Dynamics of Moving Bodies

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Abstract: Why revised study of classical mechanics which is so old and obvious, is needed? It is due to its ability to satisfactorily explain the macroscopic world. The macroscopic world or property of matter in bulk is deterministic in nature, unlike the probabilistic microscopic world and hence application of classical mechanics remains important. Only we need to modify them on light of new discoveries, to achieve a better description.

As the title suggests, we shall here deal with description of motion, their causes with Newtonian point of view, but on light of rigorous "Special Theory of Relativity". Therefore we would come to know how nature may decide the fate of moving bodies when they have a velocity comparable to *speed of light in vacuum*. We will first find quantitive description of force; equations of motion in a rest frame, and then utilizing *first principle of relativity*, extend them to any inertial frame of reference. We will extend prediction of "Special Theory of Relativity", to cases when bodies have non-zero initial velocity and then accelerate or retard. Finally we will show how the new descriptions aptly describe variable-mass system involving either mass accretion or mass ejection and hence derive "*ideal, or Tsiolkovsky rocket equation."* We shall also find centripetal acceleration and force.

Body:

"*A problem with defining force as rate of change of linear momentum*"

Let us consider a body of mass m, moving with velocity u initially, in the next time interval t it is acted by a force in the direction of motion, and at instant t+ its mass is M and velocity v. The duration't' is not infinite seminal, so we cannot use differential eqn. rather we use finite difference.

$$F \cdot t = Mv - mu \; or, v = \frac{m}{M}.u + \frac{F}{M}.t \; or, v = B.u + A.t \; where \; A = \frac{F}{M}, B = \frac{m}{M}$$

A has dimension of acceleration; B is apparently a dimension less number, and are independent of time interval t. The two other eqn. of motion are thus:

Consider a time interval t when velocity of the body is v, the velocity is assumed to be fairly constant for next $\mathbb{d}t$, the infinite seminally small distance travelled is,

$\mathbb{d}S = v\,\mathbb{d}t \; or \; \mathbb{d}S = (B.u + A.t)\,\mathbb{d}t \; or \; S = B.u.t + \frac{A}{2}.t^2$ And

$v^2 = B^2u^2 + 2A \cdot B \cdot u \cdot t + A^2t^2 \; or, v^2 = B^2u^2 + 2\text{A.S}$ Do we now have the eqn. of motion sufficient to explain motion for every possible case? Investigation leads to following observations:

a. By defining acceleration as rate of change of velocity, we have established an identity v=u+a.t which is independent of choice of v, u. However the only way to recover the identity is taking B=1, A=a implying u, v<<c.
b. M>>m, B is very small, product B.u or its higher power always tend to be negligible, even in cases when u is finitely large.

c. In cases $v \rightarrow c, F, M \rightarrow \infty$, thus A becomes indeterminate.
d. There is inconvenience as A, B are not predetermined and are functions of u, v and thus the definition goes in circle.

Hence we conclude, our hypothesis that force=rate of change of linear momentum is not sufficient; we would now find trial solutions to define force in most convenient way.

"Motion of bodies in their rest frame"

Let us prefer a reference frame R in such a way that the motion of bodies under study will always start from rest:

We imagine one such body is accelerated to a velocity v in time t. We choose a physical quantity $\alpha = v/t$. The body makes a displacement of magnitude S during time t. The relativistic mass of body as it reaches velocity v is m_1 and is related to its rest mass m_0 as $m_1 = \gamma(v) m_0 \ where \ \gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ Where c=speed of light in vacuum.

Force which acted on the body during time t is $F = m_1 v/t$. Therefore, $F = m_1 \alpha$

We can observe a particular relation between α, S and $\gamma(v)$ hold with respect to reference frame R.

By definition, work done by Force =force * displacement in direction of line of action, in rectilinear motion, displacement is always in direction of force.

So $W = m_1 \cdot \alpha \cdot S$.

By conserving energy,

$E = m_0 c^2 + W$ or, $m_1 c^2 = m_0 c^2 + \gamma(v) m_0 \alpha S$ or, $\gamma(v) m_0 c^2 = m_0 c^2 + \gamma(v) m_{0^\alpha} S$ or, $\boldsymbol{\gamma(v) = \frac{1}{1-\frac{\alpha s}{c^2}}}$. Another important result is to deduce relation between kinetic energy and linear momentum.

Let a body is moving with a uniform velocity in a single direction only. It therefore has a linear momentum p (which is the product of its relativistic mass and velocity at that instant) and kinetic energy is K.

Assuming the rest mass of the body was m_0. Its relativistic energy E is related to its linear momentum as:

$E^2 = m_0^2 c^4 + p^2 c^2$--- (a)

$also\ E = m_0 c^2 + K$ And squaring we get: $E^2 = m_0^2 c^4 + 2 m_0 c^2 \cdot K + K^2$ substituting with $K^2 = (\gamma - 1) m_0 c^2 \cdot K$ we get:

$E^2 = m_0^2 c^4 + 2 m_0 c^2 \cdot K + (\gamma - 1) m_0 c^2 \cdot K$

Therefore, $E^2 = m_0^2 c^4 + (\gamma + 1) m_0 c^2 \cdot K$ ------------------------------------ (b)

Comparing eqn a. and b. we get: $\boldsymbol{p^2 = (\gamma + 1) m_0 K}$------------------------ (c)

"Newton's equations of rectilinear motion in rest frame"

Linear momentum $p = \gamma m_0 v$ and Kinetic energy K=work done by force along direction of displacement= $m_1 \alpha S$

Using eqn c. $(\gamma m_0 v)^2 = (\gamma + 1)m_0 \cdot \gamma m_0 \alpha S\ where\ \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Or, $\boldsymbol{v^2} = \left(\mathbf{1} + \frac{\mathbf{1}}{\boldsymbol{\gamma}}\right) \boldsymbol{\alpha S}$ But $\gamma = \frac{1}{1-\frac{\alpha s}{c^2}}$ implies $1 + \frac{1}{\gamma} = 2 - \frac{\alpha S}{c^2}$

Or, $v^2 = 2\alpha S - \frac{\alpha^2 S^2}{c^2}$ --- (d)

Now linear momentum p=m_1v=F*t

Using eqn (c) $(F \cdot t)^2 = (\gamma + 1)m_0 \cdot F \cdot S$ or, $F \cdot t^2 = (\gamma + 1)m_0 S$ or, $\gamma m_0 \alpha \cdot t^2 = (\gamma + 1)m_0 S$

Or, $\boldsymbol{S} = \frac{\mathbf{1}}{\mathbf{1}+\frac{\mathbf{1}}{\boldsymbol{\gamma}}} \boldsymbol{\alpha t^2}$-- (e)

Or, $2S - \frac{\alpha S^2}{c^2} = \alpha t^2$ -- (f)

Instantaneous acceleration a=$dv/\,d\,t$ and we want to find $\frac{d^2 S}{dt^2}\ and\ v\frac{dv}{dS}$

α=v/t or, v-αt=0 or f(v, α, t)=0 therefore $dv/\,d\,t$ =$-\frac{\partial f/\partial t}{\partial f/\partial v}$ but $\partial f/\partial t = -\alpha\ and\ \partial f\ /\ \partial v = 1$ or, $\boldsymbol{dv/\,d\,t}$**= α**

