Generally Covariant Quantum Theory: non-abelian gauge theories.

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Abstract

We further investigate the new project initiated in [1, 2, 3, 4, 7, 8, 9] by generalizing non-abelian gauge theory to our setting. Given the results in [3, 9], there is not much left to do and we shall deepen our understanding of some points left open in [1] regarding the nature and presence of ghosts. Perturbative finiteness of the theory follows ad-verbatim from the analysis in [3, 9] and we shall not bother here about writing it down explicitly. Rather, our aim is to provide for a couple of new physical and mathematical insights regarding the *genesis* of the structure of quantal non-abelian gauge theory.

1 Introduction.

The aim of this paper is to *construct* "non-abelian gauge theory" from scratch using a couple of novel physical principles which constitute the substitute for results obtained in operational quantum theory using the language of quantum fields. Hence, we fill up some small points left in [1] and provide for a broad physical understanding of the necessity of the construction. This means we shall extend the "classical" vision on quantum spin [16] in Minkowski to a theory of spin valid in any curved spacetime background: we did only perform part of that analysis in [1] but so far this has been inconsequential. Now, we have to fill in the remaining fine points: obviously, I shall not *directly* rely upon any operational, nor unitarity arguments here since they do not fit in our theory. This paper is not written independently of some others and the reader who wishes to understand the triviality of our claim that the resulting theory is perturbatively finite should consult the references [1, 3, 9]. Explaining all fine points required for the construction would take me as much space as needed in those references and I advise the reader to absorb their content in the order indicated.

Therefore, I would never have written this paper but since people like to see explicit examples of how old concepts fit into your new theory, I decided to do so nevertheless albeit its content is totally trivial in light of previously obtained results. To avoid duplication, I have decided to keep it very short so that only

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the crucial points regarding the physics behind the theory are touched upon and nothing else. Entropic arguments regarding the number of Feynman diagrams with V internal vertices and n IN and m OUT vertices needed for analycity study of the defining interaction series are postponed to a book publication about this project so that I have at least still the chance to say something new there. To summarize, in section two, we study our novel rationale behind spin and present the need for gauge invariance regarding massless spin one particles. Hence, in section three, we explain the necessity of the spin zero ghost particles and their coupling to the spin-one gauge particles as well as the relevant interaction vertices. The paper is finished by some discussion in section four.

2 Spin and gauge invariance.

The content of this section shouldn't come as a surprise to those who have contemplated sometimes how to divorce the definition of spin from the global properties of Minkowski spacetime. First, let us mention that, given the superposition principle, our appraoch can still be framed into the language of linear spaces albeit this did not constitute our axiomatic starting point given that it is was not clear *what* Hilbert space to speak about. Moreover, the latter viewpoint is not crucial to the formulation of our theory given that we abandonned the language of infinite dimensional, Hermitian, linear operators on it. What concerns the internal degrees of freedom of a particle, the *finite* dimensional vectorspace viewpoint is all there is to it. Therefore, we speak again about linear spaces and linear non-unitary group operators on it and study properties of particles by means of properties of representations of the "inner subgroup" (which can always be chosen to be unitary) attached to a particle. Our reasoning is therefore *not* grounded into the unitary infinite dimensional representation theory of non-compact groups, but in the finite dimensional (unitary) representations of compact groups. Indeed, we shall elevate to a dogma that all internal degrees of freedom of an elementary particle stem from a compact internal symmetry group: such assumption being also mandatory in the operational approach [16] as we shall repeat in a while.

