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RECURRENT FORMULAS OF THE GENERALIZED FIBONACCI SEQUENCES OF FIFTH ORDER

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ABSTRACT

Coupled Fibonacci sequences involve two sequences of integers in which the elements of one sequence are part of the generalization of the other and vice versa. K. T. Atanassov was first introduced coupled Fibonacci sequences of second order in additive form. There are 32 different schemes of generalization for the Fibonacci sequences of fifth order in the case of two sequences [1]. I introduce their recurrent formulas below.

Mathematics Subject Classification: *11B39, 11B37.*

Keywords: *Fibonacci sequence, multiplicative Fibonacci sequence.*

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1. INTRODUCTION:

In the recent years much work has been done in this field but its multiplicative form is less known. The coupled Fibonacci sequence was first introduced by K. T. Atanassov and also discussed many curious properties and new direction of generalization of Fibonacci sequence in [1]. He defined and studied about four different ways to generate coupled sequences and called them coupled Fibonacci sequences (or 2-F sequences). K. T. Atanassov [1] notifies four different schemes in multiplicative form for coupled Fibonacci sequences.

2. RECURRENT FORMULAS OF THE GENERALIZED MULTIPLICATIVE FIBONACCI SEQUENCE OF FIFTH ORDER

We can construct 32 different schemes of generalized multiplicative Fibonacci sequence of fifth order in the case of two sequences. We introduce their recurrent formulas below.

Everywhere let $X_0 = C_0, Y_0 = C_1, X_1 = C_2, Y_1 = C_3, X_2 = C_4, Y_2 = C_5, X_3 = C_6, Y_3 = C_7, X_4 = C_8, Y_4 = C_9$ and assume that $n \geq 0$ is a natural number, where $C_0, C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9$ are given constants and *Z is* one of the symbols *X* or *Y*.

The different schemes are as follows:

$$T_1 : \begin{cases} X_{n+5} = X_{n+4} X_{n+3} X_{n+2} X_{n+1} X_n \\ Y_{n+5} = Y_{n+4} Y_{n+3} Y_{n+2} Y_{n+1} Y_n \end{cases}, T_2 : \begin{cases} X_{n+5} = X_{n+4} X_{n+3} X_{n+2} X_{n+1} Y_n \\ Y_{n+5} = Y_{n+4} Y_{n+3} Y_{n+2} Y_{n+1} X_n \end{cases},$$

$$T_3 : \begin{cases} X_{n+5} = X_{n+4} X_{n+3} X_{n+2} Y_{n+1} X_n \\ Y_{n+5} = Y_{n+4} Y_{n+3} Y_{n+2} X_{n+1} Y_n \end{cases}, T_4 : \begin{cases} X_{n+5} = X_{n+4} X_{n+3} X_{n+2} Y_{n+1} Y_n \\ Y_{n+5} = Y_{n+4} Y_{n+3} Y_{n+2} X_{n+1} X_n \end{cases},$$

$$T_5 : \begin{cases} X_{n+5} = X_{n+4} X_{n+3} Y_{n+2} X_{n+1} X_n \\ Y_{n+5} = Y_{n+4} Y_{n+3} X_{n+2} Y_{n+1} Y_n \end{cases}, T_6 : \begin{cases} X_{n+5} = X_{n+4} X_{n+3} Y_{n+2} X_{n+1} Y_n \\ Y_{n+5} = Y_{n+4} Y_{n+3} X_{n+2} Y_{n+1} X_n \end{cases},$$

$$T_6 : \begin{cases} X_{n+5} = X_{n+4} X_{n+3} Y_{n+2} X_{n+1} Y_n \\ Y_{n+5} = Y_{n+4} Y_{n+3} X_{n+2} Y_{n+1} X_n \end{cases}, T_8 : \begin{cases} X_{n+5} = X_{n+4} Y_{n+3} X_{n+2} X_{n+1} Y_n \\ Y_{n+5} = Y_{n+4} X_{n+3} Y_{n+2} Y_{n+1} X_n \end{cases},$$

$$T_9 : \begin{cases} X_{n+5} = Y_{n+4} X_{n+3} X_{n+2} X_{n+1} X_n \\ Y_{n+5} = X_{n+4} Y_{n+3} Y_{n+2} Y_{n+1} Y_n \end{cases}, T_{10} : \begin{cases} X_{n+5} = Y_{n+4} X_{n+3} X_{n+2} X_{n+1} Y_n \\ Y_{n+5} = X_{n+4} Y_{n+3} Y_{n+2} Y_{n+1} X_n \end{cases},$$

$$T_{11} : \begin{cases} X_{n+5} = Y_{n+4} Y_{n+3} X_{n+2} X_{n+1} X_n \\ Y_{n+5} = X_{n+4} X_{n+3} Y_{n+2} Y_{n+1} Y_n \end{cases}, T_{12} : \begin{cases} X_{n+5} = Y_{n+4} Y_{n+3} X_{n+2} X_{n+1} Y_n \\ Y_{n+5} = X_{n+4} X_{n+3} Y_{n+2} Y_{n+1} X_n \end{cases},$$

