defined by $I_{(s,Int_j)} = [x_{Inf_{(s,Int_j)}}, x_{Sup_{(s,Int_j)}}]$ with the index $j \in \{1, 2, \ldots, nbInt_s\}$. The length of the interval is $length(I_{(s,Int_j)}) = x_{Sup_{(s,Int_j)}} - x_{Inf_{(s,Int_j)}}$. The membership area of the interval $A_{I_{(s,Int_j)}}$ depends on the length of the considered interval with $h_{A_{I_{(s,Int_j)}}}$ corresponding to the height of the area ($h_{A_{I_{(s,Int_j)}}} = 1$ corresponds to a full membership) as shown in eq. (23.4).

$$A_{I_{(s,Int_j)}} = length(I_{(s,Int_j)}) * h_{A_{I_{(s,Int_j)}}} \tag{23.4}$$

  The whole area $A_{I_{(s,Int_j)}}$ is the sum of the intersected areas of intervals with the fuzzy intervals of the mapping model.

$$A_{I_{(s,Int_j)}} = \sum_{j=1}^{nbInt_s} A_{I_{(s,Int_j)}}(HD_k)$$

  with k such as $A_{I_{(s,Int_j)}} \cap A_{model_x}(HD_k) \neq \emptyset$.

  The bba transferred on HD_k results from the intersection of the interval $I_{(s,Int_j)}$ with the mapping model A_{model_x} - see eq. (23.5) - with:

  - $A_{I_{(s,Int_j)}}$ corresponding to the intersection area of the interval with the mapping model A_{model_x} (as an example DST or DSmT mapping models as described in applications section;
  - $A_{I_{(s,Int_j)}}(HD_k)$ corresponding to the intersection of the interval $A_{I_{(s,Int_j)}}$ with the fuzzy $L - R$ interval coding for the k^{th} element of the frame of discernment Θ.

$$m_{s,I_{(s,Int_j)}}(HD_k) = \frac{A_{I_{(s,Int_j)}}(HD_k)}{A_{I_{(s,Int_j)}}} \cdot m_s(I_{(s,Int_j)}) \tag{23.5}$$

  For each element of the frame of discernment HD_k, we sum the bba transferred by each interval. Finally, the resulting mapped bba of the source s for $nbInt_s$ evaluation intervals is defined by eq. (23.6):

$$m_s(HD_k) = \sum_{j=1}^{nbInt_s} m_{s,I_{(s,Int_j)}(HD_k)} \tag{23.6}$$

- **A partially reliable source with $nbInt$ imprecise evaluations**

  For a partially reliable source, the bba are discounted according the classical reliability discounting process. $m'_s(HD_k) = \alpha_s \cdot m_s(x_1)$ and $m'_s(\Theta) = (1 - \alpha_s) + \alpha_s \cdot m_s(\Theta)$. In case of a partially reliable source, the bba transferred on each element HD_k of the considered frame of discernment (corresponding to a given mapping model A_{model_x} is

$$m'_{s,I_{(s,Int_j)}}(HD_k) = \frac{A_{I_{(s,Int_j)}}(HD_k)}{A_{I_{(s,Int_j)}}} \cdot \alpha_s \cdot m_{s,I_{(s,Int_j)}}.$$

  A synthetic view of the quantitative mapping process from evaluation intervals to the mapped bba is described in Fig. 23.31 for $nbInt_s = 2$.

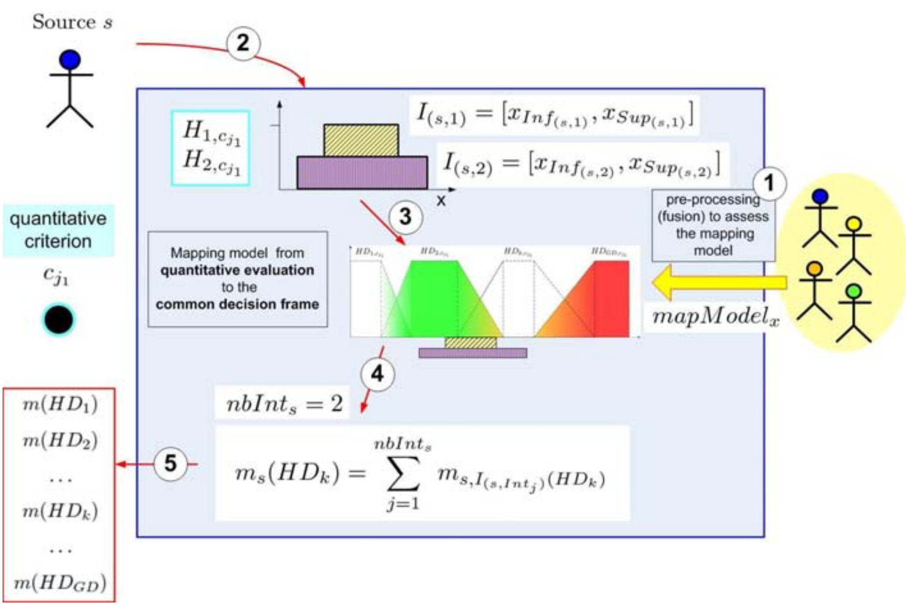


Figure 23.31: Quantitative criterion mapping: One partially confident source and a partially reliable source - Fusion

23.3.5 Step 3: Mapping qualitative criterion into a common frame

Qualitative mapping transforms an evaluation of a qualitative criterion into basic belief assignments (bba's) expressed on the common frame of discernment. At the end of the *scaling process*, the result of the evaluation of qualitative criterion c_j by the the source s is summarized in a belief interval $belInt_{(s,c_j)}$ and a weighted discounted factor α_s.

Instead of choosing only one evaluation grade, the expert can distribute his confidence between different combinations depending on the model used. Therefore, he can express the strength of his belief in the different classification levels. He can even notify that he has no information about the evaluation of the criterion by assigning his confidence to the whole set of classes (corresponding to the *total ignorance*).

Qualitative criteria correspond to criteria whose evaluation is carried out in a Boolean way. For a given criterion c_j and its g^{th} evaluation grade H_{Qualg,c_j}, a given alternative belongs or does not belong to the evaluation grade. A numerical interval $[x_{Inf}, x_{Sup}]$ can be considered as a qualitative criterion as soon as its limits cannot change. The qualitative mapping process transforms an uncertain evaluation of qualitative criteria into basic belief assignments and discounting factor compliant with the global fusion process.

The basic belief assignment elicitation for qualitative criteria is a two steps process. We consider a qualitative criterion c_j, for which a given expert or source, has to provide an evaluation according to the evaluation grades of the common frame of discernment $\Theta = \{HD_1, HD_2, \ldots, HD_k, \ldots, HD_{GD}\}$. The criterion c_j is evaluated according to the qualitative evaluation grades $\{H_{Qual_1,c_j}, H_{Qual_2,c_j}, \ldots, H_{Qual_g,c_j}, \ldots, H_{Qual_{GD},c_j}\}$. A qualitative (DST or DSmT based) *mapping model* is used to link expert's evaluation to the evaluation grades of the common frame Θ. The belief is calculated for each qualitative evaluation grade using the *importance bba* (see FIG. 23.32) and the *comparative confidence qualitative discounting factor* using a (DST or DSmT based) scaling model.

23.3.5.1 Global mapping process for qualitative criterion

As for quantitative criteria, the global process aims at build links between evaluation grades related to qualitative criteria and the element of the common frame of discernment. Each evaluation grade is assessed first according to its importance according to the decision to take (e.g. the sensitivity level) and secondly to the confidence level related to its assessment by the source. As for qualitative criteria, two frames of discernment and mapping models are considered as shown on FIG. 23.33.

The mapping process corresponds to the following steps:

- Choice of evaluation grades scaling model with regard to the acceptance (DSmT scaling model) or non-acceptance (DST scaling model) of non empty intersections between the evaluation grades;

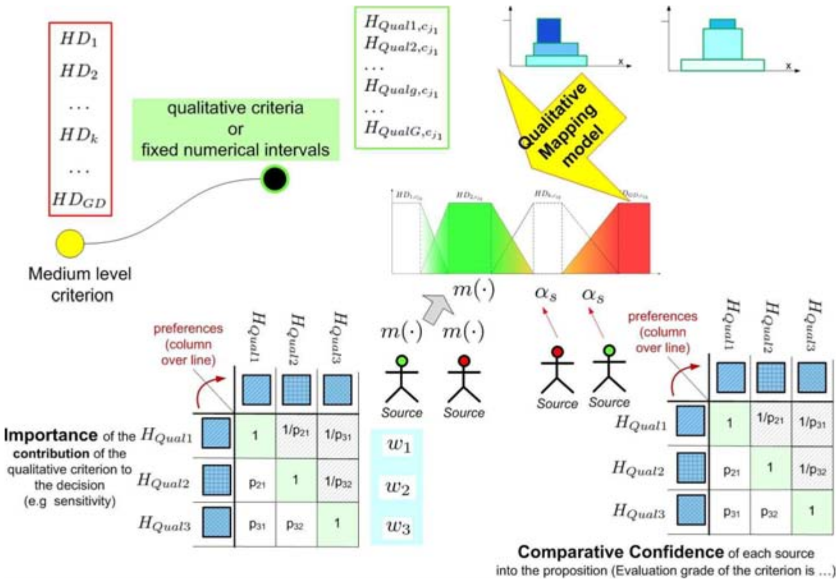


Figure 23.32: Mapping from qualitative criterion assessment to the common frame of discernment.

- Importance assessment of each evaluation grade (to calculate weights considered as equivalent to basic belief assignments);
- Confidence level assessment for the evaluation by a given source s;
- Gathering of these evaluation into a common belief interval with lower and an upper limits;
- mapping of this belief interval to the common frame of discernment.

23.3.5.2 Importance of each qualitative evaluation grade: DST or DSmT scaling

A qualitative criterion c_j is assessed according to a set of g evaluation grades denoted as $H_{(Qual_g,c_j)}$ with $g \in \{1, 2, \ldots, G\}$. These evaluation grades correspond to real situations that the source may encounter while trying to assess a real problem. As an example, the criterion $C_{[112]}$ coding for the part of global sensitivity due to the living places or infrastructures is described by a set of evaluation grades corresponding to industrial equipments ($\{Ind\}$), collectivities ($\{Col\}$) or rescue equipments ($\{Resc\}$). They respectively correspond to an increasing level of sensitivity: rescue equipment

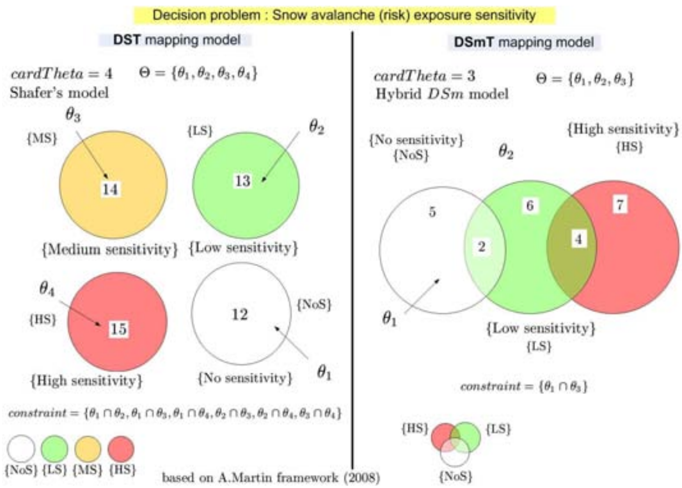


Figure 23.33: Frame of discernment corresponding to DST and DSmT mapping models.

is considered as more important than a collective equipment which is itself more important than an industrial equipment. Ranking the different evaluation grades according to their importance is handled through an AHP based pairwise comparison matrix. For each evaluation grade, weights are considered as basic belief assignments. As focal elements are singletons, according to the modeling principles, these bba's are equivalent to beliefs following eq. (23.7).

$$w_{(H_{Qual_g},c_j,s)} = m_{(H_{Qual_g},c_j,s)} = Bel_{(H_{Qual_g},c_j,s)} \tag{23.7}$$

In a real-case application, different combinations of these equipments can exist: industrial equipments such as telephonic exchanges, power plants, roads, bridges can also be considered as rescue equipments. A finest gradation in term of sensitivity can be proposed. An equipment whose contribution to sensitivity is multiple will be more sensitive than as many separate equipments: such an equipment concentrating different functions on a unique geographical point represents an higher potential of damage. The evaluation model should be able to consider this case. Therefore, two models are proposed:

- A DST based model considers that the evaluation grades are totally exclusive. This model cannot take into account the intersection of two evaluation grades;

- A DSmT based model allows intersection between the evaluation grades. Basic belief assignments put on these empty intersections correspond to the situations where equipments belong to several evaluation grades.

We could have obviously imagined modeling the intersection cases through a refinement of the initial DST model. To our point of view, the DSmT model fits in a closer way to the real case application. In such a model, the case of an equipment that would be both an industrial and an rescue equipment corresponds to an evaluation of the elements $\{Ind\}$, $\{Col\}$ and $\{Ind \cap Col\}$.

23.3.5.3 Confidence level of qualitative evaluation grade

Once the source s has chosen whether an evaluation grade was existing on the studied area, it must valuate its confidence related to its valuation. For a given qualitative criterion c_j , its evaluation grades can be partially assessed by the source s. Any evaluation attempt by the source s of the evaluation grade g of the criterion c_j corresponds to a Boolean factor denoted as $input_{(H_{Qual_g}, c_j, s)}$. This factor is important to calculate the weighted discounted factor depending on the evaluated grades.

For each evaluation grade of the criterion c_j, the source s has to valuate its confidence level through a confidence ranking interval $confRankInt$ defined in eq. (23.8) with its minimum and maximum values chosen in a "Saaty-like" ordinal scale ranging from $confRank_{min} = 1$ (no confidence at all in the valuation) to $confRank_{max} = 9$ (total confidence in the valuation).

$$confRankInt = [inputConfRank_{min}, inputConfRank_{max}] \tag{23.8}$$

These rankings are normalized to calculate lower and upper confidence index following as follows:

$$conf_{min} = \frac{inputConfRank_{min} - confRank_{min}}{confRank_{max} - confRank_{min}}$$
$$conf_{max} = \frac{inputConfRank_{max} - confRank_{min}}{confRank_{max} - confRank_{min}}$$
$$conf_{mean} = \frac{conf_{min} - conf_{max}}{2}$$

with $inputConfRank = inputConfRank_{(H_{Qual_g}, c_j, s)}$.

23.3.5.4 Belief interval

For each evaluation grade g of the criterion c_j by the source s, a belief interval $BelInt_{(H_{Qual_g}, c_j, s)}$ is derived from the confidence ranking interval $confRankInt$ and the importance bba $m_{(H_{Qual_g}, c_j, s)}$. The confidence level associated to this belief interval $\alpha_{c_j,s}$ the ratio between the importance bba weighted by the mean confidence and the maximum belief value. The final data used to map the qualitative criterion c_j are $\alpha_{c_j,s}$ and $BelInt_{c_j} = [BelInt_{min,c_j}, BelInt_{max,c_j}]$, $BelInt_{min,g} = conf_{min,g} \cdot Bel_g$, $BelInt_{max,g} = conf_{max,g} \cdot Bel_g$, $BelInt_{min,c_j} = \sum_{g=1,\dots,G} input_g \cdot BelInt_{min,g}$, $BelInt_{max,c_j} = \sum_{g=1,\dots,G} input_g \cdot BelInt_{max,g}$ with $input_g = input_{(H_{Qual_g}, c_j, s)}$, $Bel_g = Bel_{(H_{Qual_g}, c_j, s)}$, $\alpha_g = \alpha_{(H_{Qual_g}, c_j, s)}$ and $BelInt_{min,g} = BelInt_{(min, H_{Qual_g}, c_j, s)}$, (similarly with max), $BelInt_{min,c_j} = BelInt_{(min,c_j,s)}$ (similarly with max), $conf_{min,g} = conf_{(min,H_{Qual_g},c_j,s)}$ (similarly with max and mean).

23.3.5.5 Fusion order

The fusion process can be different from the hierarchical decision model. These fusion orders are parts of the description of the fusion processes in the model. Several strategies can be imagined depending whether the decision is taken by one source (see FIG. 23.34) or by several sources (see FIG. 23.35).

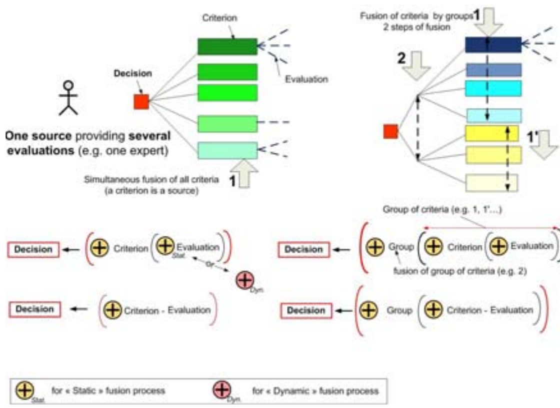


Figure 23.34: How far does the fusion process must follow the hierarchical decision model (case of one source)?

In this work, the implemented model corresponds to ($\oplus Criterion$ ($\oplus Source - Evaluation$)) depicted in FIG. 23.36) below.