Using eqn (d) $v^2 - 2\alpha S + \frac{\alpha^2 S^2}{c^2} = 0$ assuming velocity v to be implicit function of α, S we get: G(v, α, S)=0 so $\frac{dv}{dS} = -\frac{\partial G/\partial S}{\partial G/\partial v}$ now $\frac{\partial G}{\partial s} = -2\alpha + \frac{2\alpha^2 S}{c^2}$ and $\frac{\partial G}{\partial v} = 2v\ therefore\ v\frac{dv}{dS} = \alpha - \frac{\alpha^2 S}{c^2}$

$$\text{or } v\frac{dv}{ds} = \alpha\left(1 - \frac{\alpha s}{c^2}\right) \quad or, \boldsymbol{v}\frac{d\boldsymbol{v}}{d\boldsymbol{s}} = \boldsymbol{\alpha}/\boldsymbol{\gamma}\ since\ \gamma = \frac{1}{1 - \frac{\alpha S}{c^2}}$$

Using eqn (f) $2S - \frac{\alpha S^2}{c^2} - \alpha t^2 = 0$ assuming displacement S to be implicit function of α, t we get:

F(S, α,t)=0 so $\frac{dS}{dt} = -\frac{\partial F/\partial t}{\partial F/\partial s}$ now $\frac{\partial F}{\partial t} = -\alpha \cdot 2t\ and\ \frac{\partial F}{\partial S} = +2 - \frac{\alpha \cdot 2S}{c^2}$ hence,

$$\frac{d\boldsymbol{S}}{d\boldsymbol{t}} = -\frac{-2\alpha t}{+2 - \frac{2\alpha S}{c^2}} \text{ or,} \frac{dS}{dt} = \frac{\alpha}{1 - \frac{\alpha S}{c^2}} \cdot t = \boldsymbol{\gamma \alpha t}\ therefore, \frac{d^{\mathbf{2}}\boldsymbol{s}}{d\boldsymbol{t^2}} = \boldsymbol{\gamma\alpha}$$

$$since\ \gamma = \frac{1}{1 - \frac{\alpha S}{c^2}}$$

"Study of fixed mass system and a new definition of force"

Here the term "*fixed mass system*" has been used to differentiate a system from "*variable mass system*". A fixed mass system is a body or a number of bodies whose motion is only due to external Force, undergoes neither mass-accretion nor –ejection and has constant proper mass. The mass of such a system increases with velocity according to relation in "*Special Relativity*". We shall now find a trial solution of force for fixed-mass system.

The original idea of Newton in his 2^{nd} law is: force=inertial mass*acceleration. This principle cannot be merged with the Special Theory of Relativity, in which we have a maximum attainable speed equal to the speed of light in vacuum. The first issue is if a body is moving with velocity comparable to speed of light, its mass at given instant, does not correspond to its rest mass. Thus the term 'inertial mass' in definition of force needs a substitute. The second issue is relativistic force to be directly proportional to acceleration is not a valid assumption.

We like to perform a number of thought experiments before drawing any conclusion about a trial solution:

We have two bodies at rest with their rest masses in ratio 1:2. A force of magnitude F acts on both for an infinite seminally small timedt.The final velocities of the two bodies are recorded and hence instantaneous accelerations are determined. These accelerations are found in ratio 2:1.

We have two bodies moving with a uniform velocity and their relativistic masses in ratio 1:2. A force of magnitude F acts on both for an infinite seminally small timedt. The final velocities of the two bodies are recorded and hence instantaneous accelerations are determined. These accelerations are found in ratio 2:1.

1N force acts on a body at rest. Its rest mass is 1 kg. The acceleration measured is $1 m/s^2$. 1N force acts on a body moving with the "speed of light in vacuum", no acceleration is produced since mass becomes infinite in the process.

We conclude that mass is the measure of inertia, a massive body requires a higher amount of force to change its state of motion. The term "inertial mass" will be substituted with "relativistic mass".

Since we have shown force∝relatavistic mass, we now need to multiply a term in place of acceleration, to remove proportionality. Earlier when we considered a reference frame R for bodies which were initially at rest till a force acted on them, we found force=relativistic mass * α. By assuming validity of principle of relativity, we are on strong grounds to accept this as a trial solution. The physical significance of term α, and how it is related to acceleration of bodies which were initially not at rest, is the only matter of concern.

After this discussion, we define force as relativistic mass * α in rest of our work and argue for its validity due to consistency among predicted value and experimental data.

"Newton's equation of rectilinear motion in any inertial frame of reference"

We shall now attempt to construct general equations of motion when bodies not necessarily start their motions from rest. We defined F=m*a,* and v=u+at; now we would establish a relation between "*a" and "*a".

Relation between α and acceleration:

Let us choose a reference frame R1. A body is moving with an initial velocity u, when it is acted by a force F for a time t, and it accelerated to a velocity v. Let us choose another reference frame R such that the same body now appears to start its motion from rest reaching a velocity w in time t1, where t1 is the simultaneous reading by a clock in R when t time has elapsed by a clock in R1. This is possible if 'R' is moving in same direction as the body, and has a relative velocity 'u' with respect to R1 By the definition of "simultaneous events", we precisely state:

$$w = \frac{v-u}{1-\frac{uv}{c^2}}\ since\ v > u\ and\ t1 = \gamma_{(u)t}\ _{where}\ \ _{\gamma(u)} = \frac{1}{\sqrt{1-\frac{\mathrm{u}^2}{c^2}}}$$

With respect to R, from definitions alone, α=w/t1. So w=$\gamma(u)\alpha t$. By making v as subject in $w = \frac{v-u}{1-\frac{uv}{c^2}}$ we get:

$$v = \frac{u+w}{1+\frac{uw}{c^2}}$$

$$\frac{dv}{dt} = \frac{\left(1+\frac{u\omega}{c^2}\right)\left(0+\frac{dw}{dt}\right)-(u+w)\left(0+\frac{u}{c^2}\cdot\frac{dw}{dt}\right)}{\left(1+\frac{u\omega}{c^2}\right)^2}$$

Now $\frac{d\omega}{dt} = \gamma(u)\alpha\ so\ \frac{dv}{dt} = \frac{\gamma(u)\alpha\left(1+\frac{uw}{c^2}\right)-\frac{u(u+\omega)}{c^2}\gamma(u)\alpha}{\left(1+\frac{uw}{c^2}\right)^2}$ or, $\frac{dv}{dt} = \frac{\gamma(u)\alpha\cdot\left(1+\frac{u\cdot\omega}{c^2}-\frac{u^2}{c^2}-\frac{u\cdot w}{c^2}\right)}{\left(1+\frac{uw}{c^2}\right)^2}$

$$\text{or,}\frac{dv}{dt} = \frac{\alpha}{\gamma(u)\cdot\left(1+\frac{u\omega}{c^2}\right)^2}\ \text{since}\ 1-\frac{u^2}{c^2} = \frac{1}{\gamma^2(u)}$$

Therefore, acceleration of the body with respect to R1 is **a=**$\boldsymbol{\frac{\alpha}{\gamma(u)\cdot\left(1+\frac{u\omega}{c^2}\right)^2}}$

Relation between α and retardation:

Let us choose a reference frame R1. A body is moving with an initial velocity u, when it is acted by a force F for a time t in opposite direction to its motion, and it retarded to a velocity v. Let us choose another reference frame R such that the same body now appears to start its motion with a velocity w and retard to rest in time t1, where t1 is the simultaneous reading by a clock in R when t time has elapsed by a clock in R1. This is possible if 'R' is moving in same direction as the body, and has a relative velocity 'v' with respect to R1. By the definition of "simultaneous events", we precisely state:

$$w = \frac{u-v}{1-\frac{uv}{c^2}}\ since\ u > v\ and\ t1 = \gamma_{(v)t}\ _{where}\ \ _{\gamma(v)} = \frac{1}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}}$$

