DOI: 10.15415/mjis.2016.42009

Fibonacci and k Lucas Sequences as Series of Fractions

A. D. GODASE[1] AND M. B. DHAKNE[2]

[1]V. P. College, Vaijapur, Maharashtra, India
[2]Dr. B. A. M. University, Aurangabad, Maharashtra, India

E-mail: mathematicsdept@vpcollege.net

Received: July 29, 2015| Revised: September 30, 2015| Accepted: October 31, 2015
Published online: March 30, 2016

Abstract In this paper, we defined new relationship between k Fibonacci and k Lucas sequences using continued fractions and series of fractions, this approach is different and never tried in k Fibonacci sequence literature.

Keywords: k-Fibonacci sequence, k-Lucas sequence, Recurrence relation

Mathematics Subject Classification: 11B39, 11B83

1 INTRODUCTION

The Fibonacci sequence is a source of many nice and interesting identities. Many identities have been documented in [2], [3], [10], [11], [12], [13], [17]. A similar interpretation exists for k Fibonacci and k Lucas numbers. Many of these identities have been documented in the work of Falcon and Plaza[1], [4], [5], [7], [8], [9], where they are proved by algebraic means. In this paper, we obtained some new properties for k Fibonacci and k Lucas sequences using series of fraction.

2 PRELIMINARY

Definition 2.1. The $k-$Fibonacci sequence $\{F_{k,n}\}_{n=1}^{\infty}$ is defined as, $F_{k,n+1} = k \cdot F_{k,n} + F_{k,n-1}$, with $F_{k,0} = 0, F_{k,1} = 1$, for $n \geq 1$

Mathematical Journal of Interdisciplinary Sciences
Volume 4, No. 2,
March 2016
pp. 107–119

Definition 2.2. The $k-$ Lucas sequence $\{L_{k,n}\}_{n=1}^{\infty}$ is defined as, $L_{k,n+1} = k \cdot L_{k,n} + L_{k,n-1}$, with $L_{k,0} = 2, L_{k,1} = k$, for $n \geq 1$

Characteristic equation of the initial recurrence relation is,

$$r^2 - k \cdot r - 1 = 0 \tag{1}$$

Characteristic roots are

$$r_1 = \frac{k + \sqrt{k^2+4}}{2} \tag{2}$$

and

$$r_2 = \frac{k - \sqrt{k^2+4}}{2} \tag{3}$$

Characteristic roots verify the properties

$$r_1 - r_2 = \sqrt{k^2+4} = \sqrt{\Delta} = \delta \tag{4}$$

$$r_1 + r_2 = k \tag{5}$$

$$r_1 . r_2 = -1 \tag{6}$$

Binet forms for $F_{k,n}$ and $L_{k,n}$ are

$$F_{k,n} = \frac{r_1^n - r_2^n}{r_1 - r_2} \tag{7}$$

and

$$L_{k,n} = r_1^n + r_2^n \tag{8}$$

2.1 The first few members of this k Fibonacci family are

$$1,$$

$$k,$$

$$1 + k^2,$$

$$2k + k^3,$$

$$1 + 3k^2 + k^4,$$

$$3k + 4k^3 + k^5,$$

$$1 + 6k^2 + 5k^4 + k^6,$$

$$4k + 10k^3 + 6k^5 + k^7,$$

$$1 + 10k^2 + 15k^4 + 7k^6 + k^8,$$

$$5k + 20k^3 + 21k^5 + 8k^7 + k^9,$$

$$1 + 15k^2 + 35k^4 + 28k^6 + 9k^8 + k^{10},$$

$$6k + 35k^3 + 56k^5 + 36k^7 + 10k^9 + k^{11},$$

$$1 + 21k^2 + 70k^4 + 84k^6 + 45k^8 + 11k^{10} + k^{12},$$

$$7k + 56k^3 + 126k^5 + 120k^7 + 55k^9 + 12k^{11} + k^{13},$$

$$1 + 28k^2 + 126k^4 + 210k^6 + 165k^8 + 66k^{10} + 13k^{12} + k^{14},$$

$$8k + 84k^3 + 252k^5 + 330k^7 + 220k^9 + 78k^{11} + 14k^{13} + k^{15},$$

$$1 + 36k^2 + 210k^4 + 462k^6 + 495k^8 + 286k^{10} + 91k^{12} + 15k^{14} + k^{16},$$

$$9k + 120k^3 + 462k^5 + 792k^7 + 715k^9 + 364k^{11} + 105k^{13} + 16k^{15} + k^{17},$$

$$1 + 45k^2 + 330k^4 + 924k^6 + 1287k^8 + 1001k^{10} + 455k^{12} + 120k^{14} + 17k^{16} + k^{18},$$

$$10k + 165k^3 + 792k^5 + 1716k^7 + 2002k^9 + 1365k^{11} + 560k^{13} + 136k^{15} + 18k^{17} + k^{19}$$