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$$T_{13}:\begin{cases}X_{n+5}=Y_{n+4}X_{n+3}Y_{n+2}X_{n+1}X_n\\Y_{n+5}=X_{n+4}Y_{n+3}Y_{n+2}Y_{n+1}Y_n\end{cases},T_{14}:\begin{cases}X_{n+5}=Y_{n+4}X_{n+3}Y_{n+2}X_{n+1}Y_n\\Y_{n+5}=X_{n+4}Y_{n+3}Y_{n+2}Y_{n+1}X_n\end{cases},$$

$$T_{15}:\begin{cases}X_{n+5}=Y_{n+4}X_{n+3}X_{n+2}Y_{n+1}X_n\\Y_{n+5}=X_{n+4}Y_{n+3}Y_{n+2}X_{n+1}Y_n\end{cases},\ T_{16}:\begin{cases}X_{n+5}=Y_{n+4}X_{n+3}X_{n+2}Y_{n+1}Y_n\\Y_{n+5}=X_{n+4}Y_{n+3}Y_{n+2}X_{n+1}X_n\end{cases},$$

$$T_{17}:\begin{cases}X_{n+5}=X_{n+4}Y_{n+3}Y_{n+2}X_{n+1}X_n\\Y_{n+5}=Y_{n+4}X_{n+3}X_{n+2}Y_{n+1}Y_n\end{cases},T_{18}:\begin{cases}X_{n+5}=X_{n+4}Y_{n+3}Y_{n+2}X_{n+1}Y_n\\Y_{n+5}=Y_{n+4}X_{n+3}X_{n+2}Y_{n+1}X_n\end{cases},$$

$$T_{16}:\begin{cases}X_{n+5}=Y_{n+4}X_{n+3}X_{n+2}Y_{n+1}Y_n\\Y_{n+5}=X_{n+4}Y_{n+3}Y_{n+2}X_{n+1}X_n\end{cases},T_{20}:\begin{cases}X_{n+5}=X_{n+4}Y_{n+3}X_{n+2}Y_{n+1}Y_n\\Y_{n+5}=Y_{n+4}X_{n+3}Y_{n+2}X_{n+1}X_n\end{cases},$$

$$T_{21}:\begin{cases}X_{n+5}=X_{n+4}X_{n+3}Y_{n+2}Y_{n+1}X_n\\Y_{n+5}=Y_{n+4}Y_{n+3}X_{n+2}X_{n+1}Y_n\end{cases},T_{22}:\begin{cases}X_{n+5}=X_{n+4}X_{n+3}Y_{n+2}Y_{n+1}Y_n\\Y_{n+5}=Y_{n+4}Y_{n+3}X_{n+2}X_{n+1}X_n\end{cases},$$

$$T_{23}:\begin{cases}X_{n+5}=Y_{n+4}Y_{n+3}Y_{n+2}X_{n+1}X_n\\Y_{n+5}=X_{n+4}X_{n+3}X_{n+2}Y_{n+1}Y_n\end{cases},T_{24}:\begin{cases}X_{n+5}=Y_{n+4}Y_{n+3}Y_{n+2}X_{n+1}Y_n\\Y_{n+5}=X_{n+4}X_{n+3}X_{n+2}Y_{n+1}X_n\end{cases},$$

$$T_{25}:\begin{cases}X_{n+5}=Y_{n+4}Y_{n+3}X_{n+2}Y_{n+1}X_n\\Y_{n+5}=X_{n+4}X_{n+3}Y_{n+2}X_{n+1}Y_n\end{cases},T_{26}:\begin{cases}X_{n+5}=Y_{n+4}Y_{n+3}X_{n+2}Y_{n+1}Y_n\\Y_{n+5}=X_{n+4}X_{n+3}Y_{n+2}X_{n+1}X_n\end{cases},$$