With respect to R, from definitions alone, $|\alpha|$=w/t1. So w=$\gamma(v)|\alpha|t$. By making u as subject in $w = \frac{u-v}{1-\frac{uv}{c^2}}$ we get:

$$u = \frac{v+w}{1+\frac{vw}{c^2}}$$

It is noted that v=u-at, where 'a' is retardation gives: $\frac{du}{dt} = \mathrm{a}$

$$\frac{du}{dt}=\frac{\left(1+\frac{v\omega}{c^2}\right)\left(0+\frac{dw}{dt}\right)-(v+w)\left(0+\frac{v}{c^2}\cdot\frac{dw}{dt}\right)}{\left(1+\frac{v\omega}{c^2}\right)^2}$$

Now $\frac{d\omega}{dt}=\gamma(v)|\alpha|\ so\ \frac{du}{dt}=\frac{\gamma(v)|\alpha|\left(1+\frac{vw}{c^2}\right)-\frac{v(v+\omega)}{c^2}\gamma(v)|\alpha|}{\left(1+\frac{vw}{c^2}\right)^2}$

or, $$\frac{du}{dt}=\frac{\gamma(v)|\alpha|\cdot\left(1-\frac{v\cdot\omega}{c^2}-\frac{v^2}{c^2}+\frac{v\cdot w}{c^2}\right)}{\left(1+\frac{vw}{c^2}\right)^2}\ \text{or,}\ \frac{d\boldsymbol{u}}{d\boldsymbol{t}}=\frac{|\boldsymbol{\alpha}|}{\boldsymbol{\gamma(v)}\cdot\left(\mathbf{1}+\frac{\boldsymbol{v\omega}}{\boldsymbol{c^2}}\right)^{\mathbf{2}}}\ \text{ since } 1-\frac{v^2}{c^2}=\frac{1}{\gamma^2(u)}$$

Therefore, retardation of the body with respect to R1 is **a=**$\frac{|\boldsymbol{\alpha}|}{\boldsymbol{\gamma(v)}\cdot\left(\mathbf{1}+\frac{\boldsymbol{v\omega}}{\boldsymbol{c^2}}\right)^{\mathbf{2}}}$

Relation between α, S, $\boldsymbol{\gamma(u)}$ **and** $\boldsymbol{\gamma(v)}$:

A body mass m is moving with a velocity u at given instant of time, is acted by F in the direction of motion for some duration, is **accelerated** to velocity v. It is observed that it makes a **"proper" displacement S** in R during the interval force acted on it. Work done by force W=$m_1\alpha S$.

Conserving energy, $m_1c^2=mc^2+W$ or, $\gamma(v)m_0c^2=\gamma(u)m_0c^2+\gamma(v)m_0\alpha S\ or, \mathbf{1}-\frac{\boldsymbol{\alpha S}}{\boldsymbol{c^2}}=\frac{\boldsymbol{\gamma(u)}}{\boldsymbol{\gamma(v)}}$------ (g)If the force acted on **opposite direction** to motion, it would produce **retardation**, then the relation looks like:

Work done by force W=-m$|\alpha|$S.

Conserving energy, $m_1c^2=mc^2+W$ or, $\gamma(v)m_0c^2=\gamma(u)m_0c^2-\gamma(u)m_0|\alpha|S\ or, \mathbf{1}-\frac{|\boldsymbol{\alpha}|\boldsymbol{S}}{\boldsymbol{c^2}}=\frac{\boldsymbol{\gamma(v)}}{\boldsymbol{\gamma(u)}}$--- (h)

Equation g. is useful when there is **acceleration**

Squaring eqn g.

$$1-\frac{2\alpha\cdot S}{c^2}+\frac{\alpha^2\cdot S^2}{c^4}=\frac{\gamma^2(u)}{\gamma^2(v)}=\frac{1-\frac{v^2}{c^2}}{1-\frac{u^2}{c^2}}=\frac{c^2-v^2}{c^2-u^2}\ or,(c^2-u^2)-\frac{2\alpha\cdot S}{c^2}\cdot(c^2-u^2)+\frac{\alpha^2\cdot S^2}{c^2}\cdot\frac{(c^2-u^2)}{c^2}=c^2-v^2$$

$$v^2-u^2-2\alpha\cdot S\cdot\left(1-\frac{u^2}{c^2}\right)+\frac{\alpha^2\cdot S^2}{c^2}\left(1-\frac{u^2}{c^2}\right)=0$$

$$\text{or,}\ v^2-u^2-\frac{1}{\gamma_{(u)}^2}\left(2\alpha\cdot S-\frac{\alpha^2\cdot S^2}{c^2}\right)=0---\text{(i) therefore } \boldsymbol{v^2}=\boldsymbol{u^2}+\frac{\mathbf{1}}{\boldsymbol{\gamma_{(u)}^2}}\left(\mathbf{2}\boldsymbol{\alpha}\cdot\boldsymbol{S}-\frac{\boldsymbol{\alpha^2}\cdot\boldsymbol{S^2}}{\boldsymbol{c^2}}\right)$$

Using eqn (i) we get G (u, v, α, S)=0 $\frac{dv}{dS}=-\frac{\partial G/\partial S}{\partial G/\partial v}$ Now $\frac{\partial G}{\partial S}=-\frac{1}{\gamma_{(u)}^2}\left[2\alpha-\frac{\alpha^2\cdot 2S}{c^2}\right]$ and $\frac{\partial G}{\partial v}=2v$

$$so\ \boldsymbol{v}\frac{d\boldsymbol{v}}{d\boldsymbol{S}}=\frac{\alpha}{\gamma^2(u)}\cdot\left(1-\frac{\alpha\cdot S}{c^2}\right)=\frac{\alpha}{\gamma_{(u)}^2}\cdot\frac{\gamma(u)}{\gamma(v)}=\frac{\boldsymbol{\alpha}}{\boldsymbol{\gamma(u)}\cdot\boldsymbol{\gamma(v)}}$$

Equation h. is useful when there is **retardation**

Squaring eqn h.

$$1-\frac{2|\alpha|\cdot S}{c^2}+\frac{|\alpha|^2\cdot S^2}{c^4}=\frac{\gamma^2(v)}{\gamma^2(u)}=\frac{1-\frac{u^2}{c^2}}{1-\frac{v^2}{c^2}}=\frac{c^2-u^2}{c^2-v^2}\ or,$$

$$(c^2-v^2)-\frac{2|\alpha|\cdot S}{c^2}\cdot(c^2-v^2)+\frac{|\alpha|^2\cdot S^2}{c^2}\cdot\frac{(c^2-v^2)}{c^2}=c^2-u^2$$

$$u^2-v^2-2|\alpha|\cdot S\cdot\left(1-\frac{v^2}{c^2}\right)+\frac{|\alpha|^2\cdot S^2}{c^2}\left(1-\frac{v^2}{c^2}\right)=0$$

$$\text{or}, v^2-u^2+\frac{1}{\gamma_{(v)}^2}\left(2|\alpha|\cdot S-\frac{|\alpha|^2\cdot S^2}{c^2}\right)=0---\text{(i) therefore } \boldsymbol{v^2=u^2-\frac{1}{\gamma_{(v)}^2}\left(2|\alpha|\cdot S-\frac{|\alpha|^2\cdot S^2}{c^2}\right)}$$

Using eqn (i) we get G (u, v, α, S) =0 $\frac{dv}{dS}=-\frac{\partial G/\partial S}{\partial G/\partial v}$ Now $\frac{\partial G}{\partial S}=+\frac{1}{\gamma_{(v)}^2}\left[2|\alpha|-\frac{|\alpha|^2\cdot 2s}{c^2}\right]$ and $\frac{\partial G}{\partial v}=2v$

$$so\ \boldsymbol{v}\frac{d\boldsymbol{v}}{d\boldsymbol{S}}=-\frac{|\alpha|}{\gamma^2(v)}\cdot\left(1-\frac{|\alpha|\cdot S}{c^2}\right)=-\frac{|\alpha|}{\gamma_{(v)}^2}\cdot\frac{\gamma(v)}{\gamma(u)}=-\frac{|\boldsymbol{\alpha}|}{\boldsymbol{\gamma(u)\cdot\gamma(v)}}$$

Displacement in terms of initial velocity, time and acceleration:

Let us choose a reference frame R1 such that the body was moving initially with velocity u and the force acted for time t , producing an acceleration a. The resultant **"proper" displacement** is **S** (displacement measured in R moving with 'u' velocity in direction of body with respect to R1.)