2.2 k Fibonacci sequences in Encyclopaedia of Integer Sequences

$F_{k,n}$	**Classification**
$F_{1,n}$	$A000045$
$F_{2,n}$	$A000129$
$F_{3,n}$	$A006190$
$F_{4,n}$	$A001076$
$F_{5,n}$	$A052918$
$F_{6,n}$	$A005668$
$F_{7,n}$	$A054413$
$F_{8,n}$	$A041025$
$F_{9,n}$	$A099371$
$F_{10,n}$	$A041041$
$F_{11,n}$	$A049666$

3 RELATIONSHIP OF THE SEQUENCES $F_{K,N}$ AND $L_{K,N}$ AS CONTINUED FRACTIONS:

In general, a (simple) continued fraction is an expression of the form

$$[a_0, a_1, \ldots\ldots\ldots\ldots\ldots\ldots, a_n] = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \cfrac{1}{a_5 + \cfrac{1}{a_6 + \ldots}\ldots\ldots\ldots}}}}} \tag{9}$$

The letters $a_1, a_2, \ldots$ denote positive integers. The letter a_0 denotes an integer.

The expansion $\frac{F_{k,n+1}}{F_{k,n}}$ in continued fraction is written as

$$\frac{F_{k,n+1}}{F_{k,n}} = k + \cfrac{1}{k + \cfrac{1}{k + \cfrac{1}{k + \cfrac{1}{k + \cfrac{1}{k + \cfrac{1}{k + \ldots}\ldots\ldots\ldots}}}}} \tag{10}$$

Here n denotes the number of quantities equal to k.

We knew that

$$F_{k,n}^2 - F_{k,n-1}F_{k,n+1} = (-1)^{n-1} \tag{11}$$

Moreover, in general we have

$$\frac{F_{k,n+1}}{F_{k,n}} = r_1 \frac{1-(\frac{r_2}{r_1})^{n+1}}{1-(\frac{r_2}{r_1})^{n}}$$

Let, r_1 denote the larger of the root, we have

$$\lim_{n\longrightarrow\infty} \frac{F_{k,n+1}}{F_{k,n}} = r_1$$

More generaly formula (9) is written as

$$\frac{F_{k,(n+1)t}}{F_{k,nt}} = L_{k,t} - \cfrac{(-1)^t}{L_{k,t} - \cfrac{(-1)^t}{L_{k,t} - \cfrac{(-1)^t}{L_{k,t} - \cfrac{(-1)^t}{L_{k,t} - \cfrac{(-1)^t}{L_{k,t} - \cfrac{(-1)^t}{L_{k,t} - \ldots}\cdots\cdots\cdots}}}}} \tag{12}$$

Here, n denotes the number of $L_{k,t}$'s
When n increases indefinitely, we have

$$\lim_{n\longrightarrow\infty} \frac{F_{k,(n+1)t}}{F_{k,nt}} = (r_1)^t$$

As equation (9), we have relation for $L_{k,n}$

$$\frac{L_{k,(n)t}}{F_{k,(n-)t}} = L_{k,t} - \cfrac{(-1)^t}{L_{k,t} - \cfrac{(-1)^t}{L_{k,t} - \cfrac{(-1)^t}{L_{k,t} - \cfrac{(-1)^t}{L_{k,t} - \cfrac{(-1)^t}{L_{k,t} - \cdots\cdots\cdots - \cfrac{(-1)^t}{(\frac{L_{k,t}}{2})}}}}}} \tag{13}$$