The allowed spin j of a particle, given an irreducible $SL(2,\mathbb{C})$ representation, with representation space V, is determined by means of the induced $SU(2)$ sub-representation which is required to have a spin-j component. For example, the 0 representation of $SL(2,\mathbb{C})$ induces a spin-0 representation of $SU(2)$ and therefore only allows for particles of 0 spin. The Dirac representation of $SL(2,\mathbb{C})$ equals the direct sum of two irreducible spin-$\frac{1}{2}$ $SU(2)$ representations and therefore incorporates the spin degrees of freedom for two spin $\frac{1}{2}$-particles: the particle and its anti-particle. The first place where mass starts to play a role is in the case of the Lorentz representation of $SL(2,\mathbb{C})$; the latter equals the direct sum of a spin-0 and spin-1 $SU(2)$ representation and therefore determines two theories in the massive case. That of a spin-0 vector particle $d\psi$ with one local degree of freedom, and of divergenceless spin-one vectorfield V^{μ} with

$$\nabla_{\mu}V^{\mu} = 0$$

having three local degrees of freedom. For a massless particle, the situation is a little bit more complicated: here, the internal group is the two dimensional Euclidean group from which only the $U(1)$ part can be associated to internal degrees of freedom since the translations do not form a compact group. The $U(1)$ sub-representation has two "spin" zero components and one component of helicity ± 1 each. Therefore, a Lorentz covariant theory for a spin-$\frac{1}{2}$ particle needs two particles with opposite helicities and two spin 0 particles. To eliminate these spin-0 particles, we need two extra local "negative" degrees of freedom which, by themselves, satisfy Lorentz covariant laws. At least this is one viewpoint on the matter: the other being that at the level of fields we need a *one* dimensional local symmetry, which eliminates two local degrees of freedom (by means of a first class constraint and the gauge transformation associated this constraint). As it turns out, both points of view are united in the path integral approach towards non-abelian gauge theories.

3 Why is non-abelian gauge theory the way it is in our formalism?

Standard non-abelian gauge theory is constructed in a way where the transformation laws of the gauge potential, or particle polarization, $A^{\alpha}_{\mu}(x)$ are *induced* from the transformation laws of the multiplets on representation space. This means, in particular, that all interactions are constructed from the basic object

$$\mathbf{A}_{\mu} = A^{\alpha}_{\mu}(t_{\alpha})^{m}_{n}$$

by means of Lie-algebra operations as well as the trace operation between two Lie-algebra elements, where the t_{α} constitute the generators of the Lie-algebra

$$[t_{\alpha}, t_{\beta}] = i f^{\gamma}_{\ \alpha\beta} t_{\gamma}$$

and $\mathrm{Tr}(t_{\alpha}t_{\beta}) = g_{\alpha\beta}$. Here,

$$f_{\gamma\alpha\beta} = g_{\gamma\delta} f^{\delta}_{\ \alpha\beta}$$

is totally anti-symmetric in its three covariant indices and $g_{\alpha\beta}$ is positive definite. Moreover, we do not take into account interactions requiring a length scale which implies all our interaction vertices are of mass dimension four. Moreover, by the very definition of interaction, the respective vertices need to be tri- or four-valent since gauge fields contribute a mass dimension of 1, while spinorial particles a mass dimension of $\frac{3}{2}$. All these considerations leave us with the following intertwiners

$$\begin{aligned} f_{\alpha\beta\gamma}\left(\nabla_{\kappa}A^{\alpha}_{\mu}\right) A^{\beta}_{\nu}A^{\gamma}_{\lambda}g^{\kappa\nu}g^{\nu\lambda} &= -i\mathrm{Tr}\left(\nabla_{\kappa}\mathbf{A}_{\mu}\left[\mathbf{A}_{\nu},\mathbf{A}_{\lambda}\right]\right)g^{\kappa\nu}g^{\mu\lambda} \\ f_{\alpha\beta\gamma}f^{\alpha}_{\ \beta'\gamma'}A^{\beta}_{\mu}A^{\gamma}_{\nu}A^{\beta'}_{\mu'}A^{\gamma'}_{\nu'}g^{\mu\nu}g^{\mu'\nu'} &= -\mathrm{Tr}\ \left(\left[\mathbf{A}_{\mu},\mathbf{A}_{\nu}\right]\left[\mathbf{A}_{\mu'},\mathbf{A}_{\nu'}\right]\right)g^{\mu\mu'}g^{\nu\nu'} \end{aligned}$$

concerning the self interaction of the gauge particles[1]. There remain the following two vertices

$$(\mathbf{A}_{\nu})^{m}_{n}(\gamma^{a})^{i}_{j}e^{\nu}_{a}(x)\Psi_{im}\overline{\Psi}^{jn}, \ \ f_{\alpha\beta\gamma}v^{\beta}\overline{v}^{\gamma}\nabla^{\mu}A^{\alpha}_{\mu}$$