$$T_{27}:\begin{cases}X_{n+5}=Y_{n+4}X_{n+3}Y_{n+2}Y_{n+1}X_n\\Y_{n+5}=X_{n+4}Y_{n+3}X_{n+2}X_{n+1}Y_n\end{cases},\ T_{28}:\begin{cases}X_{n+5}=Y_{n+4}X_{n+3}Y_{n+2}Y_{n+1}Y_n\\Y_{n+5}=X_{n+4}Y_{n+3}X_{n+2}X_{n+1}X_n\end{cases},$$

$$T_{29}:\begin{cases}X_{n+5}=X_{n+4}Y_{n+3}Y_{n+2}Y_{n+1}X_n\\Y_{n+5}=Y_{n+4}X_{n+3}X_{n+2}X_{n+1}Y_n\end{cases},T_{30}:\begin{cases}X_{n+5}=X_{n+4}Y_{n+3}Y_{n+2}Y_{n+1}Y_n\\Y_{n+5}=Y_{n+4}X_{n+3}X_{n+2}X_{n+1}X_n\end{cases},$$

$$T_{31}:\begin{cases}X_{n+5}=Y_{n+4}Y_{n+3}Y_{n+2}Y_{n+1}X_n\\Y_{n+5}=X_{n+4}X_{n+3}X_{n+2}X_{n+1}Y_n\end{cases},T_{32}:\begin{cases}X_{n+5}=Y_{n+4}Y_{n+3}Y_{n+2}Y_{n+1}Y_n\\Y_{n+5}=X_{n+4}X_{n+3}X_{n+2}X_{n+1}X_n\end{cases},$$

The first scheme is trivial. All of the others are nontrivial; they have the following recurrent formulas for n ≥ 0:

----For $T_1: Z_{n+10}=Z_{n+9}Z_{n+8}Z_{n+7}Z_{n+6}Z_{n+5},$

----For $T_2: Z_{n+10}=\dfrac{Z^2_{n+9}Z_{n+8}Z_{n+7}}{Z^2_{n+6}Z^4_{n+5}Z^3_{n+4}Z^2_{n+3}Z_{n+2}},$

----For $T_3: Z_{n+10}=\dfrac{Z^2_{n+9}Z_{n+8}}{Z^3_{n+6}Z^3_{n+4}Z^2_{n+3}Z_{n+2}Z_n},$

----For $T_4: Z_{n+10}=\dfrac{Z^2_{n+9}Z_{n+8}Z_{n+2}Z^2_{n+1}Z_n}{Z^3_{n+6}Z^2_{n+5}Z_{n+4}},$

----For $T_5: Z_{n+10}=\dfrac{Z^2_{n+9}Z_{n+8}Z_{n+6}}{Z^2_{n+7}Z^3_{n+4}Z^2_{n+3}Z_{n+2}Z^2_{n+1}Z_n},$

----For $T_6: Z_{n+10}=\dfrac{Z^2_{n+9}Z_{n+8}Z_{n+6}Z_{n+2}Z_n}{Z^2_{n+7}Z^2_{n+5}Z_{n+4}},$

----For $T_7: Z_{n+10}=\dfrac{Z^2_{n+9}Z^2_{n+7}Z_{n+6}}{Z_{n+8}Z^3_{n+4}Z^2_{n+3}Z^3_{n+2}Z^2_{n+1}Z_n},$

----For $T_8: Z_{n+10}=\dfrac{Z^2_{n+9}Z^2_{n+7}Z_{n+6}Z_n}{Z_{n+8}Z^2_{n+5}Z_{n+4}Z_{n+2}},$

----***For*** $T_9 : Z_{n+10} = \dfrac{Z^3_{n+8} Z^2_{n+7} Z_{n+6}}{Z^3_{n+4} Z^4_{n+3} Z^3_{n+2} Z^2_{n+1} Z_n}$,

----***For*** $T_{10} : Z_{n+10} = \dfrac{Z^3_{n+8} Z^2_{n+7} Z_{n+6} Z_n}{Z^2_{n+5} Z_{n+4} Z^2_{n+3} Z_{n+2}}$,

----***For*** $T_{11} : Z_{n+10} = \dfrac{Z_{n+8} Z^4_{n+7} Z^3_{n+6} Z^2_{n+5}}{Z_{n+4} Z^2_{n+3} Z^3_{n+2} Z^2_{n+1} Z_n}$,

----***For*** $T_{12} : Z_{n+10} = \dfrac{Z_{n+8} Z^4_{n+7} Z^3_{n+6} Z^2_{n+5} Z_{n+4} Z_n}{Z_{n+2}}$,