23.3.5.6 Fusion of mapped bba of *nbSources* sources

A given criterion is identified by its criterion identification vector $CIV = [r_1, r_2, \ldots, r_n]$. s sources, denoted as s_i with $i \in \{1, 2, \ldots, s\}$, provide $nbEval_{s_i}$ interval-based evaluations. Each evaluation by the source s, denoted as $eval_{j,s}$, consists of $nbInt_{j,s}$ intervals. $nbFusedSources$ represents the total number of all the sources that are fused for the criterion (sum of all the evaluations of the sources for the given criterion) (EQ. 23.9).Several fusion processes can be proposed. The following equations concern the ($\oplus Criterion(\oplus Source - Evaluation)$) process. An example is given for the criterion $C_{[111]}$ for which two sources s_1 and s_2 provide each one evaluation (EQ. 23.10).

$$nbFusedSources_{CIV} = \sum_{s_i=1}^{s} \left(\sum_{j=1}^{nbEval_{s_i}} j \right) \tag{23.9}$$

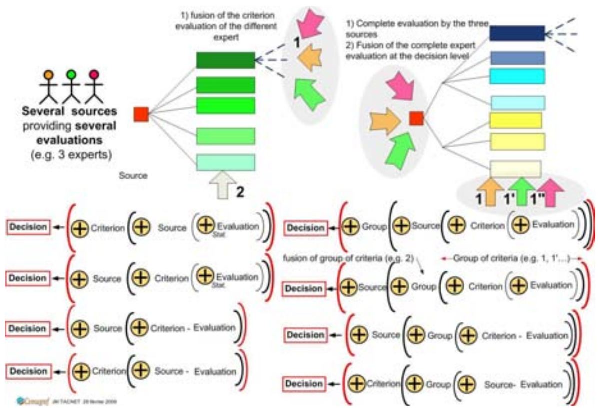


Figure 23.35: How far does the fusion process must follow the hierarchical decision model (case of several sources)?

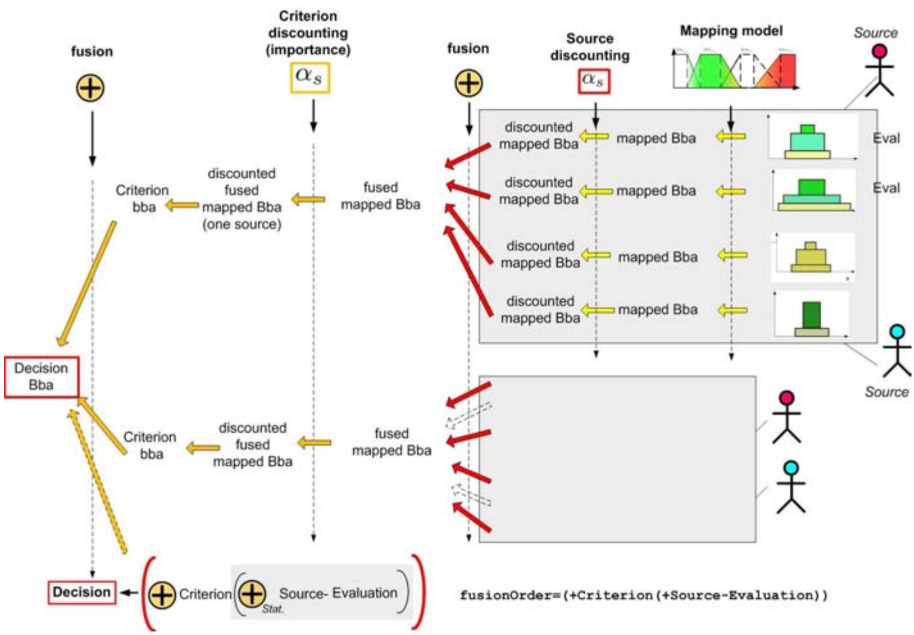


Figure 23.36: Description of the implemented fusion process.

$$m_{(C_{[111]})}(HD_k) = (m_{(C_{[111]},s_1,eval_1)} \oplus m_{(C_{[111]},s_2,eval_1)})(HD_k) \qquad (23.10)$$

In a UML standard, the fusion process can partially be represented as in Fig. 23.37 below.

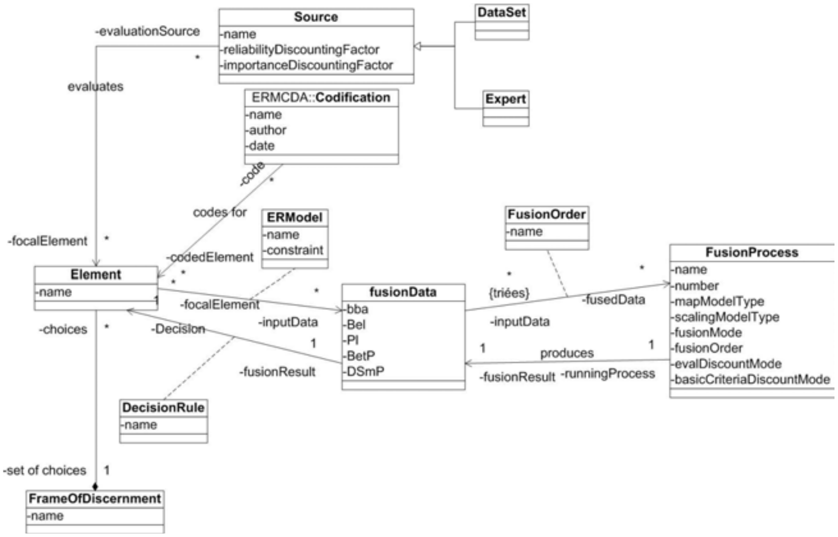


Figure 23.37: ER-MCDA framework - UML Class diagram - Principle of the fusion process.

23.3.5.7 Discounting factors for reliability and importance assessment

The classical reliability discounting factor

In a classical way, discounting factors are used to take into account the reliabilities of the sources. For each source of evidence, α_r with $r \in \{1, 2, \ldots, S\}$ represents the confidence given by the system to this source. $\alpha_r = 1$ corresponds to a totally reliable source of evidence and $\alpha_r = 0$ corresponds to totally unreliable source of evidence [Dezert 2003][10].

The AHP method can be used to calculate the discounting factors. A preference matrix using pairwise comparisons gives the relative weight of importance w_r of each source. After a normalization step based on the maximum of the weights, the discounting factor α_r can be defined as [Beynon 2005][11]:

[10] p.21
[11] p.1891

$$\alpha_r = \frac{w_r}{max(w_k)} \text{ with } k \in \{1, 2, \ldots, S\} \tag{23.11}$$

In our ER-MCDA framework, discounting factors are used at many different steps of the process:

- following the classical approach, a discounting factor is applied to the different sources providing an evaluation for qualitative or quantitative criteria. Normalization factor is used at the evaluation step of qualitative criterion to evaluate the confidence of the assessor in its judgment (*confidence qualitative discounting factor*);
- normalized weights of the basic level criteria are transformed in discounting factors (with a maximum based normalization instead of a direct use of weights – see FIG. 23.38).

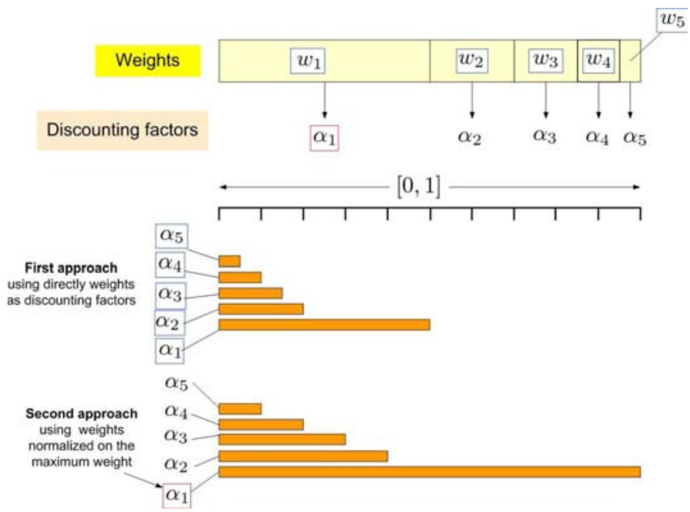


Figure 23.38: From preference weights to discounting factors for $S = 5$ sources.

The last situation may appear as a misunderstanding of the concept of discounting factor. In fact, we consider here that the evaluation of a criterion results both from its importance in the decision process and from the evaluation uncertainty. For a given criterion c_j, the pairwise comparison of qualitative evaluation grades produce weights considered as basic belief assignment related to their contribution to the sensitivity: they are named *importance bba*. This principle justifies the fact that they are used to calculate the bba in the common frame of discernment. For this criterion, the evaluator has some variable confidence about its evaluation: *"Does this evaluation* *grade really belong to the site that I am evaluating?"*. The singletons (extended with intersections in a DSmT framework) are considered to compare the levels of confidence on each of these evaluation grades. The weights are then considered as discounting factors as they reduce the importance of the previous evaluation. A fusion would not have any sense since the discernment frame and the meaning is different.

Can importance be assessed by a (new) discounting factor?

Mixing fusion and multi-criteria decision approaches can lead to the difficulty of making a difference between uncertainty and importance. This also corresponds to a classical discussion about difference between aggregation of preferences and information fusion. In an ideal framework, we do consider that fusion should mainly concern uncertain pieces of evidence and not the preferences between criteria. The final fusion step of mapped basic level criteria should be compared to an aggregation method based on the result of fusion. Nevertheless, we propose in the following section, an experimental approach to take importance into account through a new discounting factor.

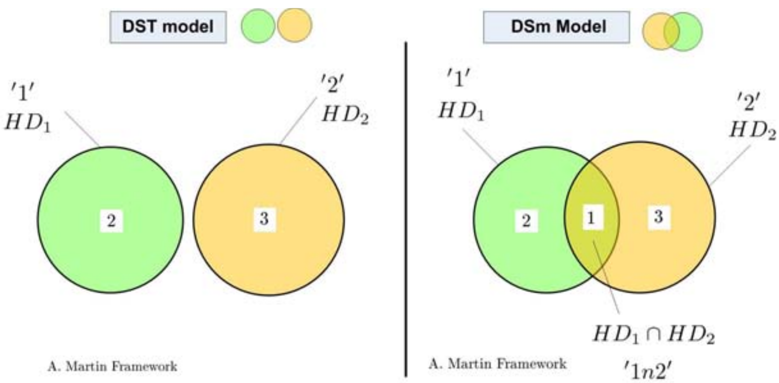


Figure 23.39: DST and DSmT models for importance discounting model.

The classical discounting method transforms a basic belief assignment $m(\cdot)$ through a discounting factor α that reduces the basic belief assignment for each focal element and increases the basic belief assignment assigned to the total ignorance Θ. In our ER-MCDA framework, the mapping process leads to mapped basic belief assignments taking into account the reliability of the different sources. During that first step (so-called mapping and scaling steps), the classical discount method is appropriate since it really corresponds to a variable level of confidence for each evaluation.

The second step of the process aims at fuse the basic level criteria evaluation according to their importance. This last fusion step produces the final basic belief assignment that can be analyzed to make a decision according different rules such as maximum of bba, maximum of credibility, etc. The question is to represent the preferences weights issued from MCDA model in a fusion model. The weights represent relative importance from one criterion to another and not relative uncertainty: a lower importance basic level criterion can be assessed in a certain way while a very important criterion can be very uncertain. Using the classical discounting factor [Beynon 2005] cannot represent this difference since it only corresponds to a reduction of the reliability.

Definition (importance discounting factor): To represent the relative importance of the basic level criteria in a same way than in a multi-criteria decision problem, we propose (following more or less some principles proposed by Smets), a specific (and experimental) importance discounting factor denoted as α_{Imp} and defined as follows: For a source $\mathcal{B}$ described by a bba $m(\cdot)$ relatively to the frame of discernment Θ and used in an ER-MCDA[12] process, the importance discounting factor $\alpha_{Imp,\mathcal{B}}$ is defined as $\alpha_{Imp} \in]0,1]$ such as for any subset $A \subset \Theta$, the importance discounted bba $m'_{Imp}(\cdot)$ is defined by the following eq. (23.12):

$$\begin{cases} m(A) \\ m(\emptyset) \end{cases} \longrightarrow \begin{cases} m'_{Imp}(A) = \alpha_{Imp} \cdot m(A), \quad \forall A \neq \emptyset \\ m'_{Imp}(\emptyset) = (1 - \alpha_{Imp}) + \alpha_{Imp} \cdot m(\emptyset) \end{cases} \tag{23.12}$$

The case where $\alpha_{Imp} = 1$ corresponds to a source $\mathcal{B}$ that has the maximum reachable relative importance value. The principle of this importance discounting factor is to reduce the basic belief assignment related to a given basic level criterion without increasing the total ignorance corresponding to $m(\Theta)$. It is therefore possible to discount a source according by using both to its reliability and its importance.

As it involves basic belief assignment on the empty set, this double discounting method should be used with fusion rules that are able to redistribute conflict and with models that make a difference between the real conflict between hypotheses and the basic belief assignment put on the empty set. The classical Dempster's rule is known to fail when conflict increases: we can expect than it will not be the best choice in our experimental model that consists in artificially transfer bba's on the empty set at the final stage of fusion.

The following examples show the principle of using this importance discounting factor in a very simple case ($Card(\Theta) = 2$) in DST and DSmT frameworks. The source c_1 is supposed to be poor reliable ($\alpha_{Rel,1} = 0.1$) but very important in the decision process ($\alpha_{ImpRel,1} = 1$) while the source c_2 is considered as fully reliable ($\alpha_{Rel,2} = 1$) but not very important in the decision process ($\alpha_{ImpRel,2} = 0.1$). Basic belief assignments correspond to the highest possible level of conflict between sources (see FIG. 23.40).

[12] Evidential reasoning - Multi-criteria decision analysis

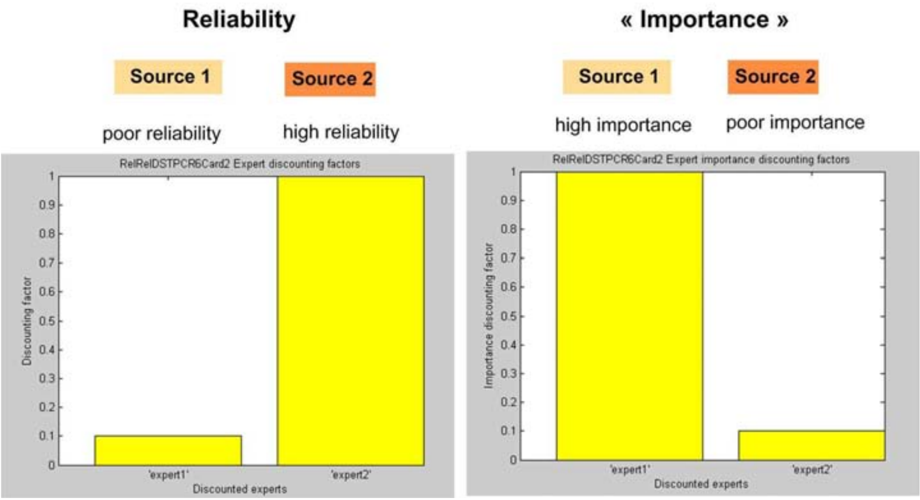


Figure 23.40: Reliability and importance of sources (experts) c_1 and c_2.

First approach: The classical discounting factor is applied twice (successively)

The Table 23.1 (for DST framework) and the Table 23.2 (for DSmT framework) of discounted criteria using a double successive reliability discounting show that no usable difference appears between the different hypotheses.

Using the classical discounting process to represent the relative importance of a criterion compared to another does not seem to be efficient to make a decision. We therefore introduce a new importance discounting factor in that simple test case.

Second (experimental) approach: The classical discounting factor is first applied, a new discounting factor is applied

The Tables 23.3 (for DST framework) and 23.4 (for DSmT framework) of discounted criteria using first a classical reliability discounting and then an importance discounting. The figure 23.41 shows the comparison with a successive discounting process based on the classical discounting factor.

Conclusion and interpretation

In our opinions, using twice the classical discounting factor to represent both reliability and importance does not provide any valuable information for decision (see left side of Fig. 23.41): the bba resulting from fusion are equal for any elements of the frame of discernment. The fusion process fails here to take importance or preference into account. Note also that the bba have been voluntarily chosen with "extreme"

Discounting steps	Source	discounting factor	$\emptyset$	HD_1	HD_2	Θ
			$\Theta = \{HD_1, HD_2\}$ Discounted bba			
initialization: **None**	c_1	none	0	0	1	0
	c_2	none	0	1	0	0
Step 1: **Reliability**	c_1	$\alpha_{Rel,1} = 0.1$	0	0	0.1	0.9
	c_2	$\alpha_{Rel,2} = 1$	0	1	0	0
Step 2: **Reliability**	c_1	$\alpha_{ImpRel,1} = 1$	0	0	0.1	0.9
	c_2	$\alpha_{ImpRel,2} = 0.1$	0.9	1	0	0
Fusion rule: Dempster's rule						
Result of fusion	$c_1 \oplus c_2$	bba	0	0.0909	0.0909	0.8182
		Bel	0	0.0909	0.0909	1
		Pl	0	0.9091	0.9091	1
		$BetP$	0	0.5	0.5	1
		$DSmP$	0	0.5	0.5	1
Fusion rule: Smets rule						
Result of fusion	$c_1 \oplus c_2$	bba	0.01	0.09	0.09	0.81
		Bel	0.01	0.1	0.1	1
		Pl	0.01	0.9	0.9	0.99
		$BetP$	0.01	0.5050	0.505	1
		$DSmP$	0.01	0.4950	0.495	0.99
Fusion rule: PCR6 rule						
Result of fusion	$c_1 \oplus c_2$	bba	0	0.095	0.095	0.81
		Bel	0	0.095	0.095	1
		Pl	0	0.905	0.905	1
		$BetP$	0	0.5	0.5	1
		$DSmP$	0	0.5	0.5	1

Table 23.1: ER-MCDA framework - **double reliability** discounting of two criteria c_1 and c_2 - **DST framework**.