Here n denotes the number of quantities equal to $L_{k,t}$.
We knew that

$$L_{k,n}^2 - L_{k,n-1}L_{k,n+1} = (-1)^n\triangle \tag{14}$$

More generaly, equations (10) and (13) are modified as

$$F_{k,nt}^2 - F_{k,(n-1)t}F_{k,(n+1)t} = (-1)^{(n-1)t}(F_{k,t})^2 \tag{15}$$

$$L^2_{k,nt} - L_{k,(n-1)t}L_{k,(n+1)t} = -(-1)^{(n-1)t}\triangle(F_{k,t})^2 \tag{16}$$

Moreover using (7) and (8), gives

$$\Delta F^2_{k,nt} = r_1^{2n+2t} + r_2^{2n+2t} - 2(-1)^{n+t} \tag{17}$$

$$\Delta L^2_{k,n} = r_1^{2n} + r_2^{2n} - 2(-1)^n \tag{18}$$

Again by subtracting (16) and (17), gives

$$\Delta(F^2_{k,n+t} - (-1)^t F^2_{k,n}) = (r_1^{2n+t} - r_2^{2n+t})(r_1^t + r_2^t) \tag{19}$$

and

$$F^2_{k,n+t} - (-1)^t F^2_{k,n} = F_{k,t}F_{k,2n+t} \tag{20}$$

Similarly, we obtain

$$L^2_{k,n+t} - (-1)^t L^2_{k,n} = \Delta F_{k,t}F_{k,2n+t} \tag{21}$$

4 SEQUENCES $F_{K,N}$ AND $L_{K,N}$ AS A SERIES OF FRACTIONS:

Theorem 4.1. For $n, k > 0$,

1.

$$\frac{F_{k,n+1}}{F_{k,n}} = \frac{F_{k,2}}{F_{k,1}} - \frac{(-1)}{F_{k,1}F_{k,2}} - \frac{(-1)^2}{F_{k,2}F_{k,3}} - \frac{(-1)^3}{F_{k,3}F_{k,4}} - \ldots\ldots\ldots\ldots - \frac{(-1)^{n-1}}{F_{k,n-1}F_{k,n}}$$

2.

$$\frac{L_{k,n+1}}{L_{k,n}} = \frac{L_{k,2}}{L_{k,1}} - \frac{(-1)^2\triangle}{L_{k,2}L_{k,1}} - \frac{(-1)^3\triangle}{L_{k,2}L_{k,3}} - \frac{(-1)^4\triangle}{L_{k,3}L_{k,4}} - \ldots\ldots\ldots - \frac{(-1)^n}{L_{k,n-1}L_{k,n}}$$

Proof. We can write expressions of $\frac{F_{k,n+1}}{F_{k,n}}$ and $\frac{L_{k,n+1}}{L_{k,n}}$ in series as

$$\begin{aligned}\frac{F_{k,n+1}}{F_{k,n}} &= \frac{F_{k,2}}{F_{k,1}} + \left(\frac{F_{k,3}}{F_{k,2}} - \frac{F_{k,2}}{F_{k,1}}\right) + \left(\frac{F_{k,4}}{F_{k,3}} - \frac{F_{k,3}}{F_{k,2}}\right) + \ldots\ldots\ldots\ldots\ldots \\ &\quad + \left(\frac{F_{k,n+1}}{F_{k,n}} - \frac{F_{k,n}}{F_{k,n-1}}\right) \\ &= \frac{F_{k,2}}{F_{k,1}} - \frac{(F_{k,2}^2 - F_{k,3}F_{k,1})}{F_{k,1}F_{k,2}} - \frac{(F_{k,3}^2 - F_{k,2}F_{k,4})}{F_{k,2}F_{k,3}} - \ldots\ldots\ldots\ldots\ldots\ldots \\ &\quad - \frac{(F_{k,n}^2 - F_{k,n+1}F_{k,n-1})}{F_{k,n-1}F_{k,n}}\end{aligned}$$

