LECTURES ON GEOMETRICAL OPTICS

***Physics for Engineering* IIT (Mains & Advanced)**

and

Medical (NEET) Entrance Exam

Dr. ANUJ KUMAR DUBEY

Ph.D. (Physics)

Physics Faculty, INSPIRE, Chandrapur, Maharashtra, India

anujkrdubey@gmail.com
+91-8486664308

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Physics for Engineering and Medical Entrance Exam
IIT (Mains & Advanced) /AIPMT

I have been involved (more than ten years) in teaching as Physics faculty for Engineering and Medical Entrance Exam such as IIT (Mains and Advanced)/AIPMT in India. These lecture notes are the lectures given at the various renowned institutions in India such as, Inspire, Maxwell Institute, Daswani Classes (KLIIT-JEE Academy), Potential Coaching Institute and Bansal Tutorials. These Lecture Notes on "Geometrical Optics" is an opportunity to present my experiences. During my interaction with Engineering and Medical aspirants I, realized that most feared topics in Physics is Geometrical Optics. Some of the reasons put forward by students behind this thought were, No spontaneous thoughts appear after reading a problem, and mind goes blank and cannot proceed in a problem. How to proceed in a problem? Which law is applicable, horrible thought appear in mind?

- *Total confusion about sign conventions*
- *How to solve problems based on Variable refractive index*
- *Confusion about apparent depth & normal shift problems*
- *No proper understandings of prism theory*
- *No proper understandings of problems related with cut lenses & silvered lenses*
- *Short cut approach in relative motion for velocity of images problems*

Very little of these Lecture notes are wholly original. When I drew up notes, I decided from outset that I would collect together the best approaches to the material known to me. So fact is that many of approaches in these Lecture notes have been borrowed from one author or another, there is a little that I have written completely afresh. My intention has been to organize the material in such a way that it is the more readily accessible to majority of the students.

(Dr. ANUJ KUMAR DUBEY)

Contents:

Lecture-1

Geometrical Optics

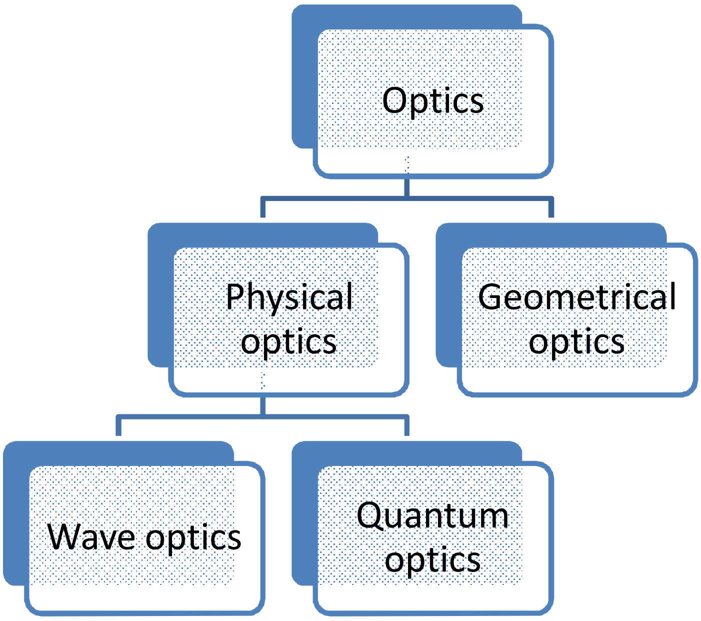
Physics for IIT - JEE

Basics of Optics

- Various theories to understand the nature of light
- Optical path & Fermat's Principle
- Derivation of Law of Reflection & Refraction by Fermat's principle
- Deviation in plane mirror
- Rotation of mirror
- Images formed by two plane mirrors in contact
- Formation of images by plane mirror
- Minimum size of plane mirror to see full height
- Power of an optical instrument

Optics:

The study of light's vision is called optics: Light is a form of energy which produces the sensation of sight in us.



Geometrical Optics:

- Based on rectilinear propagation of light
- Valid only If $\lambda_{\text{Light}} << $ Size of obstacle
- Deals with image formation reflection and refraction

Notes:

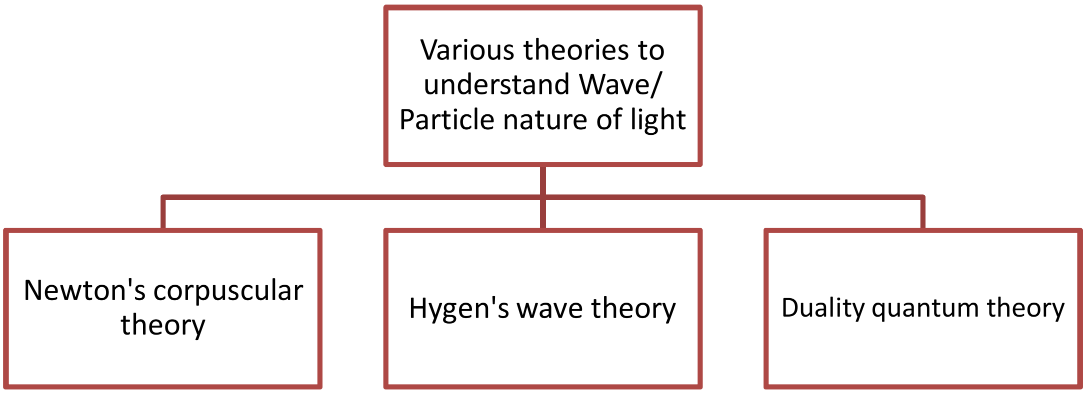
Geometrical optics can be treated as the limiting case of wave optics when size of obstacle is very much large as compared to wavelength of light under such conditions the wave nature of light can be ignored and light can be assumed to be travelling in straight line rectilinear propagation. But when size of obstacle or opening is comparable to wavelength of light rectilinear propagation no longer valid and resulting phenomenon are explained by using wave nature of light.

Wave optics:

- Light is propagated as wave motion.
- It deals with interference, diffraction & polarization.

Quantum Optics:

- Assumes that light is a stream of particles called photon.
- The concept of light particles known as photons is of importance in the study of origin spectra photoelectric effect, Compton Effect, Radiation pressure & laser.



Newton’s Corpuscular theory:

- Light is considered to be a multitude of minute particles.
- Explains rectilinear propagation of light reflection of light.
- Could not explain refraction speed of light is more in a denser medium than air which later turned to be wrong. In fact speed of light is maximum in air or vacuum.
- Also could not explain refraction, interference, polarization diffraction.

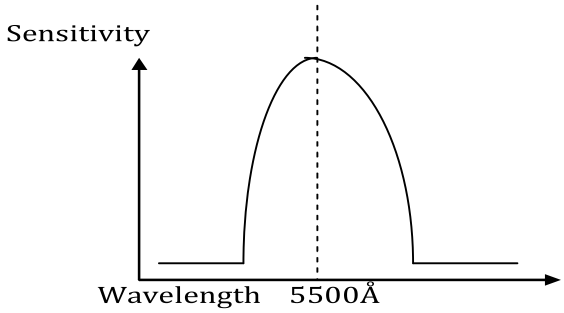
Hygen’s wave theory:

- Maxwell showed that light is an electro-magnetic wave which consist of oscillation of electric & magnetic vector perpendicular to each other & perpendicular to direction of propagation.
- Explains reflection, refraction, interference, diffraction, and polarisation.
- Could not explain photoelectric effect, Compton Effect and several other phenomenons have associated with emissions& absorption of light.

Duality quantum theory: (Einstein & Plank)

- Few phenomenons could not explain by above theorise therefore quantum theory of light is developed.
- At present it believed that light has dual nature propagate as wave and interacts with matter as a particle.

Note:

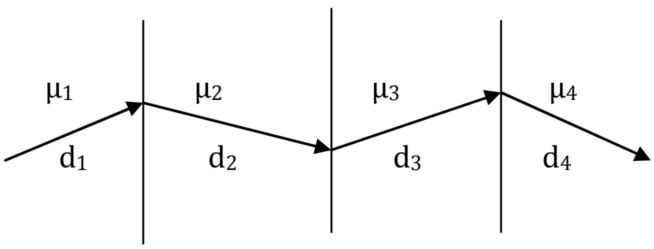


The wavelength of light typically varies from 400 nm To 700 nm. The limits of the visible spectrum are not well defined because the eye sensitivity curve approaches the axis asymptotically at both long & short wavelength.

Optical path:

It is equivalent distance travelled in air or vacuum.
Optical path = μ d , Where μ =c/v
for air or vacuum μ =c/v = c/c =1
Optical path = $\mu_1 d_1 + \mu_2 d_2 + \mu_3 d_3 + \mu_4 d_4$

$$= \sum_{i=1}^{4} \mu d$$

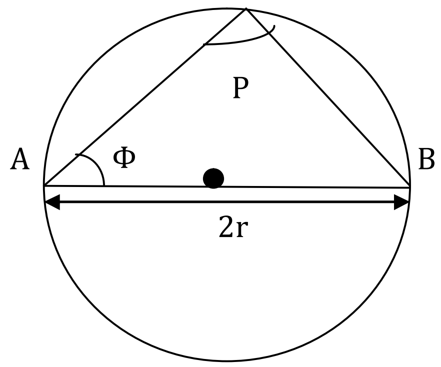


Fermat's principle: (Minimization Principle):

Light must travel along a path in which time is least or distance is Minimum.

***Illustration 1*:**

In figure light starts from point A and after reflection from the inner surface of the sphere reaches the diametrically opposite point B. Calculate the length of the hypothetical path APB and using Fermat's principle find the actual path of light. Is the path minimum?



Solution**:** A light signal is switched on from point A. How it will reach just diametrically opposite point B. Let it moves along the path APB.

$L = \mu\, AP + \mu\, PB$

$L = \mu\, 2r \cos\phi + \mu\, 2r \sin\phi = 2\mu r\,(\cos\phi + \sin\phi)$

$L = L(\phi) \rightarrow$ from minimisation principle

$dL/d\phi = 0 = 2\mu r\,(-\sin\phi + \cos\phi) = 0$

$\text{Sin}\,\phi = \cos\phi \;\Rightarrow\; \tan\phi == \Rightarrow\; \phi = 45^0$

$L_{min} = \mu 2r \cos 45^0 + \mu 2r \sin 45^0 = 2\sqrt{2}\mu r$

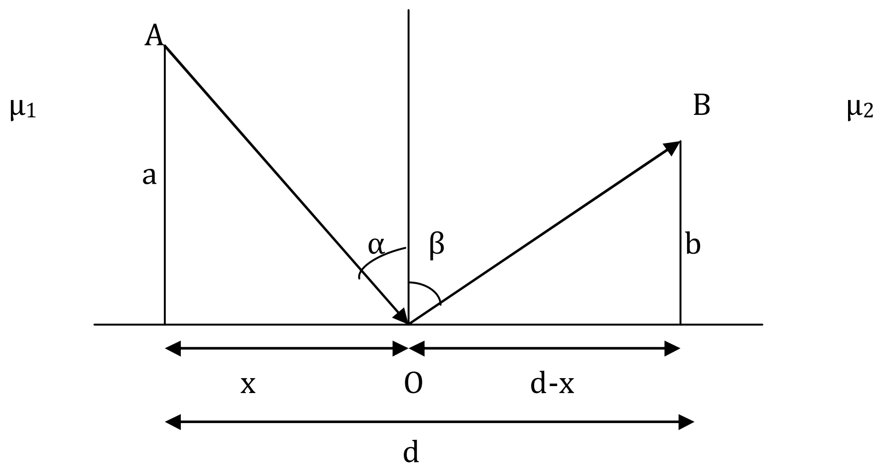
For air $\mu=1$

$L_{min} = 2\sqrt{2}r > 2r$

Answer: $L_{min} = 2\sqrt{2}\, r,\ \phi = 45^0$

Illustration 2:

Law of Reflection & Refraction from Fermat's principle:



Optical path $L = \mu_1\, AD + \mu_2 OB = \mu_1\sqrt{a^2+x^2} + \mu_2\sqrt{b^2+(d-x)^2}$

$L = L(x)$

$dL/dx = 0$ from minimization principle.

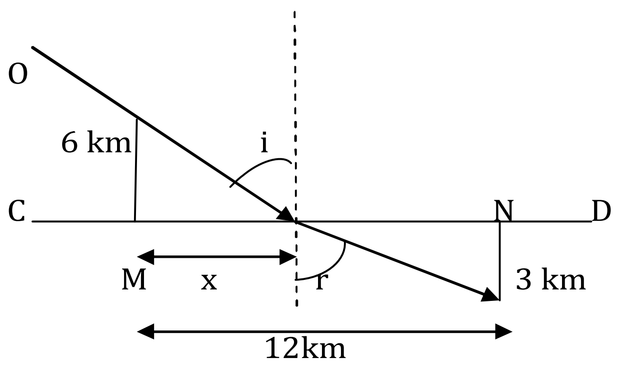
$dL/dx = \mu_1(1/2)[2x/(\sqrt{a^2+x^2})]+\mu_2(1/2)[2(d-x)(-1)]/[\sqrt{b^2+(d-x)^2}]$
$dL/dx = \mu_1[x/(\sqrt{a^2+x^2})]+ \mu_2 [(d-x)(-1)] /[\sqrt{b^2+(d-x)^2}]]$
$\mu_1 [x/\sqrt{(a^2+x^2)}] = \mu_2 (d-x) /[\sqrt{b^2+(d-x)^2}]$
$\boldsymbol{\mu_1 \sin\alpha = \mu_2 \sin\beta}$
If medium (1) & (2) is same $\mu_1 = \mu_2$
$\sin\alpha = \sin\beta \Rightarrow$ **$\alpha = \beta$ Law of reflection**
If $\mu_1 \neq \mu_2$ medium is different $\mu_1 \sin\alpha = \mu_2 \sin\beta$
$\sin\alpha/\sin\beta = \mu_2/\mu_1 = (c/v_2)/(c/v_1) = v_1/v_2$
$\boldsymbol{\sin\alpha/\sin\beta = \mu_2/\mu_1 = v_1/v_2 = \sin i/\sin r}$ **Snell's Law of refraction**
Note: it is my suggestion to use Snell's Law in this manner
$\mu_1 \sin\alpha = \mu_2 \sin\beta = \mu_3 \sin\gamma$
$\boldsymbol{\mu \sin\alpha = \text{constant}}$

Problems on optical path& Fermat's principle:

Problem 1: A man walks on the hard ground with a speed of 5 ft. /sec but he has a speed of 3ft. /sec on the sandy ground. Suppose he is standing at the border of sandy and hard ground. The man can reach the tree by walking100 ft. along the border and 120 ft on the sandy ground normal to the border.

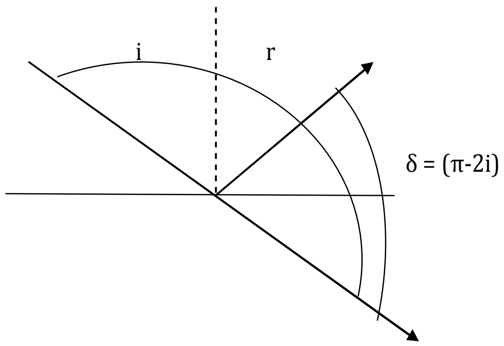
Find out the value of path which requires minimum time to reach tree.
(a) 190 or 10 (b) 180 or 20 (c) 170 or 30 (d) none

Problem 2: In figure two stations A & B in different territories are separated by a border line CD. A messenger can travel in the upper territory with speed V_a and in the lower territory with speed V_b. Several messenger start from A and follow different paths like APB having different positions of p specified by a distance x from M. It is found that the messenger who chooses x as 4.0 km reaches B in minimum time.
Answer the following question?
(1) What is the relation between V_a and V_b
(a) $V_a/V_b = \frac{1}{2}\sqrt{(73/52)}$ (b) $V_a/V_b = \frac{1}{4}\sqrt{(73/52)}$
(c) $V_a/V_b = \frac{1}{2}\sqrt{(52/73)}$ (d) None



(2) If the speed V_a and V_b are interchanged.
What will be the new value of x to give the fastest path?
(a) $X \approx 10.2$ (b) $X \approx 15.7$ (c) $X \approx 5.3$ (d) None

Deviation: Single Reflection:

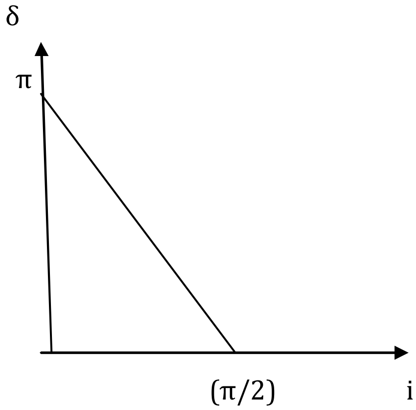


Angle between incident ray & emergent ray $\delta = \pi - 2i$

When $i=0$ (Normal incidence)
$\boldsymbol{\delta = \pi = \delta_{max}}$

When $i = \pi / 2$ (Grazing incidence)
$\boldsymbol{\delta = \pi - 2\pi / 2 = 0 = \delta_{min}}$

Graphical variation between angle of incidence and deviation is:



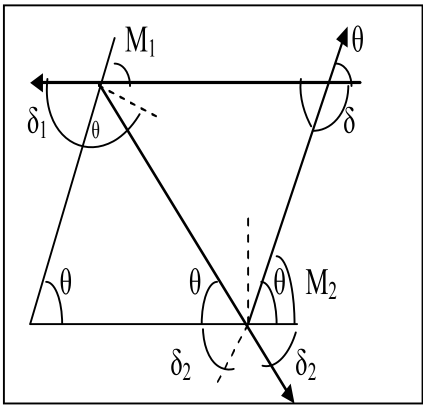
Graph between deviation & angle of incidence

***Problem 3*:** Two plane mirrors are inclined to each other such that a ray of light incident on first mirror and parallel to the second is reflected from the second mirror parallel to the first mirror.

1) The angle between the two mirrors is
(a) $\theta = 60^0$
(b) $\theta = 120^0$
(c) $\theta = 90^0$
(d) None

2) Total deviation produced in the incident ray due to the two reflections
(a) 240^0 anticlockwise or 120^0 clockwise
(b) 120^0 anticlockwise or 240^0 clockwise
(c) 60^0 anticlockwise or 120^0 clockwise
(d) 120^0 clockwise or 60^0 anticlockwise

Solution:



(1) $3\theta = 180^0$, $\theta = 60^0$

(2) From figure total deviation $\delta = \theta + \pi$
$\delta = 60 + 120 = 240^0$ anticlockwise or 120^0 clockwise
or $\delta_{net} = \delta_1 + \delta_2 = 120^0$ Anticlockwise + 120^0Anticlockwise = 240^0 Anticlockwise

***Rotation of mirror*:**

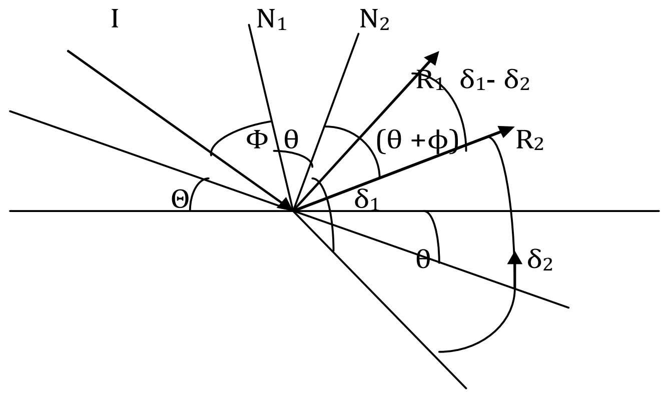
***Problem 4*:** If mirror is turned by an angle θ then angle turned by reflected ray.
(a) θ
(b) 2θ
(c) 3θ
(d) 4θ

Solution: If mirror is turned by angle θ
Thus angle turned by reflected ray $= \delta_1 - \delta_2$

$\delta_1 = \pi - 2\phi$ and $\delta_2 = \pi - 2(\theta+\phi)$

$\delta_1 - \delta_1 \cdot \delta_2 = (\pi - 2\phi) - \{\pi - 2(\theta+\phi)\}$

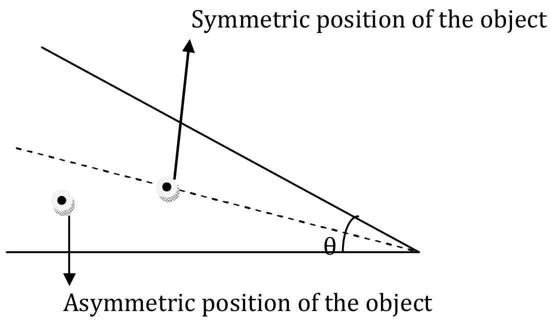
$\delta_1 - \delta_2 = \pi - 2\phi - \pi + 2\theta + 2\phi = 2\theta$



Note:

- If mirror is kept fixed & incident ray is rotated then reflected ray will rotate in opposite sense by same angle.
- If mirror and incident ray both are rotated then net rotation suffered by reflected ray will be algebraic sum of rotation suffered by reflected ray due to mirror. Rotation and incident ray rotation separated keeping sense of rotation in mind.

Images formed by two mirrors in contact:



Suppose θ is the angle between the two mirrors.

(a) **If $360^0/\theta$ is even integer.**

No of images n = $[(360/\theta) - 1]$ for all positions of the object.

(b) **If $360^0/\theta$ is odd integer.**

No of images $n = (360/\theta)$ if the object is placed Asymmetric

(when object is placed off the bisector of the mirror)
No of images n=[(360/θ) -1] if object is placed Symmetric
(when object is placed on the bisector of the mirrors)
(c) **If $360^0/\theta$ is a fraction**
No of images formed will be equal to its integral part.

Check 360/θ

Even | Odd | Fraction

n = [(360/θ) -1] (both)

Odd → Asymmetric: n= 360/θ

Odd → Symmetric: n= [(360/θ) -1]

Fraction → n = Integral part

Image formed by two mirrors in contact working procedure:

Step 1st: find the value of $360^0/\theta$
Step 2nd: check this is even no, odd no or fraction
Step 3rd:
Even no n=[(360/θ) -1]
Odd no n=[(360/θ) -1] → symmetric n=360/θ → Asymmetric
Fraction n= Integral part

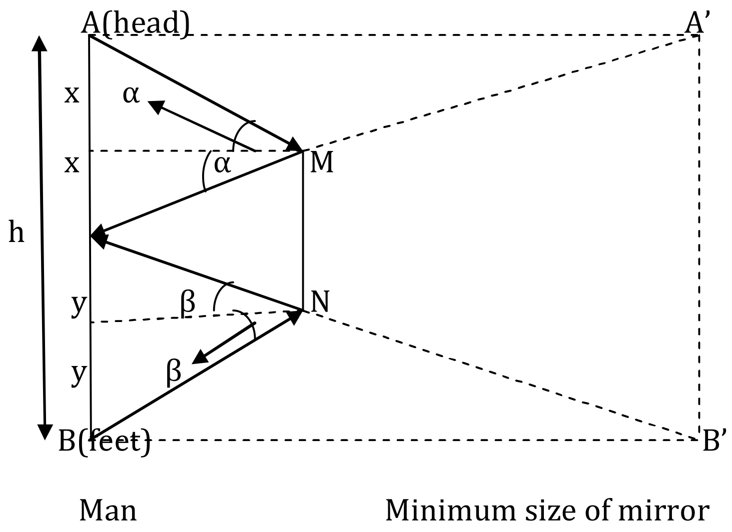
		No of images	
θ (degree)	**$360^0/\theta$**	**Asymmetric**	**Symmetric**
0	∞	∞	∞
30	12	11	11
45	8	7	7
60	6	5	5
72	5	5	4
75	4.8	4	4
90	4	3	3

Formation of image by plane mirror:

The image formed by a plane mirror has following characteristics:
(1) it is visual & erects
(2) Of the same size as the object

(3) Laterally inverted
(4) As far behind the mirror as the object in front.

Minimum size of plane mirror required to see full height of the observer himself:

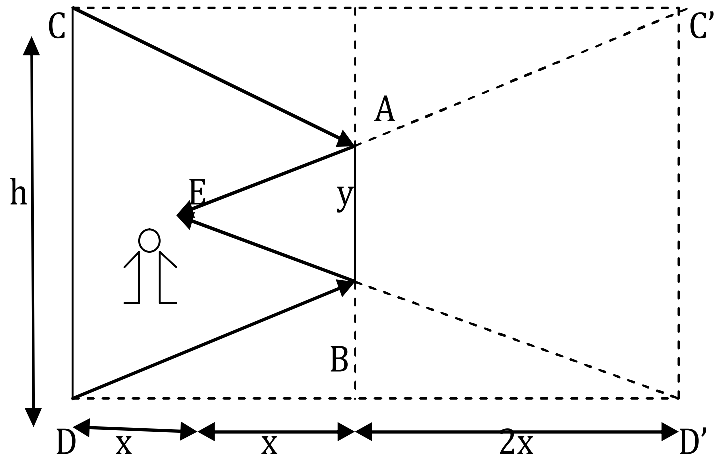


x+ y = MN = size of the mirror
2x+ 2y=h
x + y= h/2

- Thus in order to see full height, a person requires a plane mirror.
- This relation is true for any distance of observer from plane mirror.
- Also the lower edge of mirror should be kept at half of the eye level at a height y from the feet level.

Note:
(1) It should be noted that a person can see his full height by turning his head or eyes even in a smaller mirror.
(2) An observer can see the image of a tall building in a very small mirror by keeping mirror at a large distance.

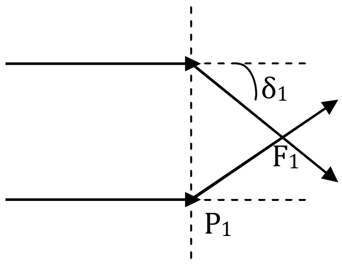
Problem 5: A man is standing exactly at the cube of the hall. He wants to see the image of his back wall in a mirror hanging on front wall. Find the minimum size of the mirror required.



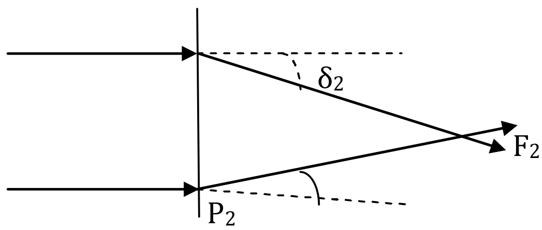
Solution: Suppose the height of the wall be h and required height of the mirror be y.
In the similar triangle EAB & EC'D' we have
y / x = h/3x
y= h/3 Answer

Power of an optical instrument:

- Powers of an optical instrument depend upon its ability to converge or diverge the light rays.
- One who can converge or diverge light rays by larger amount is more powerful than the one who converges/diverges by a lesser amount.



(a) Optical instrument

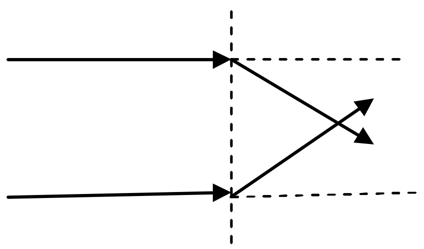


(b) Optical instrument

As from a&b it is clear that $\delta_1 > \delta_2$ so $P_1 > P_2$ i.e. the instrument in a is more powerful than instrument b.

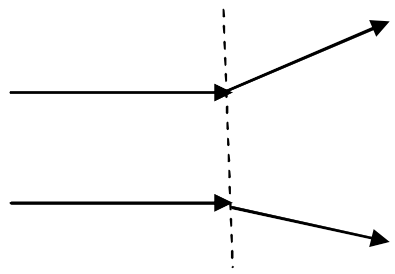
- The optical instrument which is more powerful has smaller focal length as in the figure it can be seen that $f_1 < f_2$ thus $P \propto (1/f)$
- A part from this the optical instrument which converges light rays is said to be of positive power & the one from which light rays diverge has the negative power.

Instrument → light rays convergent → +ive power
Instrument → light rays divergent → -ive power



P = +ive

(a)

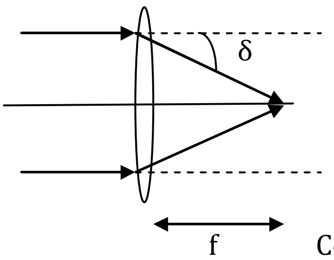


P = -ive

(b)

convergent lens has +ive power & +ive focal length. Divergent lens has –ive power & -ive focal length.

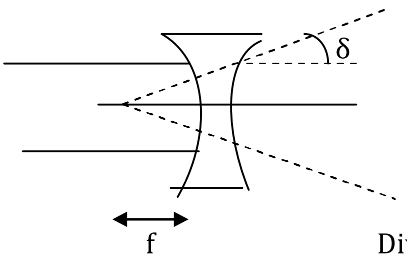
(a)



P = 1/f Convex lens

Converge → +ive power & +ive focal length

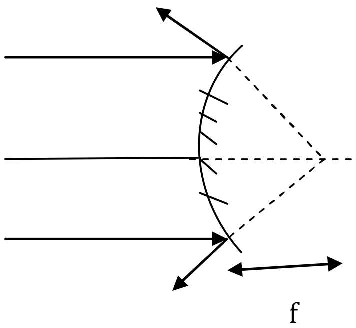
(b)



P = 1/f Concave lens

Diverge → -ive power → -ive focal length

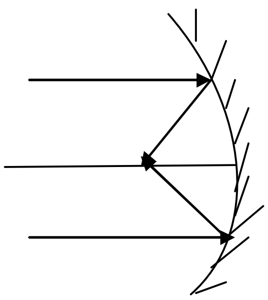
(c)



P=-1/f Convex mirror

P = -ive, f = +ive } → Diverge rays

(d)



P=-1/f Concave mirror

P =+ive, f = -ive} → Converge rays

Note: ***Power of lens=1/focal length But Power of mirror = - 1/focal length***

Lecture-2

Geometrical Optics

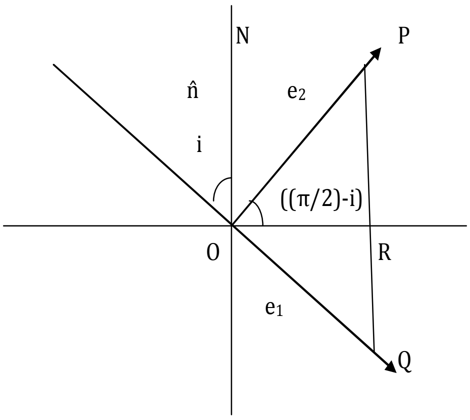
Physics for IIT - JEE

- **Vector form of Laws of Reflection**
- **Problem based on Snell's Law**
- **Variable Refractive Index Problems**

Laws of Reflection in vector form:

Problem 1: If $\hat{e}_1$=unit vector along incident ray,$\hat{e}_2$=unit vector along reflected ray & $\hat{n}$=unit vector along normal, then vector form of laws of Reflection is

a) $\hat{e}_2=\hat{e}_1+2(\hat{e}1.\,\hat{n})\,\hat{n}$ **b)** $\hat{e}_2=\hat{e}_1-2(\hat{e}_1.\,\hat{n})\,\hat{n}$
c) $\hat{e}_2=\hat{e}1+(\hat{e}1.\,\hat{n})\,\hat{n}$ **d)** $\hat{e}_2=\hat{e}_1-(\hat{e}_1.\,\hat{n})\,\hat{n}$



Solution: From vector law of addition in triangle OPQ

OP= OQ+QP

OP $\hat{e}_2$=OQ$\hat{e}_1$+2QR$\hat{n}$

$\hat{e}_2=\hat{e}_1+2QR/OP\ \hat{n}$ (OP=OQ)

sin [(π/2)-i] = cos i = PR/OP = QR/OP

$\hat{e}_2=\hat{e}_1+2\cos i\ \hat{n}$

Now $\hat{e}_1.\hat{n}=|e_1||n_1|\cos(\pi-i) = -\cos\hat{i}$

$\hat{e}_2=e_1-2(\hat{e}_1.\hat{n})\,\hat{n}$

This is vector form of law of reflection.

Problem 2: A light ray parallel to the x axis strikes the outer reflecting surface of a sphere at a point (2, 2, 0). Its centre is at the point (0, 0,-1).