$v^2=u^2+\frac{1}{\gamma_{(u)}^2}\left(2\alpha\cdot S-\frac{\alpha^2\cdot S^2}{c^2}\right)$ or, $v^2=u^2+\alpha\cdot s\cdot\frac{1}{\gamma_{(u)}^2}+\frac{1}{\gamma_{(u)}^2}\left(\alpha\cdot S-\frac{\alpha^2\cdot S^2}{c^2}\right)$

$$\text{or}, v^2=u^2+\alpha\cdot S\cdot\frac{1}{\gamma_{(u)}^2}+\alpha\cdot S\cdot\frac{1}{\gamma_{(u)}^2}\left(1-\frac{\alpha\cdot S}{c^2}\right)\ \text{or}, v^2=u^2+\alpha\cdot S\cdot\frac{1}{\gamma_{(u)}^2}+\alpha\cdot s\cdot\frac{\gamma(u)}{\gamma_{(u)}^2\gamma(v)}$$

$$\text{or}, v^2=u^2+\alpha\cdot S\cdot\left(\frac{1}{\gamma_{(u)}^2}+\frac{1}{\gamma(u)\cdot\gamma(v)}\right)----\text{(j)}$$

From our hypothesis, we have the following identities: v=u+a·t**,** and a$=\frac{\alpha}{\gamma(u)\cdot\left(1+\frac{u\omega}{c^2}\right)^2}$ Using them in eqn. j.

$u^2+2uat+a^2t^2=u^2+a\cdot\gamma(u)\cdot\left(1+\frac{u\cdot w}{c^2}\right)^2\cdot S\cdot\left(\frac{1}{\gamma^2(u)}+\frac{1}{\gamma(u)\cdot\gamma(v)}\right)$ or,

$2ut+at^2$ $=\boldsymbol{S}\cdot\left(\boldsymbol{1}+\frac{\boldsymbol{u\cdot w}}{\boldsymbol{c^2}}\right)^{\boldsymbol{2}}\cdot\left(\frac{\boldsymbol{1}}{\boldsymbol{\gamma(u)}}+\frac{\boldsymbol{1}}{\boldsymbol{\gamma(v)}}\right)$

In reference R1, displacement **s=S/ $\boldsymbol{\gamma(\mathbf{u})}$ using Lorentz contraction.**

2ut+at^2= $\boldsymbol{\gamma}(\mathbf{u})\cdot\ \boldsymbol{s}\cdot\left(1+\frac{u\cdot w}{c^2}\right)^2\cdot\left(\frac{1}{\gamma(u)}+\frac{1}{\gamma(v)}\right)$

Let, $\left(\mathbf{1}+\frac{\boldsymbol{u}\cdot\boldsymbol{w}}{\boldsymbol{c}^2}\right)^{\mathbf{2}}\cdot\left(\frac{\mathbf{1}}{\mathbf{1}}+\frac{\boldsymbol{\gamma}(\mathbf{u})}{\boldsymbol{\gamma}(v)}\right)=\boldsymbol{\beta}(\boldsymbol{u},\boldsymbol{v})\ where\ ,w=\frac{v-u}{1-\frac{uv}{c^2}},v>u$

Therefore, $\boldsymbol{\beta}(\boldsymbol{u},\boldsymbol{v})\cdot\boldsymbol{s}=\boldsymbol{2ut}+\mathbf{at^2}$where 'a' is **acceleration.**

During acceleration, we have: 2ut+at^2= $S\cdot\left(1+\frac{u\cdot w}{c^2}\right)^2\cdot\left(\frac{1}{\gamma(u)}+\frac{1}{\gamma(v)}\right)$ and a= $\frac{\alpha}{\gamma(u)\cdot\left(1+\frac{u\omega}{c^2}\right)^2}$.

$$\gamma(u)\cdot 2ut+\frac{\alpha}{\left(1+\frac{u\cdot w}{C^2}\right)^2}t^2=S\left(1+\frac{u\cdot w}{c^2}\right)^2\left(1+\frac{\gamma(u)}{\gamma(v)}\right)\ or,\frac{\gamma(u)\cdot 2ut}{\left(1+\frac{u\cdot w}{c^2}\right)^2}+\frac{\alpha t^2}{\left(1+\frac{u\cdot w}{c^2}\right)^4}=2S-\frac{\alpha S^2}{c^2}$$

$$\text{since }\ 1-\frac{\alpha S}{c^2}=\frac{\gamma(u)}{\gamma(v)}$$

$$\text{or,}\frac{\gamma(u)\cdot 2ut}{\left(1+\frac{u\cdot w}{c^2}\right)^2}+\frac{\alpha t^2}{\left(1+\frac{u\cdot w}{c^2}\right)^4}+\frac{\alpha S^2}{c^2}-2S=0\ or,F(u,v,\alpha,t,S)=0\text{ or,}\frac{dS}{dt}=-\frac{\partial F/\partial t}{\partial F/\partial S}$$

$$\text{or,}\frac{\partial F}{\partial t}=\frac{\gamma(u)\cdot 2u}{\left(1+\frac{u\cdot w}{c^2}\right)^2}+\frac{2\alpha t}{\left(1+\frac{u\cdot w}{c^2}\right)^4}\ \text{ and }\frac{\partial F}{\partial s}=\frac{2\alpha\cdot S}{c^2}-2\text{ and}\frac{dS}{dt}=\frac{\frac{\gamma(u)\cdot u}{\left(1+\frac{u\cdot w}{c^2}\right)^2}+\frac{\alpha t}{\left(1+\frac{u\cdot w}{c^2}\right)^4}}{1-\frac{\alpha\cdot S}{c^2}}$$

$$\text{or,}\frac{d\boldsymbol{S}}{d\boldsymbol{t}}=\frac{\boldsymbol{\gamma}(\boldsymbol{v})\cdot\boldsymbol{u}}{\left(\mathbf{1}+\frac{\boldsymbol{u}\cdot\boldsymbol{w}}{\boldsymbol{c}^2}\right)^2}+\frac{\boldsymbol{\gamma}(\boldsymbol{v})\cdot\boldsymbol{\alpha t}}{\boldsymbol{\gamma}(\boldsymbol{u})\cdot\left(\mathbf{1}+\frac{\boldsymbol{u}\cdot\boldsymbol{w}}{\boldsymbol{c}^2}\right)^4}\text{ since }\ 1-\frac{\alpha S}{c^2}=\frac{\gamma(u)}{\gamma(v)}\text{ and }\frac{d^2\boldsymbol{S}}{d\boldsymbol{t}^2}=\frac{\boldsymbol{\gamma}(\boldsymbol{v})\cdot\boldsymbol{\alpha}}{\boldsymbol{\gamma}(\boldsymbol{u})\cdot\left(\mathbf{1}+\frac{\boldsymbol{u}\cdot\boldsymbol{w}}{\boldsymbol{c}^2}\right)^4}$$

