The Solitary Chaos

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The present article demonstrates the generation of signal based chaos by generating the engineer's chaos as a product of sinusoids and then wave-shaping them by hyperbolic secant (soliton) or tangent (taliton) functions. The frequency ratio of the input sinusoids serves as the control parameter, and the nature of chaos is studied by observing the iterative map, bifurcation plots and phase portraits. While the phase portraits show interesting and rich patterns, the spectrum shows the new frequency components generated, thanks to the nonlinear wave-shaping.

Chaos Theory, characterized by determinism and sensitive dependence on initial conditions has been gaining prominence in recent times [1]. Earlier works have shown novel methods to generate the 'engineer's chaos', characterized by signal based control parameter, and minimalist design [2-8].

In this article, we demonstrate a novel method of generating such a signal based chaos. Specifically, we generate the engineer's chaos as a product of two sinusoids, and then feed this signal to a wave-shaping system. The wave shaping is based on the hyperbolic secant function (sech), known to be the soliton and hence the name 'Solitary Chaos" [9]. In similar fashion, we also generate chaos based on wave shaping by the hyperbolic tangent function (tanh), which is termed the 'taliton'.

We start with the basic definition of the engineer's chaos signal C(t), where, representing the inputs as sinusoids with unity amplitude, with frequency 'f' and 'rf' (and thus the frequency ratio 'r'), we then define the soliton and taliton wave-shaped signals as follows:

$$C(t) = \sin(ft) \times \sin(rft);\ S(t) = \mathrm{sech}\big(C(t)\big);T(t) = \tanh\big(C(t)\big)$$

To obtain the iterative map, we differentiate S(t) and T(t) with respect to time, and then note that for a signal X(t), its derivative X'(t) = X(i+1) – X(i) when represented as discrete samples. This enables us to obtain equations of next samples S(i+1) and T(i+1) in terms of current samples and control parameter 'r', which is thus our iterative map.

$$S(i+1) = S(i) + A(i)B(i)C(i);\ T(i+1) = T(i) + B(i)C^2(i)$$

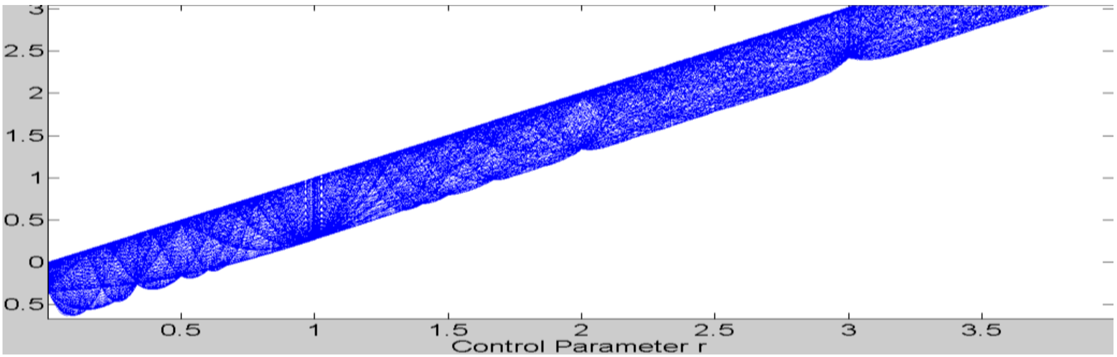
where,

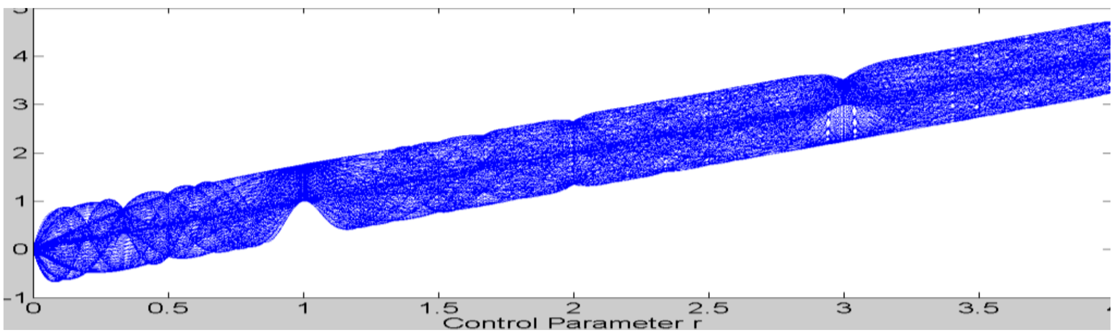
$$A(i) = -\tanh(\sin(fi)\sin(rfi))$$

$$B(i) = rfsin(fi)\cos(rfi) + fcos(fi)\sin(rfi)$$

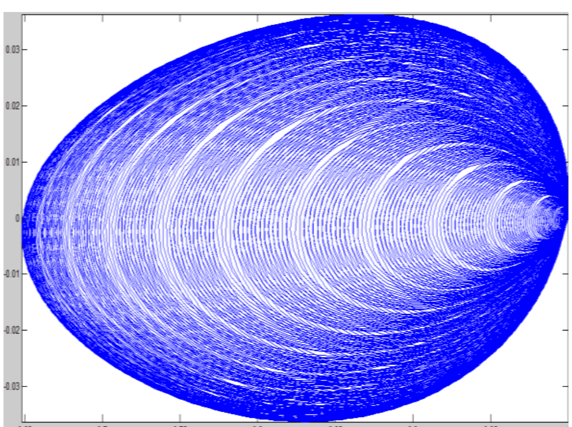
$$C(i) = \mathrm{sech}(\sin(fi)\sin(rfi))$$