1. Find the unit vector along the direction of reflected ray

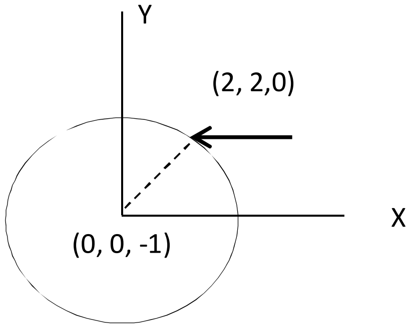
(a) (-i+8j+4k)/9 (b) (i+8j+4k)/9
(c) (-i+8j-4k)/9 (d) (-i+-j+4k)/9

2. If the unit vector along the direction of reflected ray is x**i**+y**j**+z**k.** Find the value of yz/x^2

(a) 32 (b) 28 (c) 32/9 (d) None

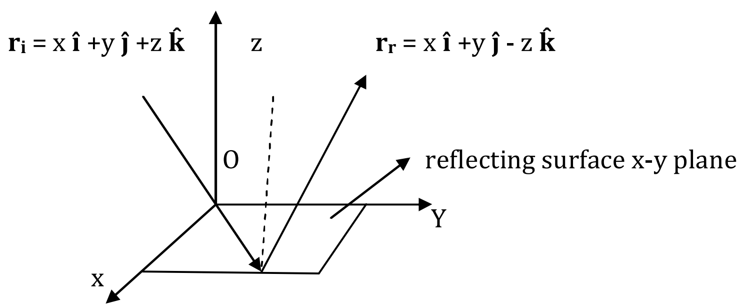
Solution:

If $\hat{e}_1$=unit vector along incident ray,$\hat{e}_2$=unit vector along reflected ray & $\hat{n}$=unit vector along normal, then vector form of laws of Reflection is $\mathbf{\hat{e}_2=e_1-2(\hat{e}_1.\hat{n})\ \hat{n}}$



$\hat{n}$ = (2i+2j+k)/3
$\hat{e}_2$=? e_1=-i
Using vector form of laws of Reflection $\mathbf{\hat{e}_2=e_1-2(\hat{e}_1.\hat{n})\ \hat{n}}$
$\hat{e}_2$= -i-2(-2/3) (2i+2j+k)/3 = -i+4 (2i+2j+k)/9= **(-i+8j+4k)/9**
According to problem the unit vector along the direction of reflected ray is x**i**+y**j**+z**k**
Now on comparisons of two equations we get the values of x, y, & z as -1/9, 8/9 & 4/9
The value of yz/x^2 **=** (8/9) (4/9) / $(-1/9)^2$ =***32***

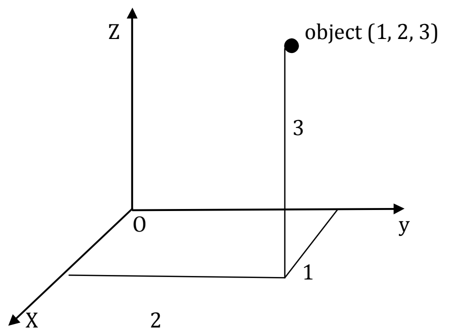
Problem 3: If x-y plane is reflecting plane than the equation of incident ray is $\mathbf{r_i}=x\hat{i}+y\hat{i}+z\hat{k}$ thenfind the equation of reflecting ray .



Solution:
Whenever reflection takes place, the component of incident ray parallel to reflecting surface remains unchanged while component perpendicular to reflecting surface(i.e along the normal)reverses in direction.The equation of reflecting ray is $\mathbf{r_r}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}-z\hat{\mathbf{k}}$.

Comprehension and analitical ability:

Problem 4:An optical image is formed due to intersection of reflected rays when reflected rays actually meet,the image so formed is called real.when reflected ray apper to meet,than image formed is called virtual.There are three plane mirrors arranged perpendicular to each forming principal co-ordinate planes namely x-y,z-y and z-x.Their common point of intersection is taken as origin.A point object having co-ordinate (1,2,3)is placed as shown in fig



(1) Which of the following is not the coordinates of images formed due to meeting of rays undergoing single reflection only
(a) (1,2,-3) (b)(1,-2,3) (c)(-1,2,3) (d)none

(2) Which of the following is not the coordinate of images formed due to meeting of rays undergoing two reflection only
(a) (-1,-2,3) (b)(1,-2,-3) (c)(-1,2,3) (d)none

(3) Which of the following is not the coordinate of images formed due to meeting of rays undergoing one reflection from each mirror
(a) (-1,-2,3) (b)(-3,-2,1) (c)(-2,-3,-1) (d)(-1,3,2)

(4) The number of images formed are
(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

Answer: 1(d) 2(c) 3(a) 4(e)

Problem 5 (AIEEE 2011): Let the x-z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $A = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is?

(a) 45^0 (b) 60^0 (c) 75^0 (d) 30^0

Solution: As refractive index for $z > 0$ and $z \leq 0$ is different x-y plane should be boundary between two media. Angle of incidence $\cos i = |[A_Z/\sqrt{A_x^2+A_y^2+A_z^2}]| = ½$
$i=60^0$ from Snell's law $\sin i / \sin r = \sqrt{3}/2$ $r = 45^0$

Problem 6: Excellent (IIT-JEE 1999): The x-y plane is the boundary between two transparent media-1 with $z \geq 0$ has a refractive index $\sqrt{2}$ and media-2 with $z \leq 0$ has a refractive index $\sqrt{3}$ .A ray of light in medium-1 given by the vector $A=6\sqrt{3}\hat{i}+8\sqrt{3}\hat{j}-10\hat{k}$is incident on the plane of separation.

(1) The angle of incidence

(a) 30^0 (b)60^0 (c)90^0 (d) none

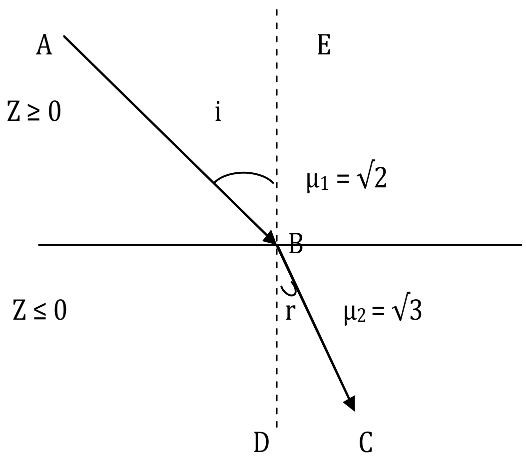
(2) The angle of refraction

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

(3) The unit vector in the direction of the refractive ray in medium-2

(a) $1/(10\sqrt{2})\,(6\hat{i}+8\hat{j}-10\hat{k})$ (b) $1/(10\sqrt{2})\,(6\hat{i}-8\hat{j}+10\hat{k})$

(c) $1/(10\sqrt{2})\,(6\hat{i}+10\hat{j}-8\hat{k})$ (d) None



Solution:

The vector of incident ray is given by

$AB = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$

In Δ ABE

AE+EB=AB

$AE = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}$, $EB = -10\hat{k}$

The angle of incidence between **AB** and **EB** can be obtained as

$\cos i = \mathbf{AB.EB}/AB.EB = [(6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k})(-10\hat{k})]/[\sqrt{(6\sqrt{3})^2 + (5\sqrt{3})^2 + (10)^2}(\sqrt{10^2})]$

$\cos i = 100/[10\sqrt{((36*3)+(64*3)+(100))}] = 100/[100\sqrt{400}] = 100/(10 \times 20) = ½$

$\cos i = ½ \quad \Rightarrow \quad i = 60^0$ Angle of incidence

By Snell's law: $\mu_1 \sin i = \mu_2 \sin r$

$\sqrt{2} \sin 60 = \sqrt{3} \sin r$

$\sqrt{2}\,(\sqrt{3}/2) = \sqrt{3} \sin r$

$(1/\sqrt{2}) = \sin r \quad \Rightarrow \quad r = 45^0$ angle of refraction

The vector of refracted ray can be written as.

BC=BD+DC

DC=A√2 $= 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}$

Unit vector along DC $= \hat{e} = (6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j})/\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2} = (6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j})/10\sqrt{3}$

$= (6\hat{i} + 8\hat{j})/10$

BC=BD+DC$= BC \cos r(-\hat{k}) + BC \sin r\, \hat{e}$

BC$/BC = \cos r\,(-\hat{k}) + \sin r\, \hat{e} = -k \cos 45^0 + \hat{e} \sin 45^0 = (1/\sqrt{2})(-\hat{k}) + (6\hat{i} + 8\hat{j})/10\sqrt{2}$

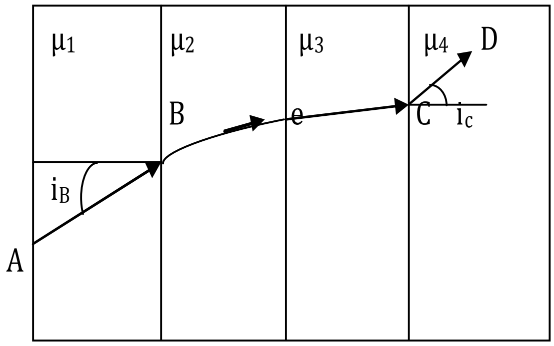
$= (1/10\sqrt{2})[-10\hat{k} + 6\hat{i} + 8\hat{j}]$

BC/BC= $(1/10\sqrt{2})[\,6\hat{i} + 8\hat{j} - 10\hat{k}]$

Unit vector in the direction of refracted ray.

Problem 7: A ray of light passes through four transparent media with refractive Indices μ_1, μ_2,μ_3,μ_4 as shown in figure. The surface of all media are parallel if the emergent ray CD is parallel to incident ray AB, we must have

(a) $\mu_1 = \mu_3$ (b) $\mu_2 = \mu_3$ (c) $\mu_3 = \mu_4$ (d) $\mu_4 = \mu_1$



Solution:

Applying Snell's law at B and C

μ sin i=constant

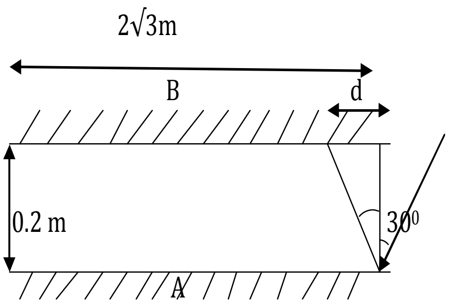
$\mu 1 \sin i_B = \mu 4 \sin i_C$

$i_B = i_C$ since AB || CB

$\mu_1 = \mu_4$

Problem 8: Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle 30^0at a point just inside one end of A the plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out

(a) 28 (b) 30 (c) 32 (d) 34



Solution:

L $=2\sqrt{3}$m

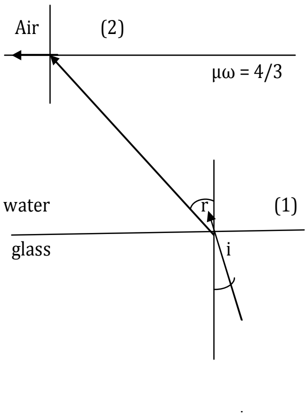
d/ 0.2 = tan 30 $\Rightarrow$ d=0.2 tan 30

d = $0.2/\sqrt{3}$

L/d = 2√3/(0.2/√3) =30

Therefore maximum number of reflection is 30

Problem 9: A ray light is incident at the glass water interface at an angle i, it emerges finally parallel to the surface of water, than the value of μg would be (a) (4/3) sin i (b) 1/ sin i (c) 4/3 (d) 1



Solution:

Apply Snell's law at surface 1 and 2

μg sin i=μa sin 90

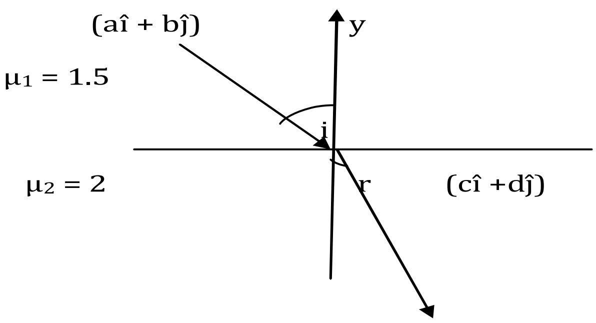
μg =1/sin i

Problem 10: The x-y plane separates two media A and B of refractive indices μ_1=1.5 and μ_2=2. A ray of light travels from A to B, its directions in to two media are given by unit vectors $\mathbf{u}_1$= a $\hat{\mathbf{i}}$+b $\hat{\mathbf{j}}$ and $\mathbf{u}_2$=c $\hat{\mathbf{i}}$+d $\hat{\mathbf{j}}$ then

(a) a/c = 4/3 (b) a/c = 3/4

(c) b/d = 4/3 (d) b/d = ¾

Solution:



$\sqrt{a^2+b^2}$=$\sqrt{c^2+d^2}$ =1 Unit vectors Magnitude unity

From Snell's law

$\mu_1 \sin i = \mu_2 \sin r$

$1.5 \sin i = 2 \sin r$

$1.5\,[a/\sqrt{(a^2+b^2)}] = 2c/\sqrt{c^2+d^2}$

$(3/2)a = 2c$, $a = (4/3)c$

$a/c = 4/3$

Variable Refractive Index:

Problem 11: IIT JEE (1995) Subjective

A ray of light travelling in air is incident at grazing angle (angle of incidence ≈ 90^0) on a long rectangular slab of a transparent media of thickness t=1.0m. The point of incidence is the origin A(0, 0). The medium has a variable refractive index given by

$n(y)=(ky^{3/2}+1)^{1/2}$ Where $k=1.0(m)^{-3/2}$, the refractive index of air is 1

1. Obtain a relation between the slope of the trajectory of the ray at appoint B(x,y) in the medium and the incidence angle at that point

   (a) $\cot\theta = dy/dx$ (b) $\tan\theta = dy/dx$ (c) $\cot\theta = (1/2)(dy/dx)$ (d) none

2. Obtain an equation for the trajectory y(x) of the ray in the medium

   (a) $y=K^2(x/4)^4$ (b) $y=kx^2$ (c) $y=k(x/4)^2$ (d) none

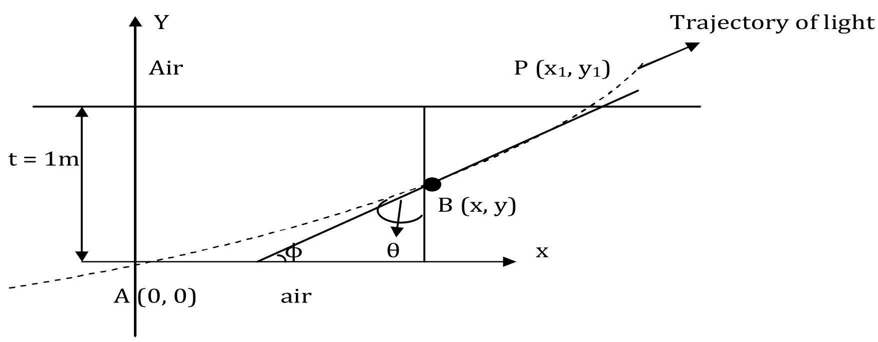
3. Determine the coordinates(x_1,y_1) of the point p where the ray intersects the upper surface of the slap-air boundary

   a) (4m, 1m) (b) (2m, 1m) (c) (1m, 1m) (d) none

4. Indicate the path of the ray subsequently.

   (a) The ray will emerge parallel to boundary (b) Perpendicular to boundary

   (c) Neither (a) nor (b (d) Data insufficient



Solution:

(a) Trajectory of the curve is shown by dotted curve the slope of the target at point B

$\tan\phi = dy/dx$

The angle of incidence at B is $\theta=(90-\phi)$

$\tan\phi=\tan(90-\theta)=\cot\theta= dy/dx$ (1)

(b) from Snell's law at A and B

$1 \sin 90=n(y) \sin\theta$ (2)

$\sin\theta= 1/n(y) = 1/[ky^{3/2} +1]^{1/2}$ (3)

$\cos\theta= \sqrt{1-\sin^2\theta} =\sqrt{1- [1/[ky^{3/2} +1]^{1/2}]}$

$\cot\theta = \cos\theta/\sin\theta =[\sqrt{1- [1/[ky^{3/2} +1]^{1/2}]}]/[1/[ky^{3/2} +1]^{1/2}]$

$\cot\theta = \sqrt{[(ky^{3/2} +1-1)/(ky^{3/2} +1)]}/[1/[ky^{3/2} +1]^{1/2}]$

$\cot\theta= \sqrt{[(ky^{3/2})/(ky^{3/2} +1)]}[[ky^{3/2} +1]^{1/2}]$

$\cot\theta= k^{1/2}y^{3/4}$ (4)

From equation (3)

$\sin\theta =1/n(y) = \cos\phi= 1/[\sqrt{1+\tan^2\phi}]$

$\cot\theta=\cot(90-\phi)=\tan\phi$

$1/n(y) = 1/[\sqrt{1+\cot^2\theta}]$

$n(y)= \sqrt{1+\cot^2\theta} = \sqrt{1 +(dy/dx)^2}$

$[ky^{3/2} +1]^{1/2} = \sqrt{1 +(dy/dx)^2}$

$(dy/dx)^2 = ky^{3/2}$

$dy/dx = k^{1/2}y^{3/2}$

$\int[dy/y]^{3/4} = \int k^{1/2}dx$

$\int y^{-3/4} = \sqrt{k} \int dx$

$[y-(3/4)]/[(-3/4)+1]=\sqrt{k}\, x +c$

$4y^{1/4}=\sqrt{k}\, x +c$

Applying boundary condition

$x=0, y=0, c=0$

$4y^{1/4}=\sqrt{k}\, x$

$y^{1/4}= (\sqrt{k}/4)\, x$

$\mathbf{y = k^2(x/4)^4}$

(c) At point P

$x=?, y=1, k=1$

$1= k^2 (x/4)^4$

$1=1(x/4)^4$

$1=(x/4)$

$x=4m$

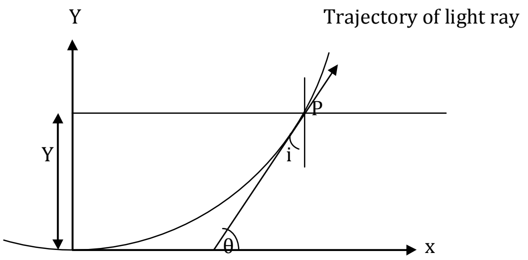
The coordinates of P (4, 1)

(d) From Snell's law $n_A \sin i_A=nP \sin i_P$, $i_A=i_P=90$

The ray will emerge parallel to the boundaries

Problem 12: Due to a vertical temperature gradient in the atmosphere the index of refraction varies as $n=n_0\sqrt{1+ay}$, where n_0 is the index of refraction at the surface and $a=2.0*10^{-6}m^{-1}$ .A person of height h=2.0 m stand is on a level surface. Beyond what distance he cannot see the runway

(a) 2000 m (b) 200 m (c) 100 m (d) 1000 m



Solution: Let O be the distant object just visible to the man. Let P be a point on the trajectory of the ray.

From figure θ=90-i (1)

Slope of tangent at point p=tan θ=dy/dx =tan(90-i)=cot i

tan θ = cot i (2)

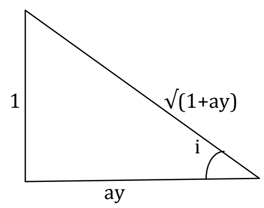
From Snell's law n sin i=constant

at the surface, y=0

$n=n_0$, $i=90^0$

$n_0 \sin 90=n \sin i$

$n_0 = n_0\sqrt{1+ay} \sin i$



sin i= $1/\sqrt{(1+ay)}$

cot i= $\sqrt{ay}$ = dy/dx

dy/ dx =ay

$\int_0^y \frac{dy}{\sqrt{y}} = \int_0^x \frac{x}{\sqrt{a}}dx$

$\int_0^y dy\, y^{-1/2} = \sqrt{a}\int_0^x dx$

$2y^{1/2} = \sqrt{a}\, x$

x= 2($\sqrt{y/a}$), On substituting y=2m ,

$X=2\sqrt{(2/9)} = 2\sqrt{(2/2*10^{-6})} =2\sqrt{10^6} =2*10^3$

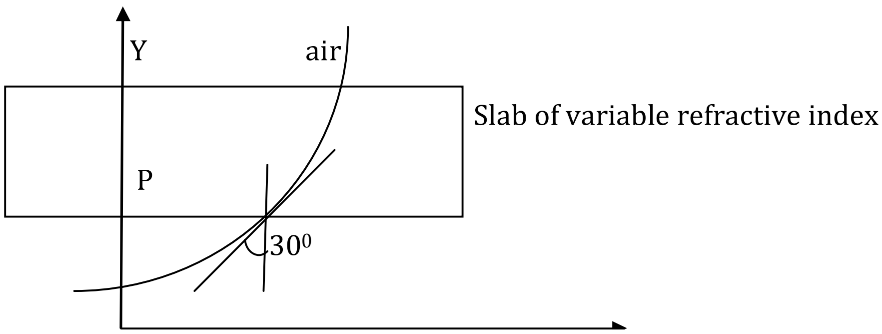
x = 2000m

Problem 13: A ray of light travelling in air is incident at angle of incident 30^0 on one surface of slab in which refractive index varies with y. The light travels along the curve $y=4x^2$ (y & x are in meter) in the slab. Find out the refractive index of the slab at

$y=1/2m$ in the slab

(a) $\mu = 3/2$ (b) $4/3$ (c) $9/8$ (d) none

Solution:



Let R.I at $y=y_1$ is μ and corresponding angle of refraction is θ

$\mu \sin \theta = 1.\sin 30$

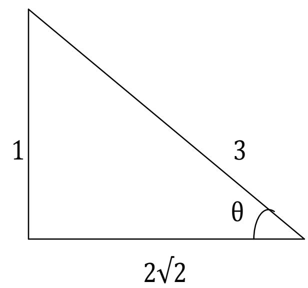
$\tan [(\pi/2)-\theta] = dy/dx = \cot \theta$, $y=4x^2$

$dy/dx = d/dx(4x^2) = 8x = \cot \theta$

$\cot \theta = 8x = 8(y/a)^{1/2} = 4y^{1/2}$

$\cot \theta = 4y^{1/2}$, at $y=1/2$

$\cot \theta = 4/\sqrt{2} = 2\sqrt{2}$,



$\sin \theta = 1/3$

$\mu \sin \theta = 1 \sin 30$,

$\mu (1/3) = 1/2$

$\boldsymbol{\mu = 3/2}$

Lecture-3

Geometrical Optics

Physics for IIT - JEE

- *The Cartesian sign convention*
- *Rules for ray diagrams*
- *Formulas related to mirrors & Magnification*
- *Relative position, size and nature of image for different positions of object in spherical mirror*

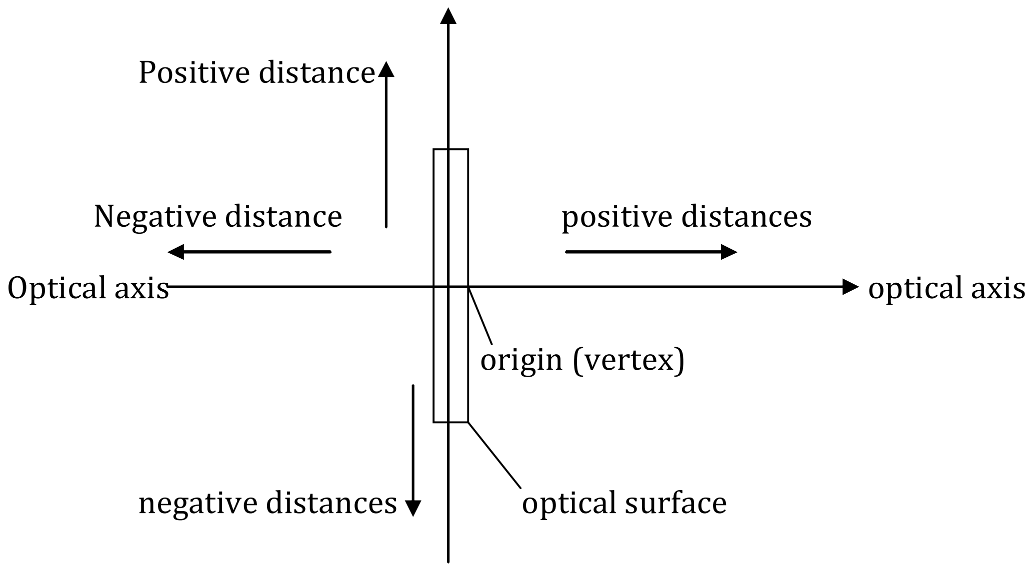
The Cartesian sign convention:

A sign convention facilitates the computations of the object and image distances, assesses the nature of image (real or virtual), its magnification and orientation.
The object is placed to the left of the optical surface (a mirror or refracting surface). The light is incident from left to right. The centre of the optical surface is called vertex. The vertex is taken as origin. The horizontal axis is called the optic axis.

(1) Distance measured to the right of the origin along optic axis are positive distances, since they are along the positive axis of the standard Cartesian coordinate system.

(2) Distances measured to the left of the origin along the optic axis are negative distances, since they are along the negative axis of the standard Cartesian coordinate system.

(3) The sign convention for magnification: Magnification is defined as the ratio of the size of image to the size of object |m| =Image size/object size.

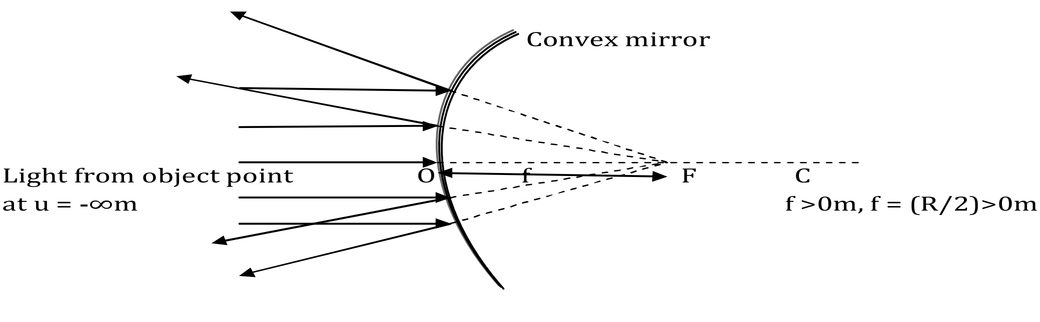


This is referred to as lateral magnification. If the magnification m is positive the image of the object is erect (upright), meaning that the image has same orientation as the object. If the magnification is negative, the image is inverted (upside down).

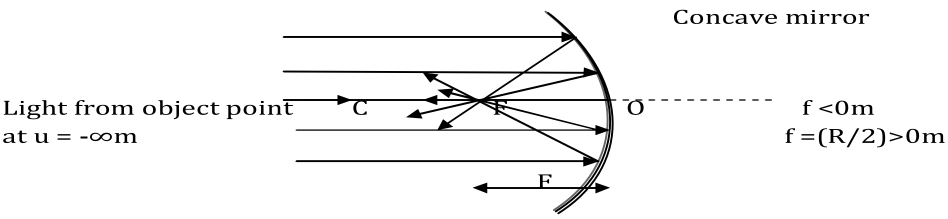
Rules for ray diagrams:

We can locate the image of any extended object graphically by drawing any two of the following four principle rays.
1. A ray, initially parallel to the principal axis is reflected through the focus of the mirror

2. A ray, initially passing through the focus is reflected parallel to the principal axis.
3. A ray passing through the so line joining point object and its image cuts principal axis at centre of curvature.
4. A ray incident at the pole is reflected symmetrically. So line joining O and I' or O' and I will cut principal axis at pole.
(i) Spherical mirrors bring paraxial rays to an approximate focus at a point on the mirror axis.
(ii) The focal length of a spherical mirror is equal to half the radius of curvature of the mirror.
The mirror equation for spherical mirror is
$(1/v) + (1/u) = (1/f) = 2/R$
when the object point O is located infinitely far away from the mirror, then $u = -\infty$ and the position of the image is called the focal length. If we substitute $u = -\infty$ into the mirror equation,
$(1/-\infty) + (1/v) = (2/R)$
since $v = f$, $0 + (1/f) = 2/f$, $f = R/2$
parallel incident rays intersect in case of concave mirror and appear to intersect in case of convex mirror.



(a)



(b)

The magnification is m = Image size/object size = $-v/u$

Note that the law of reflection is independent of the medium in which light is travelling. The mirror and magnification equation can be applied irrespective of the medium surrounding the mirror.

(1) A ray parallel to axis is reflected back through focal point as shown in fig. (a)

(2) A ray that passes through the focal point on the ray to the mirror is reflected back parallel to the mirror axis as shown in fig. (b)

(3) A ray from the object is directed towards the centre of curvature of the mirror, after reflection the ray retraces its path because it strikes the mirror along the normal to the mirror.

(4) A ray that strikes the vertex of the mirror reflects at an equal angle on the other side of the mirror axis.
Any two of these rays are sufficient to locate the position of the image.
We can assess two things from ray diagram:
(a) If the image is in front of the mirror, it is real; if the image is at the back of the mirror, the image is virtual.
(b) We are able to guess if the magnification is positive or negative and greater or less than unity.

Note: Mirror equation **1/v +1/u =1/f**

(i) Mirror equation is valid for both type of object irrespective of its position.

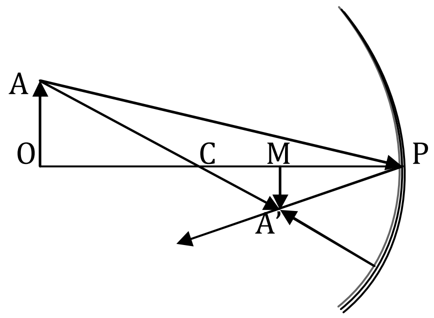
(ii) In case of spherical mirrors if object distance (x_1) are measured from focus instead of pole, then $u = f + x_1$, and $v = f + x_2$ the mirror formula reduces to
$[1/(f+x_2)] + [1/(f+x_1)] = 1/f$ or $\mathbf{x_1 x_2 = f^2}$
which is known as Newton's formula.
This formula is applicable to real objects and real images only.

(iii) In numerical problems it is convenient to use mirror equation in following form:
v = [uf/(u-f)],
u =[vf/(v-f)],
f = [uv/(u +v)]

(iv) While using mirror equation known quantities is substituted with proper sign and quantities to be calculated (unknown quantities) are not be given any sign.

Magnification: This is defined as the ratio of size of image to the size of object
magnification (m) = size of image (I)/ size of object (O)
Three types of magnification are produced by a spherical mirror which are described below:

(a) Linear magnification (Transverse magnification/Lateral magnification):



In this case, size of object and image is measured perpendicular to principal axis.

m = Height of image(H_1)/Height of object(H_0)

From similar triangles APO and A'PM

AO/A'M = PO/PM (1)

$(H_0/-H_I) = (-u/-v)$

or$(H_I/H_0) =- (v/u)$

thus **m = -v/u**

Note:

(i) The above formula is valid for convex as well as concave mirror and it is independent of nature of object (real or virtual) irrespective of its position.

(ii) In terms of focal length, magnification can be expressed as

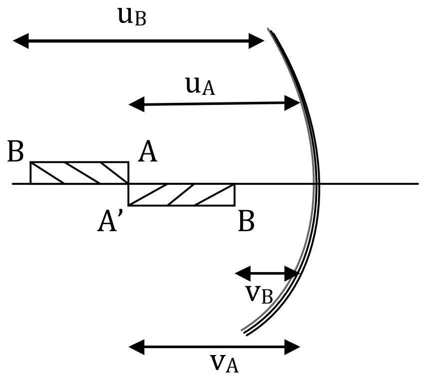
$m = f/(f-u) = (f-v)/f$

(iii) Magnification can be either positive or negative depending on the nature of the image. If m is negative, then image is inverted with respect to object and if m is positive then image is erect with respect to object.

(iv) Real image is not always inverted and virtual image is not always erect.

(b) Superficial magnification: when a small surface is placed perpendicular to the principal axis, both length and breadth are magnified in the ratio v/u. The superficial magnification is m_s = Area of image/Area of object = $[(-v/u)(-v/u)] = v^2/u^2$

(c) Axial magnification (Longitudinal magnification): In this case size of image and object is measured along principal axis



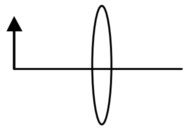
m = Length of image/Length of object
For linear object placed along the principal axis, the magnification is,
$m_L = [v_A - v_B]/[u_A - u_B]$ (2)

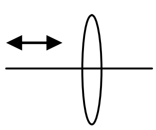
Note: For small object, $m_L = dv/du$
From mirror object, $m_L = dv/du$
From mirror equation we have $(1/v) + (1/u) = (1/f)$
Differentiating both sides, we get
$-(1/v^2)dv - (1/u^2)du = 0$
or $dv/du = -(v^2/u^2)$
Thus, **Longitudinal magnification ≈ (Transverse magnification)²**

Formulas related to mirrors

(1) The focal length of a spherical mirror of radius R is given by **f = R/2** with proper sign convention.
(2) The power of a mirror $\mathbf{P = -1/f_{(in\ m)}} = -100/f_{(in\ cm)}$
(3) Magnification: If a thin object of linear size O is situated vertically on the axis of a mirror at a distance u from the pole & its image of size I is formed at a distance v (from the pole) magnification transverse (m) = -v/u = I/O

m → -ive means image is inverted with respect to object
m → +ive means image is erect with respect to object



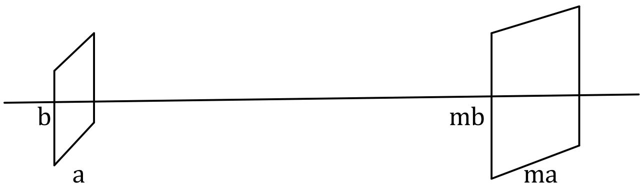


$\mathbf{M_{traverse} = -v/u}$ $\mathbf{M_{linear} = I/O = -[(v_2 - v_1)/(u_2 - u_1)] = -dv}$ For small object

-m → real image, +m → virtual image
(4) (a)In case of small linear objects: $(1/v) + (1/u) = (1/f)$
$(-dv/v^2) + (-du/u^2) = (-df/f^2) \rightarrow$ zero
$(-dv/du) = (v^2/u^2) \Rightarrow dv/du = -(v/u)^2$

$m_L = -(dv/du) = m^2$

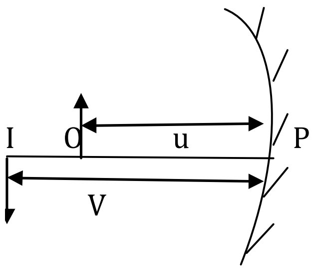
(b) If a 2D object is placed with its plane $\perp$ to principle axis. Its magnification is called **superficial magnification**

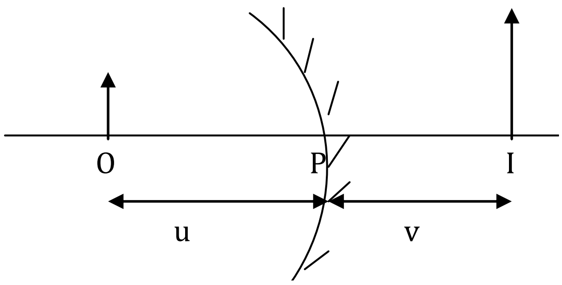


m_s = area of image/ area of object = (ma)(mb)/ ab = m^2

(c) In case of more than one optical component, the image formed by first component will act as an object for the second component & so on. So overall magnification

m = I/O = $(I_1/O)(I_2/I_1)\ldots = (m_1)(m_2(m_3)\ldots$





-m→ inverted image, real image

m = -v/u

m = -(-v)/(-u)

m = -ve

+m→ Erect image, virtual image

m = -v/u

m = -(v/-u)

Relative position, size and nature of image for different positions of object:

(1) Object is at infinity:

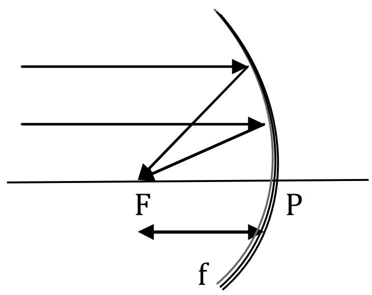


fig: object at infinity

Here, $u \rightarrow \infty$

from mirror formula, we have $(1/v) + (1/u) = (1/f)$

or $(1/v) + (1/\infty) = (1/-f)$

or $(1/v) = (1/-f)$

$\Rightarrow v = -f$

also $m = |v/u| = (-f/\infty) = 0$

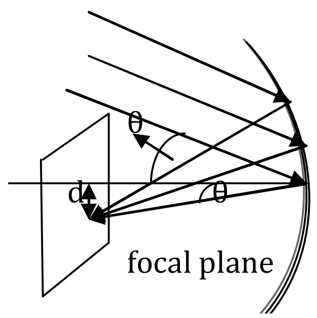
thus,

position: At F

nature: Real and inverted

size: Diminished (very small)

Note:



If rays from distant object are not parallel to principal axis, then image is formed on the focal plane.