Discounting steps	Source	discounting factor	$\emptyset$	HD_1	$HD_1 \cap HD_2$	HD_2	Θ
			$\Theta = \{HD_1, HD_2\}$ Discounted bba				
initialization: **None**	c_1	none	0	0	0	0.1	0.9
	c_2	none	0.9	0.1	0	0	0
Step 1: **Reliability**	c_1	$\alpha_{Rel,1} = 0.1$	0	0	0	0.1	0.9
	c_2	$\alpha_{Rel,2} = 1$	0	1	0	0	0
Step 2: **Reliability**	c_1	$\alpha_{ImpRel,1} = 1$	0	0	0	0.1	0.9
	c_2	$\alpha_{ImpRel,2} = 0.1$	0.9	0.1	0	0	0
Fusion rule: DSm, Smets or PCR6 rules							
Result of fusion	$c_1 \oplus c_2$	bba	0	0.09	0.01	0.09	0.81
		Bel	0	0.1	0.01	0.1	1
		Pl	0	1	1	1	1
		$BetP$	0	0.685	0.37	0.685	1
		$DSmP$	0	0.99	0.9982	0.999	1

Table 23.2: ER-MCDA framework - **double reliability** discounting of two criteria c_1 and c_2 - **DSmT framework**.

values in our example. In such a case, only the partial conflict redistribution rules

			$\Theta = \{HD_1, HD_2\}$			
			Discounted bba			
Discounting steps	Source	discounting factor	$\emptyset$	HD_1	HD_2	Θ
initialization: **None**	c_1	none	0	0	1	0
	c_2	none	0	1	0	0
Step 1: **Reliability**	c_1	$\alpha_{Rel,1} = 0.1$	0	0	0.1	0.9
	c_2	$\alpha_{Rel,2} = 1$	0	1	0	0
Step 2: **Importance**	c_1	$\alpha_{Imp,1} = 1$	0	0	0.1	0.9
	c_2	$\alpha_{Imp,2} = 0.1$	0.9	1	0	0

Fusion rule: Dempster's rule

Result of fusion	$c_1 \oplus c_2$	bba	0	1	0	0
		Bel	0	1	0	0
		Pl	0	1	0	0
		$BetP$	0	1	0	0
		$DSmP$	0	1	0	0

Fusion rule: Smets rule

Result of fusion	$c_1 \oplus c_2$	bba	0.91	0.09	0	0
		Bel	0.91	1	0	0
		Pl	0.91	0.09	0	0
		$BetP$	0.91	1	0	0
		$DSmP$	0	0.09	0	0

Fusion rule: PCR6 rule

Result of fusion	$c_1 \oplus c_2$	bba	0.486	0.095	0.014	0.4050
		Bel	0.486	0.5810	0.5	1
		Pl	0.486	0.5	0.419	0.514
		$BetP$	0.486	0.7835	0.7025	1
		$DSmP$	0	0.448	0.066	0.5140

Table 23.3: ER-MCDA framework - **Reliability** and **importance** discounting of two criteria c_1 and c_2 - **DST framework**.

manage to provide a result that can be interpreted for a decision. The analysis of the results when using the importance discounting factor at the second step of the fusion process allow to make the following conclusions (see right side of Fig. 23.41):

- the input bba issued from sources c_1 (or expert 1) and c_2 (or expert 2) are transferred on the empty set and on Θ accordingly to their relative reliability and importance;

- the bba resulting from fusion are distributed on the empty set, Θ and the focal elements. The repartition of bba on those elements provides information about information used in the fusion process. They must be interpreted in a relative way. The respectively very high value assigned to the empty set and Θ correspond to the fact that the two sources have respectively "conflicting" or "very different" importance, while the bba assigned to Θ can be classically interpreted as a comparative level of ignorance. Some limits values can probably be identified. Distance between those limits values and the calculated bba would represent the differential importance or reliability of sources;

- in that case of two highly different sources (full reliable but not important versus poor reliable but important source), the fusion process proposes to choose the most important source which is consistent in a decision context. The absolute

			$\Theta = \{HD_1, HD_2\}$				
			Discounted bba				
Discounting steps	Source	discounting factor	$\emptyset$	HD_1	$HD_1 \cap HD_2$	HD_2	Θ
initialization: **None**	c_1	none	0	0	0	0.1	0.9
	c_2	none	0.9	0.1	0	0	0
Step 1: **Reliability**	c_1	$\alpha_{Rel,1} = 0.1$	0	0	0	0.1	0.9
	c_2	$\alpha_{Rel,2} = 1$	0	1	0	0	0
Step 2: **Importance**	c_1	$\alpha_{Imp,1} = 1$	0	0	0	0.1	0.9
	c_2	$\alpha_{Imp,2} = 0.1$	0.9	0.1	0	0	0
		Fusion rule: DSm rule					
Result of fusion	$c_1 \oplus c_2$	bba	0	0.9	0.1	0	0
		Bel	0	1	0.1	0	0
		Pl	0	1	1	0	0
		$BetP$	0	1	0.55	0	0
		$DSmP$	0	1	0.999	0	0
		Fusion rule: Smets rule					
Result of fusion	$c_1 \oplus c_2$	bba	0.9	0.09	0.01	0	0
		Bel	0.9	1	0.91	0	0
		Pl	0.9	0.1	0.1	0	0
		$BetP$	0.9	1	0.955	0	0
		$DSmP$	0	1	0.099	0	0
		Fusion rule: PCR6 rule					
Result of fusion	$c_1 \oplus c_2$	bba	0.486	0.09	0.01	0.009	0.4050
		Bel	0.486	0.586	0.496	0.505	1
		Pl	0.486	0.514	0.514	0.514	0.514
		$BetP$	0.486	0.8605	0.6805	0.82	1
		$DSmP$	0	0.5136	0.5131	0.5135	0.514

Table 23.4: ER-MCDA framework - **Reliability** and **importance** discounting of two criteria c_1 and c_2 - **DSmT framework**.

value of the bba (here a very low value) and the relative bba assigned to the empty set and Θ provides additional information to interpret this result: the decision is clearly not the result from a complete consensus between sources.

This proposition must obviously be discussed and analyzed in a further way from a practical and theoretical way.

23.3.6 Decision-making

This final step corresponds to the ultimate goal of the whole process. All the more or less uncertain evaluations, provided by more or less reliable sources are fused in a unique decision criteria that has to be analyzed to make a decision. In our framework, the decision is analyzed according to the fusion parameters such as basic belief assignments, credibility, plausibility, pignistic probability assigned to the different hypotheses of the frame of discernment.

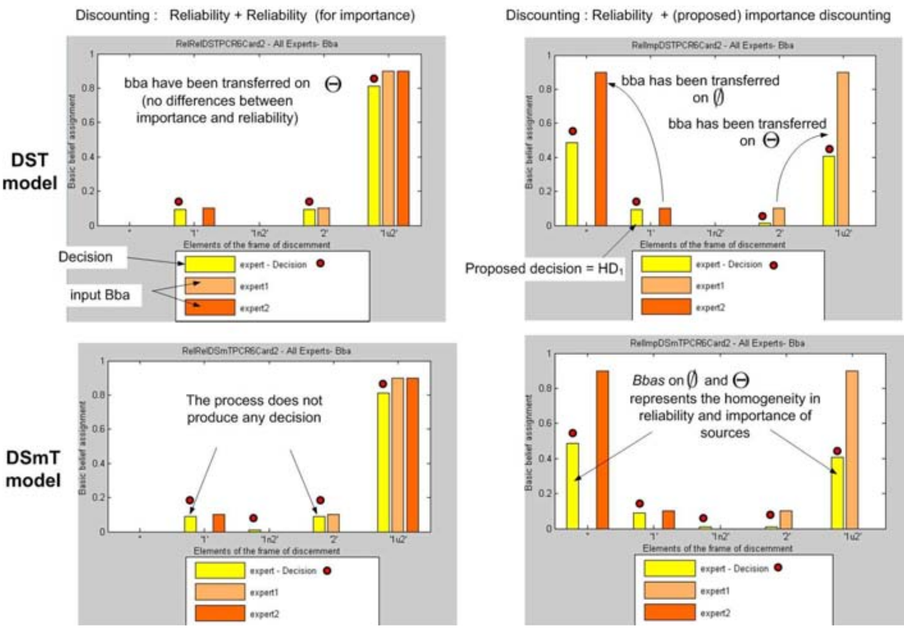


Figure 23.41: Comparison of discounting method: reliability–reliability and reliability–importance (DST and DSmT frameworks).

Making a decision on the basis of these values is a well-known problem for fusion applications [Martin and Quidu 2008, Bloch et al. 2001]. The existing applications mixing evidential reasoning and multi-criteria decision analysis also use these functions to choose a solution once the fusion is done: [Beynon et al. 2000] interprets the results according to the interval between credibility and plausibility: the smaller interval, the more certain is the alternative. There is still some place for proposition of some methods allowing to interpret the results of fusion in a more operational way with one essential objective: the decision must remain understable by the decision-makers themselves !

23.4 Applications: Sensitivity index in a multi-experts environment

We present here two applications cases:

- the first one is a simplified model corresponding to an evaluation of a sensitivity index for snow avalanches in a multi-expert framework. This case is illustrated through numerical examples applied to examples of (quantitative and qualitative) basic level criteria. The process from the evaluations to the mapped bba is illustrated through partial results.

- the second case deals with a geographic application of risk zoning maps, introducing the problem and the specificity for spatial extent of the method without any numerical results.

23.4.1 Sensitivity index in a multi-experts environment

23.4.1.1 Implementation

The DSmT framework allows coping with uncertain and imprecise information. Its main drawback is the complexity in calculations due to the huge number of elements in D^Θ (e.g. with $|\Theta| = 3$ we already get $|D^\Theta| = 19$ elements, with $|\Theta| = 3$ we get $|D^\Theta| = 167$, etc). However, not all the elements of the hyper-power set D^Θ have to be filled in and some automated routines and programs have been proposed either to encode the -power set or to implement the DSmH rule of combination [Djiknavorian and Grenier 2006].

In our application, we use a new and powerful calculation framework that allows to consider in an easy and versatile way the different models *free DSm Model* denoted ($\mathcal{M}^f(\Theta)$, the *hybrid DSm Model* $\mathcal{M}(\Theta)$ or *Shafer's Model* $\mathcal{M}^0(\Theta)$ [Martin 2009]. These different models correspond to an increasing level of constraints between the different hypotheses of the frame of discernment. Fusion routines have been encapsulated in a global framework that evaluates the multi-criteria decision model and then operates fusion of the basic level criteria. Although it was developed in MATLAB$^{\text{TM}}$, this tool has been designed according to object-oriented development principles. An UML conceptual model has been designed to describe the global process. All data are saved in hierarchical structures allowing an easy access to all steps of calculation. The data structures and internal functions can be modified to deal with other hierarchical model. Some graphical functions have been developed to help the user to interpret results.

23.4.1.2 Description of the hierarchic model

In mountainous areas and in France in particular, snow-avalanches are known to be important risks. Face to numerous avalanches prone areas, decision-makers try to determine an exposure level for any site and secondly to propose a classification based on this sensitivity level: this ranking, based on the evaluation of the hazard and vulnerability levels (FIG. 23.42) can then be used to prioritize prevention strategies implementation.

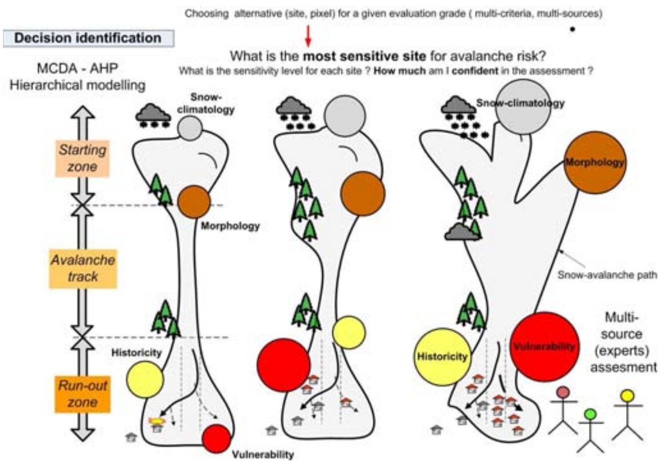


Figure 23.42: Decision context: ranking avalanche prone areas according hazard and vulnerability related criteria.

We present here a simplified version of the real existing decision support system which consists of a 6-level hierarchy [Rapin 2007] called *SSA* for *Sites Sensible Avalanches* (sensitive avalanche paths). In comparison with the original and existing framework, this application aims at merge several expert evaluations to determine the sensibility index of a snow-avalanche prone-area including imprecise and uncertain evaluations of both qualitative and quantitative criteria. The root of this 3-level hierarchical model (FIG. 23.43) corresponds to the sensitivity level of the avalanche-prone area ($C_{[1]}$). Its principles is based on the classical risk equation as presented in FIG. 23.2. This sensitivity is evaluated according to two sub-criteria corresponding to vulnerability ($C_{[11]}$) and hazard ($C_{[12]}$). The vulnerability criterion is broken down into two basic-level criteria corresponding to a permanent winter occupants ($C_{[111]}$) and living places/infrastructures ($C_{[112]}$). The hazard criterion is broken down into three basic-level criteria corresponding to morphology ($C_{[121]}$), history ($C_{[122]}$) and Snow-climatology ($C_{[123]}$). In the original model, each basic level criterion is evaluated according to a *criterion-estimator-solution* model (FIG. 23.10).

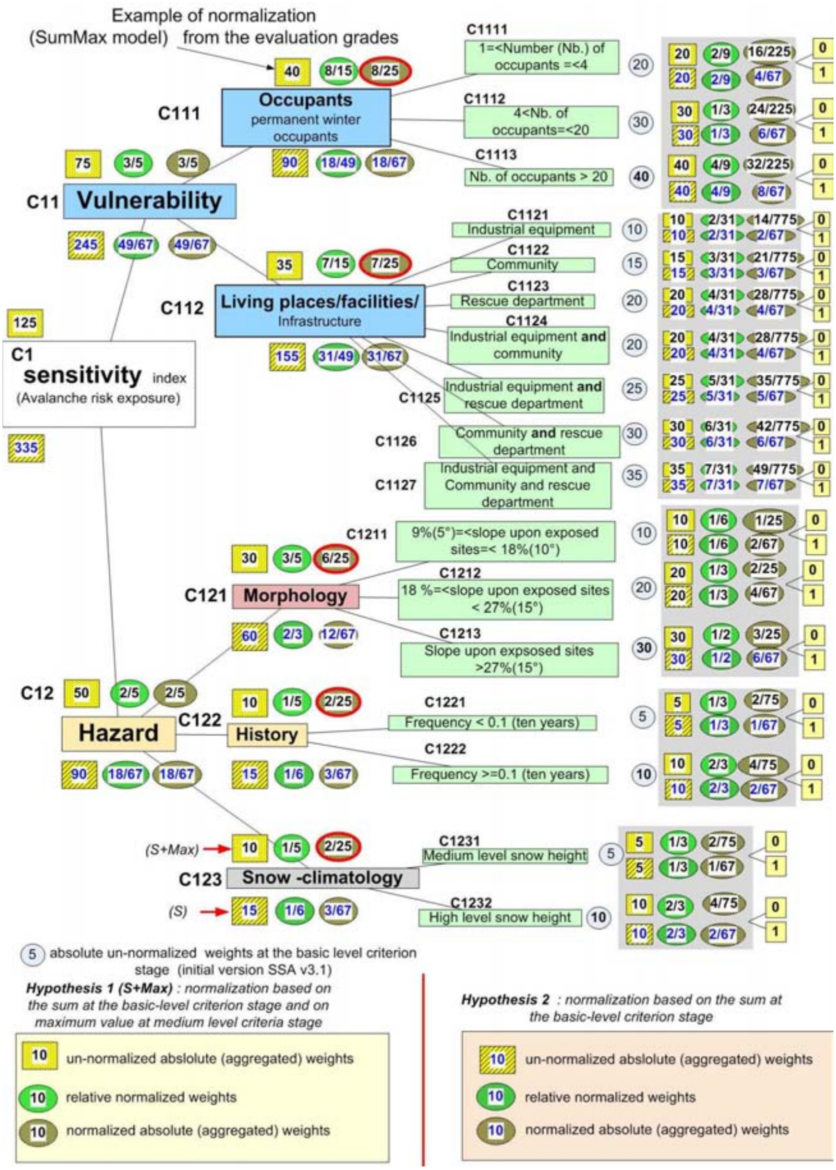


Figure 23.43: Sample simplified model of the Avalanche sensitivity framework.