Displacement in terms of initial velocity, time and retardation:

Let us choose a reference frame R1 such that the body was moving initially with velocity u and the force acted in opposite direction for time t , producing an retardation a. The resultant **"proper" displacement** is **S**. (displacement measured in R moving with 'v' velocity in direction of body with respect to R1.)

$$v^2=u^2-\frac{1}{\gamma_{(v)}^2}\left(2|\alpha|\cdot S-\frac{|\alpha|^2\cdot S^2}{c^2}\right)\text{ or,}v^2=u^2-|\alpha|\cdot s\cdot\frac{1}{\gamma_{(v)}^2}-\frac{1}{\gamma_{(v)}^2}\left(|\alpha|\cdot S-\frac{|\alpha|^2\cdot S^2}{c^2}\right)\text{ or,}$$

$$v^2=u^2-|\alpha|\cdot S\cdot\frac{1}{\gamma_{(v)}^2}-|\alpha|\cdot S\cdot\frac{1}{\gamma_{(v)}^2}\left(1-\frac{|\alpha|\cdot S}{c^2}\right)\text{ or,}$$

$$v^2=u^2-|\alpha|\cdot S\cdot\frac{1}{\gamma_{(v)}^2}-|\alpha|\cdot s\cdot\frac{\gamma(v)}{\gamma_{(v)}^2\gamma(u)}\text{ or,}v^2=u^2-|\alpha|\cdot S\cdot\left(\frac{1}{\gamma_{(v)}^2}+\frac{1}{\gamma(u)\cdot\gamma(v)}\right)$$

From our hypothesis, we have the following identities: v=u-at, and a=$\frac{|\alpha|}{\gamma(v)\cdot\left(1+\frac{v\omega}{c^2}\right)^2}$.

So, u^2-2uat+a^{2}t^2=u^2-a$\cdot$ $S\cdot\left(\frac{1}{\gamma_{(v)}^2}+\frac{1}{\gamma(u)\cdot\gamma(v)}\right)\cdot\gamma(v)\cdot\left(1+\frac{v\omega}{c^2}\right)^2$or,$\boldsymbol{2ut}-\boldsymbol{at^2}=\boldsymbol{S}\cdot\left(\mathbf{1}+\frac{\boldsymbol{v}\cdot\boldsymbol{w}}{\boldsymbol{c}^2}\right)^{\mathbf{2}}\cdot\left(\frac{\mathbf{1}}{\boldsymbol{\gamma}(\boldsymbol{v})}+\frac{\mathbf{1}}{\boldsymbol{\gamma}(\boldsymbol{u})}\right)$

In reference R1, displacement **s=S/ $\boldsymbol{\gamma}(\mathbf{v})$ by Lorentz contraction.**

$$2ut - at^2 = \boldsymbol{\gamma}(\mathbf{v}) \cdot \ \boldsymbol{s} \cdot \left(1 + \frac{v \cdot w}{c^2}\right)^2 \cdot \left(\frac{1}{\gamma(v)} + \frac{1}{\gamma(u)}\right)$$

Let, $\left(\mathbf{1} + \frac{\boldsymbol{v \cdot w}}{\boldsymbol{c^2}}\right)^{\mathbf{2}} \cdot \left(\frac{\mathbf{1}}{\mathbf{1}} + \frac{\boldsymbol{\gamma(v)}}{\boldsymbol{\gamma(u)}}\right) = \ \boldsymbol{\beta(v,u)}\ where\ w = \frac{u-v}{1-\frac{u \cdot v}{c^2}}$, $\mathrm{u} > v$

Therefore $\boldsymbol{\beta(v,u) \cdot s = 2ut}$ **$-$at^2**where ‘a’ is **retardation**.

During retardation we have, $2ut - at^2 = S \cdot \left(1 + \frac{v \cdot w}{c^2}\right)^2 \cdot \left(\frac{1}{\gamma(v)} + \frac{1}{\gamma(u)}\right)$and a= $\dfrac{|\alpha|}{\gamma(v) \cdot \left(1 + \frac{v\omega}{c^2}\right)^2}$

$$\gamma(v) \cdot 2ut - \frac{\gamma(v) \cdot |\alpha|}{\gamma(v) \cdot \left(1 + \frac{v\omega}{c^2}\right)^2} t^2 = S\left(1 + \frac{v \cdot w}{c^2}\right)^2 \left(1 + \frac{\gamma(v)}{\gamma(u)}\right)$$

$$\text{or,} \frac{\gamma(v) \cdot 2ut}{\left(1 + \frac{v \cdot w}{c^2}\right)^2} - \frac{|\alpha| t^2}{\left(1 + \frac{v \cdot w}{c^2}\right)^4} + \frac{|\alpha| S^2}{c^2} - 2S = 0 \text{or}, F(u, v, \alpha, t, S) = 0 \text{ or}, \frac{dS}{dt} = -\frac{\partial F/\partial t}{\partial F/\partial S}$$

$$\text{or,} \frac{\partial F}{\partial t} = \frac{\gamma(v) \cdot 2u}{\left(1 + \frac{v \cdot w}{c^2}\right)^2} - \frac{2|\alpha| t}{\left(1 + \frac{v \cdot w}{c^2}\right)^4} \ \text{ and } \ \frac{\partial F}{\partial s} = \frac{2|\alpha| \cdot S}{c^2} - 2 \text{and} \frac{dS}{dt} = \frac{\frac{\gamma(v) \cdot u}{\left(1 + \frac{v \cdot w}{c^2}\right)^2} - \frac{|\alpha| t}{\left(1 + \frac{v \cdot w}{c^2}\right)^4}}{1 - \frac{|\alpha| \cdot S}{c^2}}$$

$$\text{or,} \frac{d\boldsymbol{S}}{d\boldsymbol{t}} = + \frac{\boldsymbol{\gamma(u) \cdot u}}{\left(\mathbf{1} + \frac{\boldsymbol{v \cdot w}}{\boldsymbol{c^2}}\right)^{\mathbf{2}}} - \frac{\boldsymbol{\gamma(u) \cdot |\alpha| \mathbf{t}}}{\boldsymbol{\gamma(v)} \cdot \left(\mathbf{1} + \frac{\boldsymbol{v \cdot w}}{\boldsymbol{c^2}}\right)^{\mathbf{4}}} \text{ since } 1 - \frac{|\alpha| \cdot S}{c^2} = \frac{\gamma(v)}{\gamma(\mathrm{u})} \text{ and } \frac{d^{\mathbf{2}}\boldsymbol{S}}{d\boldsymbol{t^2}} = -\frac{\boldsymbol{\gamma(u) \cdot |\alpha| t}}{\boldsymbol{\gamma(v)} \cdot \left(\mathbf{1} + \frac{\boldsymbol{v \cdot w}}{\boldsymbol{c^2}}\right)^{\mathbf{4}}}$$

Where **S**= “proper displacement” in **R** and **s**=simultaneous displacement in **R1**.

“Scope for validity of equations derived”:

We describe a body in rectilinear motion and we always have a Euclidean continuum. We choose a reference frame in such a way that the body, whose motion we choose to study, is either at rest or moving with a uniform velocity just before we begin our observation. The moving body may accelerate or retard for a given period of time, after which it again moves with a uniform velocity in a single direction. The body represents a fixed mass system.