$(d/f) = \tan\theta$

or $d \approx f\,\theta$ (if θ is very small)

(ii) The object lies beyond centre of curvature:

Here $\infty > u > 2f$

or $0 < (1/u) < (1/2f)$

or {0-(1/f)} < {(-1/f) + (1/u)} < {(-1/f) + (/2f)}
or (-1/f) < (1/v) < (-1/2f)
or –f < v < -2f
also, m = |v/u| < 1

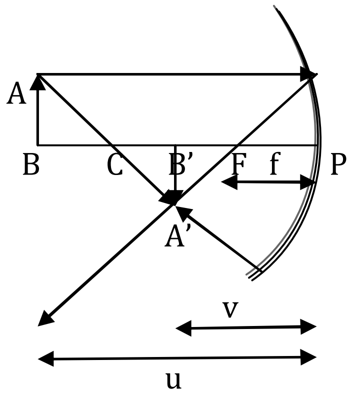


fig : Object beyond 2f

Position: At centre of curvature
Nature: Real and inverted
Size: small

(iii) Object is at centre of curvature:

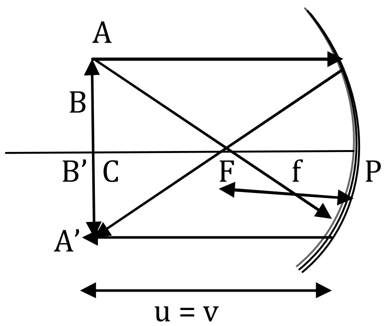


fig: object at centre of curvature

Here u = -2f
from mirror formula, we have v = uf/ (u-f) = [(-2f)(-f)]/[-2f-(-f)] = -2f
also, m = |v/u| = 1
thus,
Position: At centre of curvature

Nature: Real and inverted
Size: same

(iv) Object is between focus and centre of curvature:

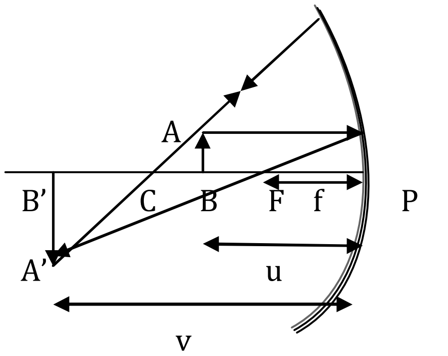
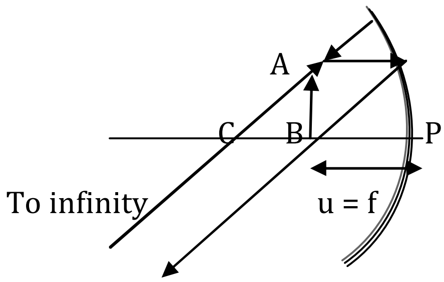


fig: Object between F and C

Here, $f < u < 2f$
or $(1/f) > (1/u) > (1/2f)$
From mirror formula, we have $(1/v) = (1/-f) + (1/u)$
since, $u > f$
$(1/v) > (1/-f) + (1/f)$
or $(1/v) > 0$
$\Rightarrow$ $v < \infty$
Again, for $u < 2f$
or $(1/v) < [(1/-f) + (1/2f)]$
$\Rightarrow$ $(1/v) < (1/2f)$
$\therefore$ $v > -2f$
Also, $m = |v/u| > 1$
Position: Beyond centre of curvature
Nature: Real and inverted
Size: Enlarged
(v) Object is kept at focus:



Here $u = -f$
From mirror formula, we get $v = [uf/(u-f)] = [(-f)(-f)]/[(-f)-(-f)] = \infty$
Also, $m = |v/u|$ = infinite

Thus,
Position: Infinite
Nature: Real and inverted
Size: Highly magnified
(vi) Object is kept between pole and focus:

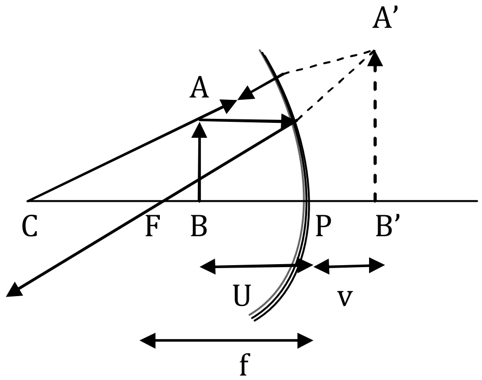
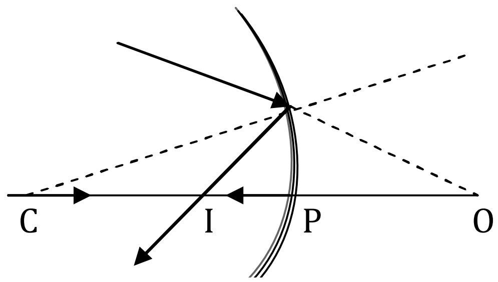


fig: object between F and pole

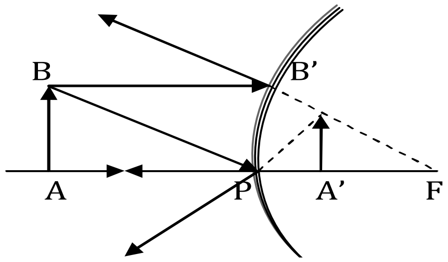
Here, $u < -f$
or $(1/u) > (-1/f)$
From mirror formula, we have $(1/v) = (1/-f) + (1/u)$
since, $(-1/u) > (1/f)$
∴ v is positive
thus,
Position: other side of mirror
Nature: virtual and erect
Size: magnified

Notes:



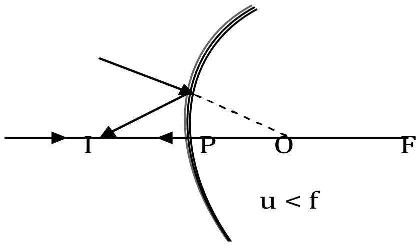
(i) Concave mirror always forms real image of a virtual object irrespective of its position. Here, u = +ive, and f = -ive
From mirror equation, we have $v = [uf/(u-f)]$
v = -ive Hence, image formed is always real.

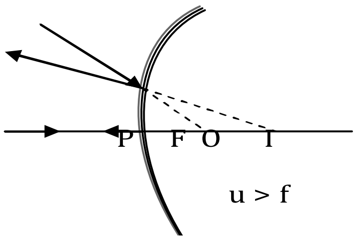
(ii)



Whatever be the position of object in front of convex mirror, image is always formed behind mirror between pole and focus, small in size, virtual and erect.

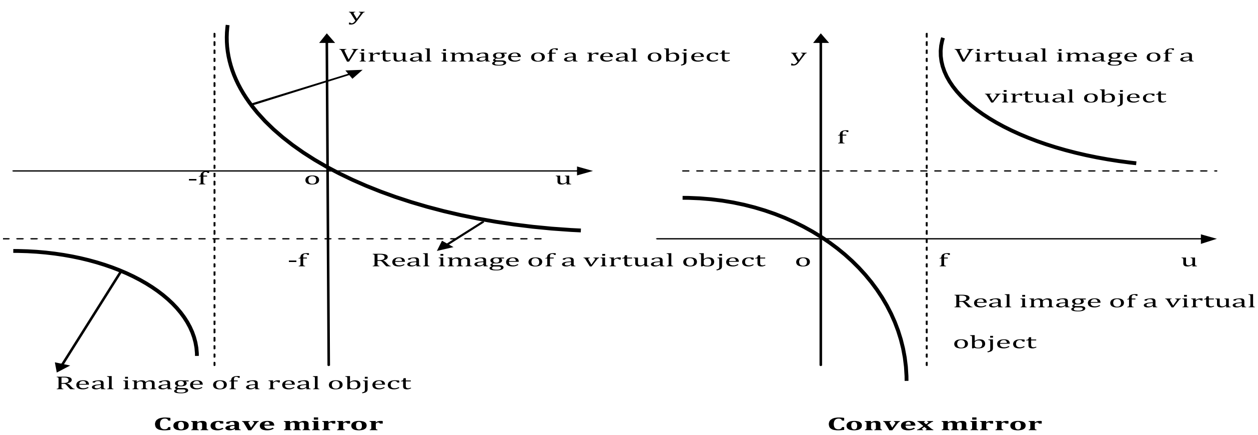
(iii)





Convex mirror can form real or virtual image of a virtual object depending on its position.

(iv) In general all situations in spherical mirror can be summarised in u-v graph as shown below:



While interpreting these graphs for numerical problems, remember following points:

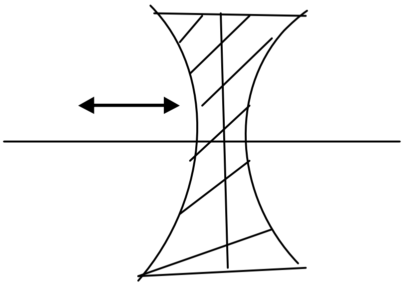
(a) Object/image before mirror is real so, v and u are negative.

(b) Object/image behind mirror is virtual so, v and u are positive.

Problem 1(IIT 1988): A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance 4 from the pole of the mirror. The size of image is approximately equal to ½

(a) $b[(u-f)/f]^{1/2}$ (b) $b[f/(f-u)]^{1/2}$ (c) $b[(u-f)/f]$ (d) $b[f/(u-f)]^2$

Solution:



Concave mirror

$(1/v) + (1/u) = (1/f)$ [mirror formula]

for concave mirror $(1/v) + (1/u) = (1/f)$

[differentiating]

$(-dv/v^2) + (-du/u^2) = (-df/f^2) = 0$

$\mathbf{dv/du = -v^2/u^2}$ (1)

$(1/v) + (1/u) = (1/f)$

$(u/v) + (u/u) = (u/f)$

$(u/v) = (u/f)-1$

$\mathbf{u/v = (u-f)/f}$ (2)

now using (2) in eq.(1)

$dv/du = -[v^2/u^2] = -[f/(u-f)]^2$

$-dv/du = -[f/(u-f)]^2$

$dv = (-du)\,[f/(u-f)]^2$

dv ⇒ size of image, du⇒ size of object = b

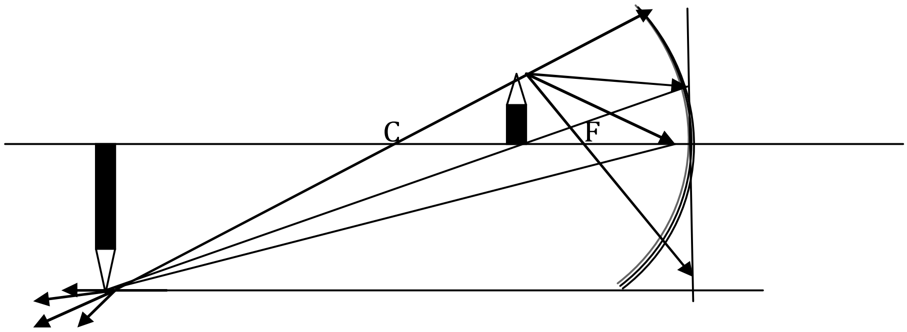
$dv = b[f/(u-f)]^2$

size of image $\mathbf{dv = b[f/(u-f)]^2}$

hence option (d)

Problem 2: A pencil is placed 20.0 cm in front of a concave spherical mirror of focal length f = 15.0 cm. Find the location of the image. State whether the image is real or virtual, erect or inverted and give its lateral magnification?

Solution:



The ray diagram shows that the image is real that, relative to the object, it is farther away from the mirror, inverted and larger.

The given parameters are

f = -15 cm

u = -20 cm

From mirror equation (1/v) = (1/f) – (1/u)

= (1/-15) – (1/-20) = -(1/60)

v = -60 cm

because the image distance is negative, the image is real and is formed in front of the mirror, the reflected rays actually pass through the image.

The lateral magnification is m = -v/u = - [(-60)/ (-20)] = -3

the lateral magnification is negative indicating that the image is inverted with respect to the object.

Problem 3: If the pencil in above Problem is at 10.0 cm in front of the concave mirror of focal length 15 cm, characterize the new image.

Solution: The ray diagram shows that the image is virtual, upright and larger relative to the object. The given parameters are u = -10 cm & f = -15 cm

From mirror equation (1/v) = (1/f) – (1/u) = (1/-15) –(1/-10) = 1/30

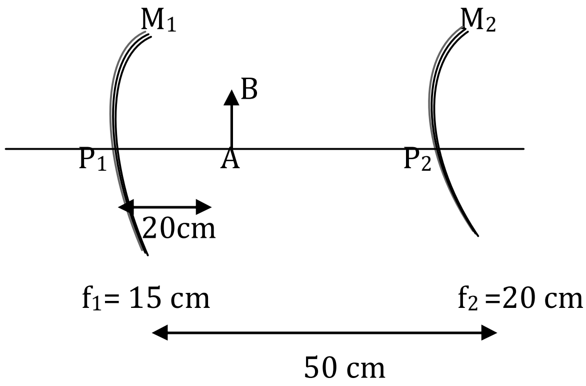
v = + 30 cm

The positive sign shows that image is behind the mirror, virtual; the rays appear to emanate from the image.

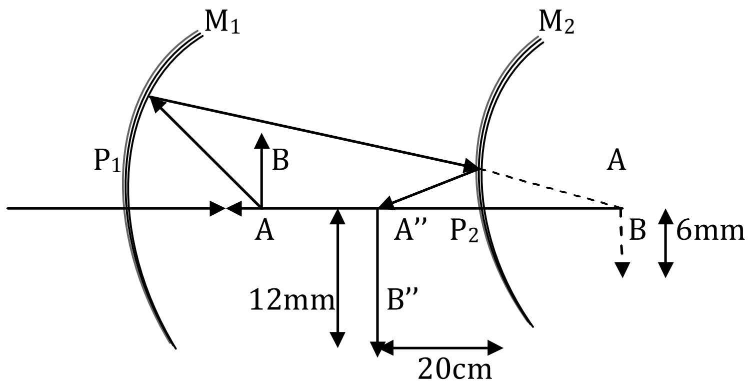
The lateral magnification is m = -v/u = - [(+30)/(-10)] = +3

The magnification is positive indicating that the image is upright; m > 1 implies that is magnified.

Problem 4: Find out position, size and nature of image of an object of height 2mm kept between two mirrors in situation as shown in figure after two successive reflection considering first reflection at concave mirror and then at convex mirror.

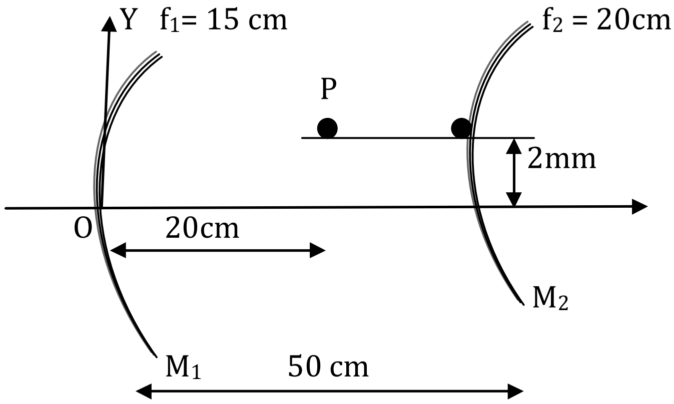


Solution: consider reflection at concave mirror (M_1) u = -20 cm, f = -15 cm
using mirror equation we get, v = uf/(u-f) =[(-20)(-15)]/[(-20)-(-15)] = -60 cm
m_1= -v/u = -(-60)/-20 = -3 (inverted)
A'B'= m_1(AB) = 6mm
The image (A'B') formed by concave mirror acts as object for convex mirror. Now, consider reflection at convex mirror (M_2).



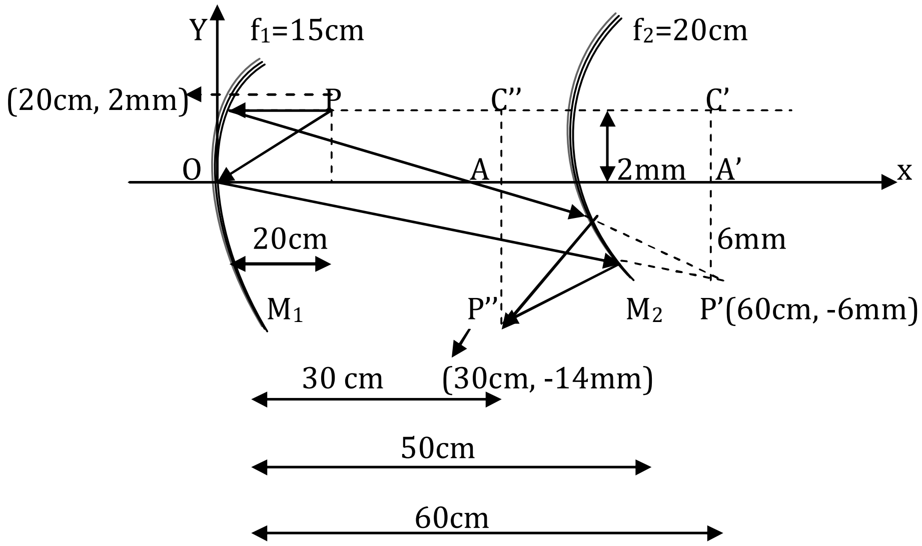
u = +10 cm, f =+20cm
v = vf/(u-f) = [(10)(20)]/[10-20] = -20 cm
m_2 = v = -v/u = -20/10 = 2 (Erect)
A''B'' = m_2(AB) = 12mm
Hence, for final image A''B''
Position: 20cm in front of convex mirror (M_2)
Nature: Real and inverted
Size: 12mm

Problem 5:



Find the co-ordinates of image of point object P formed after two successive reflections in situation as shown in figure. Considering first reflection at concave mirror and then at convex mirror.

Solution:



For reflection at concave mirror M_1

$u = -20$ cm, $f_1 = -15$ cm

$v_1 = uf_1/(u-f_1) = [(-20)(-15)]/[-20+15] = -60$ cm

Magnification $(m_1) = -v_1/u = -[(-60)/(-20)] = -3$ (Inverted)

$A'P' = m_1(AP) = 3*2 = 6$mm

For reflection at convex mirror M_2

$u = +10$ cm, $f_1 = +20$ cm

$v_2 = uf_2/(u-f_2) = [(10)(20)]/[10-20] = -20$ cm

Magnification $(m_2) = -v_2/u = -[(-20)/(10)] = 2$

$C''P'' = m_2(C'P') = 2*8 = 16$mm

so, the co-ordinate of image of point object P (30 cm, -14mm)

Problem 6: A concave mirror forms on a screen a real image of thrice the linear dimensions of the object. Object and screen are moved until the image is twice the size of the object. If the shift of the object is 6 cm then find the shift of the screen and the focal length of the mirror.

a)36 cm, 54cm, b)54 cm, 36 cm,
c)36 cm, 36 cm, d)None

Solution: Initial magnification = 3

i.e. $v_1/u_1 = 3$

$v_1 = 3u_1$

$f = u_1v_1/(u_1+v_1)$

$f= u_1(3u_1)/(u_1+3u_1) = (3/4)u_1$ (1)

In the second case, $m = 2$

i.e. $v_2/u_2 = 2$

$v_2 = 2u_2$

$f = u_2v_2/(u_2+v_2)$

$f= u_2(2u_2)/(u_2+2u_2) = (2/3)u_2$ (2)

From eq. (1) and (2) focal length is same

$(3/4)u_1 = (2/3)u_2$

or $u_2 = (9/8)u_1$ (3)

It is given that the shift of the object = 6 cm= $u_2 - u_1$

$\therefore [(9/8)u_1]-u_1 = 6$

$[(9/8)-1]u_1 = 6$

$(1/8)\ u_1 = 6$

$u_1 = 6*8 = 48$ cm

and $v_1 = 3\ u_1 = 3*48 = 144$ cm

substituting the value of u_1 and v_1 in equation (1)

$f= (3/4)u_1$

$f= (3/4)48 = 36$ cm

From equation (3)

$u_2= (9/8)u_1$

$u_2= (9/8)48 = 54$ cm

$v_2= 2u_2 = 2*54 = 108$ cm

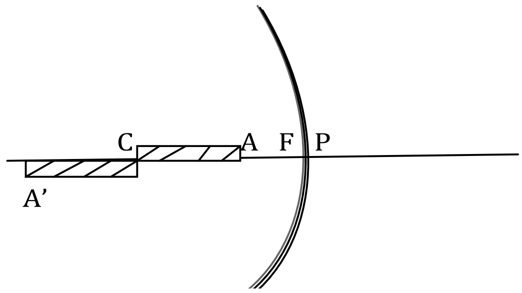
Thus, shift of the screen = $\mathbf{v_1\text{-}v_2}$

$\mathbf{v_1\text{-}v_2} = 144\text{-}108 = 36$ cm

Problem 7: A thin rod of length f/3 is placed along the principle axis of a concave mirror of focal length f such that its image just touches the rod. Calculate magnification?

Solution: Since image touches the rod, the rod must be placed with one end at centre of curvature.

Case (I):



For A:

$u = -[2f-(f/3)] = -5f/3$ $\qquad f = -f$

$v = (uf)/(u-f)$

$v= [(-5f/3)(-f)]/[(-5f/3)-(-f)] = 5f/2$

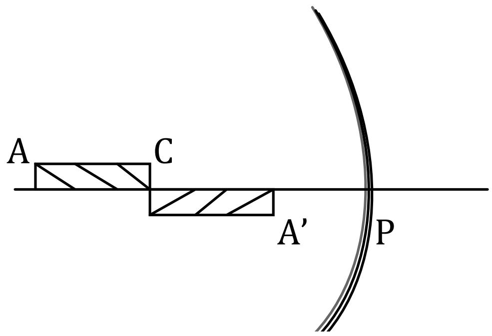
m = Length of the image/Length of the object

$m= [v_A-v_c]/[u_A-u_c]$

$m= [(-5f/2)-(-2f)]/[-5f/3-(-2f)]$

$m= -3/2$ **Ans.**

Case (II):



For A: $u = -[2f+(f/3)] = -7f/3$ $\qquad f = -f$

$v = (uf)/(u-f) = [(-7f/3)(-f)]/[(-7f/3)-(-f)] = -7f/4$

$m= [v_A-v_c]/[u_A-u_c] = [(-7f/4)-(-2f)]/[-7f/3-(-2f)]$

$m= -3/4$

Problem 8: A diamond ring is placed in front of a mirror of radius of curvature. The image is twice the size of the ring. Find the object distance of the ring.

Solution:

The first question arises: is the mirror concave or convex or either is possible? The image formed by a convex mirror is always smaller than the object; therefore the mirror must be concave. The next question arises: how many positions are there in front of a concave mirror, where the ring can be placed and produce an image that is twice the size of the object? There are two places: (1) when the object is placed between centre of curvature and the focal point, the magnified image is real and inverted. (2) When the object is between the focal point and the mirror, the magnified image is virtual and upright.

In first case the image is inverted, so the magnification is m = -2, in the second case the image is upright, so the magnification is m = +2
From mirror equation and magnification equation, $(1/v) + (1/u) = (1/f)$ and $m = -v/u$
On solving magnification equation we have $v = -mu$; substituting this expression for v in mirror equation, we get
$(1/u) + [1/(-mu)] = (1/f)$ or $u = [f(m-1)]/m$
Applying this result, we obtain
$m = -2$, $u = [f(m-1)]/m = [(-12)(-2-1)]/(-2) = -18$ cm
$m = +2$, $u = [f(m-1)]/m = [(-12)(+2-1)]/(+2) = -6$ cm
negative signs for object distances indicate that the object is real, lies in front of the mirror.

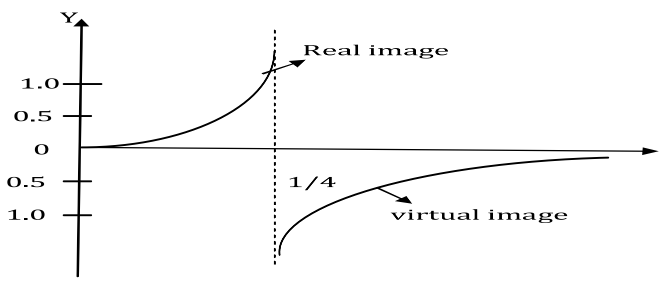
Problem 9(IIT – JEE 2007):

Statement 1: The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.
Statement 2: Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.
(a) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1
(b) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1
(c) Statement 1 is true, statement 2 is false
(d) Statement 1 is false, statement 2 is true

Solution: Laws of reflection can be applied to any type of surface.

Problem 10: When the position of an object reflected in a concave mirror of 0.25 m focal length is varied, the position of the image varies. Plot the image distance as a function of the object distance, taking the object distance from 0 to +∞

Solution:



Problem 11: An object is located 6 cm in front of a mirror. The virtual image is located 4 cm away from the mirror and diminished. Find the focal length of the mirror?

Solution:

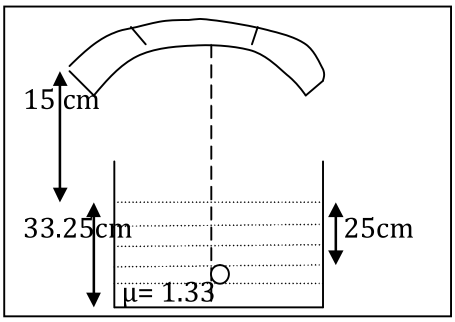
The first question arises: is the mirror concave or convex or either is possible? Since both the mirrors form virtual image if the object is within the focal point of the mirror. A convex mirror always forms a virtual image.

The second question arises that the image is diminished in size as well as virtual; do these characteristics tighter indicate a concave or convex mirror? A concave mirror produces a diminished image only when the object is located beyond the centre of curvature of the mirror. However, the image in this case is real not virtual. A convex mirror always produces an image that is virtual and smaller than the object.

The focal length of a convex mirror is positive.
The given parameters are $u = -6$ cm, $v = +4$ cm
From mirror equation,
$(1/f) = (1/v) + (1/u)$
$(1/f) = (1/4) + (1/-6) = (1/12)$
$f = +12$ cm
As expected, focal length is positive.

Problem 12 (IIT-JEE 2005): A container is filled with water (μ= 1.33) up to a height of 33.25 cm. A concave mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. The focal length of the mirror is

(a) 10 cm (b) 15 cm (c) 20 cm (d) 25 cm



Solution: Distance of object from mirror
= 15+(33.25/1.33)= 40 cm
distance of image from mirror
= 15+(25/1.33) = 33.8 cm
for the mirror, $1/v + 1/u = 1/f$

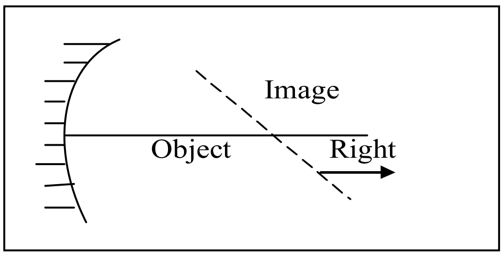
[1/(-33.8)] +[1/(-40)] =1/f
f = -18.3 cm so most suitable answer is (c)

Problem 13: A small plane mirror is placed 21 cm in front of a concave mirror of focal length 21 cm. An object is placed 42 cm in front of the concave mirror. If light from the concave mirror strikes the plane mirror, where is the final image?

Solution: First we will obtain image position from concave mirror, by mirror equation. The given parameters are
f = -21 cm
u = -42 cm
(1/v) = (1/f) – (1/u) = (1/-21) - (1/-42) = -(1/42) cm
v = -42 cm
The image is 42 cm in front of the mirror. The image formed by the concave mirror is object for the plane mirror. Now we use the mirror equation for the plane mirror with
f = ∞, u = -[42-21] cm
(1/v) + (1/-21)= (1/∞)
v = 21 cm
The position of the final image is 21 cm in front of the plane mirror.

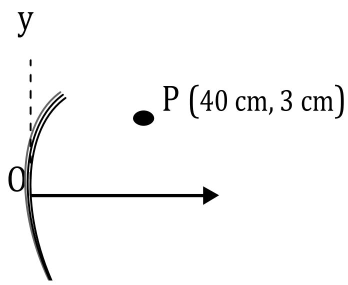
Problem 14(IIT- JEE 2007): In an experiment to determine the focal length (f) of a concave mirror by the u-v method, a student places the object pin A on the principal axis at a distance x from the pole p. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then
(a) x < f (b) f< x< 2f (c) x = 2f (d) x > 2f

Solution:



Since object and image move in opposite directions. The positioning should be as shown in the figure. Object lies between focus and centre of curvature f < x < 2f

Problem 15:



The Co-ordinates of the image of point object P formed by a concave mirror of radius of curvature 20 cm (consider paraxial rays only) as shown in the figure is

(a) 13.33 cm, -1 cm (b) 13.33 cm, +1 cm

(c) -13.33 cm, +1 cm (d) -13.33 cm, -1 cm

Solution: 1/v-1/u=1/f

1/v-1/40=1/-10

1/v=1/40-1/10

v=-40/3

u=-40, f=-10

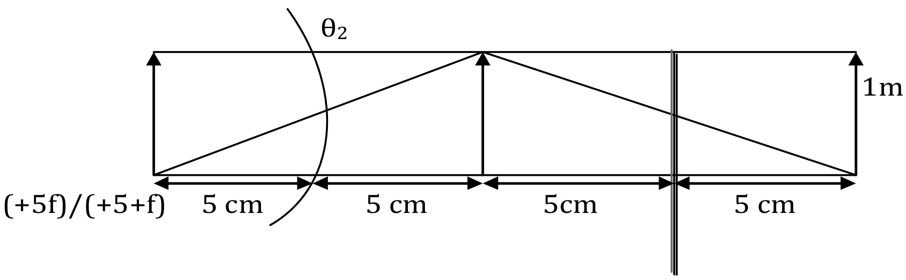
v = uf/u-f = (-40)(-10)/-40-(10)=-40/3 cm

m= h_1/3=-(40/3)/-40=-1/3

h_1=-1cm

Problem 16: *A Large convex spherical mirror in an amusement park is facing a plane mirror 10 m away a boy of height 1m standing midway between the two sees himself twice as tall as in plane mirror as in spherical one. In other words, the angle subtended at the observer by the image in the plane mirror is twice the angle subtended by the image in the spherical mirror. What is the focal length of convex mirror?*

Solution: 1/10 = θ_1= 0.1 rad.



(h_1 /1) = 5f/ (5+f) ⇒ h_1 = f/ (5+f)

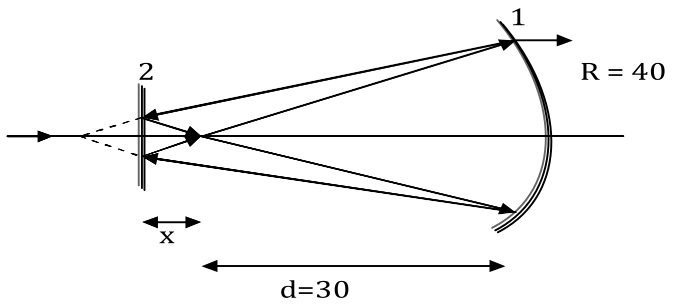
θ_2= [f/ (5+f)]/5+ [5f/ (5+f)] = f/ [25+10f] = 0.1/2

2f = 2.5 +f

f = 2.5m

Problem 17: A bright point S is on the principal axis of a concave mirror of radius R = 40 cm at d = 30 cm from its pole. At what distance (in cm) in front of the mirror should a plane mirror be placed so that after two reflections, the rays converge back at point S.