And

$$\begin{aligned}\frac{L_{k,n+1}}{L_{k,n}} &= \frac{L_{k,2}}{L_{k,1}} + \left(\frac{L_{k,3}}{L_{k,2}} - \frac{L_{k,2}}{L_{k,1}}\right) + \left(\frac{L_{k,4}}{L_{k,3}} - \frac{L_{k,3}}{L_{k,2}}\right) + \ldots\ldots\ldots\ldots\ldots \\ &\quad + \left(\frac{L_{k,n+1}}{L_{k,n}} - \frac{L_{k,n}}{L_{k,n-1}}\right) \\ &= \frac{L_{k,2}}{L_{k,1}} - \frac{(L_{k,2}^2 - L_{k,3}L_{k,1})}{L_{k,1}L_{k,2}} - \frac{(L_{k,3}^2 - L_{k,2}L_{k,4})}{L_{k,2}L_{k,3}} - \ldots\ldots\ldots\ldots\ldots\ldots \\ &\quad - \frac{(L_{k,n}^2 - L_{k,n+1}L_{k,n-1})}{L_{k,n-1}L_{k,n}}\end{aligned}$$

Using the equations (11) and (14)

$$\begin{aligned}F_{k,n}^2 - F_{k,n-1}F_{k,n+1} &= (-1)^{n-1} \\ L_{k,n}^2 - L_{k,n-1}L_{k,n+1} &= (-1)^n \triangle\end{aligned}$$

Gives

$$\frac{F_{k,n+1}}{F_{k,n}} = \frac{F_{k,2}}{F_{k,1}} - \frac{(-1)}{F_{k,1}F_{k,2}} - \frac{(-1)^2}{F_{k,2}F_{k,3}} - \frac{(-1)^3}{F_{k,3}F_{k,4}} - \ldots\ldots\ldots\ldots\ldots\ldots\ldots - \frac{(-1)^{n-1}}{F_{k,n-1}F_{k,n}}$$

And

$$\frac{L_{k,n+1}}{L_{k,n}} = \frac{L_{k,2}}{L_{k,1}} - \frac{(-1)^2\triangle}{L_{k,2}L_{k,1}} - \frac{(-1)^3\triangle}{L_{k,2}L_{k,3}} - \frac{(-1)^4\triangle}{L_{k,3}L_{k,4}} - \ldots\ldots\ldots\ldots\ldots - \frac{(-1)^n}{L_{k,n-1}L_{k,n}}$$

□

Taking limit as $\lim_{n\to\infty}$, gives

$$r_1 = \frac{1+\sqrt{k^2+4}}{2} = k + \frac{1}{1.\,k} - \frac{1}{k.\,(k^2+4)} + \ldots\ldots\ldots\ldots\ldots$$

For Fibonacci series

$$\frac{1+\sqrt{5}}{2} = 1 + \frac{1}{1.\,1} - \frac{1}{1.\,2} + \frac{1}{2.\,3} - \frac{1}{3.\,5} + \frac{1}{5.\,8} - \frac{1}{8.\,13} + \ldots\ldots\ldots\ldots$$

Now, we obtain more general relation for $F_{k,n}$ and $L_{k,n}$ as a series of fractions:-

Theorem 4.2. For $n, k > 0$,

1.

$$\frac{F_{k,(n+1)t}}{F_{k,nt}} = \frac{F_{k,2t}}{F_{k,t}} - \frac{(-1)^t F_{k,t}^2}{F_{k,t}F_{k,2t}} - \frac{(-1)^{2t} F_{k,t}^2}{F_{k,2t}F_{k,3t}} - \frac{(-1)^{3t} F_{k,t}^2}{F_{k,3t}F_{k,4t}} - \ldots\ldots\ldots - \frac{(-1)^{(n-1)t} F_{k,t}^2}{F_{k,(n-1)t}F_{k,nt}}$$

2.

$$\frac{L_{k,(n+1)t}}{L_{k,nt}} = \frac{L_{k,t}}{L_{k,0}} + \frac{\triangle F_{k,t}^2}{L_{k,0}L_{k,t}} + \frac{(-1)^t F_{k,t}^2}{L_{k,t}L_{k,2t}} + \frac{(-1)^{2t} F_{k,t}^2}{L_{k,2t}L_{k,3t}} + \ldots\ldots\ldots - \frac{(-1)^{(n-1)t} F_{k,t}^2}{L_{k,(n-1)t}L_{k,nt}}$$