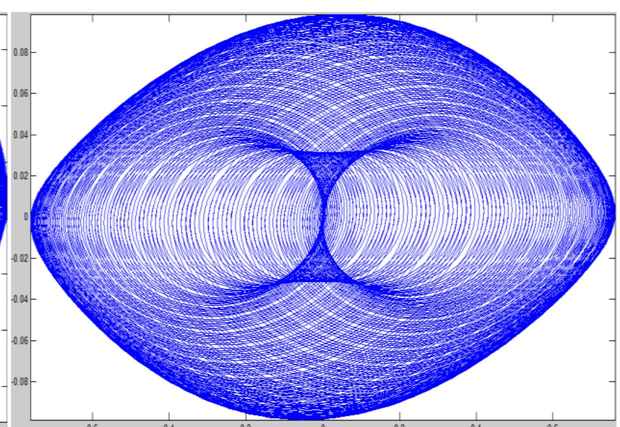
We now plot the bifurcation maps for the solitary and talitory chaos, as a plot of S(i+1) or T(i+1) as a function of 'r', as follows:



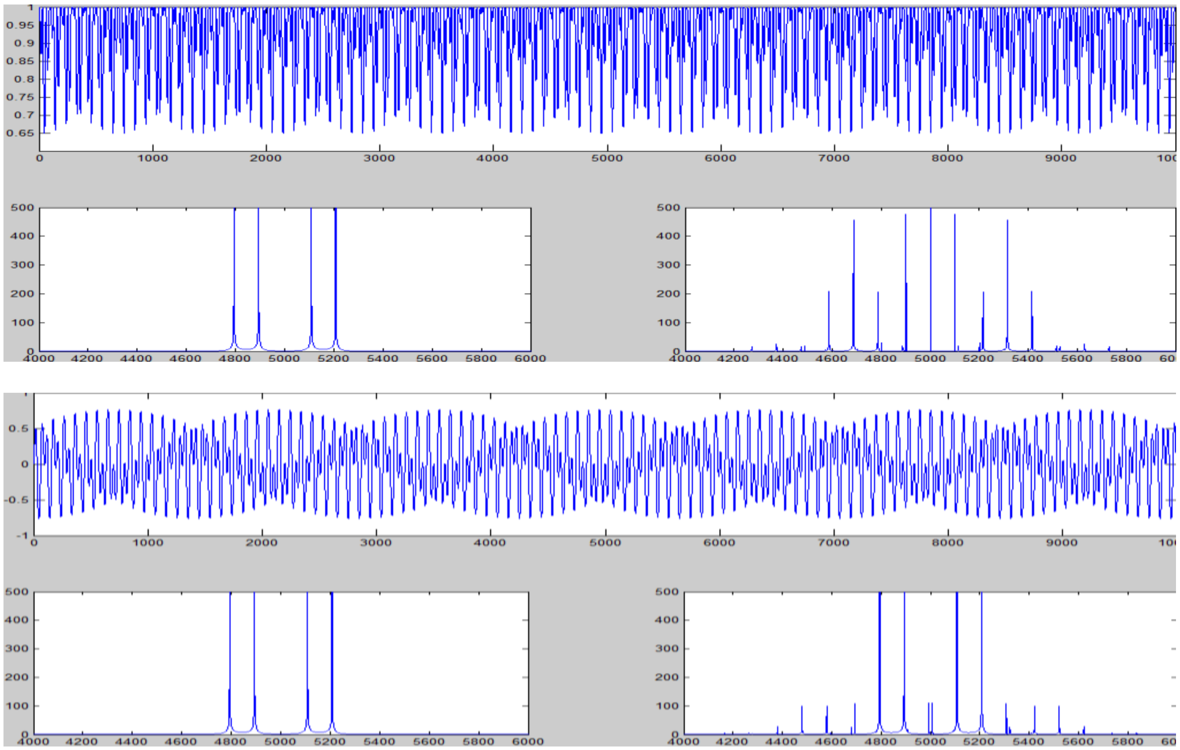


One can note the interesting patterns of sparse and dense regions in both bifurcation plots, with breaks in patterns corresponding to integer values of 'r' (r=1,2,3…). It is also noted that irrational values of 'r' such as π give rise to chaos. For this value of r, plotting S(t) and T(t) as variations of the time derivatives yield the corresponding phase portraits as follows:





One can see the presence of intricate patterns in the phase portraits in both cases, exhibiting the signatures of chaos. Finally, the waveforms and spectra of the generated chaotic signals are plotted as follows:



From the waveforms, one notes that solitary chaos is unipolar whereas talitory chaos is bipolar, oscillating on both sides of zero. From the spectra, one observes that while the engineer’s chaos (shown on the left), contains only two frequency components (namely rf+f and rf-f), the wave-shaping of both sech and tanh provide multiple orders of nonlinearity creating a horde of new frequency components.

In summary, this article demonstrates the generation of signal based chaos by generating the engineer’s chaos as a product of sinusoids and then wave-shaping them by hyperbolic secant or tangent functions. The frequency ratio of the input sinusoids serves as the control parameter, and the nature of chaos is studied by observing the iterative map, bifurcation plots and phase portraits. While the phase portraits show interesting and rich patterns, the spectrum shows the new frequency components generated, thanks to the nonlinear wave-shaping.

References

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