Note that numerical values used in this sample model are fictive and do not correspond to real numerical intervals used in the original model. The evaluation of five basic-level criteria is done accordingly to the following hypothesis:

- Winter occupants ($C_{[111]}$): this quantitative criterion is evaluated according to the number of winter occupants with 3 evaluation grades in the initial version;
- Living places ($C_{[112]}$): this qualitative criterion is evaluated according to seven evaluation grades corresponding to existing facilities or infrastructures in the studied area;
- Morphology ($C_{[121]}$): this quantitative criterion is evaluated according the slope angle;
- History ($C_{[122]}$): this quantitative criterion is evaluated to an empirical frequency;
- Snow-Climatology ($C_{[123]}$): this quantitative criterion is evaluated according snow-height.

The initial evaluation classes are used as a basis to build the mapping model used in the ER-MCDA model. In that model, classes do not exist anymore for quantitative criteria: the expert provide an evaluation on real numerical values which are then mapped into the elements of the common frame of discernment. For qualitative criteria, a specific method is proposed to consider the level of confidence of the evaluation. In a classical hierarchical AHP approach, weights are calculated for each criterion according to pairwise comparisons from the root criterion to the basic level criterion level. This principle requires having an equal number of evaluation grades for each criterion: increasing the number of evaluation grades for a given basic level criterion induces an higher weight of the basic level criterion with a classical normalization method based on sum. The initial model from which is derived our sample model had not been designed according to this principle. It was not described as a hierarchical structure and un-normalized weights had been defined directly by the experts for each evaluation grade of the basic-level criteria (e.g. 20 for the evaluation grade $C_{[1111]}$ corresponding to a class of winter occupants ranging from 1 to 4 persons). To transform these values into normalized weights and propagate them to the different levels of the hierarchy, different normalization principles can be used. In our application, based on a criterion-estimator-solution framework, we use a so-called SumMax method which is based on the following principle: un-normalized weights (at the evaluation grade) are normalized using the sum. The absolute weight of basic level criteria corresponds to the maximum of un-normalized weights of the evaluation grades. Normalization is then done on a sum basis for the other criteria levels up to the root level. The normalized weights are then calculated from the root to the basic level criteria: they are then used to calculate the importance discounting criteria (Fig. 23.44).

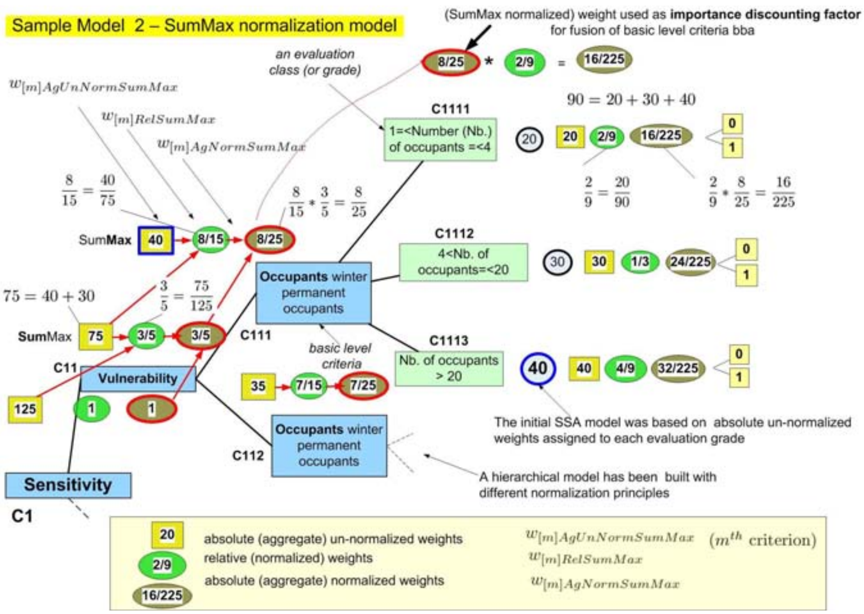


Figure 23.44: From hierarchical model to importance discounting factor.

In our context, we need to build a link between the criteria and the common frame of discernment. The first step of the process consists of mapping the evaluation of basic level criteria by the sources. For each criterion, the mapped bba of those evaluations are then fused together to get a mapped bba for each basic level criterion (FIG. 23.36). Examples of results are described in detail for an example of quantitative criterion ($C_{[111]}$) and for an example of a qualitative criterion ($C_{[112]}$) in the following sections. Only evaluation interval data and a summary table is given for the others criteria.

23.4.1.3 Example of results for the quantitative criterion $C_{[111]}$

The criterion $C_{[111]}$ is a quantitative basic level criterion which corresponds to the vulnerability due to permanent winter occupants in the area. The evaluation provided by the sources consists of numerical intervals corresponding to the number of occupants. First, each source defines numerical intervals with necessity levels (FIG. 23.45).

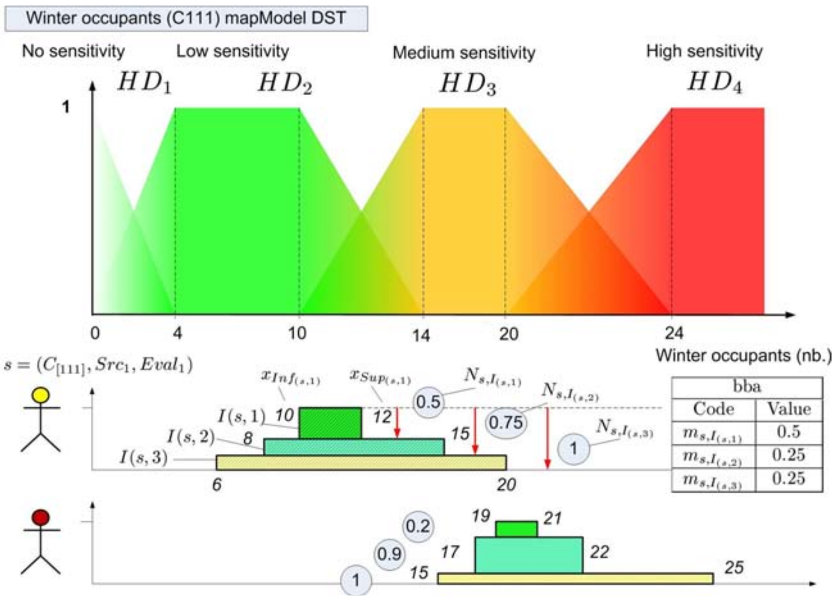


Figure 23.45: From evaluation intervals to mapped bba of intervals.

These necessity levels, interpreted as confidence levels are transformed into bba (Fig. 23.17). The bba corresponding to each evaluation interval are then transferred to each element of the frame of discernment corresponding to the chosen mapping model (DST or DSmT mapping model) according to their areas. The mapped bba for the first evaluation (including 3 intervals) of the source no. 1 is compiled in the Table 23.5 and described in a graphical way in Fig. 23.46. The principle of this calculation as it can be checked in the implemented software application is presented on (Fig. 23.47). For a given source and its evaluations intervals, different mapping processes can be applied. We only present here partial results for a DST *mapping model*.

		source s coded by $(C_{[111]}, Src_1, Eval_1)$			
		Frame of discernment - DST - $\Theta = \{NoS, LS, MS, HS\}$			
		NoS	LS	MS	HS
Int.	Code	$m_{s,I_{(s,Int)}}(HD_1)$	$m_{s,I_{(s,Int)}}(HD_2)$	$m_{s,I_{(s,Int)}}(HD_3)$	$m_{s,I_{(s,Int)}}(HD_4)$
1	$I(s,1)$	0	0.375	0.125	0
2	$I(s,2)$	0	0.1429	0.1071	0
3	$I(s,3)$	0	0.1071	0.1429	0

Table 23.5: Mapped Basic belief assignment (bba)- Criterion $C_{[111]}$ - Source 1 - Evaluation 1 - Fusion process no. 1 - DST framework.

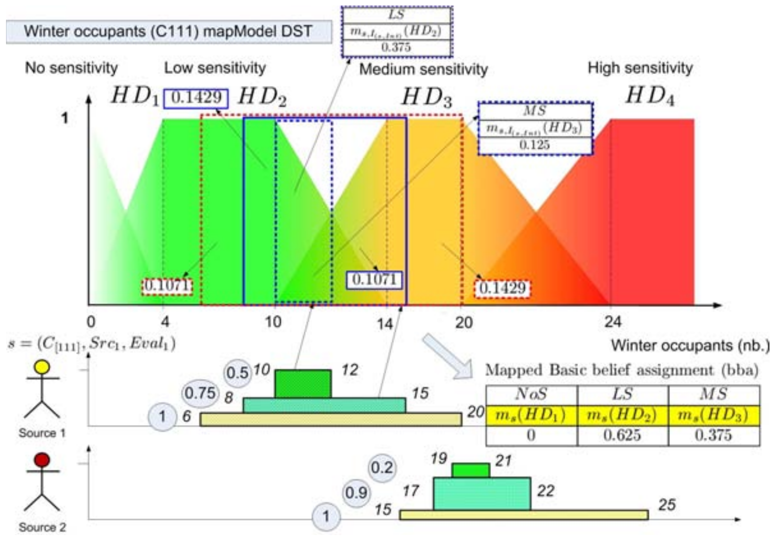


Figure 23.46: From mapped bba of evaluation to mapped bba of criterion $C_{[111]}$.

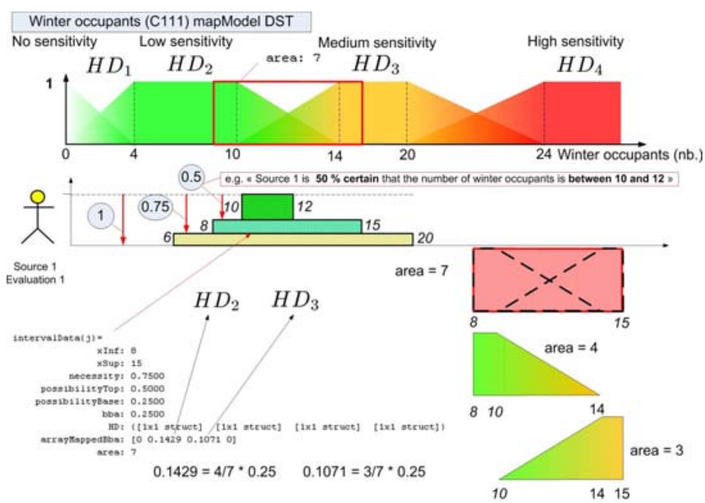


Figure 23.47: Results of mapping process of criterion $C_{[111]}$.

	Frame of discernment - DST - $Card(\Theta) = 4$						
	α	$\emptyset$	HD_1	HD_2	HD_3	HD_4	Θ
	Discounting factor	empty set	No sensitivity (NoS)	Low sensitivity (LS)	Medium sensitivity (MS)	High sensitivity (HS)	Θ
$m_1 = m_{(C_{[111]},Src_1,Eval_1)}$	0.5	0	0	0.3125	0.1875	0	0.5
$m_2 = m_{(C_{[111]},Src_2,Eval_1)}$	0.7	0	0	0	0.621	0.079	0.3
Fusion process no. 1 - Dempster-Shafer (normalized rule)							
$m_{(C_{[111]})} = m_1 \oplus m_2$		0	0	0.1223	0.6306	0.0514	0.1957
Fusion process no. 2 - Smets rule							
$m_{(C_{[111]})} = m_1 \oplus m_2$		0.2335	0	0.0937	0.4834	0.0394	0.15
Fusion process no. 3 - PCR6 rule							
$m_{(C_{[111]})} = m_1 \oplus m_2$		0	0	0.1784	0.6229	0.0487	0.15

Table 23.6: Criterion $C_{[111]}$- **Discounted (evaluation) sources** - DST Framework - Fusion order $(\oplus Criterion(\oplus Source - Evaluation))$.

Fusion of mapped bba for $C_{[111]}$

A comparison of different combination rules (DST-normalized, Smets and PCR6 rules) in a DST mapping framework can be done using the same input data taking into account un-discounted or discounted evaluation sources (see the Table 23.6) according to the user choice.

23.4.1.4 Example of results for the qualitative criterion $C_{[112]}$

The evaluation of a qualitative criterion uses both a *scaling model* to produce a belief (credibility) interval and a *mapping model* to transform this credibility interval into the common frame of discernment. The criterion $C_{[112]}$ is a qualitative criterion which corresponds to the vulnerability due to the infrastructures, facilities and collective equipments such as schools in the area. Three main categories corresponding to industrial equipment, collective or community equipments and rescue equipments. Two different scaling models (DST *scaling model* or DSmT scaling model) can be used to transform the evaluation provided by the source into a belief interval that will be further used in the qualitative mapping process (see FIG. 23.48).

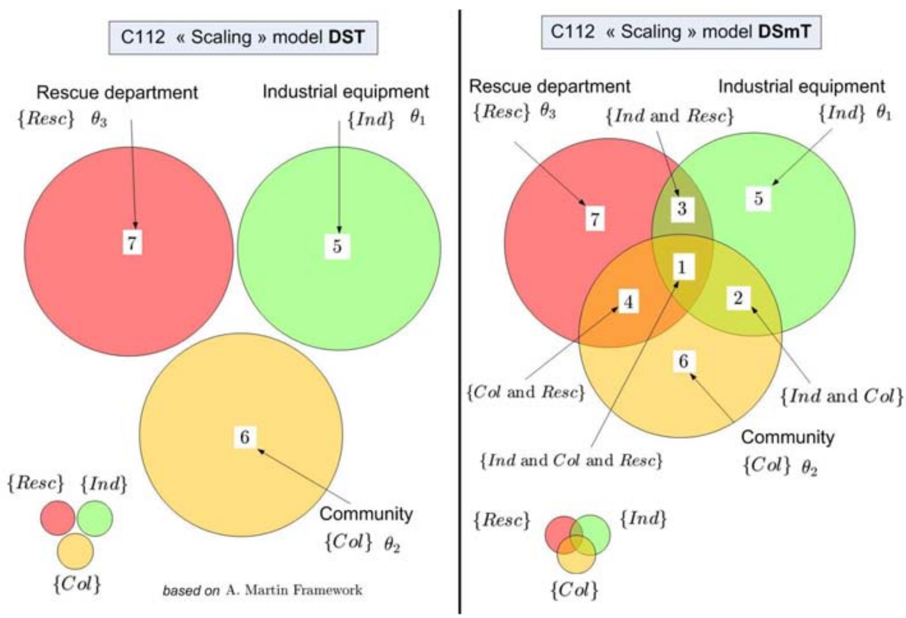


Figure 23.48: Two models for quantitative criterion "Living places" $C_{[112]}$.

We only present here partial results for a DSmT scaling model and a DST mapping model. The choice of the scaling model depends on the nature of the infrastructures that exist on the site. Some infrastructures may belong to the same time to several categories. To take this into account, we can imagine a DSmT scaling model which will be presented here. Each qualitative category is analyzed according to its importance (contribution) to the vulnerability using a pairwise comparison approach.

The weights are directly interpreted as bba's. For each combination of types of infrastructures, credibility values are calculated as shown in Fig. 23.49.

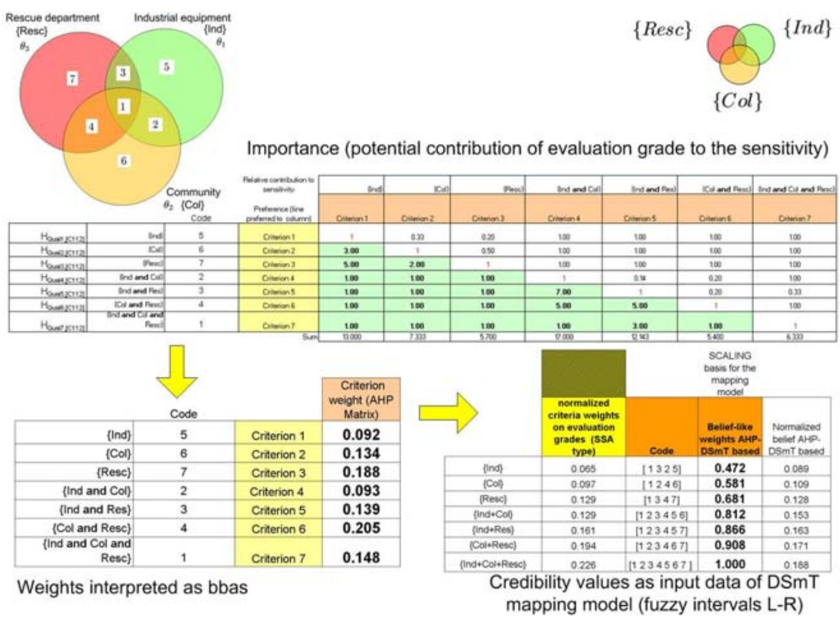


Figure 23.49: Qualitative criterion $C_{[112]}$ - DSmT scaling - Importance of the evaluation grade for mapping model.

