R-intersections and R-unions of neutrosophic cubic sets

Young Bae Jun, Florentin Smarandache, and Chang Su Kim,

***Abstract*—R-unions and R-intersections of T-external (I-external, F-external) neutrosophic cubic sets are considered. Examples to show that the R-intersection and R-union of T-external (I-external, F-external) neutrosophic cubic sets may not be a T-external (I-external, F-external) neutrosophic cubic set are provided. Conditions for the R-union and R-intersection of T-external (I-external, F-external) neutrosophic cubic sets to be a T-external (I-external, F-external) neutrosophic cubic set are discussed.**

***Index Terms*—Truth-internal (indeterminacy-internal, falsity-internal) neutrosophic cubic set, truth-external (indeterminacy-external, falsity-external) neutrosophic cubic set, R-union, R-intersection.**

I. Introduction

SMARANDACHE ([5], [6]) developed the concept of neutrosophic set as a more general platform which extends the concepts of the classic set and fuzzy set, intuitionistic fuzzy set and interval valued intuitionistic fuzzy set. We know that neutrosophic set theory is applied to various part (refer to the site http:// fs.gallup.unm.edu/neutrosophy.htm). Ali and Smarandache [1] introduced complex neutrosophic sets to handle imprecise, indeterminate, inconsistent, and incomplete information of periodic nature. Deli et al. [2] introduced the concept of bipolar neutrosophic set and its some operations.

In [3], Jun et al. introduced the notion of (internal, external) cubic sets, and investigated several properties. Jun et al. [4] extended the concept of cubic sets to the neutrosophic sets, and introduced/investigated the notions/properties of T-internal (I-internal, F-internal) neutrosophic cubic sets and T-external (I-external, F-external) neutrosophic cubic sets. As a continuation of the paper [4], we consider R-unions and R-intersections of T-external (I-external, F-external) neutrosophic cubic sets. We provide examples to show that the R-intersection and the R-union of T-external (resp. I-external and F-external) neutrosophic cubic sets may not be a T-external (resp. I-external and F-external) neutrosophic cubic set. We discuss conditions for the R-union of T-external (resp. I-external and F-external) neutrosophic cubic sets to be a T-external (resp. I-external and F-external) neutrosophic cubic set. We consider a condition for the R-intersection of T-external (resp. I-external and F-external) neutrosophic cubic sets to be a T-external (resp. I-external and F-external) neutrosophic cubic set.

F. Smarandache was with the Mathematics & Science Department, University of New Mexico, 705 Gurley Ave., Gallup, NM 87301, USA e-mail: fsmarandache@gmail.com

Y. B. Jun and C. S. Kim are with Department of Mathematics Education, Gyeongsang National University, Jinju 52828, Korea. e-mail: skywine@gmail.com (Y. B. Jun), cupncap@gmail.com (C. S. Kim)

Manuscript received mm dd, 201y; revised mm dd, 201y.

II. R-intersections and R-unions of neutrosophic cubic sets

Jun et al. [4] considered the notion of neutrosophic cubic sets as an extension of cubic sets.

Let X be a non-empty set. A neutrosophic cubic set (NCS) in X is a pair $\mathcal{A} = (\mathbf{A}, \Lambda)$ where

$$\mathbf{A} := \{\langle x; A_T(x), A_I(x), A_F(x)\rangle \mid x \in X\}$$

is an interval neutrosophic set in X and

$$\Lambda := \{\langle x; \lambda_T(x), \lambda_I(x), \lambda_F(x)\rangle \mid x \in X\}$$

is a single-valued neutrosophic set in X.

For further particulars on the notions of T (resp., I, F)-internal neutrosophic cubic sets, T (resp., I, F)-external neutrosophic cubic sets, R-union and R-intersection of neutrosophic cubic sets, we refer the reader to the the paper [4].

We know that R-intersection and R-union of T-external (resp., I-external and F-external) neutrosophic cubic sets may not be a T-external (resp., I-external and F-external) neutrosophic cubic sets as seen in the following example.