Proof. We can write expressions of $\frac{F_{k,(n+1)t}}{F_{k,nt}}$ and $\frac{L_{k,(n+1)t}}{L_{k,nt}}$ in series as

$$\begin{aligned}\frac{F_{k,(n+1)t}}{F_{k,nt}} &= \frac{F_{k,2t}}{F_{k,t}} + \left(\frac{F_{k,3t}}{F_{k,2t}} - \frac{F_{k,2t}}{F_{k,t}}\right) + \left(\frac{F_{k,4t}}{F_{k,3t}} - \frac{F_{k,3t}}{F_{k,2t}}\right) + \ldots\ldots\ldots\ldots\ldots \\ &\quad + \left(\frac{F_{k,(n+1)t}}{F_{k,nt}} - \frac{F_{k,nt}}{F_{k,(n-1)t}}\right) \\ &= \frac{F_{k,2t}}{F_{k,t}} - \frac{(F_{k,2t}^2 - F_{k,3t}F_{k,t})}{F_{k,t}F_{k,2t}} - \frac{(F_{k,3t}^2 - F_{k,2t}F_{k,4t})}{F_{k,2t}F_{k,3t}} - \ldots\ldots\ldots\ldots \\ &\quad - \frac{(F_{k,nt}^2 - F_{k,(n+1)t}F_{k,(n-1)t})}{F_{k,(n-1)t}F_{k,nt}}\end{aligned}$$

And

$$\begin{aligned}\frac{L_{k,(n+1)t}}{L_{k,nt}} &= \frac{L_{k,t}}{L_{k,0}} + \left(\frac{L_{k,2t}}{L_{k,t}} - \frac{L_{k,t}}{L_{k,0}}\right) + \left(\frac{L_{k,3t}}{L_{k,2t}} - \frac{L_{k,2t}}{L_{k,t}}\right) + \ldots\ldots\ldots\ldots\ldots \\ &\quad + \left(\frac{L_{k,(n+1)t}}{L_{k,nt}} - \frac{L_{k,nt}}{L_{k,(n-1)t}}\right) \\ &= \frac{L_{k,t}}{L_{k,0}} - \frac{(L_{k,t}^2 - L_{k,0}L_{k,2t})}{L_{k,0}L_{k,t}} - \frac{(L_{k,2t}^2 - L_{k,t}L_{k,3t})}{L_{k,t}L_{k,2t}} - \ldots\ldots\ldots\ldots\ldots \\ &\quad - \frac{(L_{k,nt}^2 - L_{k,(n+1)t}L_{k,(n-1)t})}{L_{k,(n-1)t}L_{k,nt}}\end{aligned}$$

Using the equations (15) and (16)

$$F_{k,nt}^2 - F_{k,(n-1)t}F_{k,(n+1)t} = (-1)^{(n-1)t}(F_{k,t})^2 \tag{22}$$

$$L_{k,nt}^2 - L_{k,(n-1)t}L_{k,(n+1)t} = -(-1)^{(n-1)t}\triangle(F_{k,t})^2 \tag{23}$$

Gives

$$\frac{F_{k,(n+1)t}}{F_{k,nt}} = \frac{F_{k,2t}}{F_{k,t}} - \frac{(-1)^t F_{k,t}^2}{F_{k,t}F_{k,2t}} - \frac{(-1)^{2t} F_{k,t}^2}{F_{k,2t}F_{k,3t}} - \frac{(-1)^{3t} F_{k,t}^2}{F_{k,3t}F_{k,4t}} - \ldots\ldots\ldots\ldots - \frac{(-1)^{(n-1)t} F_{k,t}^2}{F_{k,(n-1)t}F_{k,nt}}$$

And

$$\frac{L_{k,(n+1)t}}{L_{k,nt}} = \frac{L_{k,t}}{L_{k,0}} + \frac{\triangle F_{k,t}^2}{L_{k,0}L_{k,t}} + \frac{(-1)^t F_{k,t}^2}{L_{k,t}L_{k,2t}} + \frac{(-1)^{2t} F_{k,t}^2}{L_{k,2t}L_{k,3t}} + \ldots\ldots - \frac{(-1)^{(n-1)t} F_{k,t}^2}{L_{k,(n-1)t}L_{k,nt}}$$

□

Theorem 4.3. For $n, m, k > 0$,

1.

$$\frac{F_{k,n+mt}}{L_{k,n+mt}} = \frac{F_{k,n}}{L_{k,n}} + 2(-1)^n F_{k,t}\left[\frac{1}{L_{k,n}L_{k,n+t}} + \frac{(-1)^t}{l_{k,n+t}L_{k,n+2t}} + \frac{(-1)^{2t}}{L_{k,n+2t}L_{k,n+3t}} + \ldots\ldots\ldots + \frac{(-1)^{(m-1)t}}{L_{k,n+(m-1)t}L_{k,n+mt}}\right]$$