----***For*** $T_{13} : Z_{n+10} = \dfrac{Z^3_{n+8} Z^3_{n+6} Z^2_{n+5}}{Z_{n+4} Z^2_{n+3} Z_{n+2} Z^2_{n+1} Z_n}$,

----***For*** $T_{14} : Z_{n+10} = Z^3_{n+8} Z^3_{n+6} Z_{n+4} Z_{n+2} Z_n$,

----***For*** $T_{15} : Z_{n+10} = \dfrac{Z^3_{n+8} Z^2_{n+7} Z^2_{n+5}}{Z_{n+6} Z_{n+4} Z^2_{n+3} Z_{n+2} Z_n}$,

----***For*** $T_{16} : Z_{n+10} = \dfrac{Z^3_{n+8} Z^2_{n+7} Z_{n+4} Z_{n+2} Z^2_{n+1} Z_n}{Z_{n+6}}$,

----***For*** $T_{17} : Z_{n+10} = \dfrac{Z^2_{n+9} Z^3_{n+6} Z^2_{n+5}}{Z_{n+8} Z_{n+4} Z_{n+2} Z^2_{n+1} Z_n}$,

----***For*** $T_{18} : Z_{n+10} = \dfrac{Z^2_{n+9} Z^3_{n+6} Z_{n+4} Z^2_{n+3} Z_{n+2} Z_n}{Z_{n+8}}$,

----***For*** $T_{19} : Z_{n+10} = \dfrac{Z^2_{n+9} Z^2_{n+7} Z^2_{n+5}}{Z_{n+6} Z^2_{n+4} Z_{n+2} Z_n}$,

----***For*** $T_{20} : Z_{n+10} = \dfrac{Z^2_{n+9} Z^2_{n+7} Z_{n+5}}{Z_{n+8} Z^2_{n+6} Z^2_{n+4} Z_{n+2}}$,

----***For*** $T_{21} : Z_{n+10} = \dfrac{Z^2_{n+9} Z_{n+8} Z_{n+5} Z_{n+2}}{Z^2_{n+7} Z_{n+6} Z_{n+4} Z_n}$,

----***For*** $T_{22} : Z_{n+10} = \dfrac{Z^2_{n+9} Z_{n+8} Z_{n+4} Z^2_{n+3} Z^3_{n+2} Z^2_{n+1} Z_n}{Z^2_{n+7} Z_{n+6}}$,

----***For*** $T_{23} : Z_{n+10} = \dfrac{Z_{n+8} Z^2_{n+7} Z^5_{n+6} Z^4_{n+5} Z_{n+4}}{Z_{n+2} Z_{n+1}}$,

----For $T_{24}: Z_{n+10} = Z_{n+8} Z^2_{n+7} Z^5_{n+6} Z^2_{n+5} Z^3_{n+4} Z^2_{n+3} Z_{n+2} Z_n$,

----For $T_{25}: Z_{n+10} = \dfrac{Z_{n+8} Z^4_{n+7} Z^2_{n+6} Z^3_{n+5} Z_{n+4}}{Z_{n+3} Z_{n+1}}$,

----For $T_{26}: Z_{n+10} = Z_{n+8} Z^4_{n+7} Z_{n+6} Z^2_{n+5} Z^3_{n+4} Z^3_{n+3} Z_{n+2} Z_{n+1} Z_n$,

----For $T_{27}: Z_{n+10} = \dfrac{Z^3_{n+8} Z_{n+6} Z^4_{n+5} Z_{n+4} Z_{n+2}}{Z_n}$,

----For $T_{28}: Z_{n+10} = Z^3_{n+8} Z_{n+6} Z^2_{n+5} Z^3_{n+4} Z^3_{n+3} Z^2_{n+2} Z^2_{n+1} Z_n$,

----For $T_{29}: Z_{n+10} = \dfrac{Z^2_{n+9} Z_{n+6} Z^4_{n+5} Z_{n+4} Z^2_{n+3} Z_{n+2}}{Z_{n+8} Z_n}$,

----For $T_{30}: Z_{n+10} = \dfrac{Z^2_{n+9} Z_{n+6} Z^2_{n+5} Z^3_{n+4} Z^4_{n+3} Z^3_{n+2} Z^2_{n+1} Z_n}{Z_{n+8}}$,

----For $T_{31}: Z_{n+10} = \dfrac{Z_{n+8} Z^2_{n+7} Z^3_{n+6} Z^6_{n+5} Z^3_{n+4} Z^2_{n+3} Z_{n+2}}{Z_n}$,

----For $T_{32}: Z_{n+10} = Z_{n+8} Z^2_{n+7} Z^3_{n+6} Z^4_{n+5} Z^4_{n+4} Z^3_{n+3} Z^2_{n+2} Z_{n+1}$,

3. RECURRENT FORMULAS OF THE GENERALIZED FIBONACCI SEQUENCE OF FIFTH ORDER

We can construct 32 different schemes of generalized Fibonacci sequence of fifth order in the case of two sequences. We introduce their recurrent formulas below.