Solution:



For mirror 1

(1/v) + (1/u) = (1/f)   y = -30

(1/v) – (1/30) = - (1/20)   f = -20

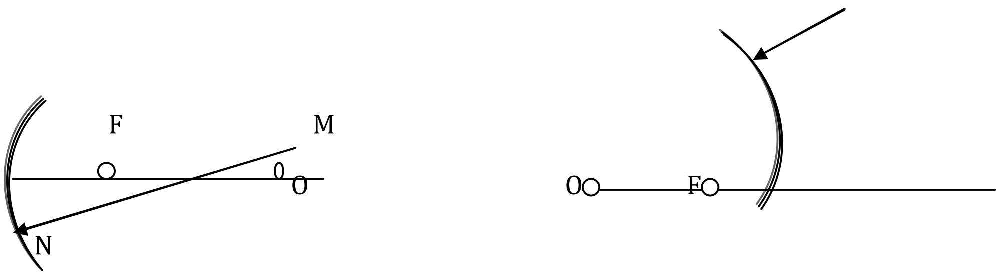
(1/v) = (1/30) – (1/20)

v = -60 cm

so x = 15cm

so distance between two mirrors = 45 cm

Problem 18:

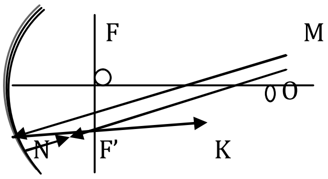


(a)   (b)

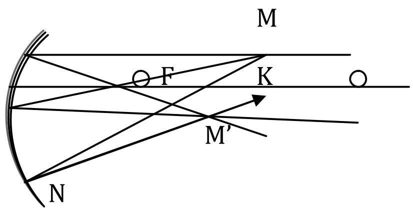
(a) A ray of light falls on a concave spherical mirror, as shown in figure (a). Trace the path of the ray further

(b) A ray of light falls on a convex mirror, as shown in figure (b). Trace the path of the ray further

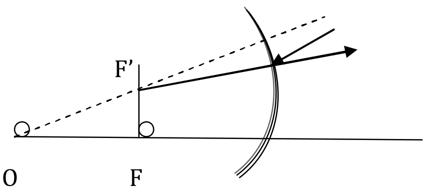
Solution: (a) consider a beam of rays parallel to the ray MN. The beam reflected from the mirror will converge at the secondary focus F, which lies in the focal plane. Drawing the ray DO ∥ MN parallel to the centre of the mirror, we find the secondary focus F'. The ray NK is the one we are looking for fig. (c)



(c)



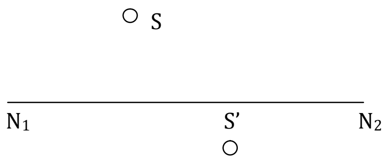
(d)



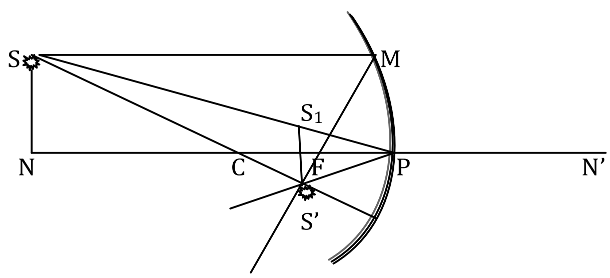
(e)

Method 2:
Choose an arbitrary point M on the ray MN and with the aid of characteristic rays construct its image M’. The required ray NK passes through this point (see in fig. (d))
(b) the paths of the rays are shown in fig. (e)

Problem 19: Point S’ is the image of a point source of light S in a spherical mirror whose optical axis is N_1N_2. Find by construction the position of the centre of the mirror and its focus.



Solution: Since the ray incident on the mirror at its pole is reflected symmetrically with respect to the major optical axis, let us plot point S_1 symmetrical to S’ and draw ray SS_1 until it intersects the axis at point P. This point will be the pole of the mirror.
The optical centres C of the mirror can obviously be found as the point of intersection of ray SS’ with axis NN’.



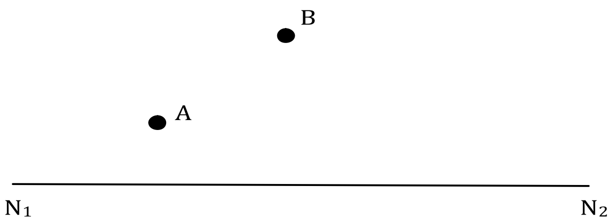
The focus can be found by the usual construction of ray SM parallel to the axis. The reflected ray must pass through focus F (lying on the optical axis of the mirror) and through S'.

Problem 20: A point source S is placed midway between two converging mirrors having equal focal length f as shown in figure. find the values of d for which only one image is formed.

Solution:

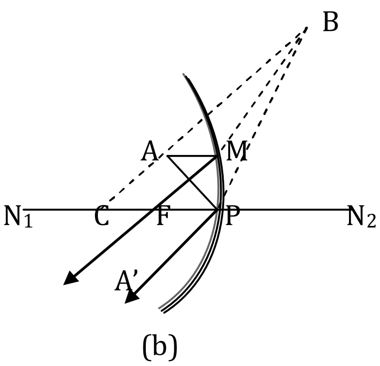
When S is placed at the common focus of mirrors, the rays after reflection from one mirror incident parallel on to the second mirror, which finally intersect at focus of the mirror. Thus there will be only one image. In this case the value of d will be 2f. When S is placed at the centre of curvature, the image will form at the same point, so in this case the value of d will be 2f +2f = 4f

Problem 21: The positions of optical axis N_1N_2 of a spherical mirror, the source and the image are known (fig. (a)) find by construction the positions of the centre of the mirror, its focus and the pole for the cases: (a) A- source, B-image; (b) B- source, A-image.

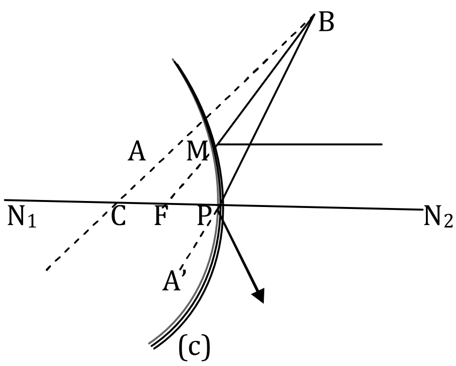


(a)

Solution:



(b)



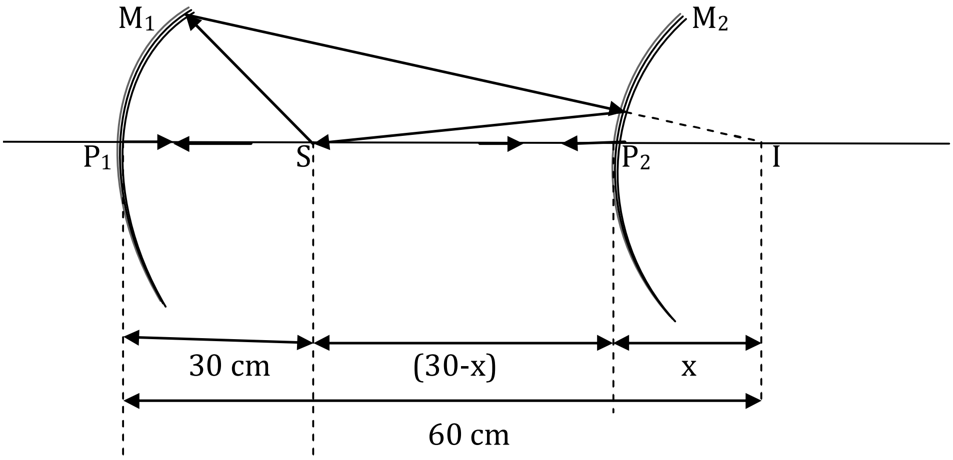
(c)

(a) Let us construct, as in the previous example, the ray BAC and find point C (optical centre of the mirror) [fig. (b)]. Pole P can be found by constructing the path of the ray APA' reflected in the pole with the aid of symmetrical point A'. The position of the mirror focus F is determined by means of the usual construction of ray AMF parallel to the axis.

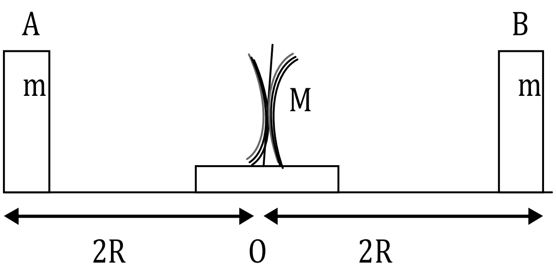
(b) The construction can also be used to find the centre C of the mirror and pole P [fig. (c)]. The reflected ray BM will pass parallel to the optical axis of the mirror.
For this reason, to find the focus, let us first determine point M at which straight line AM, parallel to the optical axis, intersects the mirror and then extend BM to the point of intersection with the axis at the focus F.

Problem 22: A converging mirror M_1, a point source S and a diverging mirror M_2 are arranged as shown in the figure. The source is placed at a distance of 30 cm from M_1. The focal length of each of the mirror is 20 cm. Consider only the images formed by a maximum of two reflections. It is found that one image is formed on the source itself.
(a) Find the distance between the mirrors
(b) Find the location of the image formed by the single reflection from M_2.

Solution: **For mirror M_1**: u = -30 cm, f = -20 cm
By mirror formula, (1/u) + (1/v) = (1/f), we have
(1/-30) + (1/v) = (1/-30)
which on solving gives, v = -60 cm
For mirror M_2: The image formed by mirror $M_1$is a virtual object for mirror M_2. Let it is a distance x from the pole of mirror M_2.
Thus u_2 = +x, v_2 = - (30-x)
Again by mirror formula. We have
(1/x) + (1/-(30-x)) = (1/20)
which on solving gives x = 10 cm or 60 cm,
x = 60 cm is not possible, thus x = 10 cm.
(a) Thus the separation between the mirrors = 60-10= 50 cm
(b) The image formed by mirror M_2 is at a distance 10 cm



Problem 23:



Two concave mirrors equal radii of curvature R are fixed on a stand facing opposite directions. The whole system has a mass m and is kept on a frictionless horizontal table. Two blocks A and B, each of mass m, are placed on the two sides of the stand. At t =0, the separation between A and the mirror is 2R and also the separation between B and the mirror is 2R. The block B moves towards the mirror at a speed v. All collisions which take place are elastic. Taking the original position of the mirrors standard system to be x = 0 and x-axis along AB, find the position of the image of A and B at;

(a) $t = (R/v)$ (b) $t = (3R/v)$ (c) $t = 5R/v$

Solution: (a) At $t = R/v$

For block A, $u = -2R$

$\therefore (1/v) + (1/-2R) = 2/-R$

or $v = -2R/3$

For block B: The distance travels by block B in time (R/v) is R

Thus $u = -R$

$(1/v) + (1/-R) = (2/-R)$

or $v = -R$

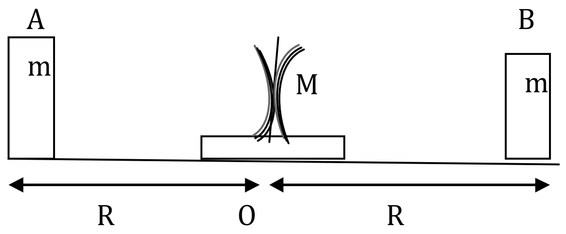
The x- coordinate of the image of the block with respect to the mirror will be +R

(b) At $t = 3R/v$

The block B will collide with the stand after time 2R/v.

After collision block B becomes at rest and mirror starts moving with the same velocity v. In the remaining time R/v, the distance moved by the mirror is R.

The position of blocks and mirror are shown in fig. (b)



(b)

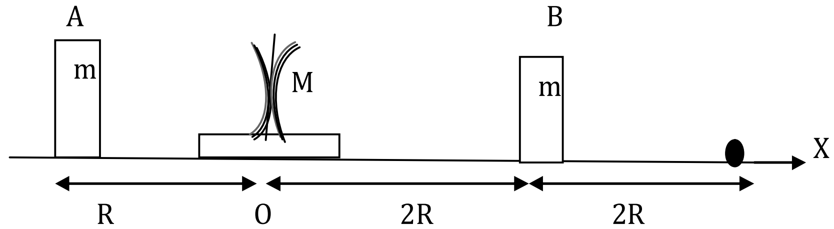
At this time the blocks lie at the centre of curvature of the respective mirrors. Their images will form at the centres of curvature. So their coordinates are:

For block A, $x = -R$

For block B, $x = +R$

(c) At $t = 5R/v$

The block B will collide to the mirror after a time (2R/v), thereafter mirror starts moving towards block A with velocity v, At t = 4R/v, the mirror will collide with block A and stops after collision. The positions of blocks and mirror are shown in fig. (c)



(c)

For block A: Its image will form on the same place.

Therefore the positions of the blocks are $x_A = -3R$

For block B: $u = -2R$

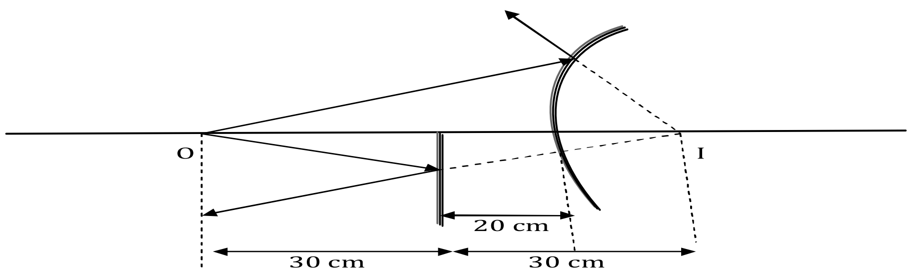
$(1/v) + (1/-2R) = (2/-R)$

v = -2R/3
The co-ordinates of B : – (2R-(2R/3)) = -4R/3

Problem 24: An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced converging lower half of the mirror is 30 cm, it is found that there is no parallax between the images formed by two mirrors.

The radius of curvature of the convex mirror is

a) 15 cm b) 20 cm c) 25 cm d) none



Solution:

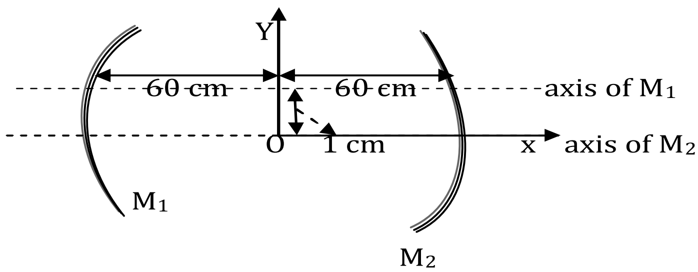
The distance of the object from the plane mirror is 30 cm and so the distance of its image is also 30 cm from the mirror.

As image formed by both the mirrors coincide, so distance of image for convex mirror is =10 cm

By mirror formula (1/u) + (1/v) = (1/f)
(1/f) = (1/10) + (1/-50)

Which on solving give f = 12.5 cm
∴ Radius of curvature R = 2f = 25 cm

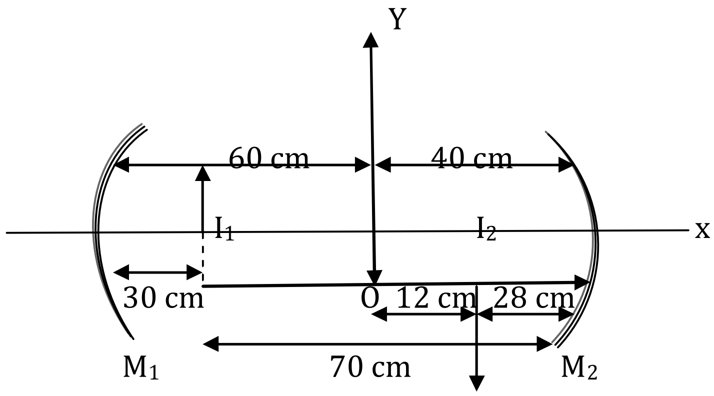
Problem 25:



(a)

Two concave mirrors each of radius of curvature 40 cm are placed such that their principal axis are parallel to each other and at a distance of 1 cm to each other. Both the mirrors are at a distance of 100 cm to each other. Consider first reflection at M_1 and then at M_2, find the coordinates of the image thus formed. Take location of object as the origin.

Solution:



Using mirror formula for first reflection:

$(1/f) = (1/v) + (1/u)$ $\Rightarrow$ $(1/-20) = (1/v) + (1/-70)$

$\Rightarrow (1/v) = (1/60) - (1/20)$ $\Rightarrow$ $v = -30$ cm

Using mirror formula for first reflection:

$(1/f) = (1/v) + (1/u)$ $\Rightarrow$ $(1/-20) = (1/v) + (1/-70)$

$(1/v) = (1/70) - (1/20) = [(2-7)/140]$

$\Rightarrow v = -(140/5) = -28$ cm

Height of $I_2 \Rightarrow m = (-30/-60) = I_2/-1 \Rightarrow I_1 = (1/2)$ cm

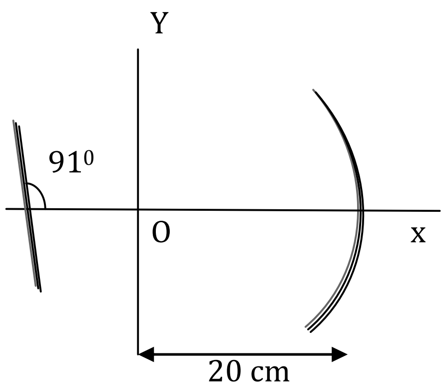
Height of first image from x-axis $= 1 + (1/2) = 3/2$ cm

Height of $I_2 \Rightarrow m = (-28/-70) = 2I_2/3$

$\Rightarrow I_2 = [3*28]/[2*70]$

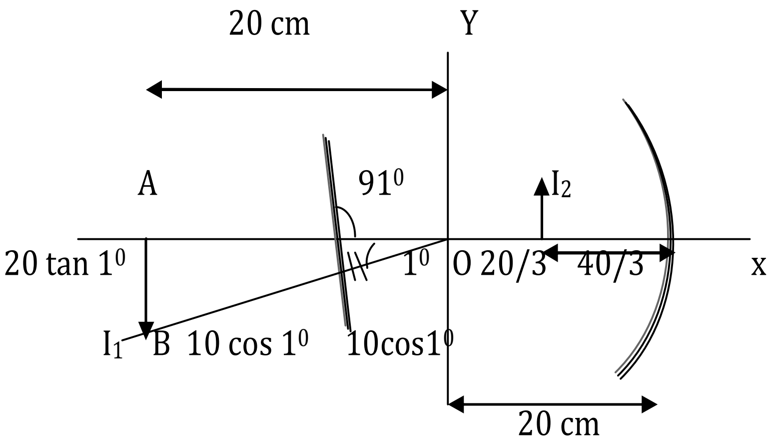
$I_2 = -0.6$ cm co-ordinate of $I_2 = (12 - 0.6)$

Problem 26:



A point object is placed at the centre of curvature of a concave mirror (take as origin). A plane mirror is also placed at a distance of 10 cm from the object as shown. Consider two reflection first at plane mirror and then at concave mirror. Find the coordinate of the image thus formed.

Solution:



Distance of I_1 from mirror = 40 cm

(1/f) = (1/v) + (1/u)

⇒ (1/-10) = (1/v) + (-1/-140)

⇒ (1/v) = (1/40) – (1/10)

⇒ (1/v) = [(1-4)/40]

⇒ v = -40/3 cm

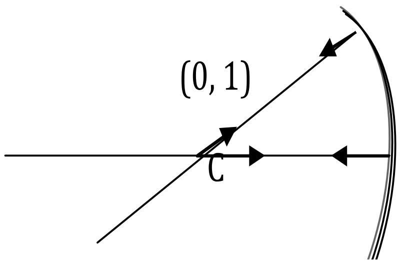
Using magnification formula m = I/O = -v/u

⇒ I/-20tan 1^0 = -[(-40/30)/-40]

⇒ I = 20 tan 1^0/3 = (20/3) tan (π/180) ≈ (20/3)(π/180) = π/27

co-ordinate of image = [(20/3), (π/27)]

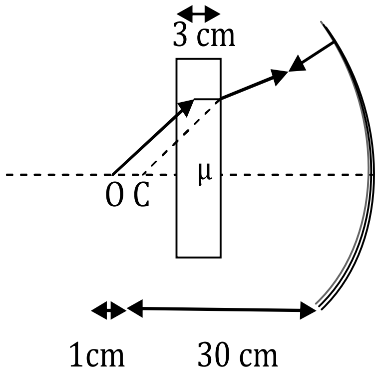
Problem 27:



The image of an object kept at a distance 30 cm in front of a concave mirror is found to coincide with itself. If a glass slab (μ = 1.5) of thickness 3 cm is introduced between the mirror and the object, then (i) Identity, in which direction the mirror should be

displaced so that the final image may again coincide with the object itself.
(ii) Find the magnitude of displacement

Solution:



(i) Since the apparent shift occurs in the direction of incident light, therefore, the mirror should be displaced away from the objects.

(ii) (ii) The magnitude of displacement is equal to the apparent shift, i.e.
$s = t\,[1 - (1/\mu)] = 3[1-(1/(3/2))] = 1$ cm

Lecture-4

Geometrical Optics

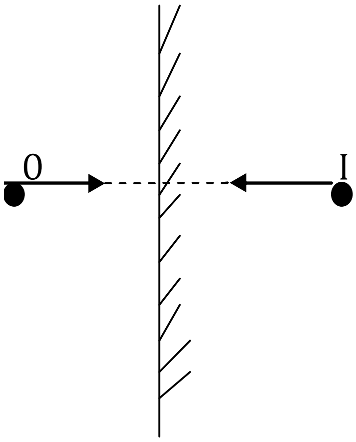
Physics for IIT - JEE

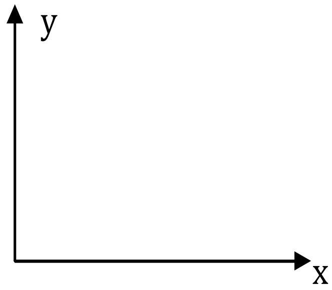
- **Velocity of image in the plane mirror**
- **Velocity of image in spherical mirror**

Velocity of image in plane mirror:

X_{OM} = x coordinate of object with respect to mirror
X_{IM} = x coordinate of image with respect to mirror
Y_{OM} = y coordinate of object with respect to mirror
Y_{IM} = y coordinate of image with respect to mirror

For plane mirror:





***X Coordinate*:**

$X_{OM} = - x_{IM}$

$(d/dt)\, X_{OM} = -(d/dt)\, \mathbf{X}_{IM}$

$(V_x)_{OM} = -(V_x)_{IM}$

$[\mathbf{V_0}-\mathbf{V_M}]_x = -[V_I - V_M]_x$

$\mathbf{V_{OX} - V_{MX}} = -V_{XI} + V_{XM}$

$\mathbf{V_{IX} = 2\, V_{MX} - V_{OX}}$

$\mathbf{(V_I)_X = 2(V_M)_X - (V_0)_X}$

***Y Coordinate*:** $Y_{OM} = Y_{IM}$

$\mathbf{(V_{OM})_Y = (V_{IM})_Y}$

Note: In nutshell, for solving numerical problems involving calculation of velocity of image of object with respect to any observer, always calculate velocity of image first with respect to mirror using following points:

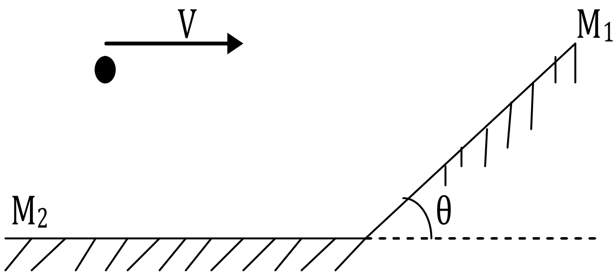
$\mathbf{(V_{IM})_{\parallel} = (V_{OM})_{\parallel}}$ **&** $\mathbf{(V_{IM})_{\perp} = -(V_{OM})_{\perp}}$

$\mathbf{(V_{IM}) = (V_{IM})_{\parallel} + (V_{IM})_{\perp}}$

Velocity of image with respect to required observer is then calculated by using equation $\mathbf{V_{AB} = V_A - V_B}$.

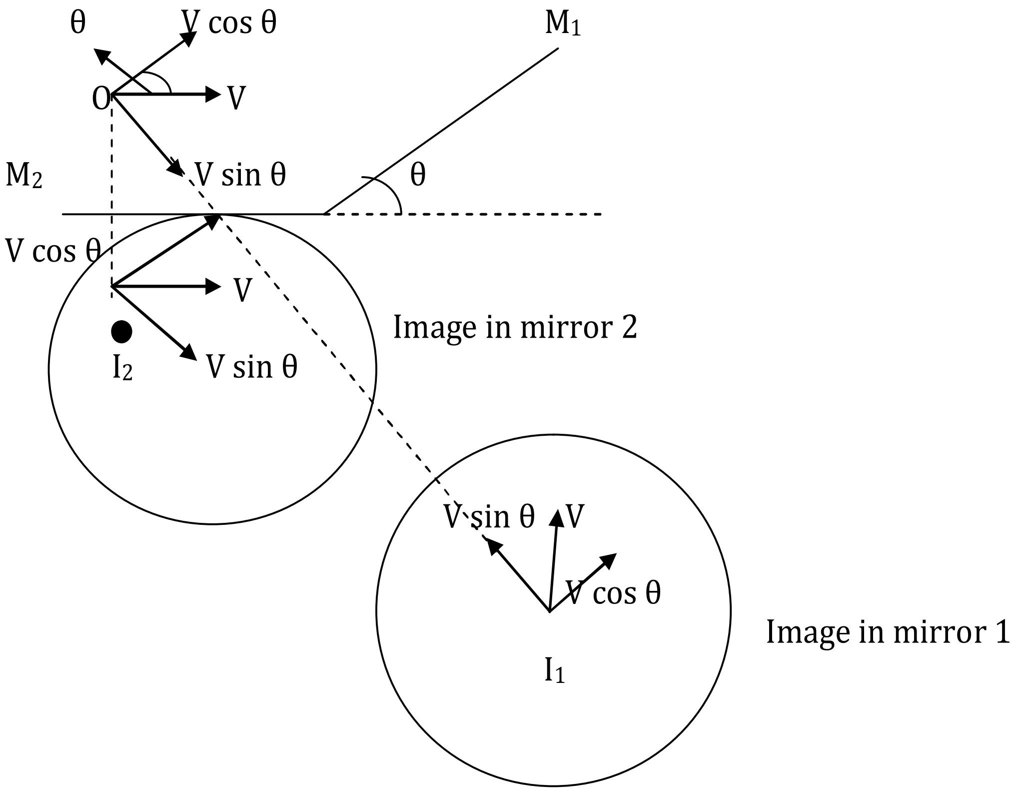
Problem 1: A point object is moving with a speed v before an arrangement of two mirrors as shown in figure find the velocity of image in mirror M_1 with respect to image in mirror M_2

a) $2v \sin\theta$ b) $2v \cos\theta$ c) $2v \tan\theta$ d) $v \sin\theta$



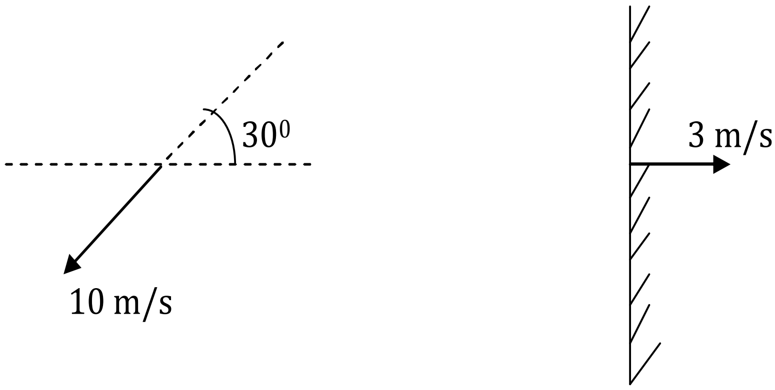
Solution:

Velocity of image in mirror M_1& M_2 is as shown in fig.



V_{12}=velocity of $I_1$with respect to I_2 = v_1-v_2 = 2v sin θ

Problem 2: A point object is moving with speed of 10m/s in front of a mirror moving with a speed of 3 m/s as shown in figure. Find the velocity of image of the object with respect to mirror, object & ground.



(1) The velocity of image of the object with respect to mirror

(a) $(5\sqrt{3}+3)\hat{i}-5\hat{j}$
(b) $(5\sqrt{3}+3)\hat{i}+5\hat{j}$
(c) $(5\sqrt{3}+3)\hat{j}-5\hat{i}$
(d) None

(2) Velocity of image of the object with respect to object

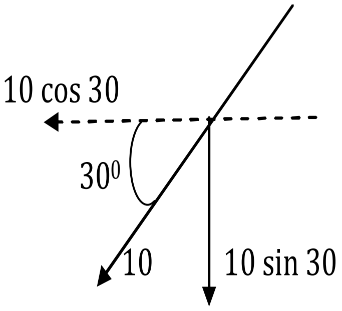
(a) $(10\sqrt{3}+6)\hat{i}$
(b) $(10\sqrt{3}+6)\hat{j}$
(c) $(10\sqrt{3}-6)\hat{i}$
(d) None

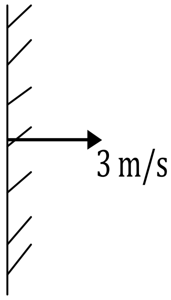
(3) The velocity of image of the object with respect to ground

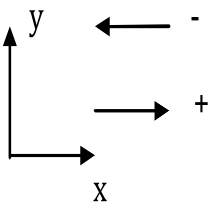
(a) $(5\sqrt{3}+6)\hat{i}-5\hat{j}$
(b) $(5\sqrt{3}+6)\hat{i}+5\hat{j}$
(c) $(5\sqrt{3}-6)\hat{i}+5\hat{j}$
(d) None

Solution: $V_{IM}=?$, $V_{IO}=?$, $V_{IG}=?$

Given: $\mathbf{V_0=-5\sqrt{3}\hat{i}-5\hat{j}} = V_{O\perp} + V_{0\|}$







$5\sqrt{3}=10\cos 30$

$V_m=3\hat{i}$

$(V_{IM})_{\perp}=(V_I-V_M)_{\perp}=-(V_{OM})_{\perp}=(V_M-V_O)_{\perp}$

$(V_{IM})_{\perp}=-(V_{OM})$

$(V_{IM})_{\perp}=(3\hat{i}+5\sqrt{3}\hat{i})$

$(V_{IM})_{\parallel}=(V_{OM})_{\parallel}=(V_0-V_M)_{\parallel}$

$(V_{IM})_{\parallel}=-5\hat{j}$

$(V_{IM})=(V_{IM})_{\parallel}+(V_{IM})_{\perp}$

$(V_{IM})=-5\hat{j}+3\hat{i}+5\sqrt{3}\hat{i}=(3+5\sqrt{3})\hat{i}=5\hat{j}$

$V_{IO}=V_I-V_O$ for this we need first velocity of image

$V_{IM}=V_I-V_M$

$V_{IM}+V_M=V_I$

$(5\sqrt{3}+3)\hat{i}-5\hat{j}+3\hat{i}=V_I$

$\Rightarrow V_I=(5\sqrt{3}+6)\hat{i}-5\hat{j}$

$V_{IO}=V_I-V_O$

$V_{IO}=(5\sqrt{3}+6)\hat{i}-5\hat{j}-(-5\sqrt{3}\hat{i}-\hat{j})=10\sqrt{3}\hat{i}+6\hat{i}+0\hat{j}$

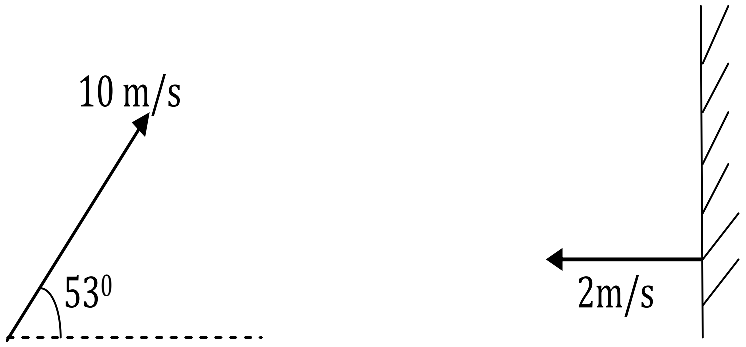
$V_{IO}=(10\sqrt{3}+6)\hat{i}$

$V_{IG}=V_I-V_G$

If ground is supposed to be at rest: $V_G=0$

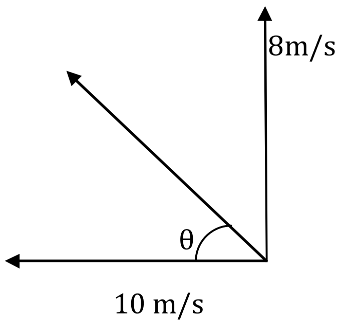
$V_{IG}=V_I=(5\sqrt{3}+6)\hat{i}-5\hat{j}$

Problem 3: Find the velocity of image of a moving particle shown in figure.