2.

$$\frac{F_{k,n+mt}}{L_{k,n+mt}} = \frac{F_{k,n}}{L_{k,n}} + 2(-1)^n F_{k,t}\left[\frac{1}{L_{k,n}L_{k,n+t}} + \frac{(-1)^t}{l_{k,n+t}L_{k,n+2t}} + \frac{(-1)^{2t}}{L_{k,n+2t}L_{k,n+3t}} + \ldots\ldots\ldots + \frac{(-1)^{(m-1)t}}{L_{k,n+(m-1)t}L_{k,n+mt}}\right]$$

Proof. We can write expressions of $\frac{F_{k,n+mt}}{L_{k,n+mt}}$ and $\frac{L_{k,n+mt}}{L_{k,n+mt}}$ in series as

$$\begin{aligned}\frac{F_{k,n+mt}}{L_{k,n+mt}} &= \frac{F_{k,n}}{L_{k,n}} + \left(\frac{F_{k,n+t}}{L_{k,n+t}} - \frac{F_{k,n}}{L_{k,n}}\right) + \left(\frac{F_{k,n+2t}}{L_{k,n+2t}} - \frac{F_{k,n+t}}{L_{k,n+t}}\right) + \ldots\ldots\ldots\ldots\ldots \\ &\quad + \left(\frac{F_{k,n+mt}}{L_{k,n+mt}} - \frac{F_{k,n+(m-1)t}}{L_{k,n+(m-1)t}}\right) \\ &= \frac{F_{k,n}}{L_{k,n}} + \frac{(F_{k,n+t}L_{k,n} - F_{k,n}L_{k,n+t})}{L_{k,n}L_{k,n+t}} + \frac{(F_{k,n+2t}L_{k,n+t} - F_{k,n+t}L_{k,n+2t})}{L_{k,n+t}L_{k,n+2t}} \\ &\quad + \ldots\ldots\ldots\ldots\ldots + \frac{(F_{k,n+mt}L_{k,n+(m-1)t} - F_{k,n+(m-1)t}L_{k,n+mt})}{L_{k,nt}L_{k,n+(m-1)t}}\end{aligned}$$

And

$$\begin{aligned}\frac{L_{k,n+mt}}{F_{k,n+mt}} &= \frac{L_{k,n}}{F_{k,n}} + \left(\frac{L_{k,n+t}}{F_{k,n+t}} - \frac{L_{k,n}}{F_{k,n}}\right) + \left(\frac{L_{k,n+2t}}{F_{k,n+2t}} - \frac{L_{k,n+t}}{F_{k,n+t}}\right) \\ &\quad + \ldots\ldots\ldots\ldots\ldots + \left(\frac{L_{k,n+mt}}{F_{k,n+mt}} - \frac{L_{k,n+(m-1)t}}{F_{k,n+(m-1)t}}\right) \\ &= \frac{L_{k,n}}{F_{k,n}} - \frac{(F_{k,n+t}L_{k,n} - F_{k,n}L_{k,n+t})}{L_{k,n}L_{k,n+t}} - \frac{(F_{k,n+2t}L_{k,n+t} - F_{k,n+t}L_{k,n+2t})}{L_{k,n+t}L_{k,n+2t}} \\ &\quad + \ldots\ldots\ldots\ldots\ldots - \frac{(F_{k,n+mt}L_{k,n+(m-1)t} - F_{k,n+(m-1)t}L_{k,n+mt})}{L_{k,nt}L_{k,n+(m-1)t}}\end{aligned}$$

Using the equations (15) and (16)

$$\begin{aligned}F_{k,nt}^2 - F_{k,(n-1)t}F_{k,(n+1)t} &= (-1)^{(n-1)t}(F_{k,t})^2 \\ L_{k,nt}^2 - L_{k,(n-1)t}L_{k,(n+1)t} &= -(-1)^{(n-1)t}\triangle(F_{k,t})^2\end{aligned}$$