Everywhere let,
$X_0 = C_0, Y_0 = C_1, X_1 = C_2, Y_1 = C_3, X_2 = C_4, Y_2 = C_5, X_3 = C_6, Y_3 = C_7, X_4 = C_8, Y_4 = C_9$, and assume that $n \geq 0$ is a natural number, where $C_0, C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9$ are given constants and Z is one of the symbols X or Y.

The different schemes are as follows:

$T_1: \begin{cases} X_{n+5} = X_{n+4} + X_{n+3} + X_{n+2} + X_{n+1} + X_n \\ Y_{n+5} = Y_{n+4} + Y_{n+3} + Y_{n+2} + Y_{n+1} + Y_n \end{cases}, T_2: \begin{cases} X_{n+5} = X_{n+4} + X_{n+3} + X_{n+2} + X_{n+1} + Y_n \\ Y_{n+5} = Y_{n+4} + Y_{n+3} + Y_{n+2} + Y_{n+1} + X_n \end{cases},$

$T_3: \begin{cases} X_{n+5} = X_{n+4} + X_{n+3} + X_{n+2} + Y_{n+1} + X_n \\ Y_{n+5} = Y_{n+4} + Y_{n+3} + Y_{n+2} + X_{n+1} + Y_n \end{cases}, T_4: \begin{cases} X_{n+5} = X_{n+4} + X_{n+3} + X_{n+2} + Y_{n+1} + Y_n \\ Y_{n+5} = Y_{n+4} + Y_{n+3} + Y_{n+2} + X_{n+1} + X_n \end{cases},$

$T_5: \begin{cases} X_{n+5} = X_{n+4} + X_{n+3} + Y_{n+2} + X_{n+1} + X_n \\ Y_{n+5} = Y_{n+4} + Y_{n+3} + X_{n+2} + Y_{n+1} + Y_n \end{cases}, T_6: \begin{cases} X_{n+5} = X_{n+4} + X_{n+3} + Y_{n+2} + X_{n+1} + Y_n \\ Y_{n+5} = Y_{n+4} + Y_{n+3} + X_{n+2} + Y_{n+1} + X_n \end{cases},$

$T_7: \begin{cases} X_{n+5} = X_{n+4} + Y_{n+3} + X_{n+2} + X_{n+1} + X_n \\ Y_{n+5} = Y_{n+4} + X_{n+3} + Y_{n+2} + Y_{n+1} + Y_n \end{cases}, T_8: \begin{cases} X_{n+5} = X_{n+4} + Y_{n+3} + X_{n+2} + X_{n+1} + Y_n \\ Y_{n+5} = Y_{n+4} + X_{n+3} + Y_{n+2} + Y_{n+1} + X_n \end{cases},$

$T_9: \begin{cases} X_{n+5} = Y_{n+4} + X_{n+3} + X_{n+2} + X_{n+1} + X_n \\ Y_{n+5} = X_{n+4} + Y_{n+3} + Y_{n+2} + Y_{n+1} + Y_n \end{cases}, T_{10}: \begin{cases} X_{n+5} = Y_{n+4} + X_{n+3} + X_{n+2} + X_{n+1} + Y_n \\ Y_{n+5} = X_{n+4} + Y_{n+3} + Y_{n+2} + Y_{n+1} + X_n \end{cases},$

$T_{11}: \begin{cases} X_{n+5} = Y_{n+4} + Y_{n+3} + X_{n+2} + X_{n+1} + X_n \\ Y_{n+5} = X_{n+4} + X_{n+3} + Y_{n+2} + Y_{n+1} + Y_n \end{cases}, T_{12}: \begin{cases} X_{n+5} = Y_{n+4} + Y_{n+3} + X_{n+2} + X_{n+1} + Y_n \\ Y_{n+5} = X_{n+4} + X_{n+3} + Y_{n+2} + Y_{n+1} + X_n \end{cases},$