“Variable mass system”:

A variable-mass system is a collection of matter whose proper mass varies with time. We shall now establish a general relation between force, as per new definition, and rate of change of linear momentum for variable-mass system.

“Relation between force and linear momentum of variable mass system during acceleration”:

Let us consider a mass m and velocity u at this instant of time, accelerates for a duration t by some mechanism, and has mass M, velocity v at next instant of time. Linear momentums at the two instant are:p1=mu, p2=Mv.

Using v=u+at, p2=Mu+Mat. We assume an intrinsic force F must have acted to produce this acceleration .Then magnitude of this force is same as D Alembert's reverse effective force but along the direction of motion.

F=Ma=$M\mathrm{a}\cdot\gamma(u)\cdot\left(1+\frac{u\omega}{c^2}\right)^2$, *p2*-p1= (M-m) u+$\frac{F\cdot t}{\gamma(u)\cdot\left(1+\frac{u\omega}{c^2}\right)^2}$, or Δp=Δm·u+$\frac{F\cdot t}{\gamma(u)\cdot\left(1+\frac{u\omega}{c^2}\right)^2}$ is the **required** relation in **finite difference**. To achieve differential eqn. we assume force acts for 'dt' amount of time, M=m+dm, p2>p1,p2=p1+dp,v=u+du, w=du . $\frac{udu}{c^2}\to 0$ for all values of u.Therefore $\left(1+\frac{u\cdot w}{c^2}\right)^2\to 1$ for all values of u.

$\mathrm{dp}=\mathrm{dm}\cdot\mathrm{u}+\frac{F\cdot dt}{\gamma(u)}\ or,\frac{\mathbf{F}}{\boldsymbol{\gamma}(\mathbf{u})}=\frac{\mathbf{dp}}{\mathbf{dt}}-\frac{\mathbf{dm}}{\mathbf{dt}}.\mathbf{u}$ is the **required** relation in **differential** form.

"Relation between force and linear momentum of variable mass system during retardation":

Let us consider a mass m and velocity u at this instant of time, retards for a duration t by some mechanism, and has mass M, velocity v at next instant of time. Linear momentums at the two instant are: p1=mu, p2=Mv.

Using v=u-at, p1=mv+mat. We assume an intrinsic force F must have acted to produce this retardation .Then this force is same as D Alembert's reverse effective force .F=-m|α|=$-m\mathrm{a}\cdot\gamma(v)\cdot\left(1+\frac{v\omega}{c^2}\right)^2$where 'a' is retardation;

So p2-p1=(M-m) v-mat, or p2-p1= (M-m) u+$\frac{F\cdot t}{\gamma(v)\cdot\left(1+\frac{v\omega}{c^2}\right)^2}$, or Δp=Δm·v+$\frac{F\cdot t}{\gamma(v)\cdot\left(1+\frac{v\omega}{c^2}\right)^2}$ is required eqn. in finite difference and differential form will be $\mathrm{dp}=\mathrm{dm}\cdot\mathrm{v}+\frac{F\cdot dt}{\gamma(v)}\ or,\frac{\mathbf{F}}{\boldsymbol{\gamma}(\mathbf{v})}=\frac{\mathbf{dp}}{\mathbf{dt}}-\frac{\mathbf{dm}}{\mathbf{dt}}.\mathbf{v}$

"*Principle of zero impulse*":

What happens if no external force is applied to a system? Conventionally impulse is equal to change of linear momentum. Zero impulse implies linear momentum of the system to be constant. But here we have defined force in a different way, but a little investigation shows the above conclusion holds.

For a **fixed mass system**, without doubt, $\frac{\mathbf{dm}}{\mathbf{dt}}=0$ so if $external\ F=0,\frac{\mathbf{dp}}{\mathbf{dt}}=0$ or $\mathbf{p}=$ const.

For a **variable mass system,** there are a number of individual bodies with different mass and linear momentum.

If such a system is imagined **isolated** from rest of the universe, we notice that sum of masses of individual bodies is constant, although some of them undergo mass-accretion or mass ejection. Thus as a whole $\frac{\mathbf{dm}}{\mathbf{dt}}=0$ where **m=sum of individual masses** and $external\ F=0,\frac{\mathbf{dp}}{\mathbf{dt}}=0$ or $\mathbf{p}=$ const. where **p=sum of individual linear momentum.**

The linear momentum is conserved!

"*Mass-accretion":*

Let us consider a mass dm with velocity u is about to collide with the main body of mass m and velocity v. After a time dt, both particles move as one body with velocity v + dv. Total linear momentum initially p1=dm.u+mv, p2=(m+dm)(v+dv)=mv+mdv+dm.v since dm.dv=0;p2-p1=mdv+dm.(v-u) or, dp=mdv+dm.(v-u)

Here the main body accelerates from v to v+dv, there is always D Alembert's Force in opposite direction of motion. We assume a force F along direction of motion equal to magnitude of D Alembert's force. Let changes in linear momentum of the main body dp*,then:

dp*=(m+dm)(v+dv)-mv=mdv+dm.v;$\frac{F}{\gamma(v)} = \frac{dp^*}{dt} - v \cdot \frac{dm}{dt}$ or $\frac{F}{\gamma(v)} = m\frac{dv}{dt} + \frac{dm}{dt} \cdot v - v \cdot \frac{dm}{dt}$ or $\frac{F}{\gamma(v)} = m\frac{dv}{dt}$

$\frac{\mathbf{dp}}{\mathbf{dt}} = m.\frac{dv}{dt} + (v-u).\frac{dm}{dt} = \frac{\boldsymbol{F}}{\boldsymbol{\gamma(v)}} + (\boldsymbol{v}-\boldsymbol{u}).\frac{\boldsymbol{dm}}{\boldsymbol{dt}}$ where 'F' is imagined to account for acceleration of the main body.

"*Mass-ejection"*:

Let us consider a main body mass m, velocity v, ejects a mass dm with velocity u, and hence accelerates to velocity v+dv in dt amount of time. Initial momentum p1=mv. Final momentum p2=(m-dm).(v+dv)+dm.u

or, p2=mv+mdv-vdm+dm.u as dm.dv=0; p2-p1=mdv+dm.u-vdm or, -dp=mdv-dm.(v-u) as p1>p2 if v>u which is necessary condition for acceleration of main body; We assume a force F along direction of motion equal to magnitude of D Alembert's force. Let changes in linear momentum of the main body dp*,then:

dp*=mv-(m-dm).(v+dv)=+dm.v-m.dv ;$\frac{F}{\gamma(v)} = \frac{dp^*}{dt} - v \cdot \frac{dm}{dt} = -\mathrm{m}.\frac{\mathrm{dv}}{\mathrm{dt}}$ or $-\frac{\mathrm{dp}}{\mathrm{dt}} = -\frac{\mathrm{F}}{\gamma(v)} - (\mathrm{v}-\mathrm{u}).\frac{\mathrm{dm}}{\mathrm{dt}}$

or, $\frac{\mathbf{dp}}{\mathbf{dt}} = \frac{\boldsymbol{F}}{\boldsymbol{\gamma(v)}} + (\boldsymbol{v}-\boldsymbol{u}).\frac{\boldsymbol{dm}}{\boldsymbol{dt}}$

Conclusion: $\frac{\mathbf{dp}}{\mathbf{dt}} = \frac{\boldsymbol{F}}{\boldsymbol{\gamma(v)}} + (\boldsymbol{v}-\boldsymbol{u}).\frac{\boldsymbol{dm}}{\boldsymbol{dt}}$ is the general eqn. for variable mass system where 'v' is initial velocity of main body, 'u' is velocity of mass accreted/ejected, 'F' is imagined force acting on main-body during acceleration, 'dp' is change of linear momentum of entire system.