Given: 10 cos 53=6, 10 sin 53=8

Solution:



$(V_I)_X = 2(V_M)_X - (V_O)_X$

$(V_I)_\perp = 2(V_M)_\perp - V_{O\perp}$

$(V_I)_X = 2(-2)-6 = -10 m/s\ \hat{i}$

$(V_I)_Y = (V_O)_y$

$(V_I)_Y = 8\hat{j}$

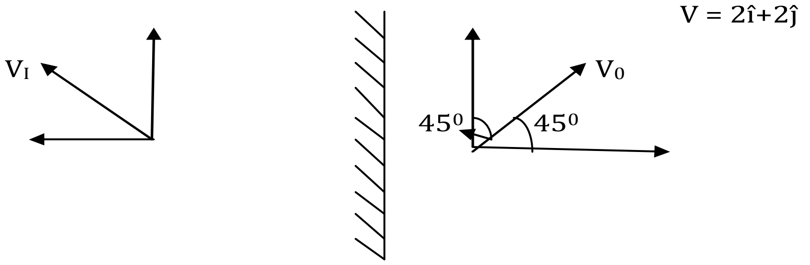
$\mathbf{V_I = -10\hat{i}+8\hat{j}}$

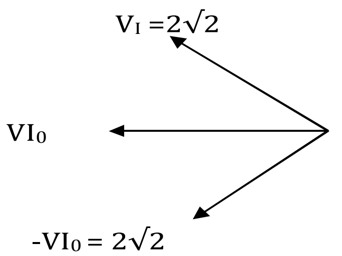
$V_I = \sqrt{8^2+6^2} = \sqrt{164}$

$\boldsymbol{\theta = \tan^{-1}(4/5)}$

Problem 4: A plane mirror is placed at origin parallel to y-axis facing the positive x-axis .an object starts from (2m,0,0) with a velocity($2\hat{i}+2\hat{j}$)m/s. The relative velocity of image with respect to object is
along
(a) Positive x-axis
(b)Negative x-axis
(c) Positive y-axis
(d) Negative y-axis

Solution:



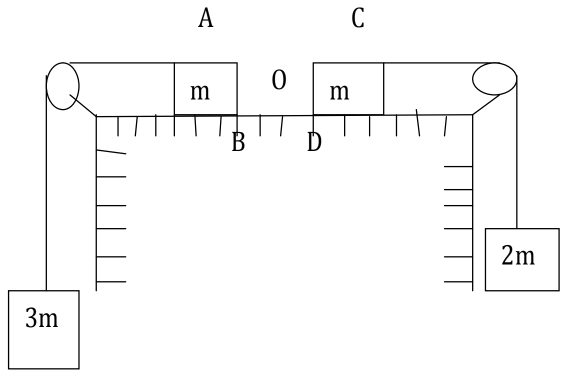


Towards negative x- axis

$\mathbf{V_I - V_O = V_{IO}}$
$V_O = 2\sqrt{2}$m/s $u = 2\sqrt{2}\sqrt{2}$
$V_I = 2\sqrt{2}$m/s $V_{IO} = 4$m/s
Hence the relative velocity of image with respect to object along –ive x-axis.

Problem 5: Two blocks each of mass m lie on a smooth table. They are attached to two other masses as shown in the figure. The pulleys and strings are light. An object O is kept at rest on the table. The sides AB &CD of the two blocks are made reflecting .the acceleration of two images formed in those two reflecting surface with respect to each other is

(a) 5g/6   (b) 5g/3   (c) g/3   (d) 17g/6



Solution: $V_I=2V_M+V_O$

$a_I= 2a_M+a_O \rightarrow$ zero

a_A =acceleration of mass AB

$T = m\, a_A$

$3mg-T=3ma_A$

$3mg=4ma_A$

$\mathbf{a_A=(3/4)g}$

A_C =acceleration of mass CD

$T = m\, a_C$

$2mg-T=2ma_C$

$2mg=3ma_c$

$\mathbf{a_c= (2/3)g}$

Acceleration of image in AB=$2a_A$= 2(3/4) g = (3/2)g

Acceleration of image in CD=$2a_C$ = 2(2/3)g = (4/3)g

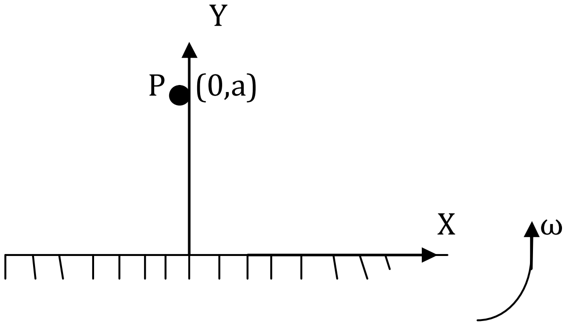
Acceleration of image in AB with respect to CD is = (3/2)g + (4/3)g =(17/6)g m/s

Problem 6: A plane mirror is placed along the xz-plane and an object P is placed at point (0, a) the mirror rotates about z-axis with constant angular velocity ω.

(1) The position of image as function of time $t < (\pi/2\omega)$
- (a) $r = a \sin 2\omega t\, \hat{i} - a \cos 2\omega t \hat{j}$
- (b) $r =- a \sin 2\omega t\, \hat{i} + a \cos 2\omega t \hat{j}$
- (c) $r = a \sin \omega t\, \hat{i} + a \cos \omega t\, \hat{j}$
- (d) $r = a \sin \omega t\, \hat{i} - a \cos \omega t\, \hat{j}$

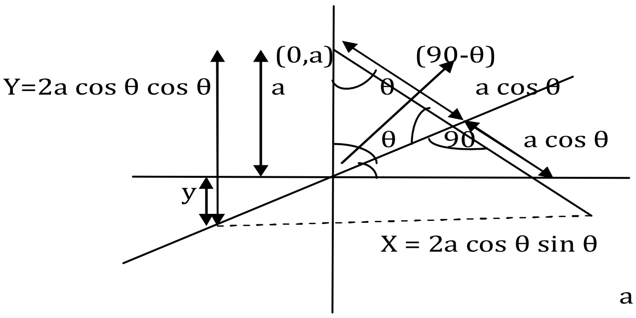
(2) the velocity of image as function of time $t<\frac{\pi}{2\omega}$
- (a) $\mathbf{V}=2\omega a[\cos 2\omega t\, \hat{i} +\sin 2\omega t\, \hat{j}]$
- (b) $\mathbf{V}=2\omega a[\cos \omega t\, \hat{i} - \sin \omega t\, \hat{j}]$
- (c) $V=\omega a\, [-\cos \omega t\, \hat{i} +\sin \omega t\, \hat{j}]$
- (d) None

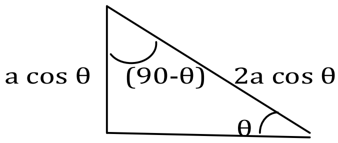
Solution:



Mirror rotates about z-axis

$\omega = \theta/t$ $\theta = \omega t$





$\sin\theta = a\cos\theta/x$

$x = 2a\cos\theta\sin\theta$

$y = 2a\cos\theta\cos\theta$

$x = 2a\cos\theta\sin\theta$

$y = -(2a\cos^2\theta - a) \quad = -a[2\cos^2\theta - 1] = -a\cos 2\theta$

$X = a\sin 2\theta$

$Y = -a\cos 2\theta$

$(r_1/v) = a\sin 2\theta\,\hat{i} - a\cos 2\theta\,\hat{j} = a\sin 2\omega t\hat{i} - a\cos 2\omega t\,\hat{j}$

$= 2a\omega\cos 2\omega t\,\hat{i} + 2a\omega\sin 2\omega t\,\hat{j}$

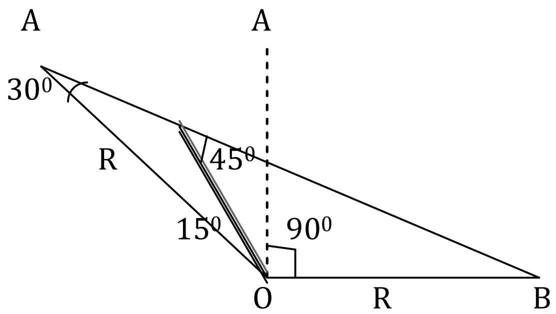
Problem 7: The mirror of length 2l makes 10 revolutions per minute about the axis crossing its midpoint O and perpendicular to the plane of the figure. there is a light source in point A and an observer in point B of the circle of radius R drawn around centre O (AOB = 90^0)

(a) Along what curve does the virtual image of the point-like light source A move?

(b) At what speed does the virtual image of A move?

(c) What is the ratio l/R if the observer B first sees the light source when the angle of the mirror is $\phi = 15^0$?

Solution:



(a) circle

(b) $\omega_M = 10(2\pi/60) = \pi/3$ rad./s

$\omega_1 = 2\ \omega_M = 2(\pi/3)$ rad./s

$v_1 = \omega_1 R = (2\pi/3)R$ m/s

(c) In $\Delta OA'C$ $\quad \sin 30^0/l = \sin 135^0/R$

$\Rightarrow \quad 1/2l = 1/R\sqrt{2}$

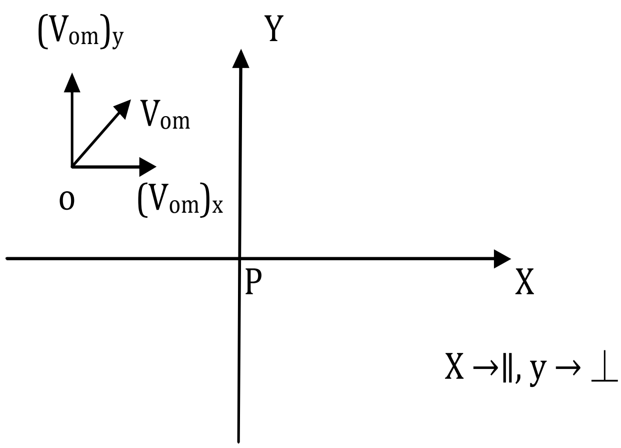
$\Rightarrow \quad l/R = 1/\sqrt{2}$

Velocity of image in spherical mirror:

$(1/v) + (1/u) = (1/f)$	$(1/v) + (1/u) = (1/f)$
$(u/u) + (u/v) = (u/f)$	$(v/v) + (v/u) = (v/f)$
$1 + (-1/m) = (u/f)$	$1 + (-m) = (v/f)$
$1 - (u/f) = 1/m$	$1 - m = (v/f)$
$(f-u)/f = 1/m$	$1 - (v/f) = m$
$m = f/(f-u)$	**$(f-v)/f = m$**

Let pole of mirror be origin of co-ordinate system X-axis be the principle axis of mirror Y-axis is $\perp$ to principle axis.

Object is placed such that incident rays travels along + X-axis.



From mirror equation:$(1/v) + (1/u) = (1/f)$

$[1/x_{Im}] + [1/x_{om}] = (1/f)$
Diff both sides with respect to t
$-(1/x^2_{IM})(dx_{IM}/dt) - (1/x^2_{OM})(dx_{OM}/dt) = 0$
$-(1/x^2_{IM})(\mathbf{V_{IM}})_x = (1/x^2_{OM})\ \mathbf{(V_{OM})_x}$
$(\mathbf{V_{IM}})_x = -[(x^2_{IM}/x^2_{OM})]\ \mathbf{V_{OM}}$
$\mathbf{(V_{IM})_x = -\ m^2\ (V_{OM})_x}$ (1)

As we know that $m = f/(f-u) = [y_{IM}/y_{OM}] \rightarrow m = I/O = v/u$
$y_{IM} = f/(f-u)\ y_{OM}$
diff with respect to t:
$(d/dt)(y_{IM}) = (d/dt)(\ f/(f-u)y_{OM}$
$(\mathbf{V_{IM}})_Y = (f/(f-u) = (\mathbf{V_{OM}})_y + (\mathbf{Y_{OM}})(d/dt)(\ f/(f-u))$
$(\mathbf{V_{IM}})_Y = (f/(f-u))(\mathbf{V_{OM}})_y + (\mathbf{Y_{OM}})[-f(d/dt)(f-u)+(f-u)(df/dt)]/[(f-u)^2]$
$(\mathbf{V_{IM}})_Y = (f/(f-u)(\mathbf{V_{OM}})_y + (\mathbf{Y_{OM}})[f/(f-u)^2\ (du/dt)]$

$\mathbf{(V_{IM})_Y = f/(f-u)(V_{OM})_y + (Y_{OM})[\ f/(f-u)^2\ (du/dt)]}$ (2)

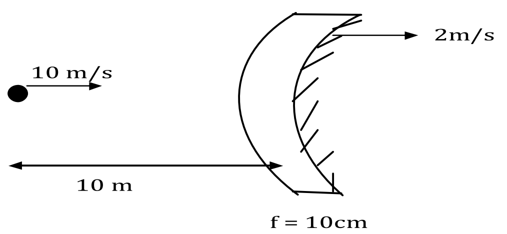
Note:
(du/dt)→ Zero, If u is constant with time.
(du/dt) = +ive, If u is increasing with time.
(du/dt) = -ive, If u is decreasing with time

Particular Case: **1st** if object is placed on the principle axis $\mathbf{Y_{OM}} = 0$
$\mathbf{(V_{IM})_Y = f/(f-u)(V_{OM})_y}$ (2a)
Case 2nd: if object is not on principle axis but moving parallel to principle axis
$\mathbf{(V_{OM})_y} = 0\ \ \mathbf{(V_{IM})_Y = -\ (Y_{OM})[(f/(f-u)^2)(du/dt)]}$ (2b)
Case 3rd: *If object is on principle axis but moving along it then* $(\mathbf{V_{IM}})_x = 0$

In general for any situation, use mirror equation for component of velocity parallel to principle axis & magnification formula for component of velocity ⊥ to principle axis.

Problem 8: find the velocity of image in situation as shown in fig.

(a) Image will move with speed of 2m/s towards mirror
(b) Image will move with speed of 4m/s towards mirror
(c) Image will move with speed of 2m/s opposite to mirror
(d) None



Solution:

m=f/(f-u) = 10/(10-(-10)) = ½

$(\mathbf{V_{IM}})_x = -m^2(\mathbf{V_{OM}})_x$

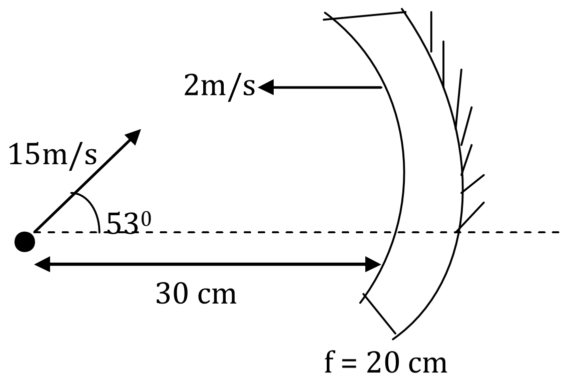
$(\mathbf{V_{OM}})=\mathbf{V_0}-\mathbf{V_m} = 10\hat{i} - 2\hat{i} = 8\hat{i}$

$(\mathbf{V_{IM}})_x = -(1/2)^2(8)\hat{i}$

$(\mathbf{V_{IM}})_x = -2i$ m/s

Hence the image will appear to be moving with a speed of 2m/s towards mirror.

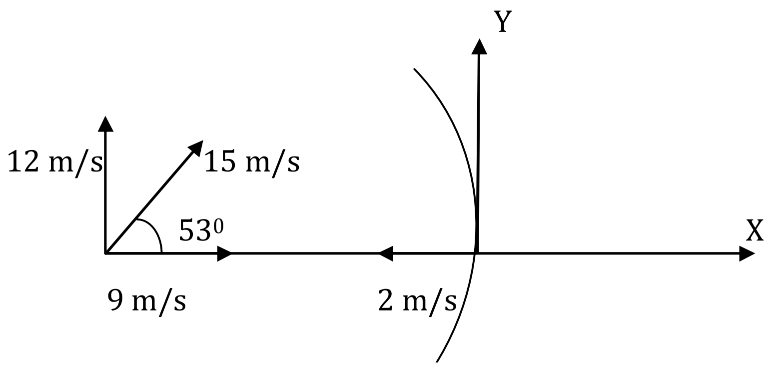
Problem 9: find the velocity of image in situation as shown in fig.



$(\mathbf{V_{IM}})_{\parallel} =? \Rightarrow -44\hat{i}$ $(\mathbf{V_{IM}})_{\perp} =? \Rightarrow -24\hat{j}$

$(\mathbf{V_{IM}}) =? \Rightarrow -44\hat{i}-24\hat{j}$ $(\mathbf{V_{IG}}) =? \Rightarrow -46\hat{i} - 24\hat{j}$

Solution:



$\mathbf{V_0}$ =velocity of object = $(9\hat{i}+12\hat{j})$ m/s

$\mathbf{V_m} = -2i$ m/s

m= f/(f-u) =-20/[-20-(-30)]= -2

for velocity component parallel to optical axis

$\mathbf{V_{IM}}_{\parallel} = -m^2(\mathbf{V_{OM}})_{\parallel}$

$= -4[(9\hat{i}+0)-(-2\hat{i})] = -4[11\hat{i}] = -44\,\hat{i}$ m/s

For velocity component perpendicular to optical axis

$(\mathbf{V_{IM}})_{\perp} = (\mathbf{V_{IM}})_y = f/(f-u)(\mathbf{V_{OM}})_y - \mathbf{Y_{OM}[f/(f-u)]^2}$ (du/dt)→zero(u is const with time)

$(\mathbf{V_{IM}})_{\perp} = (-2)\,(12\hat{j})$

$(\mathbf{V_{IM}})_{\perp} = -24\,\hat{j}$

$(\mathbf{V_{IM}}) = (\mathbf{V_{IM}})_{\parallel} + (\mathbf{V_{IM}})_{\perp}$

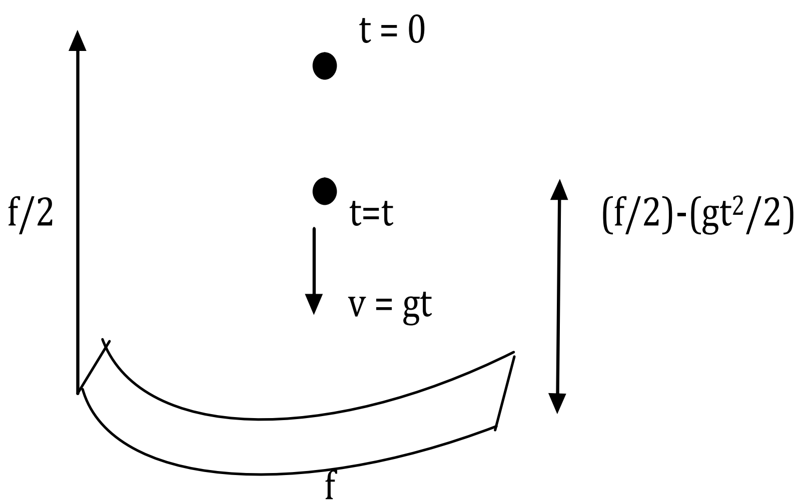
$(V_{IM}) = (-44\hat{i}-24\hat{j})$ **m/s**
$(V_{IM}) = V_I - V_M$
$V_I = (V_{IM}) + V_M = -44\hat{i}-24\hat{j} + (-2\hat{i})$
$V_{IG} = (-44\hat{i}-24\hat{j})$**m**

Problem 10: A particle is dropped along the axis from height $\frac{f}{2}$ on a concave mirror of focal length f as shown in figure the acceleration due to gravity is g. Find the maximum speed of image.

(a) $(3/4)\sqrt{3fg}$ (b) $(3/2)\sqrt{3fg}$

(c) $(3/4)\sqrt{fg}$ (d) $(3/2)\sqrt{fg}$

Solution:



$V_{IM}\,\|= -m^2\,(V_{OM})\,\|$
$m=f/(f-u) =-f/[-f-\{-(f/2)-(gt^2/2)\}] =-f/-f+[(f/2)-(gt^2/2)]$
$m=-f/[-(f/2)-(gt^2/2)] = -f/[-(f/2)+(gt^2/2)] =2f/(f+gt^2)$
$\mathbf{m = 2f/[f +gt^2]},$

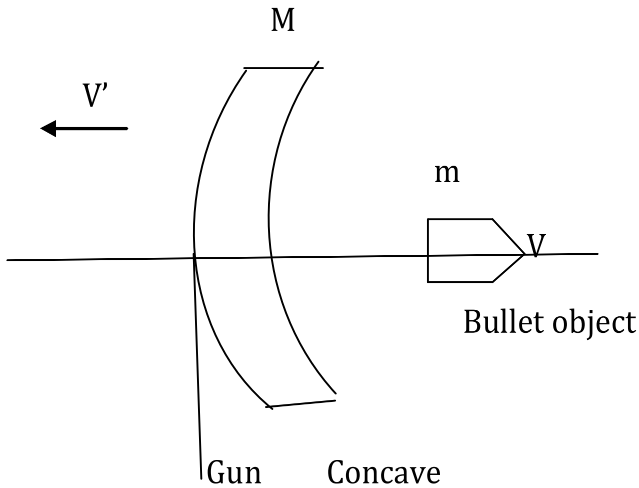
$V_{OM} = gt$
$V_{IM}\,\|= -[2f/f+gt^2]\,(gt)$
$V_{IM}\,\|= -[4f^2gt]/[f+gt^2]^2$ (1)
For obtaining the maximum speed we have to differentiate equation(1) with respect to t $V_{IM}\,\|$ is function of time
$(d\,V_{IM}/dt)\| = 0$ for velocity of image maximum.
We get $t = \sqrt{f/3g}$
$(V_{IM})_{max} = -[4f^2gt/(f+gt^2)^2$

$(V_{IM})_{max} = -(3/4)\sqrt{3fg}$

Problem 11: A gun of mass M fires a bullet of mass m with a horizontal speed V. The gun is fitted with a concave mirror of focal length f facing towards the receding bullet. Find the speed of separation of the bullet just after the gun was fired.

(a) 2V[1+(m/M)] (b) 2V [1-(m/M)] (c) V[1+m/M] (d) V[1-m/M]

Solution:



m v bullet

M V' Gun(concave)

When the bullet moves in forward direction gun with concave mirror moves in backward direction.

Applying conservation of linear momentum

MV' +mV =0

V' = - mV/M (1)

($\mathbf{V_{IM}}$ ||) = - (m^2) $\mathbf{V_{OM}}$

m = f/(f-u) when just after gun was fired, u=0 , hence m=1

($\mathbf{V_{IM}}$ ||) = - $(1)^2$ $\mathbf{V_{OM}}$ = -[$\mathbf{V_0}$ - $\mathbf{V_m}$]

($\mathbf{V_{IM}}$ ||) =- [Vî – (-m/M)Vî)] =-V[1+(m/M)]î

($\mathbf{V_{IM}}$ ||) = -V [1+(m/M)] î

As here m=1 ($\mathbf{V_{IM}}$ ||) = - ($\mathbf{V_{OM}}$)

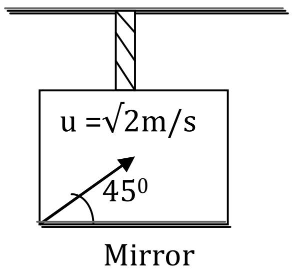
Hence the speed of separation between bullet & gun are

($\mathbf{V_{IM}}$) - ($\mathbf{V_{OM}}$) =($\mathbf{V_{IM}}$) - (-$\mathbf{V_{IM}}$) = 2$\mathbf{V_{IM}}$

Speed of separation =-2v [1+(m/M)]

Hence speed of separation between bullets as image will be 2V [1+(m/M)]

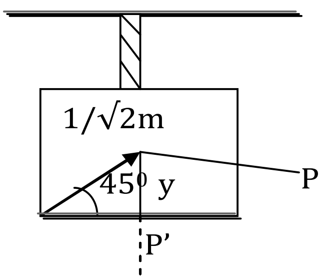
Problem 12:



(a)

An elevator at rest which is at 10th floor of a building is having a plane mirror fixed to its floor. A particle is projected with a speed √2 m/s and at 45⁰ with the horizontal as shown in the figure. at the very instant of projection, the cable of the elevator breaks and the elevator starts falling freely. What will be the separation between the particle and its image 0.5 second after instant of projection?

Solution:



(b)

$v_{E/g}$ =-gtĵ; $v_{p/g}$ = î +(1-gt) ĵ

$u \sin\theta = \sqrt{2} \sin 45^0 = 1m/s$

$u \cos\theta = \sqrt{2} \cos 45^0 = 1m/s$

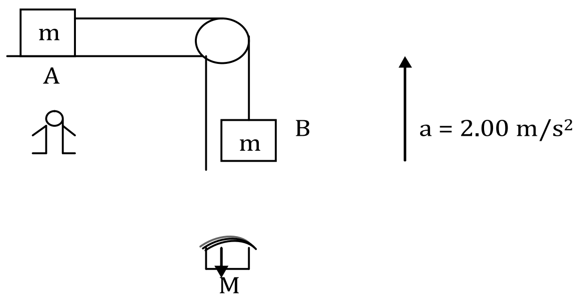
$v_{P/E}$ = î + ĵ; | $v_{P/E}$| = √2m/s

in 0.5 sec = √2(0.5) = 1/√2m

sin 45⁰= y/(1/√2); y = ½ = 0.5m

PP'=1m

Problem 13:



Consider the situation shown in fig. The elevator is going up with an acceleration of 2 m/s^2 and the focal length of the mirror is 12 cm. All the surfaces are smooth and the pulley is light. The mass-pulley system is released from rest (with respect to the elevator) at t = 0 when the distance of B from the mirror is 42 cm. Find the distance between the image of the block B and the mirror at t = 0.2s. Take $g = 10 m/s^2$.

Solution: Let us assume that the acceleration of blocks A and B is 'a' with respect to lift and a_1 is acceleration of lift.

Consider block B : *System: Block (B) Frame of reference: Lift*

$mg + ma_L - T = ma$ (1)

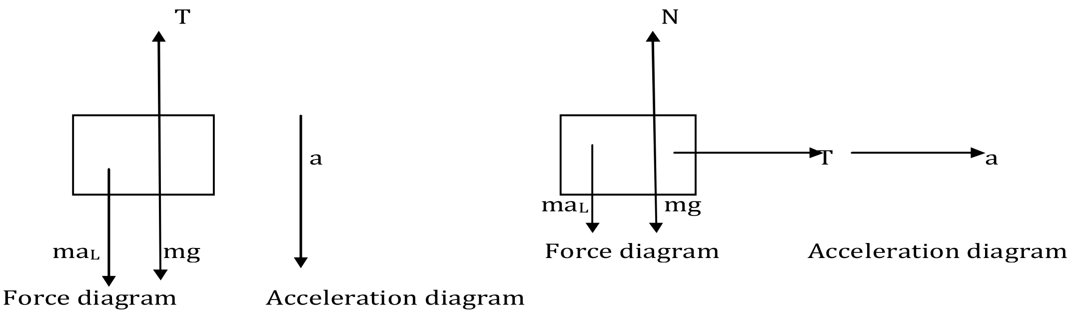
Now, consider block A: *System: Block (A) Frame of reference: Lift*

$N = mg + ma_L$ (2)

$T = ma$ (3)

on adding eq.(1) and (2), we get

$a = (g + a_L)/2 = (10+2)/2 = 6\ m/s^2$



∴ Distance fallen by block (B) = $\frac{1}{2} at^2 = \frac{1}{2} *6(0.2)^2 = 0.12m$ or 12 cm

Now, consider reflection at convex mirror u = -(42-12) = -40 cm

f = +12 cm

$v = uf/ (u-f) = [(-40)(12)]/[-40-12]= 8.75cm$

Therefore, the distance between the image of block (B) and mirror is 8.57 cm.

***Problem 14*:** A small block of mass m and concave mirror of radius R fitted with a stand, lie on a smooth, horizontal table with a separation d between them. The mirror together with its stand has a mass m. The block is pushed at t=0 towards the mirror so that it starts moving towards the mirror at speed V and collides with it. The collision is perfectly elastic.

Find the velocity of the image

1) at a time t< d/V

(a)$R^2 V/[2(d-Vt)-R]^2$

(b) $R^2 V/[2(d+ Vt)-R]^2$

(c)$R^2 V/[2(d-Vt)-R]^2$

(d) RV $[1+ R^2/[2(Vt-d)-R]^2$

2) at a time t>d/v

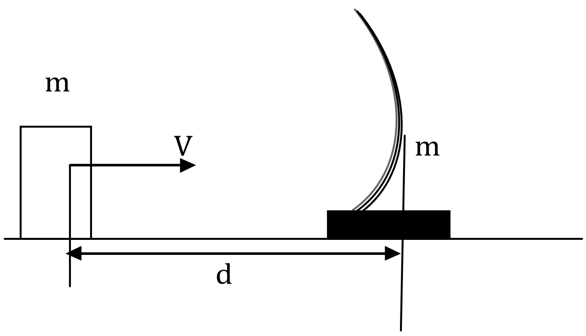
(a)$R^2 V/[2(d-Vt)-R]^2$

(b) $R^2 V/[2(d+ Vt)-R]^2$

(c)$R^2 V/[2(d-Vt)-R]^2$

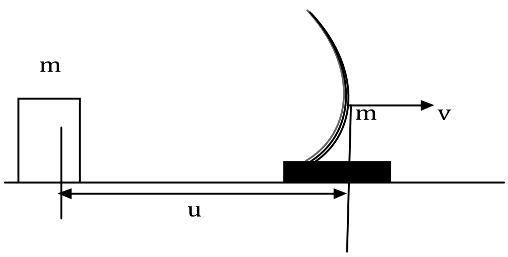
(d) RV$[1+ R^2/[2(Vt-d)-R]$

***Solution*: (1) t < (d/V)**



u = - (d- Vt)
f = -R/2
We know that $\mathbf{V}_{I/m} = -m^2 \mathbf{V}_{0/m}$
Here, m = f/ (f-u)
m = (-R/2)/[(-R/2)+(d-Vt)]
m = (-R)/[2(d-Vt)-R]
Velocity of image= $\mathbf{V}_{I/m} = -m^2 \mathbf{V}_{0/m}$
$V_{I=} R^2 V/[2(d-vt)-R]^2$

(2) t > (d/v)



Block will collide with mirror assembly after time $T = d/V$. From conservation of linear momentum, block and mirror assembly will exchange their momentum i.e., block will stop and mirror starts moving with velocity V.

$U = -V\,[t-(d/V)]$

Also, $m = f/(f-u) = [-R/2]/[(-R/2)+V(t-(d/V))]$

We know that $\mathbf{V}_{I/m} = -m^2\,\mathbf{V}_{O/m}$

$\mathbf{V}_I - \mathbf{V}m = -m^2\,\mathbf{V}_{O/m}$

Let us assume rightward direction as positive

$\mathbf{V}_I - \mathbf{V}m = -m^2(-V)$ or $\mathbf{V}_I = (1+m^2)V$

$\mathbf{V}_I = V[1+\{R^2/2(Vt-d)-R^2\}]$

Problem 15(AIEEE 2011): A car is fitted with a convex side view mirror of focal length 20cm. A second car 2.8m behind the car is overtaking the first car is a relative speed of 15m/s. The speed of the image of the second car as seen in the mirror of the first one is?

(a) 1/15 m/s (b) 10 m/s (c) 15 m/s (d) 1/10 m/s

Solution: $1/u + 1/v = 1/f$ Gives $-(1/u^2)(du/dt) - (1/v^2)(dv/dt) = 0$

$dv/dt = -(v^2/u^2)(du/dt)$ but $v/u = f/(u-f)$

$dv/dt = -[f/(u-f)]^2(du/dt)$

$= \{0.2/(-2.8-0.2)\}^{2} * 15$ $= 1/15\ ms^{-1}$ answer

Problem 16(IIT- JEE 2010): Image of an object approaching a convex mirror of radius of curvature 20m along its optical axis is observed to move from 25/3 m to 50/7 m in 30 sec. What is the speed of the object in kmh^{-1}?

(a) 3 (b) 4 (c) 5 (d) 6

Solution: using mirror formula, $[1/(+25/3)] + [1/(-u_1)] = 1/10$

or $1/u_1 = (3/25) - (1/10)$ or $u_1 = 50$ m

and $1/(+50/7) + (1/-u_2) = 1/10$

$1/u_2 = (7/50) - (1/10)$ or $u_2 = 25$ m

speed of object = $(u_1 - u_2)$ / time $= 25/30\ ms^{-1} = 3\ kmh^{-1}$

Lecture-5

Geometrical Optics

Physics for IIT - JEE

Refraction at Plane Surfaces:

- **Apparent Depth & Normal Shift**

Refraction at plane surfaces:

Snell's law $\quad \boldsymbol{\mu_1 \sin i = \mu_2 \sin r}$

Sin i/sin r $=\mu_2/\mu_1 = {}_1\mu_2$

(1) $\mu = \mu_D/\mu_R > 1 \rightarrow$ relative refractive index

(2) $\mu = \mu_D/\mu_a$ = Absolute Refractive index

(3) ${}_1\mu_2 = ½\mu_1$

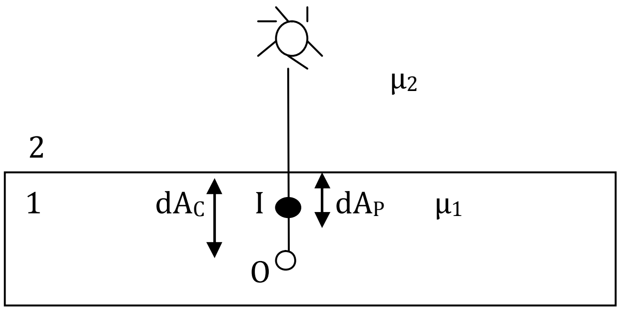
(4) In lens theory μ is used for the refractive index of material of lens relative to the medium

$\boldsymbol{\mu = \mu_{lens}/\mu_{medium}}$

$\mu > 1$, $\mu = 1$, $\mu < 1$

(5) If an object is placed in a medium of refractive index μ_1 at an actual distance dA_c from a plane boundary and when seen from a refractive index μ_2, appears to be a distance dA_p

$2\ \mu_1 = \mu_1/\mu_2 = dA_c/dA_P$



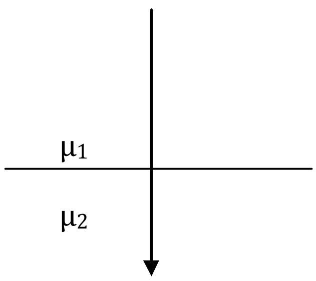
Problem 1: Which of the followings statement gives the condition of no reflected

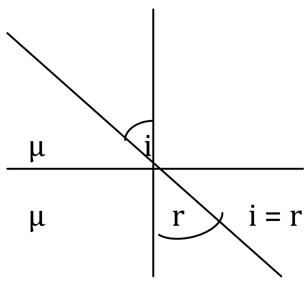
(a) If light is incident normally on the boundary

(b) If refractive index of two media are equal.