Gives

$$\begin{aligned}\frac{F_{k,n+mt}}{L_{k,n+mt}} &= \frac{F_{k,n}}{L_{k,n}} + 2(-1)^n F_{k,t}\left[\frac{1}{L_{k,n}L_{k,n+t}} + \frac{(-1)^t}{l_{k,n+t}L_{k,n+2t}} + \frac{(-1)^{2t}}{L_{k,n+2t}L_{k,n+3t}}\right. \\ &\quad \left. + \ldots\ldots + \frac{(-1)^{(m-1)t}}{L_{k,n+(m-1)t}L_{k,n+mt}}\right]\end{aligned}$$

And

$$\frac{F_{k,n+mt}}{L_{k,n+mt}} = \frac{F_{k,n}}{L_{k,n}} + 2(-1)^n F_{k,t}\left[\frac{1}{L_{k,n}L_{k,n+t}} + \frac{(-1)^t}{l_{k,n+t}L_{k,n+2t}} + \frac{(-1)^{2t}}{L_{k,n+2t}L_{k,n+3t}} + \ldots\ldots\ldots + \frac{(-1)^{(m-1)t}}{L_{k,n+(m-1)t}L_{k,n+mt}}\right]$$

□

5 CONCLUSIONS

Some new relationship between k Fibonacci and k Lucas sequences using continued fractions and series of fractions are derived, this approach is different and never tried in k Fibonacci sequence literature.

REFERENCES

1. A. D. Godase, M. B. Dhakne, *On the properties of k Fibonacci and k Lucas numbers*, International Journal of Advances in Applied Mathematics and Mechanics, 02 (2014), 100 - 106.
2. A. D. Godase, M. B. Dhakne, *Fundamental Properties of Multiplicative Coupled Fibonacci Sequences of Fourth Order Under Two Specific Schemes*, International Journal of Mathematical Archive , 04, No. 6 (2013), 74 - 81.
3. A. D. Godase, M. B. Dhakne, *Recurrent Formulas of The Generalized Fibonacci Sequences of Fifth Order*, "International Journal of Mathematical Archive" , 04, No. 6 (2013), 61 - 67.
4. A. D. Godase, M. B. Dhakne, *Summation Identities for k Fibonacci and k Lucas Numbers using Matrix Methods*, "International Journal of Mathematics and Scientific Computing" , 05, No. 2 (2015), 75 - 78.
5. A. D. Godase, M. B. Dhakne, *Determinantal Identities for k Lucas Sequence*, "Journal of New Theory" , 12 (2015), 01 - 07.
6. A. F. Horadam, *Basic Properties of a Certain Generalized Sequence of Numbers*, The Fibonacci Quarterly, 03, No. 3 (1965), 61 - 76.
7. S. Falcon, A. Plaza, *On the k Fibonacci numbers*, Chaos, Solitons and Fractals, 05, No. 32 (2007), 1615 - 1624.
8. S. Falcon, A. Plaza, *The k Fibonacci sequence and the Pascal* 2 *triangle*, Chaos, Solitons and Fractals, 33, No. 1 (2007), 38 - 49.
9. S. Falcon, A. Plaza, *The k Fibonacci hyperbolic functions*, Chaos, Solitons and Fractals Vol. 38, No.2 (2008), 409 - 20.
10. P. Filipponi, A. F. Horadam, *A Matrix Approach to Certain Identities*, The Fibonacci Quarterly, 26, No. 2 (1988), 115 - 126.

11. H. W. Gould, *A History of the Fibonacci q - Matrix and a Higher - Dimensional Problem*, The Fibonacci Quarterly , 19, No. 3 (1981), 250 - 57.
12. A. F. Horadam, *Jacobstal Representation of Polynomials*, The Fibonacci Quarterly, 35, No. 2 (1997) 137 - 148.
13. T. Koshy, *Fibonacci and Lucas numbers with applications*, Wiley - Intersection Publication (2001).
14. N. J. Sloane, The on-line encyclopedia of integer sequences 2006.
15. G. Strang, *Introduction to Linear Algebra*, Wellesley - Cambridge:Wellesley, M. A. Publication (2003).
16. S. Vajda, *Fibonacci and Lucas numbers and the Golden Section: Theory and applications*, Chichester: Ellis Horwood, (1989).
17. M. E. Waddill , *Matrices and Generalized Fibonacci Sequences*, The Fibonacci Quarterly, 55, No. 12 (1974) 381 - 386.