The qualitative mapping model is built to establish a correspondence between an interval evaluation (with a lower and an higher value of credibility) and the common frame of discernment according to the chosen mapping model (for a DST mapping model and a DSmT scaling model, see FIG. 23.50).

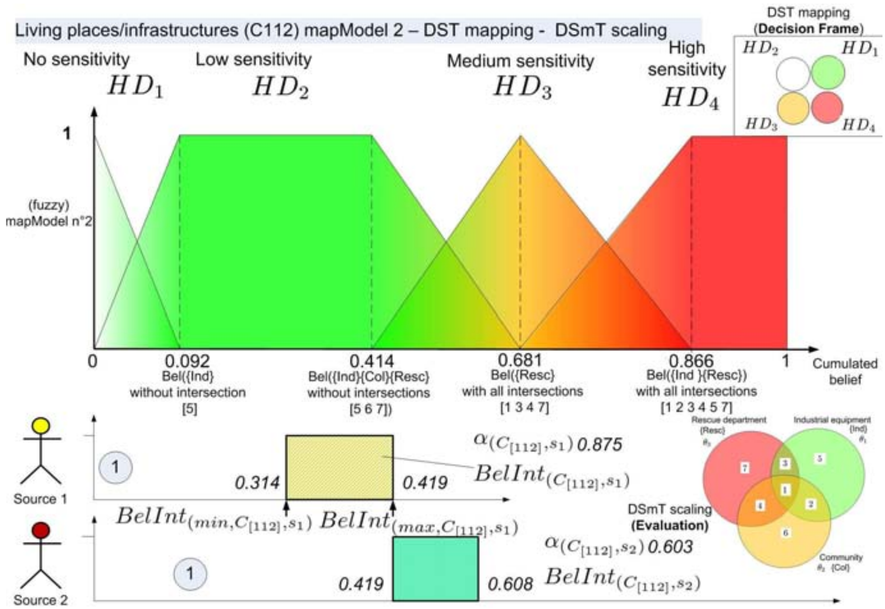


Figure 23.50: Qualitative criterion $C_{[112]}$ - DST mapping - DSmT scaling - Evaluation intervals for sources 1 and 2.

To provide an evaluation, the user chooses an input value that indicates if the chosen category exists in the zone and then a rating of its confidence level about its evaluation (FIG. 23.51). Results are a weighted belief interval and a discounting factor about this belief interval. These values are then used in the qualitative mapping process using the same principle than described for quantitative criterion.

SOURCE 1 Criterion C112 Living Places/Facilities confidence assessment

DSmT Scaling Minimum ranking value 1

Max 9

Input Evaluation boolean	Ranking from 1 to 9 (Saaty's scale)		RANKING input Minimum	RANKING input Maximum	CONFIDENCE Minimum	CONFIDENCE Maximum	CONFIDENCE Mean confidence	Maximum bba AHP DSmT based	Intervals of belief Min belief	Intervals of belief Max belief
1	$H_{Qual1,[C112]}$	{Ind}	7	9	0.75	1	0.875	0.092	0.069	0.092
0	$H_{Qual2,[C112]}$	{Col}	1	1	0	0	0	0.134	0.000	0.000
1	$H_{Qual3,[C112]}$	{Resc}	7	9	0.75	1	0.875	0.188	0.141	0.188
0	$H_{Qual4,[C112]}$	{Ind and Col}	1	1	0	0	0	0.093	0.000	0.000
1	$H_{Qual5,[C112]}$	{Ind and Res}	7	9	0.75	1	0.875	0.139	0.104	0.139
0	$H_{Qual6,[C112]}$	{Col and Resc}	1	1	0	0	0	0.205	0.000	0.000
0	$H_{Qual7,[C112]}$	{Ind and Col and Resc}	1	1	0	0	0	0.148	0.000	0.000

	Min	Max
Interval of belief	**0.314**	**0.419**

Weighted discounting factor calculation

Maximum bba *Mean confidence	Maximum bba
0.080	0.092
0.000	0.000
0.165	0.188
0.000	0.000
0.122	0.139
0.000	0.000
0.000	0.000

N= Total weighted belief	D=Total of maximum bba
0.366	0.419

Weighted discounting factor (N/D) **0.875**

DST Scaling

Min 1

Max 9

Input Evaluation boolean	Ranking from 1 to 9 (Saaty's scale)		RANKING input Minimum	RANKING input Maximum	CONFIDENCE Minimum	CONFIDENCE Maximum	CONFIDENCE Mean confidence	Maximum bba AHP DST based	Intervals of belief Min belief	Intervals of belief Max belief
1	$H_{Qual1,[C112]}$	{Ind}	6	8	0.625	0.875	0.75	0.172	0.108	0.151
0	$H_{Qual2,[C112]}$	{Col}	1	1	0	0	0	0.350	0.000	0.000
1	$H_{Qual3,[C112]}$	{Resc}	7	9	0.75	1	0.875	0.478	0.358	0.478

	Min	Max
Interval of belief	**0.466**	**0.628**

Weighted discounting factor calculation

Maximum bba *Mean confidence	Maximum bba
0.129	0.172
0.000	0.000
0.418	0.478

N= Total weighted belief	D=Total of maximum bba
0.547	0.650

Weighted discounting factor **0.842**

Figure 23.51: Qualitative criterion $C_{[112]}$ - DST and DSmT mapping - confidence levels - source 1.

			Ranking		Confidence			Max. bba	Belief interval	
$input_g$	Evaluation grades	Definition	$inputConfRank_{min}$	$inputConfRank_{max}$	$conf_{min}$	$conf_{min}$	$conf_{mean}$	Bel_g	$BelInt_{min,g}$	$BelInt_{max,g}$
1	$H_{Qual_1,C_{[112]}}$	$\{Ind\}$	7	9	0.75	1	0.875	0.092	0.069	0.092
0	$H_{Qual_2,C_{[112]}}$	$\{Col\}$	1	1	0	0	0	0.134	0	0
1	$H_{Qual_3,C_{[112]}}$	$\{Resc\}$	7	9	0.75	1	0.875	0.188	0.141	0.188
0	$H_{Qual_4,C_{[112]}}$	$\{Ind\}$ and $\{Col\}$	1	1	0	0	0	0.093	0	0
1	$H_{Qual_5,C_{[112]}}$	$\{Ind\}$ and $\{Resc\}$	7	9	0.75	1	0.875	0.139	0.104	0.139
0	$H_{Qual_6,C_{[112]}}$	$\{Col\}$ and $\{Resc\}$	1	1	0	0	0	0.205	0	0
0	$H_{Qual_7,C_{[112]}}$	$\{Ind\}$ and $\{Col\}$ and $\{Resc\}$	1	1	0	0	0	0.148	0	0
									0.314	0.419

Table 23.7: Qualitative criterion c_j - Source 1 - interval of beliefs and weighting discounting factor elicitation - DSmT scaling.

Belief interval: from scaling to mapping

A belief interval resulting from the scaling process is used as data in the mapping process of the qualitative criterion. An example of user inputs for confidence levels is given for the criterion $C_{[112]}$ in a context of so-called DSmT scaling where the evaluation grades can have non empty intersections (see the Table 23.7) and in a context of so-called DST scaling where the evaluation grades are considered as exclusive from one to each other (see Table **??**). In our application case, for a DST scaling, we get $BelInt_{(C_{[112]},s_1)} = [0.466, 0.628]$ and $\alpha_{(C_{[112]},s_1)} = 0.842$. For a DSmT scaling, we get $BelInt_{(C_{[112]},s_1)} = [0.314, 0.419]$ and $\alpha_{(C_{[112]},s_1)} = 0.875$.

23.4.1.5 Partial results for quantitative criteria: $C_{[121]}$, $C_{[122]}$ & $C_{[123]}$

The following figures describe the evaluation data interval provided by two sources for each of the basic level criteria related to the hazard evaluation in a DST mapping model for the morphology criterion $C_{[121]}$ (Fig. 23.52), the history criterion $C_{[122]}$ (Fig. 23.53) and the snoow-meteorology criterion $C_{[123]}$ (Fig. 23.54). The resulting mapped bba for each criterion and each source are then discounted and injected in a fusion process that produces a mapped bba for each criterion.

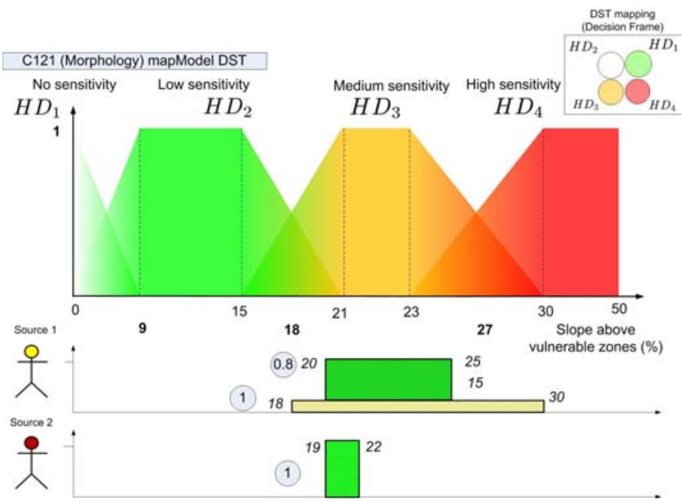


Figure 23.52: Quantitative criterion $C_{[121]}$ - DST mapping - Evaluation intervals for sources 1 and 2.

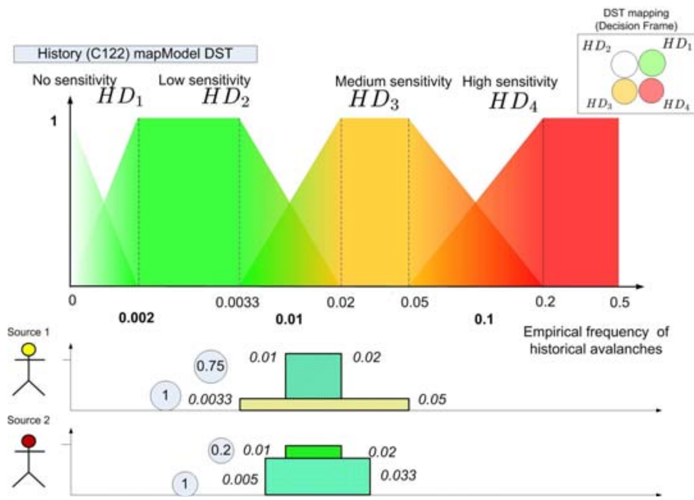


Figure 23.53: Quantitative criterion $C_{[122]}$ - DST mapping - Evaluation intervals for sources 1 and 2.

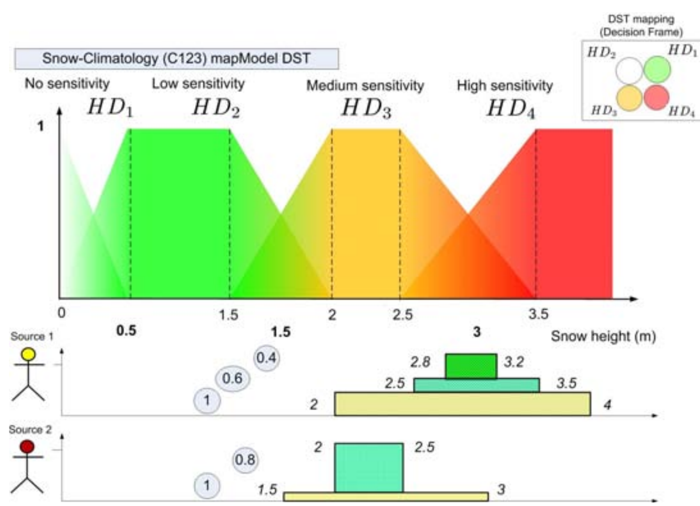


Figure 23.54: Quantitative criterion $C_{[123]}$ - DST mapping - Evaluation intervals for sources 1 and 2.

23.4.1.6 Decision level-criterion $C_{[1]}$

Fusion processes are described in an extensive way according to all the parameters chosen for fusion. On the basis on the same input data set, different simulations can be done to compare fusion rules, mapping models, ... (see FIG. 23.55). This section presents results at the decision level for different examples corresponding to a DST mapping model (fusion processes no. 1 and 3) and a DSmT mapping model (fusion processes no. 7 and 9). These results compare two different mapping models with the same fusion rule (e.g processes 1 and 7 or 3 and 9), the same mapping model with different fusion rules (e.g. processes 1 and 3 or 7 and 9).

Decision level - Fusion process - DST framework

The following tables present the results of fusion of discounted basic-level criteria:

- For the fusion process no. 1, see Table 23.8;
- For the fusion process no. 3, see Table 23.9.

The bba's in the following tables correspond to un-discounted values. The result of fusion comes from discounted bba 's. For each basic level criterion (e.g. $C_{[111]}$), the basic belief assignments correspond to the result of fusion of the discounted evaluations of the different sources (for $C_{[111]}$), this corresponds to the fusion of $m_1 = m_{(C_{[111]},Src_1,Eval_1)}$ and described in the table of FIG. 23.6. The importance discounting factors are deduced from the hierarchical decision model depending on the normalization and evaluation data input. In that example, we use the *SumMax* model (FIG. 23.43).

Decision level - Fusion process - DSmT framework

The following tables present the equivalent results to fusion process no. 1 and 3 with only changes in the mapping model (from DST model to DSmT model):

- For the fusion process no. 7, see Table 23.10;
- For the fusion process no. 9, see Table 23.11.

Note that in a DSmT model, results are the same for DST rule (to be understood as *DSm* rule) and PCR6 rules since the conflict does not exist.

Decision Model	Name	SampleModel2
	Number of (basic-level)criteria	5
	File	sampleModel2.mat

Fusion Process	Number	1	3
	Name	Fusion process 1	Fusion process 3
Evidential reasoning (ER) framework	Frame	NoS - LowS-MediumS-HighS	NoS - LowS-MediumS-HighS
	cardTheta	**4**	**4**
	ER Theory	**DST**	**DST**
Criteria and evaluations discounting	Evaluation discounting mode	discounted evaluation	discounted evaluation
	Basic criteria discounting mode	discounted basic level CriteriaSumMax	discounted basic level CriteriaSumMax
Mapping and scaling process	Mapping model type	DST map	DST map
	Scaling model type (for qualitative criteria)	DST scaling	DST scaling
Fusion parameters	Fusion rule	DST (normalized)	PCR6
	Fusion mode	Static	Static
	Fusion Order	(+Crit(+Src-Eval))	(+Crit(+Src-Eval))
	Calculation framework	A. Martin © 2008	A. Martin © 2008
Decision	Decision Rule	Max. pignistic probability	Max. pignistic probability
	Element of decision	Singletons	Singletons
	Display mode	Combination and decision	Combination and decision

Fusion Process	Number	7	9
	Name	Fusion process 7	Fusion process 9
Evidential reasoning (ER) framework	Frame	NoS - LowS-HighS	NoS - LowS-HighS
	cardTheta	**3**	**3**
	ER Theory	**DsmT**	**DsmT**
Criteria and evaluations discounting	Evaluation discounting mode	discounted evaluation	discounted evaluation
	Basic criteria discounting mode	discounted basic level CriteriaSumMax	discounted basic level CriteriaSumMax
Mapping and scaling process	Mapping model type	DSmT map	DSmT map
	Scaling model type (for qualitative criteria)	DST scaling	DST scaling
Fusion parameters	Fusion rule	DST (normalized)	PCR6
	Fusion mode	Static	Static
	Fusion Order	(+Crit(+Src-Eval))	(+Crit(+Src-Eval))
	Calculation framework	A. Martin © 2008	A. Martin © 2008
Decision	Decision Rule	Max. pignistic probability	Max. pignistic probability
	Element of decision	Singletons and focal elements	Singletons and focal elements
	Display mode	Combination and decision	Combination and decision

(+Crit(+Src-Eval)) corresponds to the fusion order (+Criterion(+Source-Evaluation))

Figure 23.55: Description of the fusion processes no. 1, 3, 7 and 9.

	Frame of discernment - DST - $Card(\Theta) = 4$						
	α	$\emptyset$	HD_1	HD_2	HD_3	HD_4	Θ
	Importance discounting factor (AHP SumMax Model)	empty set	No sensitivity (NoS)	Low sensitivity (LS)	Medium sensitivity (MS)	High sensitivity (HS)	Θ
$m_1 = m_{(\oplus C_{[111]})}$	1	0	0	0.1223	0.6306	0.0514	0.1957
$m_2 = m_{(\oplus C_{[112]})}$	0.875	0	0	0.0011	0.9164	0.0238	0.0587
$m_3 = m_{(\oplus C_{[121]})}$	0.75	0	0	0.0293	0.9536	0.005	0.012
$m_4 = m_{(\oplus C_{[122]})}$	0.25	0	0	0.1633	0.8194	0	0.0172
$m_5 = m_{(\oplus C_{[123]})}$	0.25	0	0	0.0125	0.9224	0.0453	0.0198
Fusion process no. 1 - *Dempster-Shafer (normalized rule)*							
$m_{(\oplus C_{[1]})} = m_1 \oplus m_2 \oplus m_3 \oplus m_4 \oplus m_5$		0	0	0.0059	0.9842	0.0031	0.0068

Table 23.8: Decision criteria $C_{[1]}$- **Fusion Process no.** 1 - **Discounted basic level criteria - Discounted evaluation sources** DST Framework - Fusion order ($\oplus Criterion(\oplus Source - Evaluation)$) - DST mapping - DST scaling.