(c) a & b both

(d) None of these





Apparent Depth & Normal shift:

If a point object is in one medium is observed from other medium and boundary is plane.

Snell's law: $\mu_1 \sin i = \mu_2 \sin r$

Case Ist : If object is in denser medium seen from Rarer medium

If $\mu_1 > \mu_2$, $\mu_2 = \mu_R$

Sin i = tan i If the rays OA& OB are close

Sin r = tan r enough to see the edge

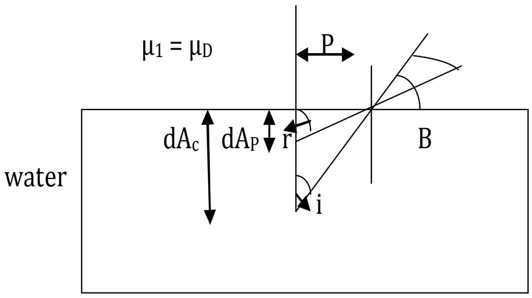
$\mu_D \sin i = \mu_R \sin r$ $\mu_D / \mu_R = \mu = dA_c / dA_p$

$\mu_D \tan i = \mu_R \tan r$ $\mathbf{dA_c = \mu\, dA_p}$

$\mu_D [P/dA_c] = \mu_R [P/dA_P]$

$\mu_D / \mu_R = dA_c / dA_p > 1$

$\mathbf{dA_c > dA_p}$



This is why an underwater object like stone or fish appears to be at depth than in reality.

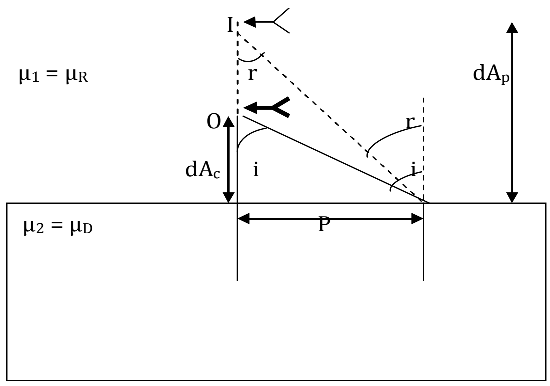
Normal shift: The distance between object & its image called normal shift

if $dA_c = t$ $dA_c / dA_p = \mu$ $\Rightarrow dA_p = t/\mu$

Normal Shift: $= dA_c - dA_p = t - t/\mu = t(1-1/\mu)$

Normal Shift $= t(1-(1/\mu))$

Case 2nd: If object in rarer medium is seen from denser medium



$\mu_1 \sin i = \mu_2 \sin r$

$\mu_R \sin i = \mu_D \sin$

$\sin i/\sin r = \mu_D/\mu_R = \mu_2/\mu_1 > 1$

$\mu_R \sin i = \mu_D \sin r$

$\mu_R \tan i = \mu_D \tan r$

$\mu_R[P/dA_c]=\mu_D [P/dA_P]$

$dA_p / dA_c = \mu_D / \mu_R > 1$

$\mathbf{dA_p > dA_c}$

if $dA_c = t$, $dA_p / dA_c = \mu$

Normal shift: $x = dA_p - dA_c = \mu\, dA_c - dA_c = (\mu-1)\, dA_c = (\mu-1)t$

$x = (\mu-1)t$ = Normal Shift $dA_p > dA_c$

If object is in denser medium seen from Rarer medium

$dA_p < dA_c$

Normal Shift = $t(1-1/\mu)$

Where $t = dA_c$

If object is in rarer medium seen from denser medium

$dA_p > dA_c$

Normal Shift = $(\mu-1)\, t$

where $t = dAc$

If there are number of liquids of different depth one over the other:

d_1	μ_1
d_2	μ_2
d_3	μ_3

$dA_c = d_1+d_2+d_3$

$dA_p = (d_1/\mu_1) + (d_2/\mu_2) + (d_3/\mu_3)$

$\mu = dA_c/dA_p = [(d_1+d_2+d_3)/ \{(d_1/\mu_1) + (d_2/\mu_2) + (d_3/\mu_3)\}]$

$\mu = [\Sigma d_i/(\Sigma d_i/u_i)]$

In case of two liquids: $d_1 = d_2$

$\mu = (d + d)/[(d/\mu_1)+(d/\mu_2)]$

$\mu = 2d/d[(1/\mu_1)+(1/\mu_2)]$

$\mu = (2\mu_1\mu_2)/(\mu_1+\mu_2)$

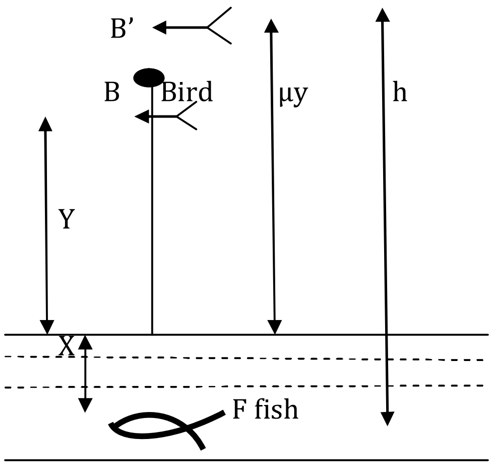
μ = Harmonic Mean

Problem 2 **:** A fish rising vertically to the surface of water in a lake uniformly at the rate of 3m/s. Observes a king fisher(bird) diving vertically towards the water at a rate of 9m/s vertically above it. If the refractive index of water is 4/3.

The actual velocity of the dive of the bird is

(a) 4 m/s (b) 4.5 m/s (c) 3 m/s (d) none

Solution:



If at any instant the fish is at depth x below the water surface while the bird is at a height y above the surface, then the apparent height of the bird above the surface as seen by fish will be

$dA_p = \mu y$ (1)

so that total apparent distance of the bird as seen by fish in water at depth will be

$h = x + \mu y$ (2)

$dh/dt = dx/dt + \mu\,(dy/dt)$ (3)

$(dy/dt) = 9$ m/s

$(dx/dt) = 3$ m/s

$dy/dt = ?$ $\mu = 4/3$

Substituting the values in eq $dh/dt = dx/dt + \mu\,(dy/dt)$ (3)

$9 = 3 + \mu\,(dy/dt)$

$dy/dt = 6/\mu = 6/(4/a) = (6*3)/4 = 9/2 = 4.5$ m/s

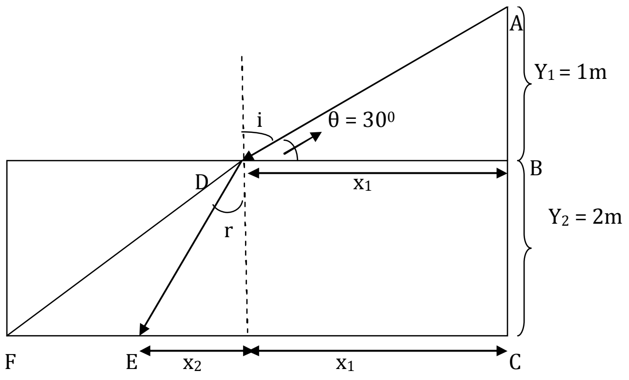
The actual velocity of the dive of the bird is

dy/dt = 4.5 m/s

Problem 3: A pole standing in a clear water pond stands in above the water surface; the pond is 2 m deep. What are the lengths of the shadows thrown by the pole on the surface and bottom of the pond if the sun is 30^0 over the horizontal refractive index of water is 4/3.

(a)$\sqrt{3}$, $\sqrt{3}$ (b) $\sqrt{3}$, $2\sqrt{3}$ (c) $\sqrt{2}$, $\sqrt{3}/2$ (d) none

Solution:



According to rectilinear propagation of light, length of Shadow on the surface will be BD = x_1

$\tan\theta = \tan 30 = y_1/x_1 = 1/x_1 1/\sqrt{3} = 1/x_1$

$X_1 = \sqrt{3}$ m (1)

However due to refraction at the surface of water, the length Of shadow of pole at the bottom will be

$CE = \sqrt{3} + x_2$ (2)

Now from Snell's Law

$\mu \sin r = 1 \sin i = (90 - \theta) = \cos\theta$

$\mu \sin r = \cos\theta$

$\sin i = \cos\theta/\mu$

$x_2/[\sqrt{x_2^2 + y_2^2}] = \cos\theta/\mu$

$x_2^2/[x_2^2 + y_2^2] = \cos^2\theta/\mu^2$

$\mu^2/\cos^2\theta = x_2^2/[x_2^2 + y_2^2] = 1 + (y_2/x_2)^2$

$\mu^2/\cos^2\theta = 1 + (y_2/x_2)^2$

$[(\mu^2/\cos^2\theta) - 1] = (y_2/x_2)^2$

$[(\mu^2 - \cos^2\theta)/\cos^2\theta] = (y_2/x_2)^2$

$x_2 = y_2 \cos\theta/[\sqrt{\mu^2 - \cos^2\theta}]$

$x_2 = 2\cos 3\theta/[\sqrt{(4/3)^2 - \cos^2 3\theta}]$

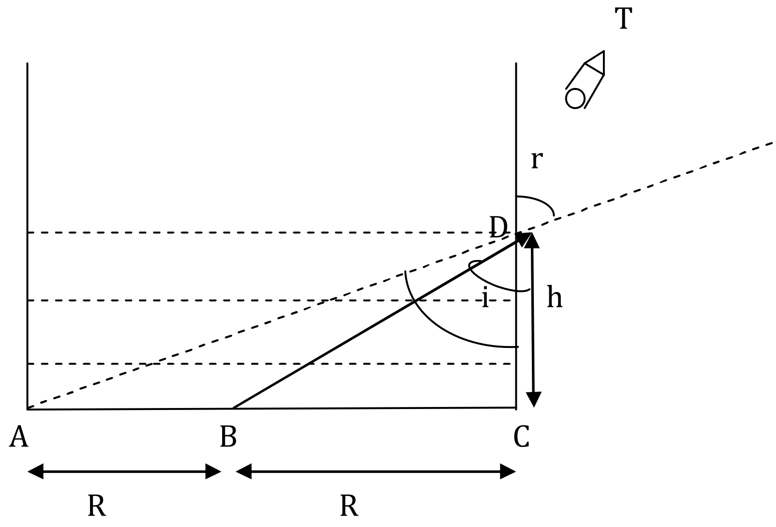
$x_2 = [2+(\sqrt{3}/2)]/[\sqrt{(16/4)-(3/4)}] = \sqrt{3}/[\sqrt{(64-27)/36}] = \sqrt{3}/[\sqrt{37/36}] = \sqrt{3}$

$X = x_1 + x_2 = 2\sqrt{3}$ m (Approx.)

Problem 4: A person looking through the telescope T just sees the point A on the rim at a cylindrical vessel when vessel is empty see figure. When the vessel is completely filled with a liquid of refractive index μ , he observes a mark at the centre of bottom , without moving the telescope or vessel. What is the height of the vessel if the radius of its cross section is R.

(a) $h = 2R\sqrt{[(\mu^2+1)/4-\mu^2]}$
(b) $h = 2R\sqrt{[(\mu^2-1)/4-\mu^2]}$
(c) $h = 2R\sqrt{[(\mu^2)/4+\mu^2]}$
(d) $h = R\sqrt{[(\mu^2+1)/\mu^2-4]}$

Solution: When filled with liquid the ray from B reaches the telescope so that



$\mu \sin i = 1 \sin r$

$\mu = R/\sqrt{[R^2+h^2]} = 2R/\sqrt{[h^2+4R^2]}$

$\mu^2/4 = [R^2+h^2]/[h^2+4R^2]$

$4/\mu^2 = [h^2+4R^2]/[h^2+R^2] = [h^2+R^2+3R^2] = 1+[3R^2/(h^2+R^2)]$

$4/\mu^2-1=[3R^2/(h^2+R^2)]$

$(4-\mu^2)/\mu^2 = 3R^2/(h^2+R^2)$

$\mu^2/(4-\mu^2) = (h^2+R^2)/3R^2 = (h^2/3R^2)+1/3$

$\mu^2/(4-\mu^2)-(1/3) = h^2/3R^2$

$[3\mu^2-(4-\mu^2)]/(4-\mu^2)3 = h^2/3R^2$

$[3\mu^2-4+\mu^2]/(4-\mu^2) = (h^2/R^2)$

$4(\mu^2-1)/(4-\mu^2) = (h^2/R^2)$

$h^2 = R^2 4(\mu^2-1)/(4-\mu^2)$

$h = 2R\sqrt{[(\mu^2-1)/(4-\mu^2)]}$

if $2R = 10$ cm, $\mu = 1.5$

$h = 8.45$cm

Problem 5: An object O is locked at the bottom of a tank containing two immiscible liquids and is seen vertically from above. The lower and upper liquids are of depth h_1&h_2 respectively and of refractive indices μ_1& μ_2 respectively. Location of the position of the image of the object o from the surface

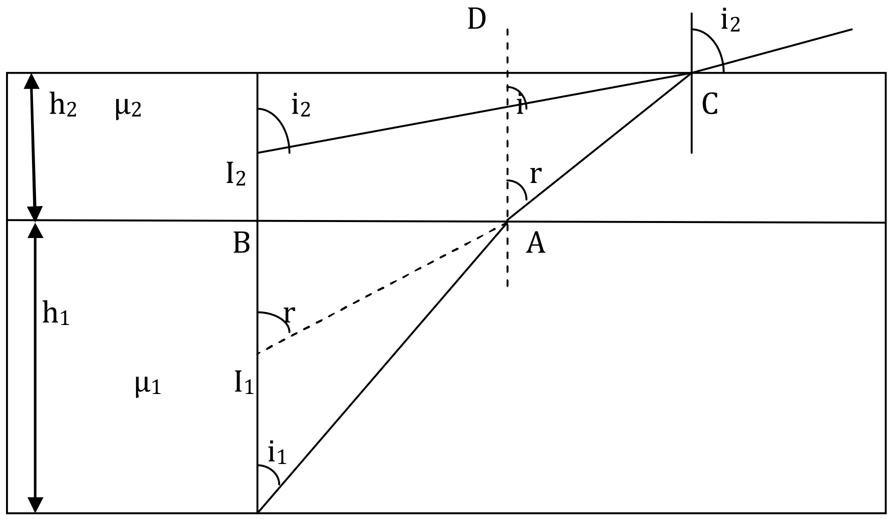
(a) $(h_1/\mu_1)+(h_2/\mu_2)$

(b) $(h_1/\mu_2)+(h_2/\mu_1)$

(c) $(h_1+h_2)/\mu_1+\mu_2$

(d) none

Solution:



As shown in the figure refraction at the 1st Surface

$\mu_1 \sin i = \mu_2 \sin r$

For small angle $\sin \theta = \tan \theta = \theta$

$\mu_1 \tan i = \mu_2 \tan r$

$\mu_1 [AB/OB] = \mu_2 [AB/BI_1]$

$\mathbf{BI_1 = (\mu_2/\mu_1)h_1}$ (1)

For refraction at the second surface

$\mu_2 \sin r = 1 \sin i_2$

$\mu_2 \tan r = \tan i_2$

$\mu_2 [CD/DI_1] = [CD/DI_2]$

$DI_2 = DI_1/\mu_2$ (2)

$DI_1 = DB + BI_1 = h_2 + (\mu_2/\mu_1) h_1$ (3)

Substituting (3) in (2)

$DI_2 = [h_2+(\mu_2/\mu_1)h_1]/\mu_2 = (h_2/\mu_2)+(h_1+\mu_1)$

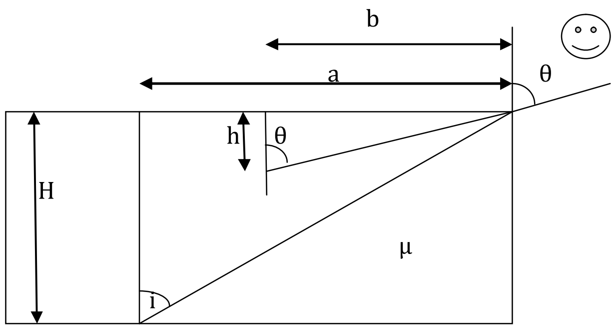
$\mathbf{DI_2 = (h_2/\mu_2)+(h_1+\mu_1)}$

Problem 6: An object o is kept at a depth H below the surface of a liquid of refractive index μ. What is apparent depth when the angle of vision at the surface is θ.

(a) $h=(H/\mu)[(\mu \cos\theta)/(\sqrt{\mu^2-\sin^2\theta})]^3$ (b) $h=(H/\mu)[(\mu \sin\theta)/(\sqrt{\mu^2-\sin^2\theta})]^3$

(c) $h=(\mu/H)[(\mu \cos\theta)/(\sqrt{\mu^2-\sin^2\theta})]^3$ (d) none

Solution:



Here it must be noted that the object when viewed from an angle θ not only shifts upwards by H-h but also laterally (a-b) with respect to its real position. If h is the apparent depth

$a = H \tan i$, $b = h \tan \theta$

$a - b = H \tan i - h \tan \theta$

a, b , H , h are independent of small variation in θ & i

$0 = H \sec^2 i \, di - h \sec^2 \theta \, d\theta$

$di/d\theta = [h \sec^2\theta / H \sec^2 i] \Rightarrow h = (H \sec^2 i / \sec^2\theta)(di/d\theta) = H(\cos^2\theta/\cos^2 i)(di/d\theta)$

(1) But as at the surface Snell's Law

$\mu \sin i = 1 \sin \theta$

$\mu \cos i \, di = \cos \theta \, d\theta$

$\mathbf{(di/d\theta) = \cos \theta / \mu \cos i}$

(2) Substituting (2) in (1) we get

$h = H (\cos^2\theta/\cos^2 i)(di/d\theta) = H (\cos^2\theta/\cos^2 i) (\cos \theta / \mu \cos i)$

$\mathbf{h = H[\cos^3\theta / \mu \cos^3 i]}$ (3)

$\mu \sin i = \sin \theta$

$\sin i = (\sin\theta)/\mu$

$\sqrt{1-\cos^2 i} = (\sin\theta)/\mu$

$1-\cos^2 i = (\sin^2\theta)/\mu$

$1-(\sin \theta)/\mu^2 = \cos^2 i$

$(\mu^2 - \sin^2\theta)/\mu^2 = \cos^2 i$

$\sqrt{(\mu^2 - \sin^2\theta)}/\mu = \cos i$ (4)

Substituting cos i in (3) we get

$h = H (\cos^3\theta/\mu \cos^3 i) = (H/\mu)[(\mu^3 \cos^3\theta)/(\mu^2 - \sin^2\theta)^{3/2}]$

$\mathbf{h = H(\mu \cos \theta)^3/\mu(\mu^2 - \sin^2\theta)^{3/2}}$

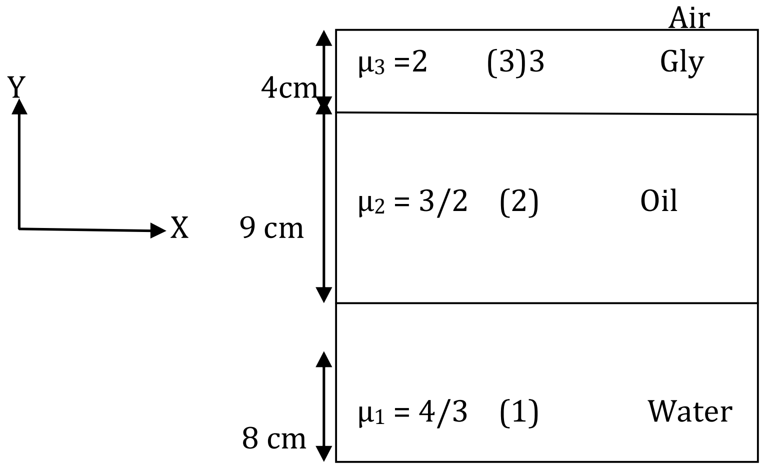
Problem 7: A tank contains three layers of immiscible liquids. The first layer is of water with refractive index 4/3 and thickness 8cm. The second layer is an oil with refractive index 3/2 and thickness 9cm. While the third layer is of glycerine with refractive index 2 and thickness 4cm. find the apparent depth of the bottom of the container.

(a) Final image is 1.4 cm below the glycerine.- air interface

(b) Final image is 1.4 cm below the oil.- glycerine interface

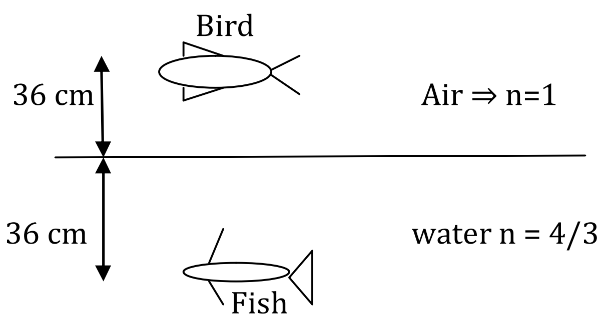
(c) Final image is 1.4 cm below the water.- oil interface

(d) None of the above



***Solution*:** A ray of light from the object undergoes refraction at three interfaces
(1) Water oil (2) oil glycerine (3) glycerine air
Water oil interface: $\mu_1 = 4/3$, $\mu_2 = 3/2$
$(\mu_1/d_1) = (\mu_2/d_2) \Rightarrow d_1 = -8$ cm
$d_2 = (\mu_2/\mu_1)d_1 = [(3/2)/(4/3)](-8) = (9/8)8 = -9$ cm
Oil glycerine interface: $(\mu_1/d_1) = (\mu_2/d_2)$, $\mu_1 = 3/2$, $\mu_2 = 2$
$(\mu_1/d_1) = (\mu_2/d_2)$, $d_1 = -(9+9) = -18$ cm
$d_2 = (\mu_2/\mu_1)d_1 = 2/(3/2)(-18) = -24$cm
Glycerine air interface*:* $\mu_1 = 2$, $\mu_2 = 1$
$d_1 = -(24+4) = -28$ cm
$d_2 = (\mu_2/\mu_1)d_1$
$d_2 = (1/2)(-28) = -14$ cm
Thus the final image is 14 cm below the glycerine air interface.

Problem 8:

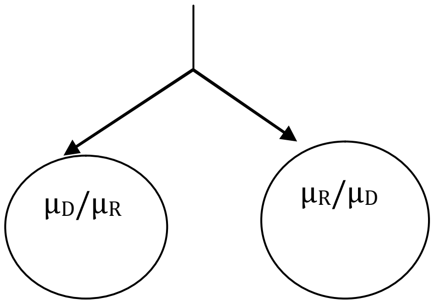


See the figure
(1) At what distance will the bird appear to fish
(a) 84 (b) 63 (c) 72 (d) none
(2) At what distance will the fish appear to bird
(a) 84 (b) 63 (c) 72 (d) none

Solution:

Concept:

Apparent depth = μy



Fish observe Bird　　　　Bird observe Fish

For fish:

$d_B = 36 + \mu\, 36 = 36[1 + (4/3)]$

$d_B = 36(7/3) = 12 * 7 = 84$ cm

For bird:

$d_f = 36 + 36\,\mu = 36(1+\mu)$

$= 36(1 + 3/4) = 36(7/4) = 63$ cm

Problem 9 (AIEEE 2011): A beaker contains water up to a height h_1 and kerosene of height h_2 above water so that the total height of (water + kerosene) is $(h_1 + h_2)$. Refractive index of water is μ_1 and that of kerosene is μ_2. The apparent shift in the position of the bottom of the beaker when viewed from above is

(a) $[1-(1/\mu_1)]\, h_2 + [1-(1/\mu_2)]\, h_1$　　(b) $[1+(1/\mu_1)]h_1+[1+(1/\mu_2)]h_2$

(c) $[1-(1/\mu_1)]h_1 +[1-(1/\mu_2)]h_2$　　(c) $[1+(1/\mu_1)]h_2-[1+(1/\mu_2)]h_1$

Solution: Apparent shift $\Delta h = [1-(1/\mu)]\, h$

so apparent shift produced by kerosene $\Delta h_1 = [1-(1/\mu_1)]\, h_1$

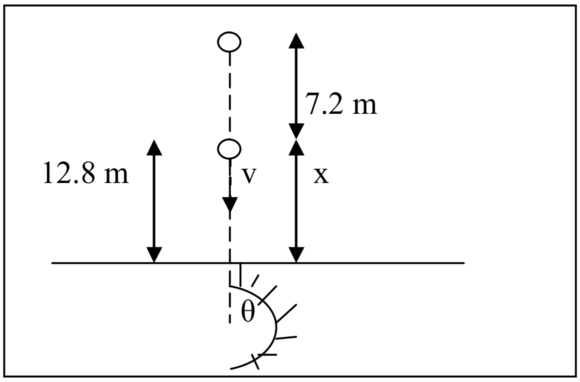
and apparent shift produced by kerosene $\Delta h_2 = [1-(1/\mu_2)]\, h_2$

$\Delta h = \Delta h_1+\Delta h_2 = [1-(1/\mu_1)]\, h_1 + [1-(1/\mu_2)]\, h_2$

Problem 10(IIT – JEE 2009): A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as

(a) 9 ms^{-1}　　(b) 12 ms^{-1}　　(c) 16 ms^{-1}　　(d) 21.33 ms^{-1}

Solution: $v = \sqrt{2gh} = \sqrt{2 * 10 * 7} = 12 ms^{-1}$



In this case when eye is inside water $X_{app} = \mu x$

$d X_{app} / dt = \mu(dx/dt)$ or $v_{app} = \mu v = (4/3) * 12 = 16 ms^{-1}$

Lecture-6

Geometrical Optics

Physics for IIT - JEE

Refraction at Plane Surface:

- **Critical Angle**
- **Total Internal Reflection**

Total Internal Reflection (TIR):

The phenomenon:

In case of refraction of light from Snell's law $\mu_1 \sin i = \mu_2 \sin r$

If light is passing from denser to rarer medium through a plane boundary.

Then $\mu_1 = \mu_D$, $\mu_2 = \mu_r$

$\mu_D/\mu_r = \mu$

$\mu_D \sin i = \mu_r \sin r$

$\mu_D/\mu_r = \sin i / \sin r = \mu > 1$

$\text{Sin } i = \sin r/\mu$

$\text{Sin } i \ \alpha \ \sin r$

$\angle i \ \alpha \ \angle r$

$\mu_D \sin i = \mu_r \sin r$

if $r=90$, $i=i_c$

$\mu_D \sin i_c = \mu_r \sin 90$

$\mu_D \sin i_c = \mu_r$

$\mu_D/\mu_r = 1/\sin i_c = \mu$

$\text{Sin } i_c = 1/\mu$

$\text{Sin}^{-1}(1/\mu) = i_c \longrightarrow$ Critical angle

* so as angle of incidence increases angle of refraction also increases.

* for certain value of i < 90, r will becomes 90^0 , the value of angle of incidence for which angle of refraction becomes 90^0 is called critical angle.

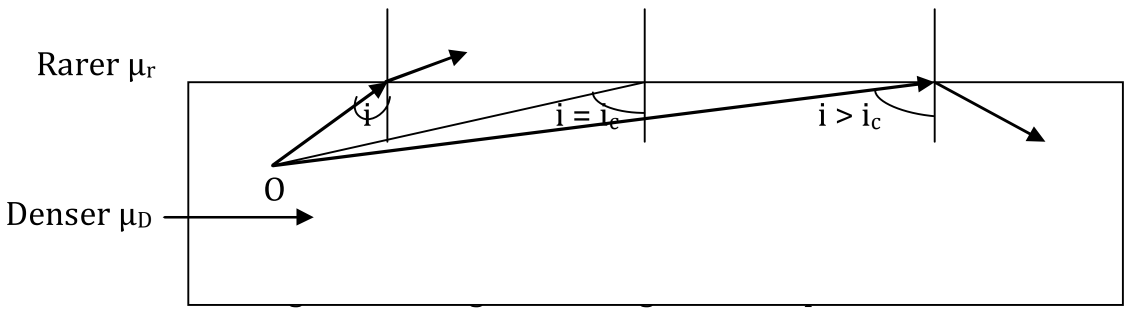


Fig. ***Illustrating critical angle & TIR phenomenon***

If *$i < i_c$ → refraction, $i = i_c$ →critical Angle, $i > i_c$ →total internal reflection*

$\mu_D \sin i = \mu_r \sin r$

$(\mu_D/\mu_r) \sin i = \sin r$

$\mu \sin i = \sin r$

$\sin i / \sin i_c = \sin r$

* so if $i > i_c$, $\sin r > 1$, which is no meaning, r is imaginary simply this situation implies that refracted ray does not exist.

* so the total light incident on the boundary will be reflected back in to the same medium from the boundary. This phenomenon is called total internal reflection. (T.I.R)

Note:

1. For total Internal reflection to take place light must be propagating from denser to rarer medium.
2. When light is passing from denser to rarer medium. Total internal reflection will take place only if angle of incidence is greater than a certain value called critical Angle. Given by $\theta_c = \sin^{-1}(\mu)$ with $\mu = \mu_D$
3. In case of T.I.R i.e. (100 %) incident light is reflected back into the same medium there is no loss of intensity. While in case of reflection from mirrors or refraction from lances there is some loss of intensity as all light can never be reflected or refracted.
4. Images formed by T.I.R. are much brighter than formed by mirrors or lenses.
5. $\theta_c = \sin^{-1}(1/\mu), \mu = c/v, \mu \propto (1/v) \propto (1/\lambda)$
   for a given pair of media, critical angle depends on wavelength of light used. i.e. greater is the wavelength of light, lesser will be the value of μ & so greater will be the value of critical angle.
6. Critical angle is maximum for red & minimum for violet
   $\sin \theta_c = (1/\mu)$, $F_R - F_V = \omega F_y$, $1/f = (\mu-1)[(1/R_1)-(1/R_2)]$,
   $\delta_V - \delta_R \rightarrow$deviation
7. For a given light, critical angle depends on the nature of pair of media. Lesser is the value of μ, greater will be the critical angle & vice versa. Glass-air → $\theta_c = 42^0$
   Water-air → $\theta_c = 49^0$
   Glass- water → $\theta_c = 63^0$
   more gap in refractive index less critical angle

Some examples of TIR:

(1)*Shining of air bubble* ***(2) Sparking of diamond***
(3) Optical fibre ***(4) Mirage's & looming***
(5) Duration's of sun's visibility

Problem 1: A beam of light consisting of red, green & blue colours is incident on a right – angled prism as shown in figure for the above red, green & blue wavelength are 1.39,1.44 & 1.47 respectively.

Which colour will be separated from the other by prism

(a) red (b) green (c) blue (d) all (e) none

Solution: At face AB i=0, So r=0 i.e. no refraction will take place. So light will be incidence on face AC at an angle of incidence of 45^0. The face AC will not transmit the light for which $i>\theta_c$

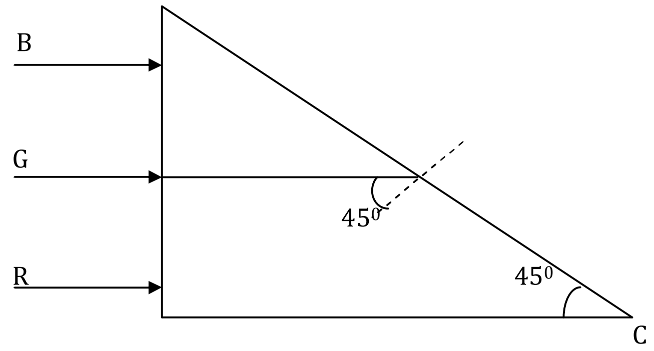
Sin i > sin θ_c, Sin 45 > $(1/\mu) \Rightarrow 1/\sqrt{2} > (1/\mu)$

$\mu = \sqrt{2} = 1.41$

$\mu_R=1.39 < 1.41$

$\mu_g=1.44> 1.44$ will not transmitted

$\mu_b=1.47> 1.47$

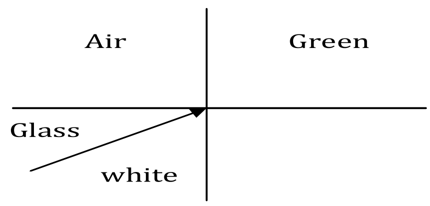


Now as $\mu_R < \mu$ while μ_G & $\mu_B > \mu$, so red colour will be transmitted through the face AC while green & blue will be reflected. So the prism will be separated red colour from green & blue.