$$T_{13}:\begin{cases}X_{n+5}=Y_{n+4}+X_{n+3}+Y_{n+2}+X_{n+1}+X_n\\Y_{n+5}=X_{n+4}+Y_{n+3}+Y_{n+2}+Y_{n+1}+Y_n\end{cases},\ T_{14}:\begin{cases}X_{n+5}=Y_{n+4}+X_{n+3}+Y_{n+2}+X_{n+1}+Y_n\\Y_{n+5}=X_{n+4}+Y_{n+3}+Y_{n+2}+Y_{n+1}+X_n\end{cases},$$

$$T_{15}:\begin{cases}X_{n+5}=Y_{n+4}+X_{n+3}+X_{n+2}+Y_{n+1}+X_n\\Y_{n+5}=X_{n+4}+Y_{n+3}+Y_{n+2}+X_{n+1}+Y_n\end{cases},\ T_{16}:\begin{cases}X_{n+5}=Y_{n+4}+X_{n+3}+X_{n+2}+Y_{n+1}+Y_n\\Y_{n+5}=X_{n+4}+Y_{n+3}+Y_{n+2}+X_{n+1}+X_n\end{cases},$$

$$T_{17}:\begin{cases}X_{n+5}=X_{n+4}+Y_{n+3}+Y_{n+2}+X_{n+1}+X_n\\Y_{n+5}=Y_{n+4}+X_{n+3}+X_{n+2}+Y_{n+1}+Y_n\end{cases},\ T_{18}:\begin{cases}X_{n+5}=X_{n+4}+Y_{n+3}+Y_{n+2}+X_{n+1}+Y_n\\Y_{n+5}=Y_{n+4}+X_{n+3}+X_{n+2}+Y_{n+1}+X_n\end{cases},$$

$$T_{19}:\begin{cases}X_{n+5}=X_{n+4}+Y_{n+3}+X_{n+2}+Y_{n+1}+X_n\\Y_{n+5}=Y_{n+4}+X_{n+3}+Y_{n+2}+X_{n+1}+Y_n\end{cases},\ T_{20}:\begin{cases}X_{n+5}=X_{n+4}+Y_{n+3}+X_{n+2}+Y_{n+1}+Y_n\\Y_{n+5}=Y_{n+4}+X_{n+3}+Y_{n+2}+X_{n+1}+X_n\end{cases},$$

$$T_{21}:\begin{cases}X_{n+5}=X_{n+4}+X_{n+3}+Y_{n+2}+Y_{n+1}+X_n\\Y_{n+5}=Y_{n+4}+Y_{n+3}+X_{n+2}+X_{n+1}+Y_n\end{cases},\ T_{22}:\begin{cases}X_{n+5}=X_{n+4}+X_{n+3}+Y_{n+2}+Y_{n+1}+Y_n\\Y_{n+5}=Y_{n+4}+Y_{n+3}+X_{n+2}+X_{n+1}+X_n\end{cases},$$

$$T_{23}:\begin{cases}X_{n+5}=Y_{n+4}+Y_{n+3}+Y_{n+2}+X_{n+1}+X_n\\Y_{n+5}=X_{n+4}+X_{n+3}+X_{n+2}+Y_{n+1}+Y_n\end{cases},\ T_{24}:\begin{cases}X_{n+5}=Y_{n+4}+Y_{n+3}+Y_{n+2}+X_{n+1}+Y_n\\Y_{n+5}=X_{n+4}+X_{n+3}+X_{n+2}+Y_{n+1}+X_n\end{cases},$$

$$T_{25}:\begin{cases}X_{n+5}=Y_{n+4}+Y_{n+3}+X_{n+2}+Y_{n+1}+X_n\\Y_{n+5}=X_{n+4}+X_{n+3}+Y_{n+2}+X_{n+1}+Y_n\end{cases},\ T_{26}:\begin{cases}X_{n+5}=Y_{n+4}+Y_{n+3}+X_{n+2}+Y_{n+1}+Y_n\\Y_{n+5}=X_{n+4}+X_{n+3}+Y_{n+2}+X_{n+1}+X_n\end{cases},$$

$$T_{27}:\begin{cases}X_{n+5}=Y_{n+4}+X_{n+3}+Y_{n+2}+Y_{n+1}+X_n\\Y_{n+5}=X_{n+4}+Y_{n+3}+X_{n+2}+X_{n+1}+Y_n\end{cases},\ T_{28}:\begin{cases}X_{n+5}=Y_{n+4}+X_{n+3}+Y_{n+2}+Y_{n+1}+Y_n\\Y_{n+5}=X_{n+4}+Y_{n+3}+X_{n+2}+X_{n+1}+X_n\end{cases},$$