[1]The only other two remaining options $\mathrm{Tr}\left(\nabla_{\kappa}\mathbf{A}_{\mu}\left[\mathbf{A}_{\nu},\mathbf{A}_{\lambda}\right]\right)g^{\kappa\mu}g^{\nu\lambda}$ and $\mathrm{Tr}\ \left(\left[\mathbf{B}_{\mu\nu\lambda},\mathbf{A}_{\kappa}\right]\right)Z^{\mu\nu\lambda\kappa}$ vanish by means of symmetry. Types such as $\mathrm{Tr}\ \left(\left[\left[\mathbf{A}_{\mu},\mathbf{A}_{\nu}\right],\mathbf{A}_{\lambda}\right]\mathbf{A}_{\kappa}\right)Z^{\mu\nu\lambda\kappa}$ can be expressed in terms of the previous cases.

where the last vertex is constructed from

$$\mathbf{v} = v^{\alpha} t_{\alpha}$$

as

$$-i\mathrm{Tr}\left([\mathbf{v}, \overline{\mathbf{v}}]\, \nabla^{\mu} \mathbf{A}_{\mu}\right).$$

Therefore, just out of completeness, we should supplement our theory with a spin zero particle and anti-particle transforming in the adjoint representation of the symmetry group with Fermionic statistics due to the anti-symmetry of the commutator. In [1] we argued that the relevant two point functions for such particle had to be given by

$$W_{a}^{\alpha\beta}(x,y) = \overline{\theta}(x)\theta(y)W(x,y)g^{\alpha\beta}, \; W_{p}^{\alpha\beta}(x,y) = \theta(x)\overline{\theta}(y)W(x,y)g^{\alpha\beta}$$

and in calculating Feynman diagrams, integration over the Grassmann coordinates should occur. There is however a deeper reason to introduce these ghosts than mere completeness which is that precisely as many "negative" local degrees of freedom are needed to kill the spin zero modes in the propagator

$$W_{\mu\nu'}^{\alpha\beta}(x,y) = g^{\alpha\beta} g_{\mu\nu'}(x,y) W(x,y).$$

The associated multiplication terms $\nabla^{\mu} A_{\mu}^{\alpha}$ are then seen as a "gauge condition" eliminating those degrees of freedom.

Hence, we are left with precisely the same four interaction vertices as in standard non-abelian gauge theory. Moreover, by rescaling the Lie algebra generators $t_{\alpha} \to \lambda t_{\alpha}$, suitably defining the interaction constant $\tilde{g}$ of the theory and by redefining the Grasmann numbers $\theta \to \lambda' \theta$ we obtain that they are of standard textbook form.

4 Some final remarks.

Given that we have clarified the remaining fine points left open in [1], we can now place some further comments. First of all, by literally the same methods as in [3, 9] the theory is perturbatively finite. It remains of course to investigate the analycity of the series and for this entropic arguments regarding the number of Feynman diagrams with V internal vertices and n-IN and m-OUT vertices are required. In case one would import a length scale in the interaction vertices, many more vertices could be written down (including ghost-Fermi interactions) leading to gravitational deviations from the standard model. We postpone these avenues for future work. Defining the graviton theory rigorously is a bit harder to do given that it concerns an infinite series of interaction vertices but we shall adress this issue in a forthcoming publication. I cannot emphasize enough the fact that our work shows that relativistic particle theory is alive and well and that no strings are needed to obtain finite results. The implementation of the idea of friction and the subsequent violation of unitarity being sufficient for our purposes.

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