	Frame of discernment - DST - $Card(\Theta) = 4$						
	α	$\emptyset$	HD_1	HD_2	HD_3	HD_4	Θ
	Discounting factor (AHP SumMax Model)	empty set	No sensitivity (NoS)	Low sensitivity (LS)	Medium sensitivity (MS)	High sensitivity (HS)	Θ
$m_1 = m_{(\oplus C_{[111]})}$	1	0	0	0.1784	0.6229	0.0487	0.15
$m_2 = m_{(\oplus C_{[112]})}$	0.875	0	0	0.001	0.9102	0.0368	0.052
$m_3 = m_{(\oplus C_{[121]})}$	0.75	0	0	0.0393	0.9363	0.0143	0.01
$m_4 = m_{(\oplus C_{[122]})}$	0.25	0	0	0.1902	0.7973	0	0.0125
$m_5 = m_{(\oplus C_{[123]})}$	0.25	0	0	0.0098	0.8386	0.1392	0.0125
Fusion process no. 3 - PCR6 Rule							
$m_{(\oplus C_{[1]})} = m_1 \oplus m_2 \oplus m_3 \oplus m_4 \oplus m_5$		0	0	0.0196	0.8279	0.0047	0.1478

Table 23.9: Decision criteria $C_{[1]}$ - **Fusion Process no. 3 - Discounted basic level criteria - Discounted evaluation sources** DST Framework -($\oplus Criterion(\oplus Source - Evaluation)$) - DST mapping - DST scaling.

	Frame of discernment - DSmT - $Card(\Theta) = 3$							
	α	$\emptyset$	HD_1	$HD_1 \cap HD_2$	HD_2	$HD_2 \cap HD_3$	HD_3	Θ
	Discounting factor (AHP SumMax Model)	empty set	No sensitivity (NoS)	$NoS \cap LS$	Low sensitivity (LS)	$LS \cap HS$	High sensitivity (HS)	Θ
$m_1 = m_{(\oplus C_{[111]})}$	1	0	0	0	0.3337	0.2238	0.2925	0.15
$m_2 = m_{(\oplus C_{[112]})}$	0.875	0	0	0	0.3068	0.2995	0.3417	0.052
$m_3 = m_{(\oplus C_{[121]})}$	0.75	0	0	0	0.3706	0.3841	0.2354	0.01
$m_4 = m_{(\oplus C_{[122]})}$	0.25	0	0	0	0.8638	0.0989	0.0248	0.0125
$m_5 = m_{(\oplus C_{[123]})}$	0.25	0	0	0	0.2752	0.3986	0.3138	0.0125
Fusion process no. 7 - Dempster-Shafer (normalized rule)								
$m_{(\oplus C_{[1]})} = m_1 \oplus m_2 \oplus m_3 \oplus m_4 \oplus m_5$		0	0	0	0.0868	0.8563	0.0532	0.0037

Table 23.10: Decision criteria $C_{[1]}$ - **Fusion Process no. 7 - Discounted basic level criteria - Discounted evaluation sources** DSmT Framework - Fusion order $(\oplus Criterion(\oplus Source - Evaluation))$ -DSmT mapping - DST scaling.

	Frame of discernment - DSmT - $Card(\Theta) = 3$							
	α	$\emptyset$	HD_1	$HD_1 \cap HD_2$	HD_2	$HD_2 \cap HD_3$	HD_3	Θ
	Discounting factor (AHP SumMax Model)	empty set	No sensitivity (NoS)	$NoS \cap LS$	Low sensitivity (LS)	$LS \cap HS$	High sensitivity (HS)	Θ
$m_1 = m_{(\oplus C_{[111]})}$	1	0	0	0	0.3337	0.2238	0.2925	0.15
$m_2 = m_{(\oplus C_{[112]})}$	0.875	0	0	0	0.3068	0.2995	0.3417	0.052
$m_3 = m_{(\oplus C_{[121]})}$	0.75	0	0	0	0.3706	0.3841	0.2354	0.01
$m_4 = m_{(\oplus C_{[122]})}$	0.25	0	0	0	0.8638	0.0989	0.0248	0.0125
$m_5 = m_{(\oplus C_{[123]})}$	0.25	0	0	0	0.2752	0.3986	0.3138	0.0125
Fusion process no. 9 - PCR6 rule								
$m_{(\oplus C_{[1]})} = m_1 \oplus m_2 \oplus m_3 \oplus m_4 \oplus m_5$		0	0	0	0.0868	0.8563	0.0532	0.0037

Table 23.11: Decision criteria $C_{[1]}$ - **Fusion Process no.** 9 - **Discounted basic level criteria - Discounted evaluation sources** DSmT Framework - $(\oplus Criterion(\oplus Source - Evaluation))$ - DSmT mapping - DST scaling.

23.4.1.7 Examples of implementation

An integrated framework has been developed using MATLAB^{TM} and the calculation routines developed by [Martin 2009]. All data are saved in structures corresponding to the UML conceptual modeling principles (the application is not an object application but only an object-oriented framework - see Fig. 23.56).

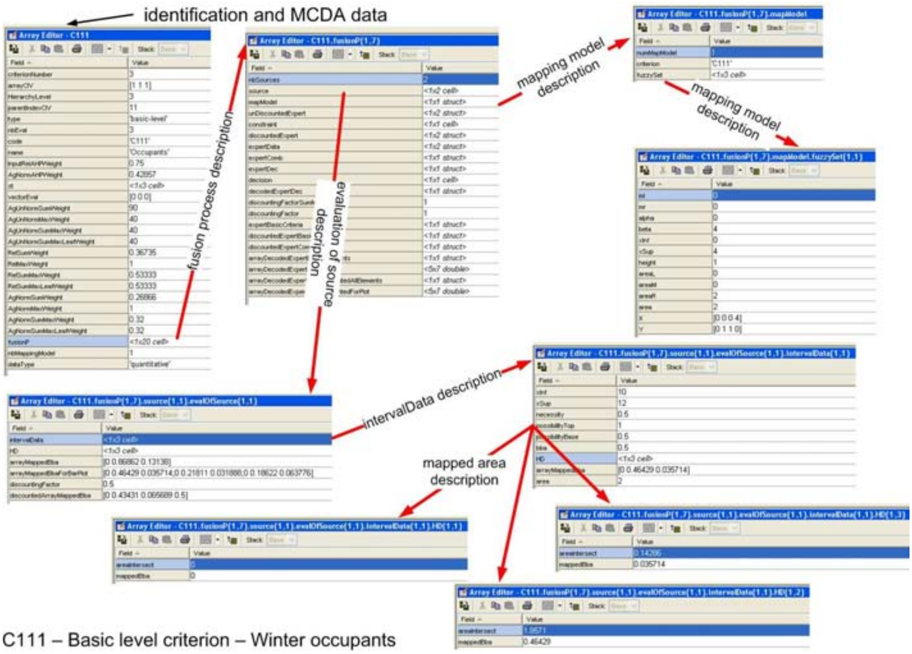


Figure 23.56: Quantitative criterion $C_{[111]}$ - Fusion process no. 7 - Data structures from global identification to evaluation level.

In addition to the calculation framework, some graphical functions have been added to facilitate the use and interpretation of results (see Fig. 23.57 and Fig. 23.58).

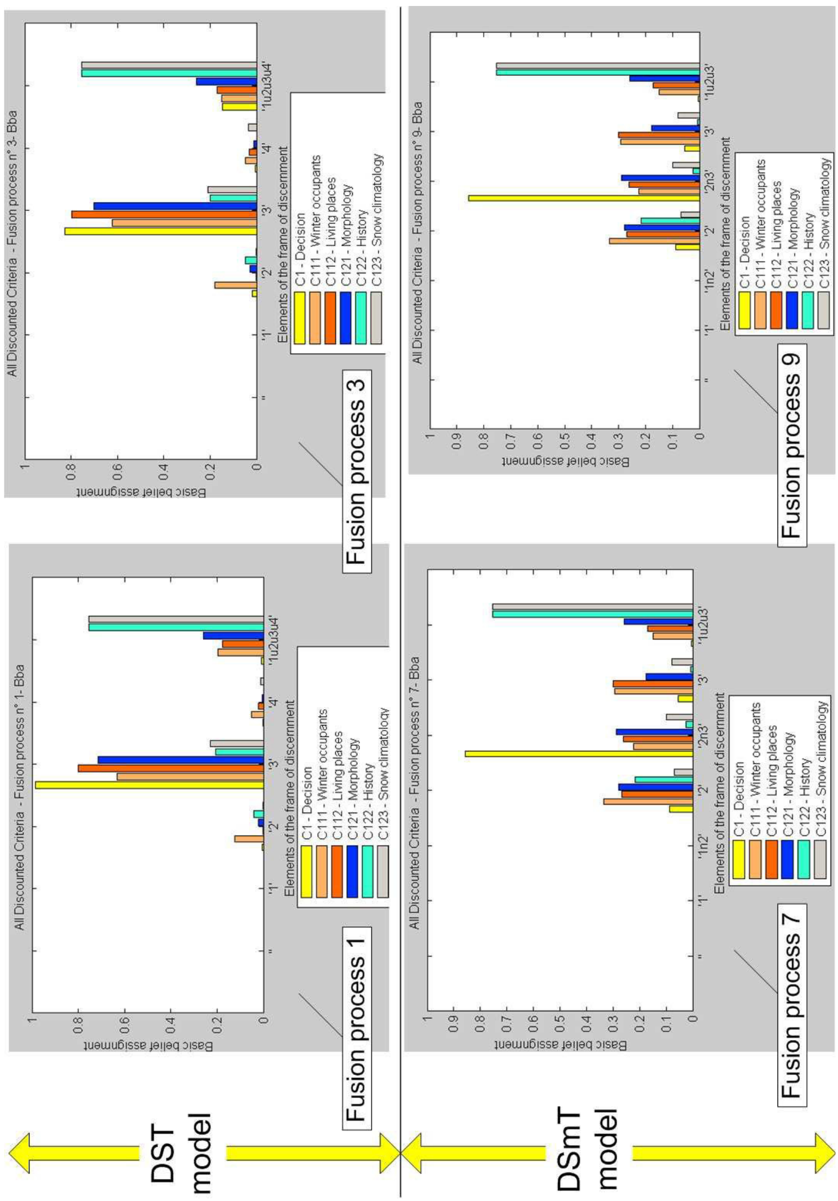


Figure 23.57: Comparison of results for the fusion processes no. 1, 3, 7 and 9.

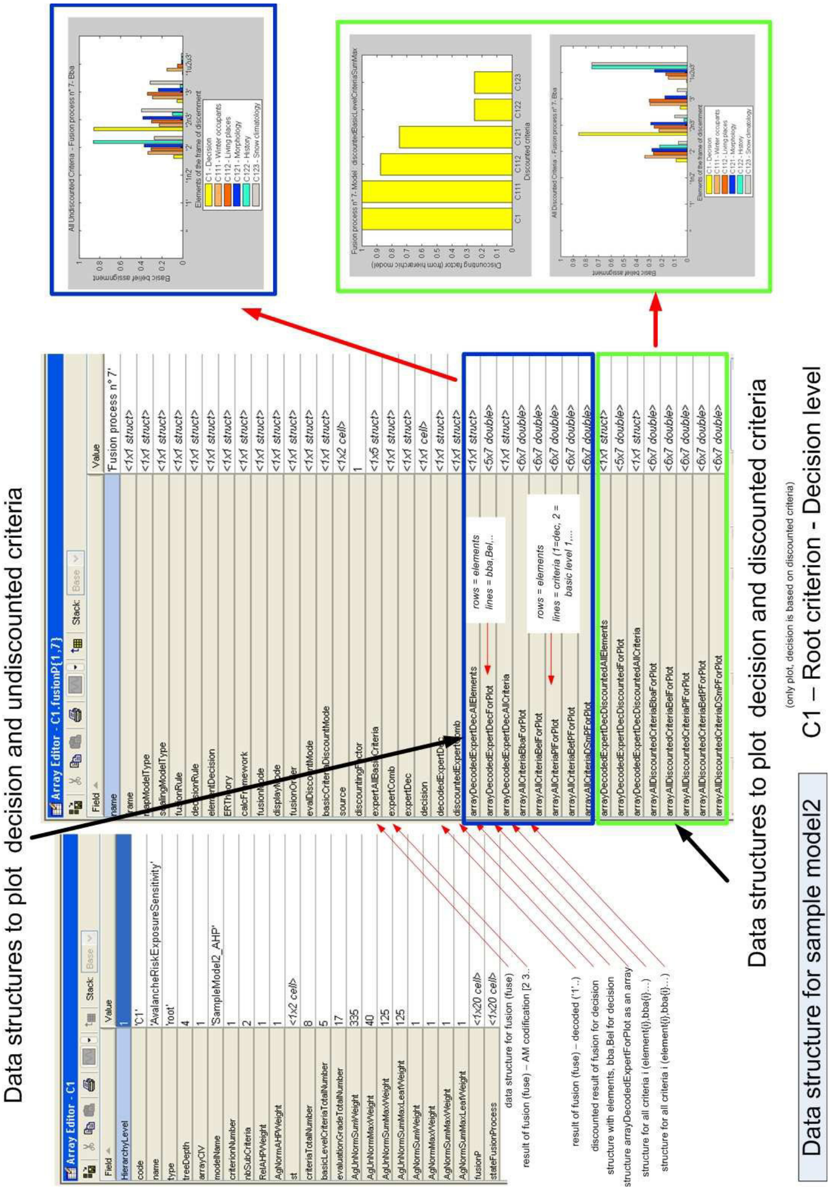


Figure 23.58: Root decision criterion $C_{[1]}$ - Fusion process no. 7 - Data structures for results plot.

23.5 Discussion

23.5.1 Mixing uncertainty, imprecision, importance and traceability

Considering both uncertainty, imprecision, importance and traceability of the expertise process is the ultimate goal of a mixed ER-MCDA framework based on decision-aid methods and formal theories for uncertainty management. The purpose is both to aid decision and to describe how far the different sources and evaluations contribute to the final result: is the decision based on certain evaluations of non important criteria and/or based on uncertain evaluations of important criteria?

Through the literature review, two main approaches can be identified. From one hand, decision-aid science and specially the multi-criteria decision analysis community introduces uncertainty management in its traditional framework. This mainly consists of considering uncertain assessment of decision criteria through interval-based, fuzzy or evidence theory based approaches. On the other hand, "new" uncertainty theories (possibilities, evidence theory) develop applications with obvious decision purposes. Criterion decision based on fused information are proposed in those different frameworks.

In our approach, the Analytic Hierarchy Process (AHP) is used as a conceptual tool to model the problem, to elicit preferences and subjective basic belief assignments (bba) to be used in the fusion process. Using information fusion in a multi-criteria decision analysis framework requires that the model analyst should be able to assess each criterion according to common scale and/or evaluation grades. In the proposed model, these evaluation grades are considered as elements of the frame of discernment. Under this assumption of a common frame of discernment, the information fusion and specially its new developments such as DSmT and fusion rules for conflict situations offer interesting abilities to help to make a decision in the natural hazard context. Uncertain evaluations of quantitative criteria are fused either at the design model stage or at the evaluation stage (fusing the different experts sources). As decision depends on fusion process, choosing ad-hoc combination rules is essential: the combination rules must remain efficient when the conflict level is very high, e.g. when the classical combination rules of DST fails to propose acceptable results.

Our approach has explored some developments of these ideas while trying to consider limits and drawbacks of each of methods and theories. Indeed, if the principle of a joined application of evidential reasoning and Multi-Criteria Decision Analysis (MCDA) is an interesting perspective, some questions remain, as described in the following section.

23.5.2 Advantages and lacks of the ER-MCDA framework

In comparison with existing approaches, we consider that this framework offer the following advantages:

- First, it allows to trace uncertainty and imprecision for both quantitative and qualitative criteria. In comparison with existing approaches mixing multi-criteria decision analysis and evidential reasoning, the information sources, mainly resulting from expert assessment, are fully described. The expert judgements are identified both for modeling and evaluation steps. Links between criteria evaluation and the fusion process an specially the choice of the frame of discernment are elicitated through so-called quantitative and qualitative mapping processes;

- A formal description and conceptual modeling are proposed. They describe both the decision model design and the belief function theory framework. A comparison is proposed to model the same problem using the classical Dempster-Shafer framework (DST) based on exhaustive and exclusive hypothesis, and the more recent Dezert-Smarandache framework (DSmT)which relaxes those constraints;

- In our application, using advanced and recent fusion rules (such as PCR6 rule) allow more realistic decisions. "Ad-hocity" of fusion rules depending on the class problem is still a research question;

- Importance (related to the preference concept) and reliability should be considered as two different concepts in any model. A method of a specific discounting method is proposed but has to be studied in a theoretical way.