"*Relativistic energy and linear momentum"*:

A body is moving uniformly with velocity u and its relativistic mass is m_u so its linear momentum is $p_u = m_u u$. The body is accelerated to a velocity v and its relativistic mass is m, so its linear momentum is p=mv. We assume that rest mass of the body is m_0. Using momentum-energy relation in Special Relativity we write:

$$m_u^2c^4 = m_0^2c^4 + p_u^2c^2 \; and \; m^2c^4 = m_0^2c^4 + p^2c^2$$

$$E = mc^2 \text{ or } E^{\mathbf{2}} = m^2c^4 = m_0^2c^4 + p^2c^2 \text{or}, E^{\mathbf{2}} = m_u^2c^4 - p_u^2c^2 + p^2c^2$$

Therefore $\boldsymbol{E^2 = m_u^2c^4 + (p^2 - p_u^2)c^2}$-- (k)

This eqn. is valid for a fixed mass system. Next we shall consider a variable-mass system. Let the main body of such a system has at a instant, proper mass m0, linear momentum is $p_u = m_u u$ and at a different instant, proper mass m'0, linear momentum is p=mv. Using momentum-energy relation in Special Relativity we write:

$$m_u^2c^4 = m_0^2c^4 + p_u^2c^2 and \; m^2c^4 = m'^2_0c^4 + p^2c^2$$

$E = mc^2$ or $E^{\mathbf{2}} = m^2c^4 = m'^2_0c^4 + p^2c^2$. Let, $m'^2_0c^4 - m_0^2c^4 = M_0^2c^4 \text{for mass} - \text{oblation and} = -M_0^2c^4 \text{ for mass} -$ ejection.

$$\text{or}, E^{\mathbf{2}} = p^2c^2 \pm M_0^2c^4 + m_0^2c^4 \text{ or}, E^{\mathbf{2}} = m_u^2c^4 - p_u^2c^2 + p^2c^2 \pm \; M_0^2c^4 \,.$$

Therefore $\boldsymbol{E^2} = (\boldsymbol{m_u^2} \pm \boldsymbol{M_0^2})\boldsymbol{c^4} + (\boldsymbol{p^2} - \boldsymbol{p_u^2})\boldsymbol{c^2}$ ---------------- (I). sign is positive for mass-oblation between instants and negative for mass-ejection.

"Ideal or Tsiolkovsky rocket equation":

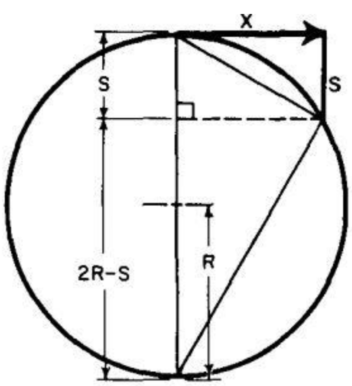
Using formula for mass-ejection we have: $\frac{\mathbf{dp}}{\mathbf{dt}} = \frac{\boldsymbol{F}}{\gamma(v)} + (\boldsymbol{v} - \boldsymbol{u}).\frac{\boldsymbol{dm}}{\boldsymbol{dt}}$ where 'dm' is mass-ejected by main body, dp is change of linear momentum of whole system in 'dt', 'F'=imagined force acting on main body to account for acceleration. **If external F=0, $\frac{\mathbf{dp}}{\mathbf{dt}} = \mathbf{0}$,** in according with **principle of zero impulse.** $\frac{F}{\gamma(v)} = -\mathrm{m}.\frac{\mathrm{dv}}{\mathrm{dt}}$.

Since, velocity of ejected propellant is opposite to direction of acceleration of the rocket the scalar equivalent of above eqn takes form:

$$+\mathrm{m}.\frac{\mathrm{dv}}{\mathrm{dt}} + (v-u).\frac{dm}{dt} = 0\ or, -(v-u)\frac{dm}{m} = dv\ or, -(v-u)\int_{m_0}^{m_1} \frac{dm}{m} = \int_{v_0}^{v_1} dv$$

or, $(v-u)\cdot\ln\frac{m_0}{m_1} = v_1 - v_0$ or, $\boldsymbol{\Delta v} = \boldsymbol{V}\cdot\mathbf{ln}\frac{\boldsymbol{m_0}}{\boldsymbol{m_1}}$ where **V=effective exhaust velocity=v-u, Δv=change in velocity of rocket**, **m0,m1** are **mass** of the rocket at different **instant**. This is the required **Tsiolkovsky rocket equation**.

"Centripetal acceleration and force":



In given fig.' x'=distance travelled horizontally if centripetal force is absent.

'S'=vertical fall; 'R'=radius of circular loci if centripetal balances centrifugal force.

What must be orientation of force to produce circular motion? A body is moving with uniform velocity v on straight line, and we apply a Force at right angle to motion. It now moves in both horizontal and vertical directions simultaneously. The locus of the body will be a circle. The force we applied is the centripetal force. To measure the centripetal acceleration, we must first measure the vertical fall S.

Using plane geometry, $\frac{x}{s} = \frac{2R-s}{x}\ or, x^2 - 2Rs + s^2 = 0$ or, $\mathrm{s} = \mathrm{R} \pm \sqrt{R^2 - x^2}$

$$\text{or}, \mathrm{s} = \mathrm{R}\left[1 - \sqrt{1 - \left(\frac{x}{R}\right)^2}\right] \text{ since } \mathrm{s} < R\ we\ neglect\ the\ other\ factor\ .$$

'x=v.t', by taking t sufficiently small x/R becomes very small, truncated $s = \frac{x^2}{2\mathrm{R}} = \frac{v^2}{2\mathrm{R}}.t^2$ Now, $\frac{d^2s}{dt^2} = \gamma\alpha\ or, \alpha = \frac{v^2}{R}$
Since final velocity at maximum vertical fall<<c due to sufficiently small't'; $\gamma \to 1$. Centripetal force is hence
$\mathbf{F} = \mathbf{M}.\frac{\mathbf{v^2}}{\mathbf{R}}\ where\ M = \gamma(v)m_0$ if 'm0' is rest mass of the body.

CONCLUSION:

How should we know that a hypothesis is correct? The ultimate weapon to test an idea is experiment. A hypothesis is drawn with aid of reasoning and fact, it is next utilized to make predictions, if the predictions remain consistent to facts already discovered, it survives for more prediction until replaced by a new one. Adding relativistic corrections to Newtonian dynamics, should meet this criteria. Here we discuss some tips on the topic:

Since, this is Newtonian mechanics, by symmetry of problems; we have chosen **rectangular coordinate system**.

"Sign convention of force": Force=relativistic mass**a.* During **acceleration**, **relativistic mass** is calculated at **final velocity** and at **initial velocity** during **retardation**. Force in opposite direction is arbitrarily assigned a '-ve' value and absolute value of '*a' is considered. 'α' has however same dimensions as acceleration M^0LT^{-2} and its units are N/kg in S.I. and dyne/gm in C.G.S. system of units. 1 N/kg=100 dyne/gm.*

"*Physical nature of a"*: Suppose we see an body at acceleration from a non-zero initial velocity. If we imagine a corresponding rest frame for the event, then we must see the body start from rest and accelerate by *'a'.* We memorize this saying ***'a'*** is **"rest-frame acceleration"**.

"*Derivative of velocity, displacement"*: In dynamics the differential equation for Newton's laws of motion: $\frac{d\boldsymbol{v}}{d\boldsymbol{t}} = \frac{d^2\boldsymbol{S}}{d\boldsymbol{t}^2} = \boldsymbol{v}\frac{d\boldsymbol{v}}{d\boldsymbol{S}}$=acceleration. However **relativistic ally** the three derivatives have **separate physical existence**.