$$T_{29}:\begin{cases}X_{n+5}=X_{n+4}+Y_{n+3}+Y_{n+2}+Y_{n+1}+X_n\\Y_{n+5}=Y_{n+4}+X_{n+3}+X_{n+2}+X_{n+1}+Y_n\end{cases},\ T_{30}:\begin{cases}X_{n+5}=X_{n+4}+Y_{n+3}+Y_{n+2}+Y_{n+1}+Y_n\\Y_{n+5}=Y_{n+4}+X_{n+3}+X_{n+2}+X_{n+1}+X_n\end{cases},$$

$$T_{31}:\begin{cases}X_{n+5}=Y_{n+4}+Y_{n+3}+Y_{n+2}+Y_{n+1}+X_n\\Y_{n+5}=X_{n+4}+X_{n+3}+X_{n+2}+X_{n+1}+Y_n\end{cases},\ T_{32}:\begin{cases}X_{n+5}=Y_{n+4}+Y_{n+3}+Y_{n+2}+Y_{n+1}+Y_n\\Y_{n+5}=X_{n+4}+X_{n+3}+X_{n+2}+X_{n+1}+X_n\end{cases},$$

The first scheme is trivial. All of the others are nontrivial; they have the following recurrent formulas for n ≥ 0:

----For $T_1 : Z_{n+10} = Z_{n+9} + Z_{n+8} + Z_{n+7} + Z_{n+6} + Z_{n+5},$

----For $T_2 : Z_{n+10} = 2Z_{n+9} + Z_{n+8} - Z_{n+6} - 4Z_{n+5} - 3Z_{n+4} - 2Z_{n+3} - Z_{n+2} + Z_n,$

----For $T_3 : Z_{n+10} = 2Z_{n+9} + Z_{n+8} - 3Z_{n+6} - 3Z_{n+4} - 2Z_{n+3} - Z_{n+2} - Z_n,$

----For $T_4 : Z_{n+10} = 2Z_{n+9} + Z_{n+8} - 3Z_{n+6} - 2Z_{n+5} - Z_{n+4} + Z_{n+2} + 2Z_{n+1} + Z_n,$

----For $T_5 : Z_{n+10} = 2Z_{n+9} + Z_{n+8} - 2Z_{n+7} + 2Z_{n+6} - 4Z_{n+4} - 2Z_{n+3} - Z_{n+2} - 2Z_{n+1} - Z_n,$

----For $T_6 : Z_{n+10} = 2Z_{n+9} + Z_{n+8} - 2Z_{n+7} + Z_{n+6} - 2Z_{n+5} - Z_{n+4} + Z_{n+2} + Z_n,$

----For $T_7 : Z_{n+10} = 2Z_{n+9} - Z_{n+8} + 2Z_{n+7} + Z_{n+6} - Z_{n+4} - 2Z_{n+3} - 3Z_{n+2} - 2Z_{n+1} - Z_n,$

----For $T_8 : Z_{n+10} = 2Z_{n+9} - Z_{n+8} + 2Z_{n+7} + Z_{n+6} - 2Z_{n+5} - Z_{n+4} - Z_{n+2} + Z_n,$

----For $T_9 : Z_{n+10} = 3Z_{n+8} + 2Z_{n+7} + Z_{n+6} - Z_{n+4} - 4Z_{n+3} - 3Z_{n+2} - 2Z_{n+1} - Z_n,$

----For $T_{10} : Z_{n+10} = 3Z_{n+8} + 2Z_{n+7} + Z_{n+6} - 2Z_{n+5} - Z_{n+4} - 2Z_{n+3} - Z_{n+2} + Z_n,$

----For $T_{11} : Z_{n+10} = Z_{n+8} + 4Z_{n+7} + 3Z_{n+6} + 2Z_{n+5} - Z_{n+4} - 2Z_{n+3} - 3Z_{n+2} - 2Z_{n+1} - Z_n,$

----For $T_{12} : Z_{n+10} = Z_{n+8} + 4Z_{n+7} + 3Z_{n+6} + Z_{n+4} - Z_{n+2} + Z_n,$

----For $T_{13} : Z_{n+10} = 3Z_{n+8} + 3Z_{n+6} + 2Z_{n+5} - Z_{n+4} - 2Z_{n+3} - Z_{n+2} - 2Z_{n+1} - Z_n,$

----For $T_{14} : Z_{n+10} = 3Z_{n+8} + 3Z_{n+6} + Z_{n+4} + Z_{n+2} + Z_n$,

----For $T_{15} : Z_{n+10} = 3Z_{n+8} + 2Z_{n+7} - Z_{n+6} + 2Z_{n+5} - Z_{n+4} - 2Z_{n+3} - Z_{n+2} - Z_n$,