The lacks or remaining questions related to the proposed framework are described as following:

- The difference between fusion and aggregation of preferences remains an important subject of debate. Fusion and multi-criteria decision analysis cannot be used in the same conditions. In that sense, some hypothesis of pre-existing models mixing MCDA and evidential reasoning such as DS-AHP Beynon2000 and other variants can immediatly be critized according to the way in which they mix weights (corresponding to preferences) and fusion process. The fusion process should be compliant with the nature of combined information: it is recognized that aggregating preferences and fusing pieces of uncertain evidence should involve different fusion methods Bloch2001 but no definitive and practical classification is available;

- Basic belief assignment elicitation is an essential part of process. The subjective evaluation of *bba* for qualitative criteria using the AHP process can be critized. At least, it allows to trace the hypothesis and choices of the evaluating experts;

- At the present stage of development, reliability of sources are chosen in a very arbitrary and subjective way. Multi-criteria approaches can be imagined upstream from the fusion process to characterize this expert reliability according to experience, backgrounds ... Tacnet2006b;

- The fact that we consider a unique frame of discernment is also questionable: with such a principle, we force the sources to provide an evaluation that will be compliant with the common frame of discernment through a so-called mapping process. It may also be argued that the decision is too strongly influenced by the chosen hierarchical model. This framework requires to define mapping processes to evaluate all the criteria in the chosen frame of discernment. Sensitivity analysis should be done to analyze whether the choice of this mapping models influences the final result for fusion;

- In this present version, the framework provide information for decision but not a real decision. Different alternatives or choices are described in a finer way than with usual MCDA methods with regards to their uncertainty level. The final result has still to be analyzed to produce a decision as in any fusion problem. The further development will probably involve decision-aid method (total aggregation or outranking methods) using result of fusion to make a decision;

23.5.3 The question of the validation

As it involves a fusion process, the proposed ER-MCDA framework does not avoid the difficult question of validation. How can a fusion system be validated and evaluated (what does it mean) ? Bloch2001 analyse the way to propose such a validation as following. The validation should concern the problem modeling, the data input, the fusion in itself and the outputs of the system. In most applications, the proposed decision-aid systems propose solutions but do not check with a real and pre-existing choice. This situation also includes applications dealing with simplified testing cases without any real need for decision (choosing a car, a master course, a candidate). Nevertheless, this remains an important question and we humbly recognize that no satisfying answer exists in our application domain of natural hazards at the moment. We only describe here some principles to implement such a validation.

Testing the sources is the first necessary step to evaluate the experts reliability according to, as an example, their tendency toward overconfidence. Finally, in order to judge the value of the outputs, the easiest situation corresponds to cases where it is possible to make a comparison between a collection of input examples for which expected answers of the fusion process are known by experience or expertise. When validation results arrive after the fusion was done, it is much more difficult to make a conclusion and decide whether the process is inappropriate or the information provided is not sufficient. Finally, the last but not the least way to validate the global decision and/or fusion results is to check that its principles are understood and useful for the end-users who are supposed to use it as a decision tool . . ..

From a thematic point of view related to natural hazards management, validation in a decision context (normative vs. empirical approach) also remains a problem. In industrial contexts, experimental data are more easily available to validate models and decision-aid tools. When dealing with expert approaches, it remains quite difficult to validate the result of the proposed methodology since the solution is never unique and fully certain. Should we consider the existing result as the target for the decision-aid system, given that all the hypotheses are not always fully argued and justified in an explicit way ? For risk zoning maps, we cannot consider one result as a reference that should be obtained by the compared method. The intrinsic value of such a map is in fact difficult to establish. A satisfactory zoning map would correspond to a situation where no unexpected damage occur. A zoning map can be considered as right as long as no event has occurred in a way that had not been planned during its design. Therefore, a way to validate the process can consist of making a list of required quality criteria for expertise processes and to analyze if the proposed methodology is able to improve the existing implementation framework. We are able to measure the validity of the result only when the reference event (considered as rare most of time) occurs. A priori validation is therefore quite difficult. In our case, we consider than a formal elicitation of the reasoning process and the uncertainty level linked to these information in a recognized theoretical framework is already a valuable result. Being able to explicit how the decision was taken (with or without conflicts between experts) and on which initial basis (scientific hypothesis, field data, historical data . . .) it was founded are already two important step towards the validation of an expertise result.

23.5.4 Towards an improved ER-MCDA framework

Neither the framework based on multi-criteria decision analysis or fusion seem able to propose alone an ideal framework to make a decision when several more or less reliable sources provide uncertain and imprecise evaluations on heterogeneous and conflicting criteria. Reaching a compromise respecting the preferences of decision-makers seems as necessary than evaluating and considering the truth associated to their evaluations. At the end, despite of some known difficulties, mixing evidential reasoning and multi-criteria decision-aid methods remains a promising perspective. For further developments, we think at the end that an improved decision framework should use fusion results as inputs data for a multi-criteria partial aggregation method (or outranking method) such as ELECTRE Roy1985 and its more recent variants (Fig. 23.59).

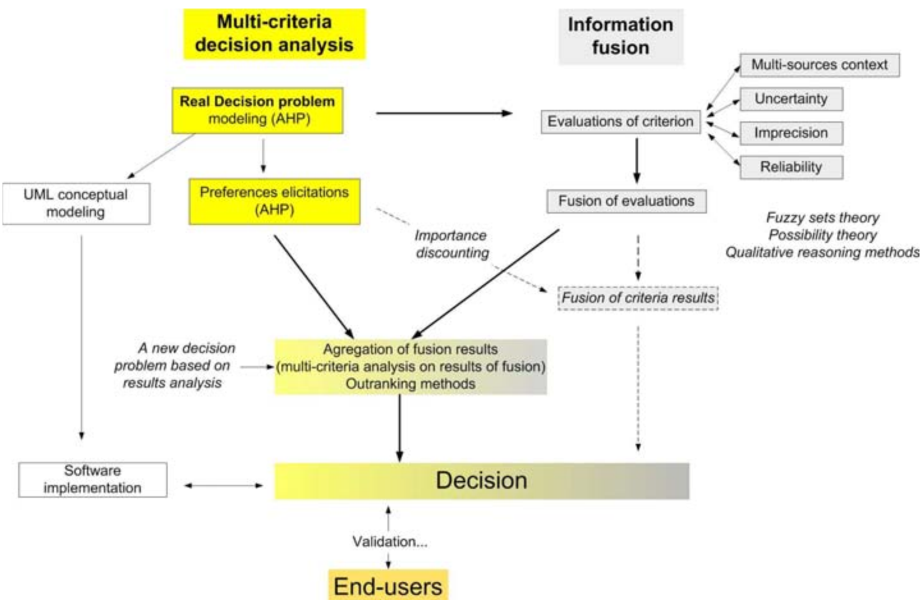


Figure 23.59: The ideal ER-MCDA framework: fusion at the evaluation level, multi-criteria decision analysis for problem modeling and decision making.

23.6 Conclusion

Searching for the best of MCDA and evidential reasoning

The natural hazards risk management process is indeed a real complex decision framework where uncertainty and imprecision come both from the different steps of the risk analysis, its actors and the information sources. Mixing multi-criteria decision analysis (MCDA) and evidential reasoning (ER), using some recent developments such as new fusion rules and theoretical framework such as Dezert-Smarandache Theory (DSmT) is a very attractive objective. This first link between the belief function theory, multi-criteria decision making for natural risks management in mountains areas appears as an encouraging research development direction. From one side, multi-criteria methods consider (more or less depending on their hypothesis) the complexity of the real world, the non-rational behaviour of decision maker, the un-comparability of choices to help the decision-maker. On the other side, belief function (or Evidence) theory is a powerful and versatile framework for human reasoning under uncertainty. Departing from a real, therefore complex, decision problem, this work proposes an operational methodology to integrate those two approaches at different steps of the reasoning process. Improving the elicitation of these levels of imprecision and uncertainty obviously induce more complexity in the risk management framework. As against, it is a possible way to increase the risk awareness in the population and decision makers: experts judgements are not the absolute truth.

Implementation is possible: a first practical framework to improve

A dilemma when trying to imagine a framework that deals with decision and uncertainty is to propose an application whose principles, input and results can be understood by the decision makers. On this basis, introducing an uncertainty level in existing decision-aid methods could be roughly and immediately considered as useless according to the previous objective since fusion calculation can quickly induce high complexity. Though its recognized abilities to represent human theory under uncertainty, the belief function theory (or Evidence Theory) still remains difficult to implement. This applies to the classical Dempster-Shafer Theory (DST) but also to the recent DSmT. Last developments on fusion calculation moderate these traditional drawbacks. From a software programming implementation point of view, this framework implies to handle a great amount of data which needs to be structured. Mixing multi-criteria decision analysis and fusion applications produce more informational results than the classical individual approaches. Data models and conceptual modeling of this kind of problem have been proposed as a basis for further development. The formal description of both hierarchical model and uncertain evaluation also allows to make some links with information systems. Such methodologies issued from software engineering appear as valuable tools to describe the problematic, its components but also to prepare a further integration in a database management system (DBMS). The global methodology contributes therefore to help decision but also to improve the traceability of reasoning process which is an important requirement and domain of progress in the natural hazards risk management context. The principles of the method remain quite simple and we consider that it can be easily understood by the decision-makers and experts. Graphical synthetic results are proposed as examples to help the decision. All this remain a prototype and for decision purposes, there are still work to be done to design and realize a full friendly-user application. To our point of view, one of the advantage of this framework is first to elicit the reasoning hypothesis chosen by the experts along their decision process with respect to the conflict and ignorance levels associated to their evaluations. This concern as much the alternatives evaluations than the models used to make transformation from one framework to another (e.g. the so-called "scaling" and "mapping" models used to transform qualitative and quantitative evaluations into a common frame of discernment.

Remaining issues and further developments

Main issues to use DST and DSmT in the natural hazards expertise context remain:

- the use of the results for decision purpose with optimistic, pessimistic or compromise point of views;
- fusion order according to (or not) the hierarchical framework of the multi-criteria approach;
- choice of fusion rules according to their ability to take conflict into account;
- choice and evaluation of discounting factors related to the different information sources. A multi-criteria approach can be useful to determine these discounting factors;
- results validation.

Main difficulties come from the choice of the frame of discernment, the conflict management and aggregation techniques. This approach extends some existing mixed application of evidential reasoning and multi-criteria decision models. We show that DSmT provides a versatile tool able to consider imprecise and uncertain information with some advantages such as conflict management and paradoxical information. In our framework, deterministic models such as snow-avalanches modeling tools would be considered as common sources. Assessing the reliability of such model corresponds to an important research issue: it comes as much from the modeling hypotheses than from data uncertainty. To handle this uncertainty, some new approaches mixing probabilistic and possibilistic frameworks and called "hybrid methods" have been proposed by Baudrit et al. recently. In the natural hazards context, data are often lacking or incomplete. Those approaches should be developed to characterize the uncertainty coming from modeling in the global expertise process. Other multi-criteria decision frameworks could be tested in order to compare this framework with partial aggregation techniques such as Electre-based method. Outranking methods should be used to produce a decision. This could include comparison between differences of credibility, plausibility, pignistic probability, etc. An improved ER-MCDA framework, and a further way for development, could include fusion process at the evaluation level and multi-criteria decision analysis at the initial stage of problem modeling and at the ultimate stage for decision making.

Guidelines for further developments

From a theoretical point of view, the question of ad-hocity of fusion or aggregation methods according to the problem still requires additional research. Efficient fusion techniques are necessary to have a global assessment of situation and to help to take the right decision, and an efficient decision-making support system will help in the risk prevention against natural hazards. The model proposed in this work is a first attempt to introduce the global problematic of information fusion for natural hazards risk assessment. Of course, some developments for improving these frameworks in relationship with the fusion and decision-aid methods community are under progress in several directions. For example, a deep parametric analysis must be carried out to precisely estimate the importance discounting and reliability factors of all the sources before extending this ER-MCDA approach to the full-criteria real case application. From a thematic point of view, the global methodology is not strictly limited to the snow-avalanche domain: it can be used in others contexts of natural hazards where expertise is required such as torrential floods, rockfalls, etc, as well. Many ways are possible to improve this approach, say by a better choice and comparative analysis of decision rules and on the model choices specially for geographical aspects. To be used in a practical way, numeric tools will be also required. The model has to be plugged with DBMS systems that use information. New developments about qualitative combination rules proposed in DSmT have not yet been tested and could also be used.

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23.8 References

[Allais 1953] M. Allais, *Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école américaine*, Econometrica, Vol. 21, pp. 503–546, 1953.

[Ancey 2006] C. Ancey, *Dynamique des avalanches*, Presses polytechniques universitaires romandes (Lausanne, Suisse) - Cemagref (Antony, France), 2006.

[Apel et al. 2004] H. Apel, A. Thieken, B. Merz and G. Bloschl, *Flood risk assessment and associated uncertainty*, Natural Hazards and Earth System Science, pp. 295–308, 2004.

[Barbolini and Savi 2001] M. Barbolini, F. Savi, *Estimate of uncertainties in avalanche hazard mapping*, Annals of Glaciology, pp. 299–305, 2001.

[Baudrit et al. 2005a] C. Baudrit, D. Guyonnet and D. Dubois, *Postprocessing the hybrid method for addressing uncertainty in risk assessments*, Journal of Environmental Engineering, 131(12):1750–1754, 2005.

[Baudrit et al. 2007] C. Baudrit, D. Guyonnet and D. Dubois, *Joint propagation of variability and imprecision in assessing the risk of groundwater contamination*, Journal of Contaminant Hydrology, 93(1-4):72–84, 2007.

[Baudrit et al. 2005b] C. Baudrit, C. Mercat-Rommens and E. Chojnacki, *Influence de la représentation mathematique de la connaissance sur les résultats de calcul de risque*, rapport dei/sesure no. 2005-14, Technical report, IRSN (Institut de Radioprotection et de Sûreté Nucléaire), Le Vesinet, 2005.

[Beynon et al. 2000] M. Beynon, B. Curry and P. Morgan, *The Dempster-Shafer theory of evidence: an alternative approach to multicriteria decision modelling*, Omega, 28(1):37–50, 2000.

[Beynon 2002] M. Beynon, *DS/AHP method: A mathematical analysis, including an understanding of uncertainty*, European Journal of Operational Research, 140(1):148–164, 2002.

[Beynon 2005] M. Beynon, *Understanding local ignorance and non-specificity within the DS/AHP method of multi-criteria decision making*, European Journal of Operational Research, 163(2):403–417, 2005

[Binaghi et al. 1998] E. Binaghi, L. Luzi, P. Madella, F. Pergalani and A. Rampini, *Slope instability zonation: A comparison between certainty factor and fuzzy Dempster-Shafer approaches*, Natural Hazards, 17(1):77–97, 1998.

[Bloch et al. 2001] I. Bloch et al., *Fusion: General concepts and characteristics*, International Journal of Intelligent Systems, 16(10):1107–1134, 2001.

[Brans et al. 1984] J.-P. Brans, B. Mareschal and P. Vincke, *Promethee, a new family of outranking methods in multicriteria analysis*, Operations research '84', pp. 408–421, Elsevier Science Publishers, North Holland, 1984.

[Bryson et al. 1994] N. Bryson, O. K. Ngwenyama and A. Mobolurin, *A qualitative discriminant process for scoring and ranking in group support systems*, Information Processing and Management, 30(3):389–405, 1994.

[Climaco 2004] J. C. NClimaco, *A critical reflection on optimal decision*, European Journal of Operational Research Management of the Future MCDA: Dynamic and Ethical Contributions, 153(2):506–516, 2004.

[Corgne et al. 2006] S. Corgne, L. Hubert-Moy, J. Dezert and G. Mercier, *Application of DSmT for Land Cover Change Prediction*, Chapter 17 in [Smarandache and Dezert 2004], pp. 371–382, 2006.

[Curt 2008] C. Curt, *Evaluation de la performance des barrages en service basée sur une formalisation et une agrégation des connaissances - Application aux barrages en remblai- Thèse de doctorat*, Ph.D. thesis, Université Blaise Pascal - Clermont 2, Clermont-Ferrand, 2008

[Dezert 2003] J. Dezert, *Fondations pour une nouvelle théorie du raisonnement plausible et paradoxal. application à la fusion d'informations incertaines et conflictuelles*, Rapport no RT 1/06769/DTIM, Technical report, ONERA, Département Traitement de l'Information et Modélisation, 2003.

[Dezert and Smarandache 2006a] J. Dezert, F. Smarandache, *Introduction to the fusion of quantitative and qualitative beliefs*, Information & Security An International Journal, 20:9–49, 2006.

[Dezert and Smarandache 2009] J. Dezert, F. Smarandache, *Chapter 1: An introduction to DSmT*, in Advances and applications of DSmT for Information Fusion, Collected works, Volume 3, 2009.

[Djiknavorian and Grenier 2006] P. Djiknavorian, D. Grenier, *Chapitre 15: Reducing DSmT hybrid rule complexity through optimization of the calculation algorithm*, in [Smarandache and Dezert 2006a], pp. 365–430, 2006.

[Dorren and Heuvelink 2004] L. Dorren, G. Heuvelink, *Effect of support size on the accuracy of a distributed rockfall model*, International Journal of Geographical Information Science, pp. 595–609, 2004.