Problem 2: When light is incident at the glass and air as shown in the figure. If green light is just T I. Reflected than the emergent ray in air contains

(a) Yellow, orange, red (b) Violet, indigo, blue

(c) All colours (d) None

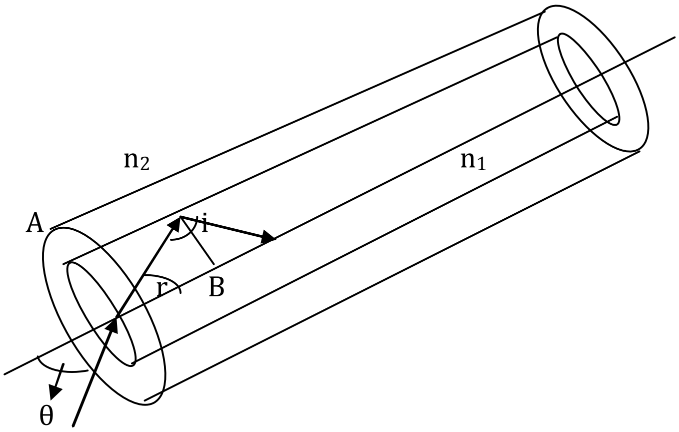


Optical fibre*:*

Problem 3: A particular optical fibre consist of a glass core (index of refraction n_1) surrounded by a cladding (index of refraction n_2). $n_1 > n_2$. Suppose a beam of light enters the fibre from air at an angle θ with the fibre axis as shown in figure. Show that the greatest possible value θ for which a ray can be propagated down the fibre is given by.

(a) $\theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}$ (b) $\theta = \sin^{-1}\sqrt{n_2^2 + n_1^2}$

(c) $\theta = \cos^{-1}\sqrt{n_1^2 - n_2^2}$ (d) $\theta = \cos^{-1}\sqrt{n_1^2 + n_2^2}$



Solution: The situation is shown in figure. The light will be propagated down the fibre if it does not emerge from curved surface. i.e. at P.

$i > \theta_c$, $\sin i > \sin \theta_c$

$\mu = \mu_D/\mu_r = n_1/n_2$, $\sin \theta_c = (1/\mu) = n_2/n_1$

$\sin i > n_2/n_1$ (1)

now from Snell's law

$1 \sin \theta = n_1 \sin r$

$r + i = 90$ $r = 90 - i$

$\sin \theta = n_1 \sin (90 - \theta) = n_1 \cos i$

$\cos i = \sin \theta / n_1$

$\sin i = \sqrt{1 - \cos^2 i} = \sqrt{1 - (\sin^2 \theta / n_1^2)}$ (2)

Substituting the value of sin i from equation (2) in equation (1)

$\sin i = n_2/n_1 = \sqrt{1 - (\sin^2 \theta / n_1^2)}$

$(n_2/n_1)^2 = 1 - (\sin^2 \theta / n_1^2)$

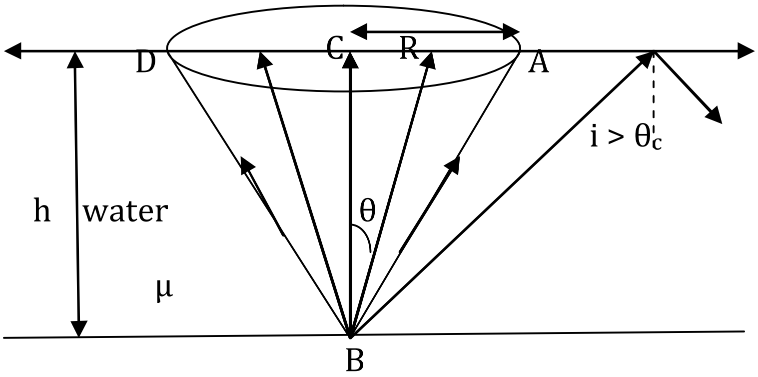
$[(n_2^2 - n_1^2)/n_1^2] \leq - (\sin^2 \theta / n_1^2)$

$(n_2^2 - n_1^2) \leq - \sin^2\theta$

$\boldsymbol{\theta \leq \sin^{-1}\sqrt{n_1^2 - n_2^2}}$ (3)

Problem 4: A bulb is placed at a depth of $2\sqrt{7}$ m in water and floating opaque disc is placed over the bulb so that the bulb is not visible from surface. What is the minimum diameter of the disc $\mu = 4/3$ (water)

(a) 6m (b) 12m (c) 10m (d) none



Solution:

As shown in figure light from bulb will not emerge out of water at the edge of the disc

$i > \theta_c$, $\sin i > \sin \theta_c$ (1)

Now if R is the radius of disc & h is the height/depth of bulb from it.

Sin i =$[R/\sqrt{R^2 + n^2}]$,

$\sin \theta_c = 1/\mu$

$[R/\sqrt{R^2 + n^2}] = 1/\mu$

or R >$[\mu/\sqrt{\mu^2 - 1}]$,

$h = 2\sqrt{7}$, $\mu = 4/3$

$R_{min} > [(2\sqrt{7})/(4/3)^2 - 1] = 6m$

Hence diameter of the disc = 2R=2*6=12m

Problem 5: A ray of light in a transparent medium falls on a surface separating the medium from air atom angle of incidence 45^0 the ray undergoes TIR. If n is the refractive index of the medium with respect to air, select the possible values of n from the following.

(a) 1.3 (b) 1.4 (c) 1.5 (d) 1.6

Solution: For T.I.R. to take place $\sin i > \sin \theta_c$

Sin 45^0 $> \sin \theta_c$

$1/\sqrt{2} > 1/n$

$\sqrt{2} = 1.414$

n > 1.414

the possible values of n can be 1.5 or 1.6

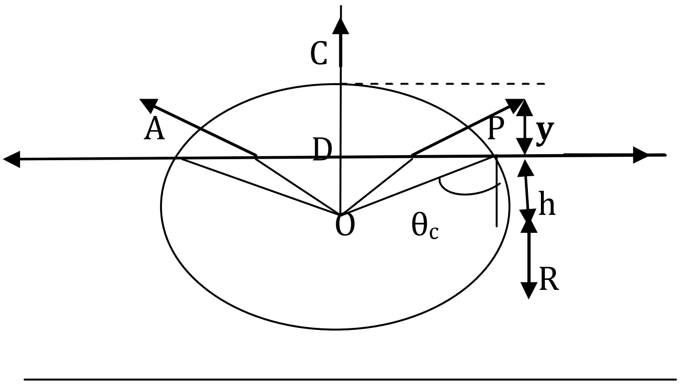
Problem 6: A point source of light is placed at a distance h below the surface of a large and deep lake. Show that the fraction f of the light energy that escapes directly from the water surface is independent of n and is given by

(a) $f = \frac{1}{2}[1 - \sqrt{1 - (1/n)^2}]$ (b) $f = \frac{1}{2}[1 + \sqrt{1 - (1/n)^2}]$

(c) $f = \frac{1}{2}[1 - \sqrt{1 + (1/n)^2}]$ (d) $f = \frac{1}{2}[1 + \sqrt{1 + (1/n)^2}]$

Solution:

As due to total internal reflection, light will be reflected back into water if $i > \theta_c$, so only the position of incident light will escape which pass through the core of angle $\theta = 2\theta_c$.



$$\cos \theta_c = h/r$$

So the fraction of light Escaping

f = area of cap /area of sphere

$f = 2\pi Ry/4\pi R^2$

$f = 2\pi y/4\pi R = y/2R$

$f = [(1/2)\ (R-h)/R]$

$f = (1/2)[1-(h/R)]$

$f = (1/2)\ [1-(h/R)]$

$f = (1/2)\ [1- \cos \theta_c]$

$f = (1/2)[1- \sqrt{1- \sin^2\theta_c}]$

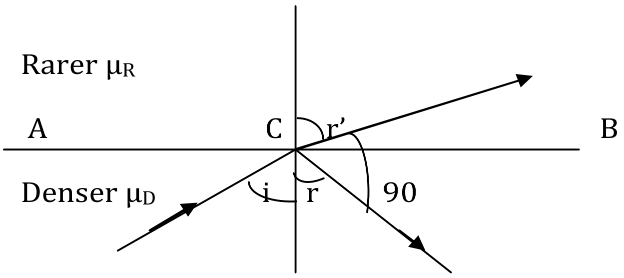
$\mathbf{\sin \theta_c = 1/n}$

$f = (1/2)\ [1- \sqrt{1-(1/n)^2}]$

$\mathbf{f = \frac{1}{2}[1-\sqrt{1- (1/n)^2}]}$

Problem 7: A ray of light from a denser medium strikes a rarer medium at an angle of incidence i, if the reflected and refracted rays are mutually $\perp$ to each other. What is the value of critical angle

(a) $\sin^{-1} \tan i$ (b) $\sin^{-1} \cot i$ (c) $\sin^{-1} \cos i$ (d) none



Solution: The situation in accordance with given problem is shown in figure applying Snell's law at the boundary at c.

$\mu_D \sin i = \mu_r \sin r'$ $r' + r = 90$

$\mu_D \sin i = \mu_r \sin(90-r)$

$\mu_D \sin i = \mu_r \cos r$ but i=r

from reflection

$\mu_D \sin i = \mu_r \cos i$

$\mu_D / \mu_r = \cos i / \sin i = \cot i$

$1/\mu = \tan i$

but by definition $\theta_c = \sin^{-1}(1/\mu) \Rightarrow \theta_c = \sin^{-1}(1/\mu)$

$\boldsymbol{\theta_c = \sin^{-1}(\tan i)}$

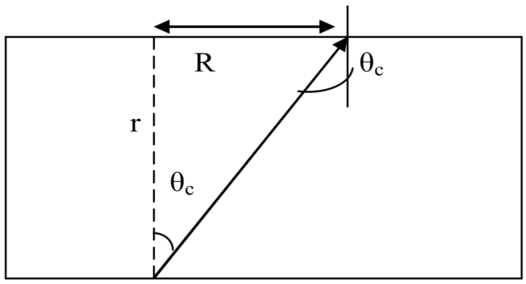
incident < reflected & refracted

reflected ray & refracted ray are $\perp$ to each other $r' + r = 90$

Problem 8(IIT –JEE 2010): A large glass slab ($\mu = 5/3$) of thickness 8cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

(a) 6 cm   (b) 7 cm   (c) 8 cm   (d) 9 cm

Solution: $(R/t) = \tan\theta_c$ or $R = t(\tan\theta_c)$

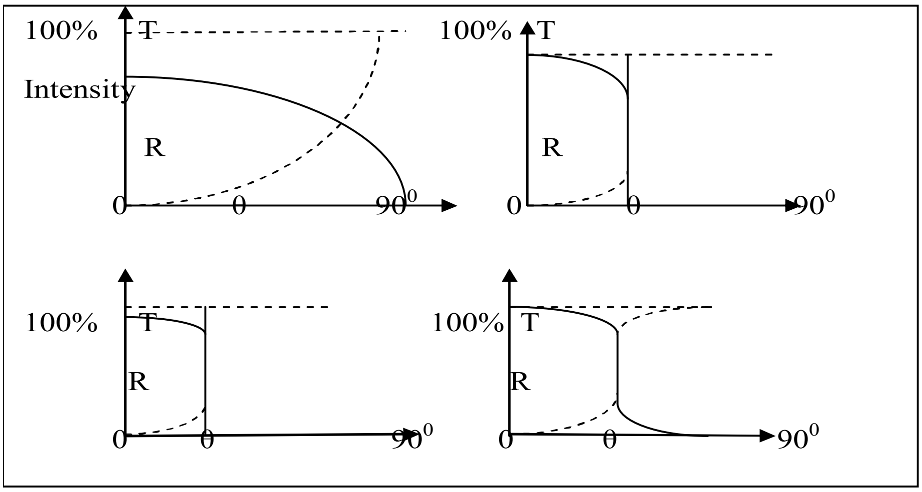


But $\sin\theta_c = 1/\mu = 3/5$

$\tan\theta_c = ¾$ so $R = ¾\, t = ¾\,(8cm) = 6$ cm

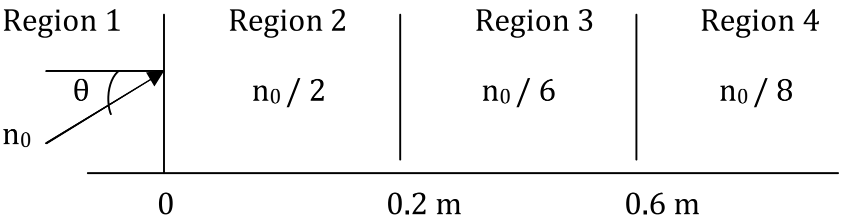
Hence the answer is 6.

Problem 9(IIT-JEE 2011): A light ray travelling in glass medium on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is?



Solution: After critical angle reflection will be 100% and transmission is 0%. Options (b) and (c) satisfy this condition. But option (c) is the correct option. Because in option (b) transmission is given 100% at $\theta = 0^0$, which is not true.
So correct answer is(c).

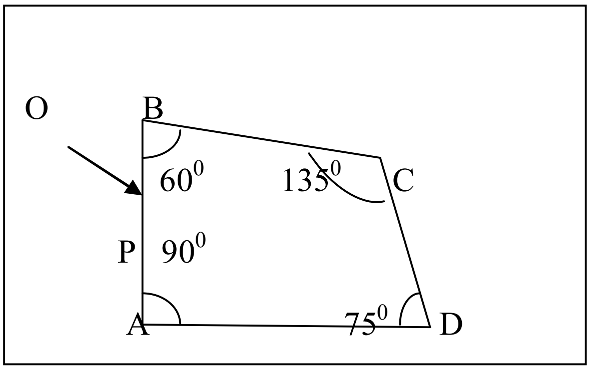
Problem 10(IIT – JEE 2008): A light beam is travelling from region 1 to region 4 (refer figure). The refractive indexes in region 1, 2, 3, 4 are n_0. $n_0 / 2$, $n_0/6$, $n_0/8$ respectively. The angle of incidence θ for which the beam just misses entering region 4 is



(a) $\sin^{-1}(3/4)$ (b) $\sin^{-1}(1/8)$ (c) $\sin^{-1}(1/4)$ (d) $\sin^{-1}(1/3)$

Solution: Critical angle from region 3 to region 4 $\sin\theta_c = (n_0/8)/(n_0/6) = 3/4$
now applying Snell's law in region 1 and region 3 $n_0 \sin\theta = (n_0/6)\sin\theta_c$
$\sin\theta = 1/6 \sin\theta_c = (1/6)(3/4) = 1/8$
so $\theta = \sin^{-1}(1/8)$

Problem 11(IIT – JEE 2010):

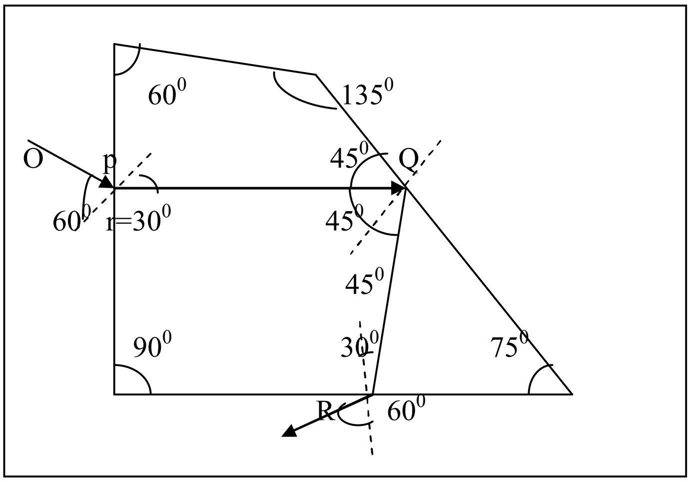


A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60^0. If the refractive index of the material of the prism is$\sqrt{3}$, which of the following is (are) correct?
(a) The ray gets totally internally reflected at face CD
(b) The ray comes out through face AD
(c) The angle between the incident ray and the emergent ray is 90^0
(d) The angle between the incident ray and the emergent ray is 120^0

Solution: $\sqrt{3} = \sin 60^0 / \sin r$ so $r = 30^0$
$\theta_c = \sin^{-1}(1/\sqrt{3})$ $\sin\theta_c = 1/\sqrt{3} = 0.577$

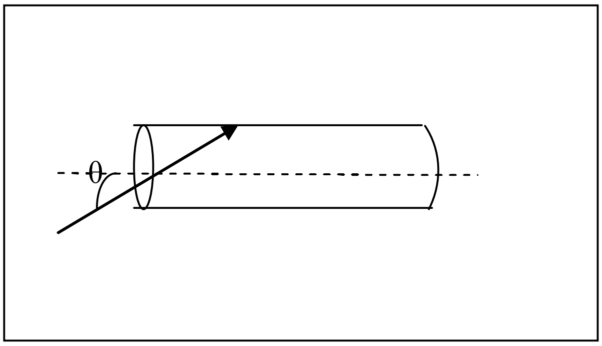
At point Q, angle of incidence inside the prism is i = 45^0

Since sin i= $1/\sqrt{2}$ is greater than sin $\theta_c = 1/\sqrt{2}$, Ray gets totally internally reflected at face CD. Path of ray of light after point Q is shown in figure.



From the figure, we can see that angle between incident ray OP and emergent ray RS is 90^0 therefore, correct options are (a), (b) and (c).

Problem 12(AIEEE 2009):



A transparent solid cylindrical rod has a refractive index of $2/\sqrt{3}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure. The incident angle θ for which the light ray grazes along the wall of the rod is

(a) $\sin^{-1}(1/2)$ (b) $\sin^{-1}(\sqrt{3}/2)$ (c) $\sin^{-1}(2/\sqrt{3})$ (d) $\sin^{-1}(1/\sqrt{3})$

Solution: $\sin c = \sqrt{3}/2$ (1)

$\sin r = \sin(90^0 - c) = \cos c = ½$

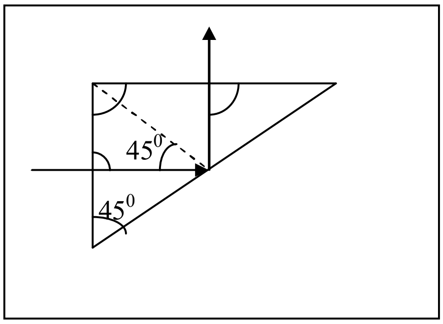
$\sin\theta / \sin r = \mu_2/\mu_1$

$\sin\theta = (2/\sqrt{3})*(1/2)$

$\theta = \sin^{-1}(1/\sqrt{3})$:

Problem 13(AIEEE 2004): A light ray is incident perpendicular to one face of a 90^0 prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45^0, we conclude that the refractive index n

(a) $n< 1/\sqrt{2}$ (b) $n > \sqrt{2}$ (c) $n> 1/\sqrt{2}$ (d) $n< \sqrt{2}$



Solution: For total internal reflection from glass-air interface, critical angle c must be less than angle of incidence. $C< i$ or $C < 45^0$ $(\because \angle i= 45^0)$

but $n= 1/ \sin c$ $\Rightarrow$ $c = \sin^{-1}(1/n)$ $\therefore \sin^{-1} (1/n) < 45^0$

$1/n < \sin 45^0$ $\Rightarrow$ $n > 1/ \sin 45^0$ $n > 1/(1/\sqrt{2})$ $\Rightarrow$ $n > \sqrt{2}$

Lecture-7

Geometrical Optics

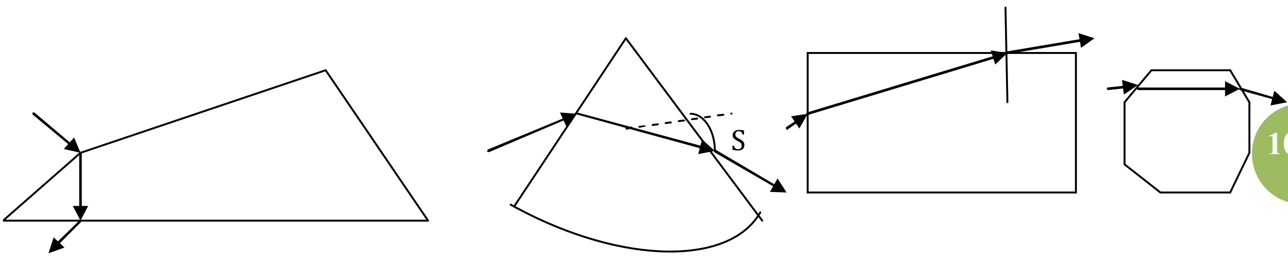
Physics for IIT - JEE

Prism Theory:

- Refraction through prism
- Deviation in prism
- Maximum deviation in prism
- Minimum deviation in prism
- Condition of no emergence: (TIR) in prism
- Dispersion of light & Causes of dispersion
- Angular dispersion & Dispersive power
- Dispersion without deviation
- Deviation without dispersion

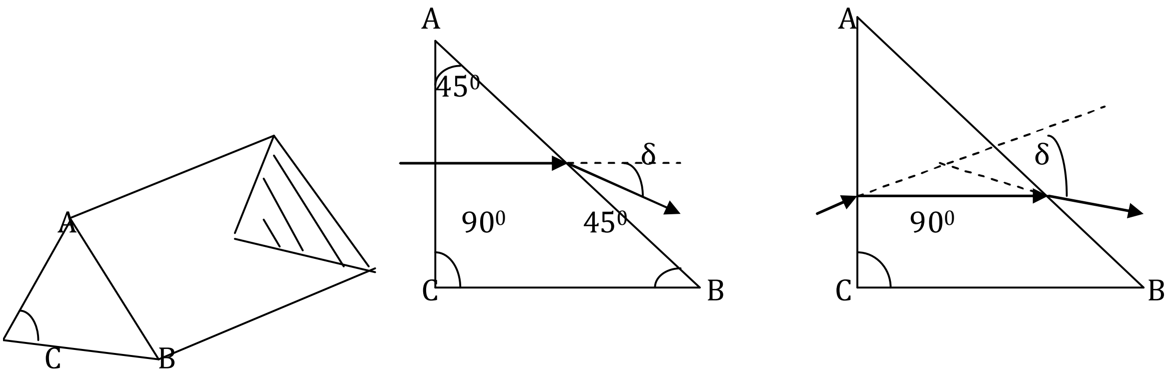
Prism: Basic facts:

Prism is a transparent medium bounded by any number of surfaces in such a way that the surfaces from which light emerges are plane and non-parallel.



(A) (B) (c) (d)

Generally equilateral, right angled isosceles or right angled prisms are used



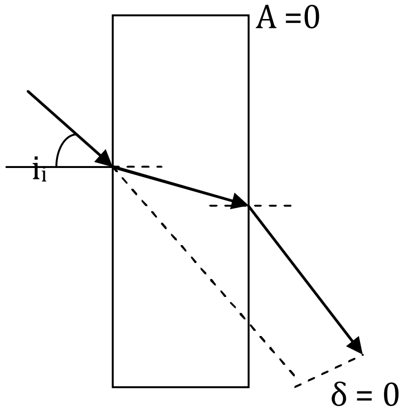
Equivalent prism (A) Right angled isosceles prism (B) Right angle prism (c)

Note:

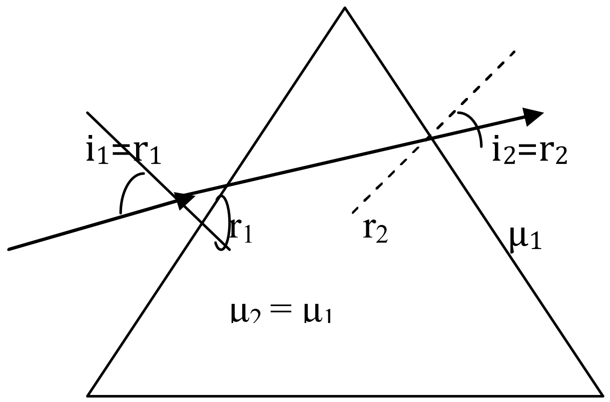
(1) Angle of prism or refractive angle of prism means the angle between the faces on which light is incident and from which it emerges in above fig. A is the angle of prism.

(2) Angle of deviation means the angle between emergent and incident rays i.e. the angle through which incident ray turns in passing through a prism. It is represented by δ in figure.

(3) If the faces of a prism on which light is incident and from which it emerges are parallel as in figure then the angle of prism will be zero and as incident ray will emerges parallel to itself deviation will also be zero. i.e. The prism will act as transparent plate.



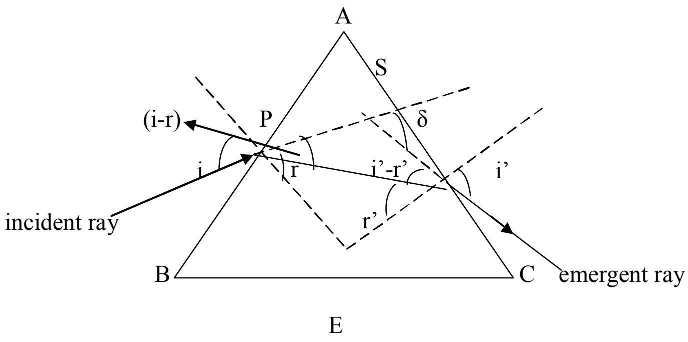
(4) If μ of the material of the prism is equal to that of surroundings, no refraction at its face will take place & light will pass through it an deviated i.e. $\delta = 0$



Hence $i_1 = r_1$ & $i_2 = r_2$

Deviation in prism:

Refraction through prism: Consider a mono chromatic ray of light incident an angle i on the face AB of the prism. It gets refracted at an angle r into the prism. After this the ray incident on other face AC of the prism at an angle r' and then finally emerges from this face with angle i' see figure.



From □ APEQ

A +E + 90 + 90 = 360

A+E = 180 (1)

r + r'+ E = 180 (2)

From Δ PQE

from (1) & (2) we get

A= r + r' (3)

From Δ PQS $\quad i - r + i' - r' = \delta$

(i + i) – (r + r') = δ deviation angle (4)

For minimum deviation i=i', r=r', $\delta = \delta_m$

$A = 2r \Rightarrow r = A/2$ from equation (3)

$2i - 2r = \delta_m$ from equation (4)

$2i - A = \delta_m$

$= (A + \delta_m)/2$

from Snell's law

$\mu = \sin i / \sin r = \sin\{(A + \delta_m)/2\} / \sin(A/2)$

Deviation produced by small angled prism or

thin prism. $\quad \sin i / \sin r = \mu = i/r$

$\mu = \{(A + \delta_m)/2\} / (A/2)$

$\mu = (A + \delta_m)/A$

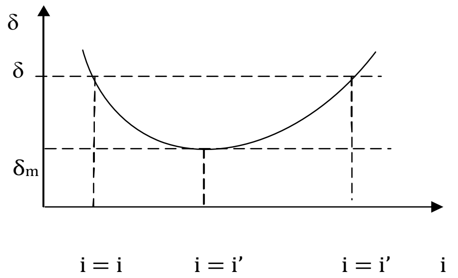
$\mu A = A + \delta_m$

$(\mu - 1) A = \delta_m$

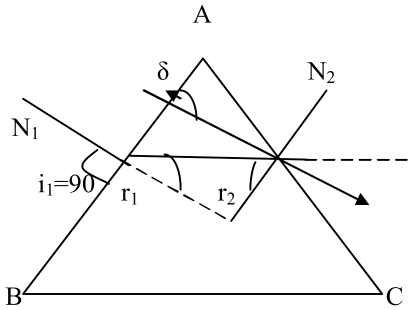
$\delta_v = (\mu_v - 1) A$

$\delta_R = (\mu_R - 1) A$ for deviation of different colours in white light

$\delta_y = (\mu_y - 1) A$



Maximum deviation in prism: $r_1+r_2 = A$, $r_2 = A - r_1 = A-\theta_c$



$\delta = i_1+i_2-(r_1+r_2)$ (1)

$\delta = i_1+i_2- A$

$\delta = \delta_{max}$ where $i_1= 90^0$ (Grazing incidence)

$\delta_{max} = 90+i_2-A$ (2)

However when $i_1=90$, Snell's law (At AB)

$1 \sin 90 = \mu \sin r_1$

$1/ \mu = \sin r_1$

$r_1 = \sin^{-1}(1/\mu)$ which is critical angle

$\mathbf{r_1= \theta_c = \sin^{-1}(1/\mu)}$

so at surface AC $\mu \sin r_2 = 1. \text{Sin } i_2$

$\text{Sin } i_2 = \mu \sin (A-\theta_c)$

$\mathbf{i_2= \sin^{-1}[\mu \sin (A-\theta_c)]}$

This is the value of angle of emergence in maximum deviation condition.

Minimum deviation in prism:

$1 \sin i= \mu \sin r$

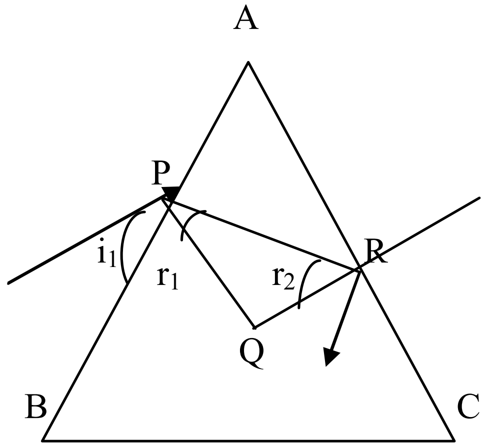
$i= \sin^{-1} \mu \sin r$

$\mathbf{i = \sin^{-1} \mu \sin (A/2)}$

In this situation angle of emergence or incidence will be obtained by applying formula.

$\mathbf{i = \sin^{-1} \mu \sin (A/2)}$

Condition of no emergence: (TIR) in prism:



The light will not emerge out of a prism for all values of angle of incidence if at face AB for i_1= max =90,at face AC $r_2 > \theta_c$ (1)

now Snell's law at face AB

$1\sin 90 = \mu \sin r_1$

$\sin r_1 = (1/\mu)$

$r_1 = \sin^{-1}(1/\mu) = \theta_c$ (2)

$r_2 > \theta_c$

$r_1 + r_2 > r_2 + \theta_c$

$r_1 + r_2 > \theta_c + \theta_c$

$r_1 + r_2 > 2\theta_c$

$\mathbf{A > 2\theta_c}$ (3)

$A/2 > \theta_c$

$\sin A/2 > \sin \theta_c$

$\sin A/2 > 1/\mu \Rightarrow$

$\mu > 1/ [\sin (A/2)] \Rightarrow$

$\boldsymbol{\mu > \operatorname{cosec} (A/2)}$

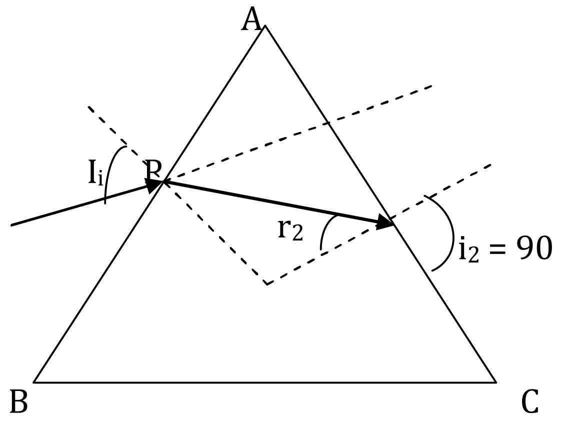
$A/2 = \sin^{-1}(1/\mu)$

$\mathbf{A > 2 \sin^{-1}(1/\mu)}$ (4)

A ray of light will not emerge out of a prism whatever be the angle of incidence if $A>2\theta_c$ i.e. if $\mu>$ cosec A/2.

Condition of grazing Emergence:

If a ray can emerge out of a prism, the value of angle i_i for which angle of emergence i_2 = 90 is called condition of grazing emergences out of face AC i.e. TIR does not take place at it



$r_2 < \theta_{critical}$ (1)

but as in prism $r_1+r_2 = A$

$r_1 = A-(\angle\theta_c)$

i.e. $r_1 > A-\theta_c$ (2)

Now from Snell's law at face AB we have

$i \sin i_1 = \mu \sin r_1$

which in light of equation (2) gives

$\sin i_1 > \mu \sin(A-\theta_c)$

i.e. $\sin i_1 > \mu[\sin A \cos \theta_c - \cos A \sin \theta_c]$

$\sin i_1 > \mu[\sin A \sqrt{1-\sin^2 \theta_c} - \cos A \sin \theta_c]$

$\sin i_1 > \sqrt{\mu^2-1} \sin A - \cos A$

as $\sin \theta_c = 1/\mu$

$i_1 > \sin^{-1}[\sqrt{\mu^2-1} \sin A - \cos A]$

$(i_1)_{min} = \sin^{-1}[\sqrt{\mu^2-1} \sin A - \cos A]$ (3)

i.e. light will emerge out of a prism only if angle of incidence is greater than $(i_1)_{min}$ given by equation (3) in this situation deviation will be given by
$\delta = (i_1+90-A)$ with i_1 given by equation (3)

Problem 1(IIT – JEE 2008): Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60^0). In the position of minimum deviation, the angle of refraction will be

(a) 30^0 for both the colours (b) greater for the violet colour
(c) greater for the red colour (d) equal but not 30^0 for both the colours

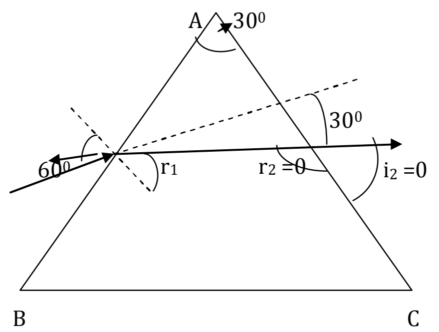
Solution: At minimum deviation ($\delta=\delta_m$)

$r_1 = r_2 = A/2 = 60^0/2 = 30^0$ (For both colours)

Problem 2: A ray of light is incident at an angle of 60^0 on one face of a prism which has an angle of 30^0. The ray emerging out of the prism makes an angle of 30^0 with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of material of the prism.