Example 2.1: Let $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ be neutrosophic cubic sets in $X = [0, 1]$ where

$\mathbf{A} = \{\langle x; [0.5, 0.7], [0.2, 0.4], [0.3, 0.5]\rangle \mid x \in [0, 1]\}$,
$\Lambda = \{\langle x; 0.6, 0.7, 0.8\rangle \mid x \in [0, 1]\}$,
$\mathbf{B} = \{\langle x; [0.6, 0.7], [0.6, 0.8], [0.7, 0.9]\rangle \mid x \in [0, 1]\}$,
$\Psi = \{\langle x; 0.5, 0.9, 0.8\rangle \mid x \in [0, 1]\}$.

Then $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ are I-external neutrosophic cubic sets in $X = [0, 1]$. The R-union $\mathcal{A} \cup_R \mathcal{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \wedge \Psi)$ of $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ is given as follows:

$\mathbf{A} \cup \mathbf{B} = \{\langle x; [0.6, 0.7], [0.6, 0.8], [0.7, 0.9]\rangle \mid x \in [0, 1]\}$,
$\Lambda \wedge \Psi = \{\langle x; 0.5, 0.7, 0.8\rangle \mid x \in [0, 1]\}$,

and it is not an I-external neutrosophic cubic set in $X = [0, 1]$.

We provide a condition for the R-union of two T-external (resp., I-external, F-external) neutrosophic cubic sets to be T-external (resp., I-external, F-external).

Theorem 2.2: Let $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ be I-external neutrosophic cubic sets in X such that

$$\max\left\{\min\{A_I^+(x), B_I^-(x)\}, \min\{A_I^-(x), B_I^+(x)\}\right\} \le (\lambda_I \wedge \psi_I)(x) < \min\left\{\max\{A_I^+(x), B_I^-(x)\}, \max\{A_I^-(x), B_I^+(x)\}\right\}$$

for all $x \in X$. Then the R-union $\mathcal{A} \cup_R \mathcal{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \wedge \Psi)$ is an I-external neutrosophic cubic set in X.

Proof. For any $x \in X$, let

$$a_x := \max\left\{\min\{A_I^+(x), B_I^-(x)\}, \min\{A_I^-(x), B_I^+(x)\}\right\}$$

and

$$b_x := \min\left\{\max\{A_I^+(x), B_I^-(x)\}, \max\{A_I^-(x), B_I^+(x)\}\right\}.$$

Then $b_x = A_I^-(x)$, $b_x = B_I^-(x)$, $b_x = A_I^+(x)$, or

$b_x = B_I^+(x)$. It is possible to consider the cases $b_x = B_I^-(x)$ and $b_x = B_I^+(x)$ only because the remaining cases are similar to these cases. If $b_x = B_I^-(x)$, then $A_I^+(x) \leq B_I^-(x)$ and so

$$A_I^-(x) \leq A_I^+(x) \leq B_I^-(x) \leq B_I^+(x).$$

Thus $a_x = A_I^+(x)$, and so

$$(A_I \cup B_I)^-(x) = B_I^-(x) = b_x > (\lambda_I \wedge \psi_I)(x)$$

Hence

$$(\lambda_I \wedge \psi_I)(x) \notin \left((A_I \cup B_I)^-(x), (A_I \cup B_I)^+(x)\right).$$

If $b_x = B_I^+(x)$, then $A_I^-(x) \leq B_I^+(x) \leq A_I^+(x)$ and thus $a_x = \max\{A_I^-(x), B_I^-(x)\}$. Suppose that $a_x = A_I^-(x)$. Then

$$B_I^-(x) \leq A_I^-(x) = a_x \leq (\lambda_I \wedge \psi_I)(x) < b_x = B_I^+(x) \leq A_I^+(x).$$

It follows that

$$B_I^-(x) \leq A_I^-(x) < (\lambda_I \wedge \psi_I)(x) < B_I^+(x) \leq A_I^+(x) \quad (1)$$

or

$$B_I^-(x) \leq A_I^-(x) = (\lambda_I \wedge \psi_I)(x) < B_I^+(x) \leq A_I^+(x). \quad (2)$$