"*Laws of conservation*": Conservation of energy, linear momentum is not violable in nature, but only appears to hold if viewed from a single frame of reference.

"*Vector algebra"*: Scalar multiplied with a vector is also a vector. ***'a'*** is thus a **vector***.* In cases involving a number of forces in different direction, the resultant force can be determined from **resultant** *'a'* using **"Superposition Principle".**

"*Rest frame"*: **Rest frame** for an event is **unique**. We may show that ***'a'*** for an event is **same**, although the other physical quantities have different values for different inertial frame of references.

"*Relativistic mass"*: **"The gravitational mass at any instant of time is relativistic mass**". W**eighing** a body gives its **mass**; as acceleration due to gravity is independent. Even **'photons'** appear to have **gravitational mass**; even though its rest-mass is zero and can be thought as **relativistic mass**.

"Space-time interval": We presumed in our work that if there is event 1 initially, a force of same nature must work to produce event2 finally after some time interval. Chronological order of occurrence of these events is maintained, independently in inertial reference frames we choose. This demands **time-like** intervals.

We shall next find some results, to show the current hypothesis is consistent to known facts.

MISCELLANEOUS:

"Kinetic Energy":

We consider a reference frame R such that the body is initially at rest and under the effect of a force accelerates to a velocity v. Kinetic energy of the body is equal to the work done by this force.

Any force is defined as F=m_1α where m_1=relativistic mass of the body at velocity v; α=acceleration

Work done by the force W= m_1αS where S=displacement of the body when force acted on it.

Using $v^2 = \left(1+\frac{1}{\gamma}\right)\alpha S$ we get its kinetic energy as **K=1**$/\left(\boldsymbol{1}+\frac{\boldsymbol{1}}{\boldsymbol{\gamma}}\right)$ $\mathbf{m_1}\boldsymbol{v}^{\boldsymbol{2}}$.

If we substitute v^2 for $c^2\left(1-\frac{1}{\gamma^2}\right)$, and m_1 for γm_0 we get K=$(\gamma-1)m_0c^2$ which is the expression given by Special Theory of Relativity.

"Mass phenomenon": Mass of a fixed mass system increases with velocity, so during acceleration mass increases. What happens to this mass, if it retards? Using eqn. k, we find the relativistic mass at final velocity due to retardation is same as it should be if the body had been accelerated to this velocity from rest. The excess mass simply vanishes!

"Instantaneous acceleration due to a force on a moving body":We consider a reference frame R1 such that the body is moving with uniform velocity u, when a force F acts on it for an infinite seminally small duration of time dt, due to which it accelerates to a velocity v=u+du. We assume its relativistic mass is m when it is moving with the final velocity. So α=F/m. And we know that acceleration of the body with respect to R1 is a=$\frac{\alpha}{\gamma(u)\cdot\left(1+\frac{u\omega}{c^2}\right)^2}$. $Here\ w=\frac{v-u}{1-\frac{uv}{c^2}}=\frac{u+du-u}{1-\frac{u(u+du)}{c^2}}=\frac{du}{1-\frac{u^2}{c^2}-\frac{u\,du}{c^2}}\ and\ \frac{uw}{c^2}=\frac{u\frac{du}{c^2}}{1-\frac{u^2}{c^2}-\frac{u\,du}{c^2}}$

$$so\ 1+\frac{uw}{c^2}=\frac{1-\frac{u^2}{c^2}-\frac{u\,du}{c^2}+u\frac{du}{c^2}}{1-\frac{u^2}{c^2}-\frac{u\,du}{c^2}}\ or,1+\frac{uw}{c^2}=\frac{1-\frac{u^2}{c^2}}{\left(1-\frac{u^2}{c^2}\right)-\frac{u\,du}{c^2}}$$

Maximum possible value of u/c is 1. However du/c is very very small number. So $\frac{udu}{c^2}\to 0$ for all values of u.

Therefore $\left(1+\frac{u\cdot w}{c^2}\right)^2\to 1$ for all values of u.Hence we conclude **instantaneous acceleration** of the body with respect to R1 is given by**a** $=\boldsymbol{\frac{\alpha}{\gamma(u)}=\frac{F}{\gamma(u)\cdot m}}$

"Instantaneous retardation due to a force on a moving body":We consider a reference frame R1 such that the body is moving with uniform velocity u, when a force F acts on it for an infinite seminally small duration of time dt, due to which it retards to a velocity u=v+dv. We assume its relativistic mass is m when it is moving with the final velocity. So α=-F/m. And we know that retardation of the body with respect to R1 is a=$\frac{|\alpha|}{\gamma(v)\cdot\left(1+\frac{v\omega}{c^2}\right)^2}$. $Here\ w=\frac{u-v}{1-\frac{uv}{c^2}}=\frac{v+dv-v}{1-\frac{v(v+dv)}{c^2}}=\frac{dv}{1-\frac{v^2}{c^2}-\frac{v\,dv}{c^2}}\ and\ \frac{vw}{c^2}=\frac{v\frac{dv}{c^2}}{1-\frac{v^2}{c^2}-\frac{v\,dv}{c^2}}$

$$so\ 1+\frac{vw}{c^2}=\frac{1-\frac{v^2}{c^2}-\frac{v\,dv}{c^2}+v\frac{dv}{c^2}}{1-\frac{v^2}{c^2}-\frac{v\,dv}{c^2}}\ or,1+\frac{uw}{c^2}=\frac{1-\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\right)-\frac{v\,dv}{c^2}}$$

Maximum possible value of v/c is 1. However dv/c is very very small number. So $\frac{vdv}{c^2}\to 0$ for all values of v.

Therefore $\left(1+\frac{v\cdot w}{c^2}\right)^2\to 1$ for all values of v.Hence we conclude **instantaneous retardation** of the body with respect to R1 is given by **a**$=\boldsymbol{\frac{|\alpha|}{\gamma(v)}=-\frac{F}{\gamma(v)\cdot m}}$

In light of the above two results, we should define: *"**1N force** is the amount of force which when acts on a mass of **1kg** at **rest**, produces an instantaneous acceleration of **1m/s^2** in its direction."* Force applied to produce any other acceleration is measured by answering how many times a greater force than 1N has been applied. If we would use force=rate of change of linear momentum, 1N force is the force which produces a change of linear momentum by 1kgm/s.

Two bodies having their **relativistic masses** in ratio **n: 1** are moving with same uniform velocity. They are acted by a force of magnitude F. Find the ratio of their instantaneous accelerations.

Let **instantaneous accelerations** are a1 and a2 respectively of bodies having relativistic masses m1 and m2.Given: m1:m2=n: 1.Now a1:a2=$\frac{F}{\gamma(u)\cdot m1}:\frac{F}{\gamma(u)\cdot m2} = \mathrm{m2:m1} = \mathbf{1:n}$Ans: The instantaneous accelerations due to force on bodies moving with same velocity and having relativistic masses in ratio n: 1 are in ratio 1: n.

“g-force in an accelerated laboratory”: Let S is an inertial reference frame, according to which, a laboratory accelerated from ‘u’ to ‘v’ by amount ‘a’ in +ve Y-axis. Static observers in the laboratory would experience an acceleration *‘a’* in –ve Y’ direction. The same is true for retardation but $|\boldsymbol{\alpha}|$ would be along +ve Y’ axis.

These results are extremely satisfactory and hence the hypothesis is acceptable.

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