[Dubois et al. 2000] D. Dubois, H. NGuyen and H. Prade, *Possibility theory, probability and fuzzy sets: Misunderstandings, bridges and gaps*, The Handbook of Fuzzy Sets Series, Kluwer Academic Publishers, Boston, MA, U.S.A., pp. 343–438, 2000.

[Dubois et al. 2001] D. Dubois, M. Grabisch, H. Prade and Ph. Smets, *Using the transferable belief model and a qualitative possibility theory approach on an illustrative example: The assessment of the value of a candidate*, International Journal of Intelligent Systems, 16(11):1245–1272, 2001.

[Dubois and Prade 2004] D. Dubois, H. Prade, *On the use of aggregation operations in information fusion processes*, Fuzzy Sets and Systems, 142(1):143–161, 2004.

[Dubois and Prade 2006] D. Dubois, H. Prade, *Chapitre 3 - Représentations formelles de l'incertain et de l'imprécis*, in Concepts et méthodes pour l'aide à la décision 1 - Outils de modélisation, Hermès - Lavoisier, Paris, France, 2006.

[Dyer 2005] J. Dyer, *Maut - Multiattribute Utility theory*, International Series in Operations Research and Management Science, Springer, New-York, 2005.

[Dweiri 1999] F. Dweiri, *Fuzzy development of crisp activity relationship charts for facilities layout*, Computers & Industrial Engineering, 36(1):1–16, 1999.

[Ellsberg 1961] D. Ellsberg, *Risk, ambiguity and the savage axioms*, Quaterly Journal of Economics, (no. 75):643–669, 1961.

[Eckert et al. 2008a] N. Eckert, E. Parent, T. Faug and M. Naaim, *Bayesian optimal design of an avalanche dam using a multivariate numerical avalanche model*, Stochastic Environmental Research and Risk Assessment, pages 1–19, 2008.

[Eckert et al. 2008] N. Eckert, E. Parent, T. Faug and M. Naaim, *Optimal design under uncertainty of a passive defense structure against snow avalanches: From a general Bayesian framework to a simple analytical model*, Natural Hazards and Earth System Science, 8(5):1067–1081, 2008.

[Fenton and Wang 2006] N. Fenton, W. Wang, *Risk and confidence analysis for fuzzy multicriteria decision making*, Knowledge-Based Systems, 19(6):430–437, 2006

[Forman 1993] E. H. Forman, *Facts and fictions about the analytic hierarchy process*, Mathematical and Computer Modelling, 17(4-5):19–26, 1993.

[Forman and Selly 2002] E. Forman, M. A. Selly, *Decision by Objectives*, World Scientific Publishing, Singapore, 2002.

[Fowler 2000] M. Fowler, *UML Distilled, Second Edition*, Addison-Wesley, 2000.

[Gajdos et al. 2008] T. Gajdos, H. Takashi, J.-M Tallon and J.-C. Vergnaud, *Attitude toward imprecise information*, Journal of Economic Theory, 140:27–65, 2008.

[Gamma et al. 1995] E. Gamma, R. Helm, R. Johnson and J. Vlissides, *Design Patterns: Elements of reusable Object-Oriented Software*, Addison-Wesley, 1995.

[Guitouni and Martel 1998] A. Guitouni, J.-M. Martel, *Tentative guidelines to help choosing an appropriate mcda method*, European Journal of Operational Research, 109(2):501–521, 1998.

[Haenni 2002] R. Haenni, *Are alternatives to dempster's rule of combination real alternatives? comments on "About the belief function combination and the conflict management problem" - Lefevre et al.*, Information Fusion, 3(3):237–239, 2002.

[Hua et al. 2008] Z. Hua, B. Gong and X. Xu, *A DS-AHP approach for multi-attribute decision making problem with incomplete information*, Expert Systems with Applications, 34(3):2221–2227, 2008.

[Jabeur and Martel 2005] K. Jabeur, J.-M. Martel, *La décision de groupe: application de méthodes de surclassement de synthèse*, Bulletin du Groupe de Travail Européen "Aide multicritère à la décision", Série no. 3 (no. 11 - Printemps 2005), 2005.

[Jabeur and Martel 2006] K. Jabeur, J.-M. Martel, *An ordinal sorting method for group decision-making*, European Journal of Operational Research, Corrected Proof, 2006.

[Keeney and Raiffa 1976] R. Keeney, H. Raiffa, *Decisions with multiple objectives; preferences ans values trade-offs*, J. Wiley and Sons, New York, 1976.

[Klinke and Renn 2002] A. Klinke, O. Renn, *A new approach to risk evaluation and management: Risk-based, precaution based, and discourse based strategies*, Risk Analysis, 22(6):1071–1094, 2002.

[Kuo et al. 2006] M.-S. Kuo, G.-S.NLiang and W.-C.Huang, *IExtensions of the multicriteria analysis with pairwise comparison under a fuzzy environment*, International Journal of Approximate Reasoning, 43(3):268–285, 2006.

[Lacroix 2006] F. Lacroix, *Préface*, in Dynamique des avalanches, Presses polytechniques universitaires romandes - Cemagref, Lausanne, Suisse - Antony, France, 2006.

[Lin et al., 2004] G. F. Lin, L. H. Chen and J. N. Lai, *Reliability-based delineation of debris-flow deposition areas*, Natural Hazards, 32(3):395–412, 2004.

[Linkov et al. 2006] I. Linkov et al., *From comparative risk assessment to multi-criteria decision analysis and adaptive management: Recent developments and applications*, Environment International Environmental Risk Management - the State of the Art, 32(8):1072–1093, 2006.

[Magne and Vasseur 2006] L. Magne, D. Vasseur, *Chapitre 9: le risque et sa gestion entre incertitude et ajustement de règles*, in Risques industriels. complexité, incertitude et décision: une approche interdisciplinaire, Collection EDF R&D, Lavoisier, Paris, 2006.

[Martin and Quidu 2008] A. Martin, I. Quidu, *Decision support with belief functions theory for seabed characterization*, In Proc. of the 11th International Conference on Information Fusion, FUSION 2008, Cologne, Germany, July 2008.

[Martin and Osswald 2006] A. Martin, C. Osswald, *A new generalization of the proportional conflict distribution rule stable in terms of decision*, in [Smarandache and Dezert 2006a], 2006.

[Martin 2009] A. Martin, *Implementing general belief function framework with a practical codification for low complexity*, in Advances and Applications of DSmT for Information Fusion, Volume 3, 2009.

[M.A.T.E. and M.E.T.L. 1997] M.A.T.E. and M.E.T.L., *Plans de prévention des risques naturels prévisibles (ppr) - guide général*, Technical report, Ministère de l'Aménagement du Territoire et de l'Environnement - Ministère de l'Equipement, des Transports et du Logement, 1997.

[Matos 2007] M. A. Matos, *Decision under risk as a multicriteria problem*, European Journal of Operational Research, 181(3):1516–1529, 1007.

[Merad et al. 2004] M. M. Merad, T. Verdel, B. Roy and S. Kouniali, *Use of multicriteria decision-aids for risk zoning and management of large area subjected to mining-induced hazards*, Tunnelling and Underground Space Technology, 19(2):125–138, 2004.

[Mortureux 2001] Y. Mortureux, *La sûreté de fonctionnement: méthodes pour maîtriser les risques*, Ref. AG 4 670 - 17 p., 2001.

[Mustajoki et al. 2005] J. Mustajoki, R. Haimailaiinen and A. Salo, *Decision support by interval smart/swing - incorporating imprecision in the smart and swing methods*, Decision Sciences, 36(2):317–339, 2005.

[Omrani et al. 2007] H. Omrani, L. Ion-Boussier and P. Trigano, *A new approach for impacts assessment of urban mobility*, WSEAS Transactions on Information Science and Applications, 4(3):439–444, 2007.

[Ozdemir and Saaty 2006] M. S. Ozdemir, T. L. Saaty, *The unknown in decision making: What to do about it*, European Journal of Operational Research, 174(1), pp. 349–359, 2006.

[Pan 2008] N. F. Pan, *Fuzzy AHP approach for selecting the suitable bridge construction method*, Automation in Construction, 17(8), pp. 958–965, 2008.

[Parsons 2001] S. Parsons, *Qualitative approaches for reasoning under uncertainty*, Massachusetts Institute of Technology Press, Cambridge, MA, U.S.A., 2001.

[Pigeon, 2005] P. Pigeon, *Geographie critique des risques*, Economica, Paris, 2005.

[Rapin 2007] F. Rapin, *Outils de classification des sites à risques d'avalanche - manuel pour le renseignement - ref: Manuel - v 3.1 - 4 avril 2007*, Technical report, Cemagref - Unité ETNA (Erosion Torrentielle Neige Avalanches), Grenoble, 2007.

[Roy 1968] B. Roy, *Classement et choix en présence de points de vue multiples (la méthode electre)*, Revue Frand'Informatique et de Recherche Opérationnelle, Vol. 8, pp. 57–75, 1968.

[Roy 1985] B. Roy, *Méthodologie Multicritère d'Aide à la Décision*, Collection Gestion - Série Production et techniques quantitatives appliquées à la gestion, Economica, Paris, 1985.

[Roy 1989] B. Roy, *Main sources of inaccurate determination, uncertainty and imprecision in decision models*, Mathematical and Computer Modelling, 12(10-11):1245–1254, 1989.

[Roy 1990] B. Roy, *Decision-aid and decision-making*, European Journal of Operational Research, 45(2-3):324–331, 1990.

[Rumbaugh et al. 1999] J. Rumbaugh, G. Booch and I. Jacobson, *The Unified Modelling Language Reference Manual*, Addison-Wesley , Pearson Education, Inc., 1999.

[Saaty 1980] T. L. Saaty, *The analytic hierarchy process*, McGraw-Hill, New York, 1980.

[Saaty 1982] T. L. Saaty, *Decision-Making for leaders: the analytical hierarchy process for decisions in a complex word*, Life-time learning publications, Belmont, CA, 94002 U.S.A., 1982.

[Saaty 1990] T. L. Saaty, *How to make a decision: The analytic hierarchy process*, European Journal of Operational Research, 48(1):9–26, 1990.

[Saaty and Tran 2007] T. L. Saaty, L. T Tran, *On the invalidity of fuzzifying numerical judgments in the analytic hierarchy process*, Mathematical and Computer Modelling, 46(7-8):962–975, 2007.

[Saaty and Vargas 1987] T. L. Saaty, L. G.Vargas, *Uncertainty and rank order in the analytic hierarchic process*, European Journal of Operational Research, 32(1):107–117, 1987.

[Salo and Haimailaiinen 2001] A. A Salo, R. P. Haimailaiinen, *Preference ratios in multiattribute evaluation (prime)-elicitation and decision procedures under incomplete information*, IEEE Transactions on Systems, Man, and Cybernetics Part A: Systems and Humans, 31(6), pp. 533–545, 2001.

[Salo and Hamalainen 1995] A. A Salo, R. P. Haimailaiinen, *Preference programming through approximate ratio comparisons*, European Journal of Operational Research, 82(3):458–475, 1995.

[Salo 1996] A. A. Salo, *On fuzzy ratio comparisons in hierarchical decision models*, Fuzzy Sets and Systems, 84(1):21–32, 1996.

[Scharlig 1985] A. Scharlig, *Décider sur plusieurs critères - Panorama de l'aide à la décision multicritère*, Diriger l'entreprise, Presses polytechniques et universitaires romandes, Lausanne, 3 ème edition, 1985.

[Sentz and Ferson 2002] K. Sentz, S. Ferson, *Combination of evidence in Dempster-Shafer theory*, SAND TR 2002-0835, Technical report, Sandia National Laboratories, U.S.A., 2002.

[Shafer 1976] G. Shafer, *A mathematical theory of Evidence*, Princeton University Press, 1976.

[Smarandache and Dezert 2004] F. Smarandache, J. Dezert, (Editors), *Advances and Applications of DSmT for Information Fusion (Collected Works)*, Vol. 1, American Research Press, Rehoboth, U.S.A., 2004.

[Smarandache and Dezert 2006a] F. Smarandache, J. Dezert, (Editors), *Advances and Applications of DSmT for Information Fusion (Collected Works)*, Vol. 2, American Research Press, Rehoboth, U.S.A., 2006.

[Smarandache and Dezert 2006b] F. Smarandache, J Dezert, *Proportional Conflict Redistribution Rules for Information Fusion*, in [Smarandache and Dezert 2006a], 2006.

[Soundappan et al. 2004] P. Soundappan et al., *Comparison of evidence theory and Bayesian theory for uncertainty modeling - alternative representations of epistemic uncertainty*, Reliability Engineering & System Safety, 85 (1-3), pp. 295–311, 2004.

[Stewart 2005] T. J. Stewart, *Dealing with uncertainties in MCDA - chapter 11*, Vol. 78 of International Series in Operations Research and Management Science, Springer, 2005.

[Tacnet et al. 2005a] J.-M. Tacnet, D. Laigle, F. Berger, L. Dorren and J. Nedellec, *Expertise des risques naturels gravitaires en montagne et estimation du risque résiduel: un exemple de contribution et des limites actuelles des systèmes d'information*, In Colloque SIRNAT (Systèmes d'information et risques naturels), Montpellier, France, March 10-11, 2005.

[Tacnet et al. 2005] J.-M. Tacnet, R. Burnet, P. Ravel, P. Maurel and J. Lievois, *Information systems and avalanche risk prevention plans (p.p.r.): contribution* *to improvement of data and reasoning processes traceability*, Technical report (in French), Cemagref - ONF/RTM - Ministry for Ecology and Sustainable development (MEDD) / Directorate for risks and pollution prevention (DPPR), 2005.

[Tacnet et al. 2006] J.-M. Tacnet, S. Escande, F. Rapin, R. Burnet, M. and Batton-Hubert, *Methodology of snow avalanche post-event field investigations: tools, difficulties and perspectives*, In International Snow Science Workshop Proceedings (ISSW) 2006, pp. 530–539, Telluride, CO, U.S.A., Sept. 30 – Oct. 6, 2006.

[Tacnet and Richard 2008] J.-M. Tacnet, D. Richard, *Irasmos european project - wp5 - risk management framework in France*, Technical report, Cemagref, 2008.

[Vaidya and Kumar 2006] O. S. Vaidya, S. Kumar, *Analytic hierarchy process: An overview of applications*, European Journal of Operational Research, 169(1):1–29, 2006.

[Van Der Most and Wehrung 2005] H. VanNDer Most, M. Wehrung, *Dealing with uncertainty in flood risk assessment of dike rings in the Netherlands*, Natural Hazards, 36(1-2):191–206, 2005.

[Veyret 2004] Y. Veyret, *Géographie des risques naturels en France - De l'aléa à la gestion*, HATIER, Paris, 2004.

[Von Neumann and Morgenstern 1967] J. Von Neumann, O. Morgenstern, *Theory of games and economic behavior*, Princeton University Press, Third Edition, 1967.

[Vrijling et al. 1998] J. K. Vrijling, W. Van Hengel and R. J. Houben, *Acceptable risk as a basis for design*, Reliability Engineering & System Safety, 59(1):141–150, 1998.

[Wadsworth and Hall 2007] R. Wadsworth, J. Hall, *Setting site specific critical loads: An approach using endorsement theory and Dempster-Shafer*, Water, Air, and Soil Pollution: Focus, 7(1-3):399–405, 2007.

[Wang et al. 1996] J. Wang, J. B. Yang and P. Sen, *Multi-person and multi-attribute design evaluations using evidential reasoning based on subjective safety and cost analyzes*, Reliability Engineering and System Safety, 52(2):113–128, 1996.

[Wang and Elhag 2006] Y.-M. Wang, T. M. S. Elhag, *An approach to avoiding rank reversal in AHP*, Decision Support Systems, 42(3):1474–1480, 2006.

[Wang et al. 2006] Y.-M. Wang, J. B. Yang and D. L. Xu, *Environmental impact assessment using the evidential reasoning approach*, European Journal of Operational Research, 174(3):1885–1913, 2006.

[Weichselgartner and Bertens 2000] J. Weichselgartner, J. Bertens, *Natural disasters: Acts of god, nature or society? on the social relation to natural hazards*, In Congrès Risk analysis II - International conference on computer simulation in risk analysis and hazard mitigation No; 2, pp. 3–12, Bologna, Italy, October, 2000.

[Wong and Lingras 1994] S. Wong, P. Lingras, *Representation of qualitative user preference by quantitative belief functions*, IEEE Transactions on Knowledge and Data Engineering, 6(1):72–78, 1994.

[Yang 2001] J. B. Yang, *Rule and utility based evidential reasoning approach for multiattribute decision analysis under uncertainties*, European Journal of Operational Research, 131(1):31–61, 2001.

[Yang et al. 2006] J. B. Yang, Y. M. Wang, D. L. Xu and K. S. Chin, *The evidential reasoning approach for mada under both probabilistic and fuzzy uncertainties*, European Journal of Operational Research, 171(1):309–343, 2006.

[Yang and Xu 2002] J. B. Yang, D. L. Xu, *On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty*, IEEE Transactions on Systems, Man, and Cybernetics Part A: Systems and Humans, 32(3):289–304, 2002.

[Zadeh 1965] L. A. Zadeh, *Fuzzy sets*, Information and Control, 8:338–353, 1965.

[Zadeh 1978] L. A. Zadeh, *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets and Systems, 1:3–28, 1978.