The case (1) induces a contradiction. The case (2) implies that

$$(\lambda_I \wedge \psi_I)(x) \notin \left((A_I \cup B_I)^-(x), (A_I \cup B_I)^+(x)\right)$$

since $(\lambda_I \wedge \psi_I)(x) = A_I^-(x) = (A_I \cup B_I)^-(x)$. Now, if $a_x = B_I^-(x)$, then

$$A_I^-(x) \leq B_I^-(x) = a_x \leq (\lambda_I \wedge \psi_I)(x) < b_x = B_I^+(x) \leq A_I^+(x).$$

Hence we have

$$A_I^-(x) \leq B_I^-(x) < (\lambda_I \wedge \psi_I)(x) < B_I^+(x) \leq A_I^+(x) \quad (3)$$

or

$$A_I^-(x) \leq B_I^-(x) = (\lambda_I \wedge \psi_I)(x) < B_I^+(x) \leq A_I^+(x). \quad (4)$$

The case (3) induces a contradiction. The case (4) implies that

$$(\lambda_I \wedge \psi_I)(x) \notin \left((A_I \cup B_I)^-(x), (A_I \cup B_I)^+(x)\right).$$

Therefore the R-union $\mathcal{A} \cup_R \mathcal{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \wedge \Psi)$ is an I-external neutrosophic cubic set in X.

Similarly, we have the following theorems.

Theorem 2.3: Let $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ be T-external neutrosophic cubic sets in X such that

$$\max\left\{\min\{A_T^+(x), B_T^-(x)\}, \min\{A_T^-(x), B_T^+(x)\}\right\} \leq (\lambda_T \wedge \psi_T)(x) < \min\left\{\max\{A_T^+(x), B_T^-(x)\}, \max\{A_T^-(x), B_T^+(x)\}\right\}$$

for all $x \in X$. Then the R-union $\mathcal{A} \cup_R \mathcal{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \wedge \Psi)$ is a T-external neutrosophic cubic set in X.

Theorem 2.4: Let $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ be F-external neutrosophic cubic sets in X such that

$$\max\left\{\min\{A_F^+(x), B_F^-(x)\}, \min\{A_F^-(x), B_F^+(x)\}\right\} \leq (\lambda_F \wedge \psi_F)(x) < \min\left\{\max\{A_F^+(x), B_F^-(x)\}, \max\{A_F^-(x), B_F^+(x)\}\right\}$$

for all $x \in X$. Then the R-union $\mathcal{A} \cup_R \mathcal{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \wedge \Psi)$ is an F-external neutrosophic cubic set in X.

Corollary 2.5: Let $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ be external neutrosophic cubic sets in X. Then the R-union of $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ is an external neutrosophic cubic set in X when the conditions in Theorems 2.2 2.3 and 2.4 are valid.

The following examples show that the R-intersection of two T-external (resp., I-external, F-external) neutrosophic cubic sets may not be T-external (resp., I-external, F-external).

Example 2.6: Let $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ be neutrosophic cubic sets in $X = [0, 1]$ where

$\mathbf{A} = \{\langle x; [0.2, 0.4], [0.5, 0.7], [0.3, 0.5]\rangle \mid x \in [0, 1]\}$,
$\Lambda = \{\langle x; 0.1, 0.4, 0.8\rangle \mid x \in [0, 1]\}$,
$\mathbf{B} = \{\langle x; [0.6, 0.8], [0.4, 0.7], [0.7, 0.9]\rangle \mid x \in [0, 1]\}$,
$\Psi = \{\langle x; 0.3, 0.3, 0.8\rangle \mid x \in [0, 1]\}$.

Then $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ are T-external neutrosophic cubic sets in $X = [0, 1]$. The R-intersection $\mathcal{A} \cap_R \mathcal{B} = (\mathbf{A} \cap \mathbf{B}, \Lambda \vee \Psi)$ of $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ is given as follows:

$$\mathbf{A} \cap \mathbf{B} = \{\langle x; [0.2, 0.4], [0.4, 0.7], [0.3, 0.5]\rangle \mid x \in [0, 1]\},$$
$$\Lambda \vee \Psi = \{\langle x; 0.3, 0.4, 0.8\rangle \mid x \in [0, 1]\},$$

and it is not a T-external neutrosophic cubic set in $X = [0, 1]$.