----For $T_{16} : Z_{n+10} = 3Z_{n+8} + 2Z_{n+7} - Z_{n+6} + Z_{n+5} - Z_{n+3} + Z_{n+1}$,

----For $T_{17} : Z_{n+10} = 2Z_{n+9} - Z_{n+8} + 3Z_{n+6} + 2Z_{n+5} - Z_{n+4} - Z_{n+2} - 2Z_{n+1} - Z_n$,

----For $T_{18} : Z_{n+10} = 2Z_{n+9} - Z_{n+8} + 3Z_{n+6} + Z_{n+4} + 2Z_{n+3} + Z_{n+2} + Z_n$,

----For $T_{19} : Z_{n+10} = 2Z_{n+9} - Z_{n+8} + 2Z_{n+7} - Z_{n+6} + 2Z_{n+5} - Z_{n+4} - Z_{n+2} - Z_n$,

----For $T_{20} : Z_{n+10} = 2Z_{n+9} - Z_{n+8} + 2Z_{n+7} - Z_{n+6} + Z_{n+4} + 2Z_{n+3} + Z_{n+2} + 2Z_{n+1} + Z_n$,

----For $T_{21} : Z_{n+10} = 2Z_{n+9} - Z_{n+8} - 2Z_{n+7} - Z_{n+6} + 2Z_{n+5} - Z_{n+4} + Z_{n+2} - Z_n$,

----For $T_{22} : Z_{n+10} = 2Z_{n+9} + Z_{n+8} - 2Z_{n+7} - Z_{n+6} + Z_{n+4} + 2Z_{n+3} + 3Z_{n+2} + 2Z_{n+1} + Z_n$,

----For $T_{23} : Z_{n+10} = Z_{n+8} + 2Z_{n+7} + 5Z_{n+6} + 4Z_{n+5} + Z_{n+4} - Z_{n+2} - 2Z_{n+1} - Z_n$,

----For $T_{24} : Z_{n+10} = Z_{n+8} + 2Z_{n+7} + 5Z_{n+6} + 2Z_{n+5} + 3Z_{n+4} + 2Z_{n+3} + Z_{n+2} + Z_n$,

----For $T_{25} : Z_{n+10} = Z_{n+8} + 4Z_{n+7} + Z_{n+6} + 4Z_{n+5} + Z_{n+4} - Z_{n+2} - Z_n$,

----For $T_{26} : Z_{n+10} = Z_{n+8} + 4Z_{n+7} + Z_{n+6} + 2Z_{n+5} + 3Z_{n+4} + 2Z_{n+3} + Z_{n+2} + 2Z_{n+1} + Z_n$,

----For $T_{27} : Z_{n+10} = 3Z_{n+8} + 2Z_{n+6} + 3Z_{n+5} + Z_{n+4} + Z_{n+2} - Z_n$,

----For $Z_{n+1} \quad = 3Z_{n\theta 8} + Z_{n+6} + 2Z_{n+5} + 3Z_{n+4} + 2Z_{n+3} + 3Z_{n+2} + 2Z_{n+1} + Z_n$,

----For $T_{29} : Z_{n+10} = 2Z_{n+9} - Z_{n+8} + Z_{n+6} + 4Z_{n+5} + Z_{n+4} + 2Z_{n+3} + Z_{n+2} - Z_n$,

----For $T_{30} : Z_{n+10} = 2Z_{n+9} - Z_{n+8} - Z_{n+6} + 2Z_{n+5} + 3Z_{n+4} + 4Z_{n+3} + 3Z_{n+2} + 2Z_{n+1} + Z_n$,

----For $T_{31} : Z_{n+10} = Z_{n+8} + 2Z_{n+7} + 3Z_{n+6} + 6Z_{n+5} + 3Z_{n+4} + 2Z_{n+3} + Z_{n+2} - Z_n$,

----For $T_{32} : Z_{n+10} = Z_{n+8} + 2Z_{n+7} + 3Z_{n+6} + 4Z_{n+5} + 5Z_{n+4} + 4Z_{n+3} + 3Z_{n+2} + 2Z_{n+1} + Z_n$.

CONCLUSION

In this paper I introduced recurrent formulas for coupled Fibonacci sequences of fifth order under different schemes. The proofs for these facts can be shown by induction, using methods similar to those in [2] or [3].An open problem is the construction of an explicit formula for each of the schemes given above**.**

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