We provide a condition for the R-intersection of two T-external (resp., I-external, F-external) neutrosophic cubic sets to be T-external (resp., I-external, F-external).

Theorem 2.7: Let $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ be T-external neutrosophic cubic sets in X such that

$$\max\left\{\min\{A_T^+(x), B_T^-(x)\}, \min\{A_T^-(x), B_T^+(x)\}\right\} < (\lambda_T \vee \psi_T)(x) \leq \min\left\{\max\{A_T^+(x), B_T^-(x)\}, \max\{A_T^-(x), B_T^+(x)\}\right\}$$

for all $x \in X$. Then the R-intersection $\mathcal{A} \cap_R \mathcal{B} = (\mathbf{A} \cap \mathbf{B}, \Lambda \vee \Psi)$ is a T-external neutrosophic cubic set in X.

Proof. For any $x \in X$, let

$$c_x := \max\left\{\min\{A_T^+(x), B_T^-(x)\}, \min\{A_T^-(x), B_T^+(x)\}\right\}$$

and

$$d_x := \min\left\{\max\{A_T^+(x), B_T^-(x)\}, \max\{A_T^-(x), B_T^+(x)\}\right\}.$$

Then $d_x = A_T^-(x)$, $d_x = B_T^-(x)$, $d_x = A_T^+(x)$, or $d_x = B_T^+(x)$. It is possible to consider the cases $d_x = A_T^-(x)$ and $d_x = A_T^+(x)$ only because the remaining cases are similar to these cases. If $d_x = A_T^-(x)$, then

$$B_T^-(x) \leq B_T^+(x) \leq A_T^-(x) \leq A_T^+(x).$$

Thus $c_x = B_T^+(x)$, and so

$$B_T^-(x) = (A_T \cap B_T)^-(x) \leq (A_T \cap B_T)^+(x) = B_T^+(x) = c_x < (\lambda_T \vee \psi_T)(x).$$

Hence $(\lambda_T \vee \psi_T)(x) \notin ((A_T \cap B_T)^-(x), (A_T \cap B_T)^+(x))$. If $d_x = A_T^+(x)$, then $B_T^-(x) \leq A_T^+(x) \leq B_T^+(x)$ and thus $c_x = \max\{A_T^-(x), B_T^-(x)\}$. Suppose that $c_x = A_T^-(x)$. Then

$$B_T^-(x) \leq A_T^-(x) = c_x < (\lambda_T \vee \psi_T)(x) \leq d_x = A_T^+(x) \leq B_T^+(x).$$

It follows that

$$B_T^-(x) \leq A_T^-(x) < (\lambda_T \vee \psi_T)(x) < A_T^+(x) \leq B_T^+(x) \quad (5)$$

or

$$B_T^-(x) \leq A_T^-(x) < (\lambda_T \vee \psi_T)(x) = A_T^+(x) \leq B_T^+(x). \quad (6)$$

The case (5) induces a contradiction. The case (6) implies that

$$(\lambda_T \vee \psi_T)(x) \notin \left((A_T \cap B_T)^-(x), (A_T \cap B_T)^+(x)\right)$$

since $(\lambda_T \vee \psi_T)(x) = A_T^+(x) = (A_T \cap B_T)^+(x)$. Now, if $c_x = B_T^-(x)$, then

$$A_T^-(x) \leq B_T^-(x) = c_x < (\lambda_T \vee \psi_T)(x) \\ \leq d_x = A_T^+(x) \leq B_T^+(x).$$

Hence we have

$$A_T^-(x) \leq B_T^-(x) < (\lambda_T \vee \psi_T)(x) < A_T^+(x) \leq B_T^+(x) \quad (7)$$

or

$$A_T^-(x) \leq B_T^-(x) < (\lambda_T \vee \psi_T)(x) = A_T^+(x) \leq B_T^+(x). \quad (8)$$

The case (7) induces a contradiction. The case (8) induces

$$(\lambda_T \vee \psi_T)(x) \notin \left((A_T \cap B_T)^-(x), (A_T \cap B_T)^+(x)\right).$$

Therefore the R-intersection $\mathcal{A} \cap_R \mathcal{B} = (\mathbf{A} \cap \mathbf{B}, \Lambda \vee \Psi)$ is a T-external neutrosophic cubic set in X.

Similarly, we have the following theorems.

Theorem 2.8: Let $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ be I-external neutrosophic cubic sets in X such that

$$\max\left\{\min\{A_I^+(x), B_I^-(x)\}, \min\{A_I^-(x), B_I^+(x)\}\right\} \\ < (\lambda_I \vee \psi_I)(x) \\ \leq \min\left\{\max\{A_I^+(x), B_I^-(x)\}, \max\{A_I^-(x), B_I^+(x)\}\right\}$$

for all $x \in X$. Then the R-intersection $\mathcal{A} \cap_R \mathcal{B} = (\mathbf{A} \cap \mathbf{B}, \Lambda \vee \Psi)$ is an I-external neutrosophic cubic set in X.

Theorem 2.9: Let $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ be F-external neutrosophic cubic sets in X such that

$$\max\left\{\min\{A_F^+(x), B_F^-(x)\}, \min\{A_F^-(x), B_F^+(x)\}\right\} \\ < (\lambda_F \vee \psi_F)(x) \\ \leq \min\left\{\max\{A_F^+(x), B_F^-(x)\}, \max\{A_F^-(x), B_F^+(x)\}\right\}$$

for all $x \in X$. Then the R-intersection $\mathcal{A} \cap_R \mathcal{B} = (\mathbf{A} \cap \mathbf{B}, \Lambda \vee \Psi)$ is an F-external neutrosophic cubic set in X.

Corollary 2.10: Let $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ be external neutrosophic cubic sets in X. Then the R-intersection of $\mathcal{A} = (\mathbf{A}, \Lambda)$ and $\mathcal{B} = (\mathbf{B}, \Psi)$ is an external neutrosophic cubic set in X when the conditions in Theorems 2.7, 2.8 and 2.9 are valid.

III. Conclusion

We have considered the R-union and R-intersection of T-external (I-external, F-external) neutrosophic cubic sets. We have provided examples to show that the R-intersection and R-union of T-external (resp. I-external and F-external) neutrosophic cubic sets may not be a T-external (resp. I-external and F-external) neutrosophic cubic set. We have discussed conditions for the R-union and R-intersection of T-external (resp. I-external and F-external) neutrosophic cubic sets to be a T-external (resp. I-external and F-external) neutrosophic cubic set. Based on this paper, we will study conditions for the R-intersection of two neutrosophic cubic sets to be both an α-external neutrosophic cubic set and an α-internal neutrosophic cubic set for $\alpha \in \{T, I, F\}$. Also, we will consider conditions for the R-union and R-intersection of two α-internal neutrosophic cubic sets to be an α-external neutrosophic cubic set for $\alpha \in \{T, I, F\}$.

Acknowledgment

The authors would like to thank the anonymous reviewers for their valuable suggestions.

References

[1] M. Ali, and F. Smarandache, Complex neutrosophic set, Neural Computing and Applications, 25 (2016), 1–18. DOI: 10.1007/s00521-015-2154-y.

[2] I. Deli, M. Ali and F. Smarandache, Bipolar neutrosophic sets and their application based on multi-criteria decision making problems, Proceeding of the 2015 International Conference on Advanced Mechatronic Systems, Beijing, China, August 22-24, 2015. IEEE Xplore, DOI: 10.1109/ICAMechS.2015.7287068

[3] Y. B. Jun, C. S. Kim and K. O. Yang, Cubic sets, Ann. Fuzzy Math. Inform. 4(1) (2012), 83–98.

[4] Y. B. Jun, F. Smarandache and C. S. Kim, Neutrosophic cubic sets, New Mathematics & Natural Computation (in press).

[5] F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Reserch Press, Rehoboth, NM, 1999.

[6] F. Smarandache, Neutrosophic set-a generalization of the intuitionistic fuzzy set, Int. J. Pure Appl. Math. 24(3) (2005), 287–297.