(a) $\mu = \sqrt{2}$ (b) $\mu = \sqrt{3}$ (c) $\mu = \sqrt{4/3}$ (d) none

Solution:



According to given problem $A = 30^0$, $i_1 = 60^0$, $\delta = 30^0$

In a prism $\delta = (i_1 + i_2) - (r_1 + r_2)$

$\delta = (i_1 + i_2) - A$

$30 = 60 + i_2 - 30$

$\mathbf{i_2 = 0}$

So the emergent ray is $\perp$ to the face from which it emerges

Now as $i_2 = 0$, $r_2 = 0$ but $r_1 + r_2 = A$

$r_1 + 0 = 30$

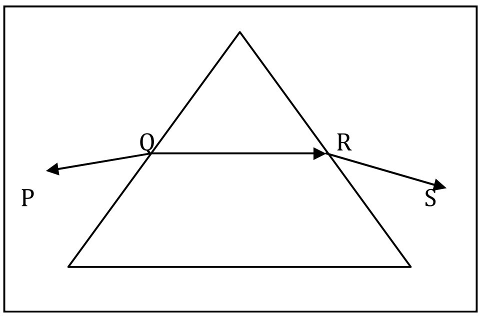
$\mathbf{r_1 = 30}$

So at first face $1 \sin 60 = \mu \sin 30$

$\sqrt{3}/2 = \mu(1/2)$

$\boldsymbol{\mu = \sqrt{3}}$ Answer

Problem 3 (IIT – JEE 2004):



A ray of light is incident on an equilateral glass prism placed on a horizontal table. For minimum deviation which of the following is true?

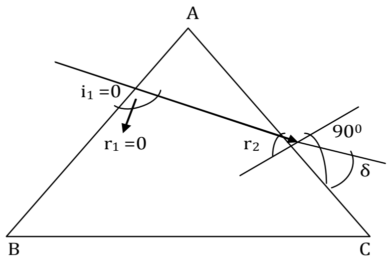
(a) PQ is horizontal (b) QR is horizontal
(c) RS is horizontal (d) Either PQ or RS is horizontal

Solution: during minimum deviation the ray inside the prism is parallel to the base of the prism in case of an equilateral prism.

Problem 4: A ray of light falls normally on a refracting face of a prism of refractive index 1.5. Find the angle of the prism if the ray just fails to emerge from the prism

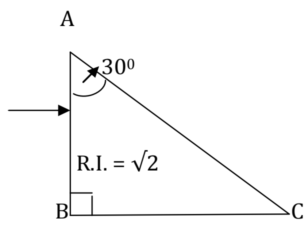
(a) $\sin^{-1}(2/3) = 42^0$ (b) $\sin^{-1}(3/2)$ (c) $\cos^{-1}(2/3)$ (d) none

Solution:



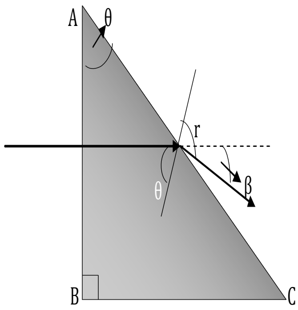
At first face of the prism as $i_1=0$, $\sin 0 = 0$
Snell's law $1 \sin i_1 = 1.5 \sin r_1$ i.e. $\mathbf{r_1 = 0}$
and as for a prism $r_1 + r_2 = A$
$0 + r_2 = A$ so $\mathbf{r_2 = A}$ (1)
But as second face as the ray just fails to emerge i.e. $r_2 = \theta_c$ (2)
$A = r_2 = \theta_c$
$\theta_c = \sin^{-1}(1/\mu)$ critical angle
$\theta_c = \sin^{-1}(1/1.5) = \sin^{-1}(2/3) = 42^0$
$\mathbf{A = 42^0}$
Note: Here $\delta = i_1 + i_2 - A = 0 + 90 - 42 = 48^0$
$\boldsymbol{\delta = 48^0}$

Problem 5:



A light ray is incident on face AB of a right angled prism as shown in fig. The refractive index of prism is $\sqrt{2}$. Now, the second face AC is rotated to increase the angle of prism. Plot deviation versus angle of prism graph.

Solution:



Critical angle (C) = $\sin^{-1}(1/\mu) = \sin^{-1}(1/\sqrt{2}) = 45^0$

Now, at any instant, angle of incidence is same as prism angle, initially θ is 30^0, as θ is increased so the angle of incidence at second face will increase and ray is transmitted out till $\theta < 45^0$

Applying Snell's law at interface AC, we get

$\sqrt{2}\sin\theta = 1.\ \text{Sin } r$ or $r = \sin^{-1}(\sqrt{2}\sin\theta)$

Deviation (β) = $r - \theta = \sin^{-1}(\sqrt{2}\sin\theta) - \theta$

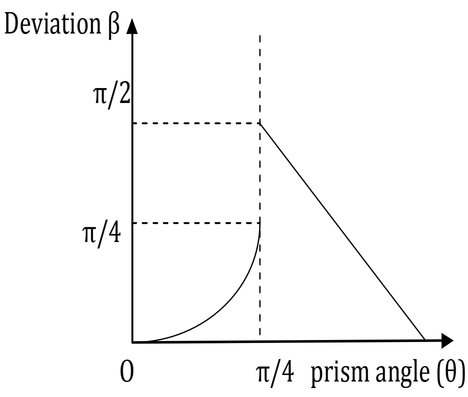
This is a non-linear function Also $\theta \to 45^0$

$\beta = \sin^{-1}(\sqrt{2}\sin 45^0)-(\pi/4) = [(\pi/2)-(\pi/4)] = \pi/4$

When prism angle is increased above 45^0, TIR will take place and deviation is given by

$\beta = \pi - 2\theta$, As $\theta \to (\pi/2)$, $\beta = 0$

The graph between deviation and angle of prism is as shown in fig.

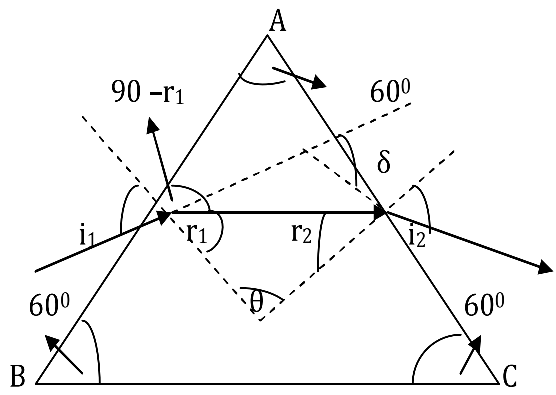


Problem 6: A ray of light undergoes deviation of 30^0 when incident on an equilateral prism of refractive index $\sqrt{2}$. What is the angle subtended by the ray inside the prism with the base of the prism.

(a) 30^0 (b) 60^0

(c) 90^0 (d) zero

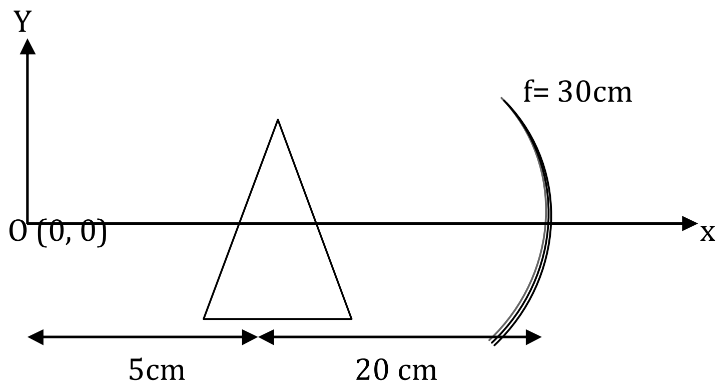
Solution:



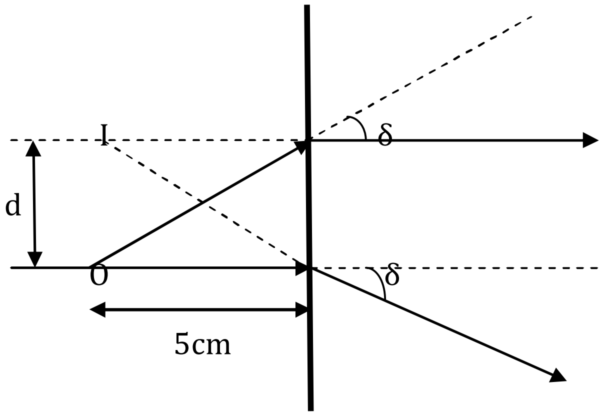
Here $\delta = 30^0$, $A = 60^0$
so if the prism had been in a minimum deviation
$\mu = \sin\{(A+\delta_m)/2\}/ \sin(A/2)$
$\mu = \sin\{(60+30)/2\}/ \sin(60/2)$
$\mu = \sin 45^0/ \sin 30^0$
$\mu = (1/\sqrt{2})*2 = \sqrt{2}$
Refractive index of prism is also given as $\sqrt{2}$,
the prism is in the position of minimum deviation.
This in turn implies that $r_1 = r_2 = r = A/2 = 60/2 = 30^0$
So the angle subtended by the ray inside the prism with surface AB is equal to
$90-r_1 = 90-30 = 60^0$
As base also subtends an angle of 60^0 with the face AB.

Hence the ray inside the prism is parallel to the base.
i.e. the angle subtended by the ray inside the prism with the base is zero.

Problem 7: Find the co-ordinates of image of the point object 'o' formed after reflection from concave mirror as shown in fig. Assuming prism to be thin and small in size of prism angle 2^0 Refractive index of prism material is (3/2).



Solution:



Consider image formation through prism. All incident rays will be deviated by

$\delta = (\mu\text{-}1)A = [(3/2)\text{-}1]2^0 = 1^0 = (\pi/180)$ rad

Now as prism is thin so object and image will be in same plan as shown in fig.

It is clear $d/5 = \tan \delta = \delta$ (∵ δ is very small)

or $d = (\pi/36)$ cm

Now this image will act as an object for concave mirror

$u = -25$ cm, $f = -30$ cm

$v = uf/(u-f) = 150$ cm

Also, $m = -v/u = +6$

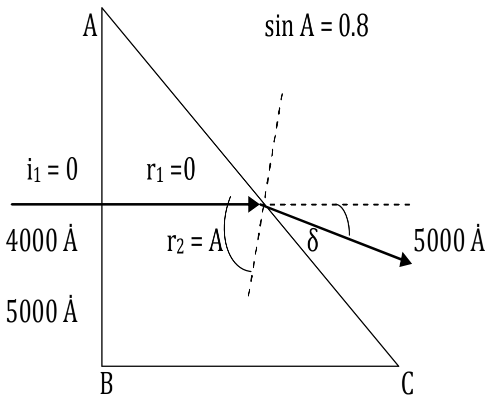
∴ Distance of image from principal axis = $(\pi/36)6 = \pi/6$ cm

Hence, co-ordinates of image formed after reflection from concave mirror are (175cm, $\pi/6$ cm)

Problem 8: IIT-JEE Based on causes of dispersion (TIR): A beam of light which contains radiations of wavelengths 5000 Å and 4000 Å is incident normally on a prism as shown in figure. The refractive index of the prism as a function of wavelength is given by $\mu(\lambda) = 1.20 + (b/\sqrt{2})$

where λ is in Å and b is a +ive constant. The value of b is such that the condition for total internal reflection at the face AC is just satisfied for one wavelength and not for other. Find the value of constant b and deviation produced by the prism.

Solution:



According to given problem with increase in wavelength, μ decreases and as $\theta_c = \sin^{-1}(1/\mu)$ longer wavelength will have shorter μ and greater critical angle. Now as condition of TIR is satisfied only for one wavelength (and not for other), the wavelength must be shorter one i.e. 4000 Å

[As if the condition of TIR is satisfied for longer wavelength, i.e. $r_2 > (\theta_c)_{longer}$, it will automatically be satisfied for shorter wavelength also as $(\theta_c)_{shorter} < (\theta_c)_{longer}$ and no light will be transmitted] and hence $\mu = 1.20 + b/\surd 2$ (1)

Now as in prism $r_1 + r_2 = A$ & as here $i_1 = r_1 = 0$, $r_2 = A$ (2)

but condition of TIR for λ = 4000 Å at face AC required $r_2 = \theta_c$ (3)

so from equation (2) & (3)

$A = \theta_c$ i.e. $\sin A = \sin \theta_c$ (4)

$0.8 = \sin \theta_c = 1/\mu$, **$\mu = 1/0.8 = 1.25$**

Substituting the value of μ from eq. (4) in eq. (1)

$\mu = 1.20 + (b/\surd 2) = 1.20 + (b/(4000)^2)$

i.e. $b = 8*10^5$ $(\text{Å})^2$

$\mu = 1.20 + [8*10^5 / (500)^2] = 1.20 + 0.032 = 1.232$

And hence for light of wavelength 5000 Å at face AC from Snell's law

$\mu \sin r_2 = 1. \text{Sin } i_2,$

$1.232 \sin A = \sin i_2$ ($\because r_2 = A, \mu = 1.232$)

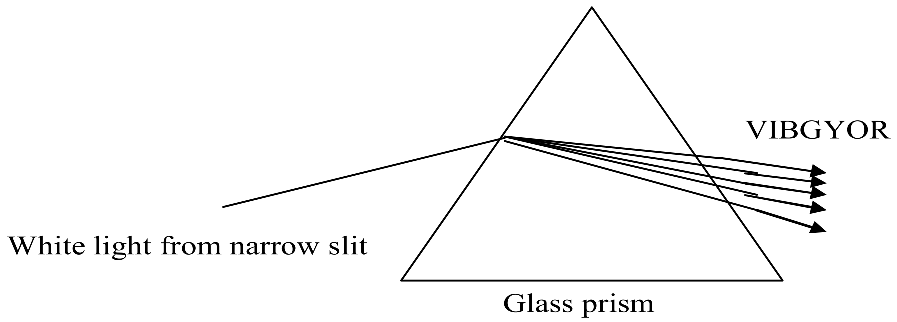
$\sin i_2 = 1.232*0.8$ ($\therefore \sin A = 0.8$)

$i_2 = \sin^{-1}(0.9856) = 80.265^0$

Now as in prism $\delta = i_1 + i_2 - A$ here $i_1 = 0$, $A = \sin^{-1}(0.8) = 53.13^0$

$\delta = 27.135^0$

Dispersion of light:



* In Rig-Veda it is mentioned that light is made of many colours.

* In 1665 Sir Isaac Newton showed that natural light actually consists of seven colours. All the colours of light mixed together appear white.

* The splitting of light (white) in to its constitutions is called dispersion of light VIBGYOR The band of colours formed on a screen due to dispersion is called spectrum.

* Theoretically each wavelength is associated with its own colour; there are infinite numbers of colours in natural light. Our eyes can differentiate only six colours indigo & violet cannot be differentiable. So in further study we consider only six colours in the spectrum of white light in wavelength range (4000 – 7000 Å).

Causes of dispersion:

$\mu = A + (B/\lambda^2)$ *Cauchy's equation*

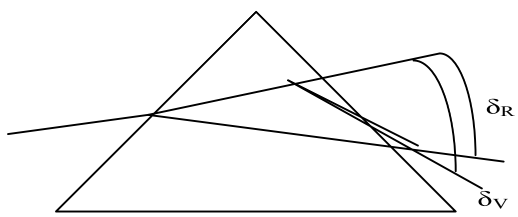
Where μ= refractive index of any medium

λ = wavelength of light, μ depends on λ , A& B are constants. $\lambda_R > \lambda_v$ hence $\mu_v > \mu_R$

hence $\delta_V > \delta_R$ by $\delta = (\mu-1)$

Hence deviation of violet colour is maximum is at the lower end &

red colour is the upper end of spectrum.



Angular dispersion: $\theta = \delta_v - \delta_R$

Dispersive power:

$\omega = \theta/\delta_y$= angular dispersion/mean deviation $\omega = (\delta_v - \delta_R)/\ \delta_y = (\mu_v - \mu_R)/(\mu_y - 1)$

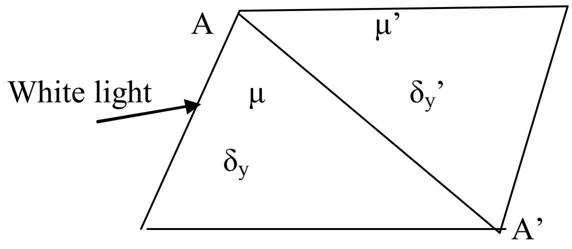
Combination of prisms: prism produces both

Deviation Dispersion

prism cannot give deviation without dispersion and vice versa when white light is incident on it. So combination of prism (two) can do so...

Dispersion without deviation Deviation without dispersion

Dispersion without deviation:



Consider two prisms of angles A& A' ,

index μ &μ' ,deviation by intermediate colours δ_y & δ_y'

$\delta_y = (\mu_y-1)A$, $\delta_y'= (\mu_y'-1)A'$

Total deviation produced by two prism's is zero.

$\delta_y + \delta_y' = 0$

$(\mu_y-1)A + (\mu_y'-1)A'=0$

$A'= - [((\mu_y-1) A) / (\mu_y'-1)]$ (1)

The negative sign indicates the two prisms must be placed with their angles oppositely. But the combination will produce some dispersion.

By 1st prism dispersion produced $\delta_v - \delta_R$

By **2**nd prism dispersion produced $\delta_v' - \delta_R'$

Total dispersion produced $D= \delta_v - \delta_R + \delta_v' - \delta_R'$

$= (\mu_v - \mu_R) A+(\mu_v' - \mu_R')A'$ (2)

$D = (\mu_v - \mu_R) A+(\mu_v' - \mu_R')[- (\mu_y-1)/(\mu_y'-1)]A$ substituting (1)

$D = (\mu_v - \mu_R) A-[\{(\mu_v' - \mu_R')/ (\mu_y'-1)\}(\mu_y-1)A]$ 3(a)

$D= [\{(\mu_v - \mu_R) / (\mu_y-1)\} (\mu_y-1)A] - [\{(\mu_v' - \mu_R')/ (\mu_y'-1)\} (\mu_y-1)A]$

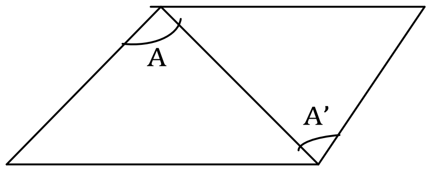
$D= \omega A(\mu_y-1) - \omega' A(\mu_y-1)$

$D= (\omega-\omega') A (\mu y-1)$ 3(b)

Deviation without dispersion: Total dispersion $D= (\delta v - \delta_R) + (\delta_v' - \delta_R') =0$

$(\mu_v - \mu_R) A+(\mu_v' - \mu_R')A' = 0$

$A' = - [(\mu_v - \mu_R)/ (\mu_v' - \mu_R')] A'=0$ (4)



Negative sign indicates two prisms must be placed with their angle oppositely. But deviation : $\delta = \delta_y + \delta_y'$

$\delta = (\mu_y-1)A + (\mu_y'-1)[-(\mu_v - \mu_R)/(\mu_v' - \mu_R')]A$

$\delta = (\mu_y-1)A[1- \{(\mu_y'-1)/(\mu_v' - \mu_R')\}*\{(\mu_v - \mu_R)/(\mu_y-1)\}]$ 6(a)

$\delta = (\mu_y-1)\,A\,[1-(\omega/\omega')]$ Two identical prisms of same material Placed in contact will give light without deviation & dispersion.

Problem 9: A thin prism $P_1$with angle 4^0 & made from glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. What is the angle of prism P_2.

(a) $+3^0$ (b) $+ 4^0$ (c) 5^0 (d) none

Solution: given $A_1= 4^0$, $\mu_1 = 1.54$, $\mu_2 = 1.72$

the angle of second prism for no total deviation:

$A_2 = -[(\mu_1-1)/(\mu_2-1)]A_1 = [(1.54-1)/(1.72-1)]*4= -3*$

Hence, the angle of the second prism should be 3^0 & it should be placed opposite to the first.

Problem 10: White light is passed through a prism of angle 5^0. If the angle refractive indices for red and blue colours are 1.641 & 1.659 respectively .Find

(1) The angle of dispersion between them (2) dispersive power of prism

Solution: (1) As for small angle of prism $\delta = (\mu-1)\,A$

$\delta_v = (1.659-1) *5^0 = 3.295^0$

$\delta_R = (1.641-1) *5^0 = 3.205^0$

$\theta = \delta_v - \delta_R = 3.295 - 3.205 = 0.090^0$

(2) As $\mu_{av} = (\mu_v - \mu_R)/2 = (1.659 + 1.641)/2 = 1.650$

so dispersive power of the prism

$\omega = (\mu_v - \mu_R)/ (\mu-1) = (1.659 - 1.641)/(1.650-1) = 0.081/(1.650 - 1)$

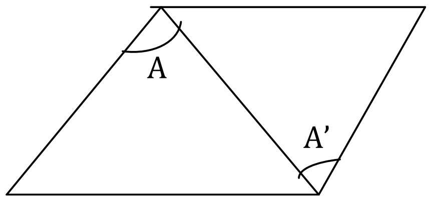
$\omega = 0.018/0.650$

$= 0.0277$

Problem 11 (IIT 2001): The refractive indices of crown glass for blue and red lights are 1.51 and 1.49 respectively and those of the flint glass are 1.51 and 1.49 respectively. An isosceles prism of angle 6^0 is made of crown glass. A beam of white light is incident at a small angle on this prism. The other flint glass isosceles prism is combine with the crown glass prism such that there is no deviation of the incident light.

Determine the angle of the flint glass prism. Calculate the net dispersion of the combined system.

Solution:



For combination of prisms

net deviation = $\delta_1 + \delta_2 = (n_y - 1)A + (n'_y - 1)A$

where δ_1 and δ_2 are deviations produced by individual prisms.

Net dispersion = $(n_v - n_R) A + (n'_v - n'_R) A$

According to problem net deviation is zero

hence $(n_y - 1)A = -(n'_y - 1)A'$

$\mathbf{A' = -[(n_y - 1)A / (n'_y - 1)]}$

Negative sign implies that second prism is inverted relative to first

$n_y = (n_v + n_R)/2$, $n'_y = (n'_v - n'_R)/2$

$A' = -[(n_y - 1)A / (n'_y - 1)]$

$n_y = (1.51 + 1.49)/2$

$n'_y = (1.77 + 1.73)/2$, $A = 6^0$ on substituting numerical values of n_y, n'_y, A

we get $\mathbf{A' = -4^0}$

Net dispersion = $(n_v - n_R)A - (n'_v - n'_R)A'$ = $\mathbf{0.04^0}$ **ans.**

Lecture-8

Geometrical Optics

Physics for IIT - JEE

Defects of images (Aberration)

Chromatic Abberation:

- *Achromatism*
- *The achromatic doublet*
- *Achromatism by separated doublet*

Monochromatic Abberation:

- *Spherical*
- *Coma*
- *Astigmatism*
- *Curvature*
- *Distortion*

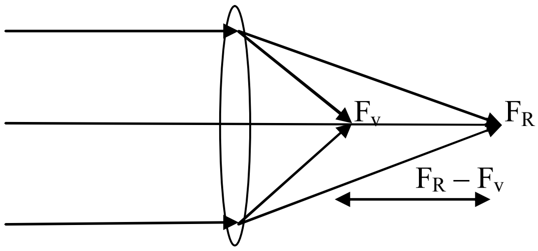
Defects of images (Aberration):

- The equation & relations derived for lenses hold for paraxial light rays or for the rays making small angles with the optic axis.
- In practise however lenses are used to form images of points. Which are off the axis also if light coming from an object is not monochromatic; a number of overlapped coloured images are formed by the lens.
- Thus in actual practise the image of a point object is not sharp & white. This defect of lens is called aberration.
- The coloured object formed by a lens of white light is called chromatic aberration.

Chromatic Aberration:

$$\mu = A + (B/\lambda^2)$$

- λ is the different for different colours of white light.
- μ is different for different colours of white light.
- Hence the focal length of a lens is different for different colours. It is longest for red & shortest for violet colour is called axial or longitudinal chromatic aberration.



$\delta_f = f_R - f_v$

δ_f = longitudinal or axial chromatic aberration for thin lens, the expression for chromatic aberration can be easily derived.

1st Approach: The focal length of a thin lens is

$1/f = (\mu-1)[(1/R_1)-(1/R_2)]$ (1)

$-df/f^2 = \delta\mu[(1/R_1)-(1/R_2)]$ (2)

dividing equation (2) & (1) -

$(df/f^2) / 1/f = \delta\mu/(\mu-1)$ If a small change in μ gives a small change in f.

$-(df/f^2) f = \delta\mu/(\mu-1)$

$[df/f] = -\delta\mu/(\mu-1) = \omega$

$[df] = -\delta\mu/(\mu-1) f$

This represents the axial chromatic aberration of lens.

If μ_v & μ_R represents the refractive indices of the red & violet colours

$f_R - f_v = -[(\mu_R-\mu_v)/(\mu-1)]fy$

$f_R - f_v = [(\mu_v-\mu_R)/(\mu_y-1)]fy$

$[(\mu_v-\mu_R)/(\mu_y-1)]$ = dispersive power of a lens material

$f_R - f_v = \omega f_y$ **Expression for chromatic aberration**

2^{nd} Approach:

$1/f_R = (\mu_R-1)[(1/R_1)-(1/R_2)]$
$1/f_v = (\mu_v-1)[(1/R_1)-(1/R_2)]$
$1/f_y = (\mu_y-1)[(1/R_1)-(1/R_2)]$
$(1/f_R)-(1/f_v) = (\mu_R-\mu_v)[(1/R_1)-(1/R_2)]$
$(f_v-f_R)/f_R f_v = (\mu_R-\mu_v)[(1/R_1)-(1/R_2)]$
$(f_R-f_v)/f_R f_v = [(\mu_v-\mu_R)/(\mu_y-1)][(1/R_1)-(1/R_2)]$
$(f_R-f_v)/f_y^2 = [(\mu_v-\mu_R)/(\mu_y-1)][1/f_y]$ $(f_R f_v = f_y^2)$
$(f_R - f_v) = \omega f_y$

Achromatism:

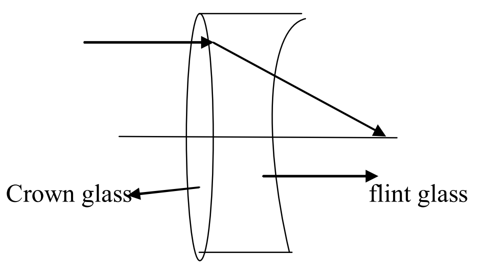
Minimization or removal of chromatic aberration is called Achromatism.

- This can be possible by using two lenses of opposite nature.
- The system of two lenses which is free from chromatic aberration is called achromatic doublet.

The achromatic doublet:

Consider two lenses of focal lengths f_1&f_2 & dispersive powers ω_1 & ω_2 are put in contact.
f is the focal length of the combination.
Then $1/f=1/f_1+1/f_2$
$-df/f^2 = -df_1/f_1^2 - df_2/f_2^2$ for **Achromatism** $df = 0$
$df_1/f_1^2 + df_2/f_2^2 = 0$
$(\omega_1/f_1) + (\omega_2/f_2) = 0$ This is the required condition of Achromatism.

Case1^st^: If $\omega_1 = \omega_2$
$[1/f_1+1/f_2]\,\omega=0$
$\omega\neq0$, Properties of the material of the prism.
$[1/f_1+1/f_2]=0$
$1/f = 0$
$f = \infty$
Lens behaves like a plane mirror.
Hence both lenses have opposite material.



Case 2nd: $(\omega_1/f_1) = -(\omega_2/f_2)$

ω_1 & ω_2 are the properties of the material of the prism. Hence it must be positive. This indicates both the lenses must be of opposite nature. If one is concave then other is convex.

Concave: Diverging **Flint glass** Convex: Converging **Crown glass**

Achromatism by separated doublet:

Problem 1: Consider two convex lenses of focal lengths f_1&f_2 are made from same material, are separated by a suitable distance d. The focal length of the combination is given by $1/f = 1/f_1+1/f_2-d/f_1f_2$, the value of d for which combination is free from chromatic aberration is given by

a) $(f_1+f_2)/2$ b) $(f_1-f_2)/2$ c) (f_1+f_2) d) none

Solution: Consider two convex lenses of focal lengths f_1&f_2 separated by a suitable distance d. The focal length of the combination is given by: $1/f_1+1/f_2-d/f_1f_2 = 1/f$ (1)
differentiate above equation

$-df_1/f_1^2 - df_2/f_2^2 - d[1/f_1(-df_2/f_2^2)+1/f_2(df_1/f_1^2)] = (-1/f^2)df$ {df=0 for Achromatism}

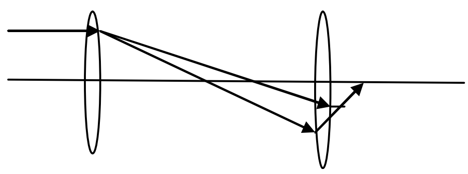
$(df_1/f_1)1/f_1+(df_2/f_2)1/f_2 - d[(df_2/f_2)1/f_1f_2+(df_1/f_1)1/f_1f_2]=0$

$(\omega_1/f_1) + (\omega_2/f_2) - d[(\omega_2/f_1f_2) + (\omega_1/f_1f_2)] = 0$

$(\omega_1 f_2+\omega_2 f_1)/f_1f_2 = d(\omega_2+\omega_1)/f_1f_2$

which gives $\mathbf{d = (\omega_1 f_1+\omega_2 f_2)/(\omega_1+\omega_2)}$

In case when lenses are of same material, $\boldsymbol{\omega_1 = \omega_2 = \omega}$ Then $\mathbf{d = (f_1+f_2)/2}$



Thus two lenses of same nature can be free from chromatic aberration if they are placed at a separation $\mathbf{d = (f_1+f_2)/2}$

Problem 2: Achromatism of Hygen's Eyepiece:

Hygen's Eyepiece consist of two Plano convex lenses L_1&L_2 of same material having focal length +3f & +f separated by a distance d from each other. Their convex side face the incident light. Hygen's' Eyepiece is achromatic when lenses are placed coaxially at distance

a) d=f /2 b) d=f c) d=2f d) d=3f

Solution: As we know that in case of Achromatism by separated doublet separation distances between two lenses is given by : $\mathbf{d = (\omega_1 f_1+\omega_2 f_2)/(\omega_1+\omega_2)}$

In case when lenses are of same material, $\boldsymbol{\omega_1 = \omega_2 = \omega}$ Then $\mathbf{d = (f_1+f_2)/2}$
in this problem f_1=3f & f_2=f
$d = (f_1+f_2)/2=3f+f/2=2f$
Hygen's Eyepiece satisfies the condition of Achromatism and has the same focal length for all colours. This makes Hygen's Eyepiece apparently free from lateral chromatic aberration.

Problem 3: The focal lengths of two convex lenses of same material are 20 cm & 30 cm. What should be the separation between them so that they form an achromatic combination?

a) 25 cm b) 30 cm c) 40 cm d) None

Solution: $d = (f_1+f_2)/2 = (20+30)/2 = 25$ cm

Problem 4: The dispersive power of crown and flint glasses are 0.02&0.04 respectively. Find the focal length of the two components of an achromatic doublet of focal length 20 cm.

Solution: An achromatic doublet (telescope objective) consists of two lenses in contact, a convex lens of crown glass and a concave lens of flint glass. If f_1 & f_2 be the focal length and ω_1 & ω_2 the dispersive powers of the convex and concave components, then the condition of Achromatism is

$\mathbf{(\omega_1/f_1) + (\omega_2/f_2) = 0}$
here $\boldsymbol{\omega_1}$=0.02 & $\boldsymbol{\omega_2}$=0.04
$(0.02/f_1) + (0.04/f_2) = 0$
$f_2=-2f_1$
Focal length of combination of two thin lenses in contact $1/f_1 + 1/f_2 = 1/F$
here F=20 cm & $f_2=-2f_1$
$1/f_1 + 1/(-2f_1) = 1/20$
$1/f_1 - 1/2f_1 = 1/20$
$1/2f_1 = 1/20$
$\mathbf{f_1 = 10}$ **cm**
$\mathbf{f_2}=-2f_1=$ **- 20 cm**

Problem 5: A convex lens made of material 'A' is combined with a concave lens made of material 'B' so as to form an achromatic doublet. If an object of height 6 cm is placed 30 cm in front of the doublet, it forms an erect image of size 2 cm. Find the focal lengths of the component lenses, given that the ratio of dispersive powers of materials A & B is 2:1.

(a) 15, -7.5 cm (b) -7.5, 15cm (c) 1.5, 15cm (d) none

Solution: The erect & small size of the image shows that doublet should be of diverging nature. For erect image
m= v/u = I/O {m=-v/u}
m = (v)/(u) = I/O = 2/6
v/u =1/3 , v = u/3

$u = -30cm$, $v = -10cm$

By the lens formula $1/v - 1/u = 1/f$

$[1/(-10)] - [1/(-30)] = 1/f$ $\qquad f_{combination} = -15$ cm

$1/f_A + 1/f_B = 1/f$

$1/f_A + 1/f_B = -1/15$

$1/f_A[1+(f_A/f_B)] = -1/15$

From condition of Achromatism $(\omega_A/f_A) + (\omega_B/f_B) = 0$,

$\omega_A/\omega_B = - f_A/f_B = 2$ (given in problem)

$1/f_A[1+(-2)] = -1/15$

$-1/f_A = -1/15$ gives $f_A = 15$

$f_A/f_B = -2$, $15/f_B = -2$ gives $f_B = -7.5$ cm

Hence option (a) 15; -7.5 cm is only correct.

Problem 6: An equiconvex lens of crown glass and an equiconcave lens of flint glass make an achromatic system. The radius of curvature of convex lens is 0.54m. If the focal length of the combination for mean colour is 1.50mand the refractive indices for the crown glass are $\mu_R = 1.53$ & $\mu_v = 1.55$. Find the dispersive power of flint glass

(a) 1/18 = .055 (b) 2/18 (c) 3/18 (d) none

Solution: As for a lens $1/f = (\mu-1)[(1/R_1)-(1/R_2)]$

$R_1 = R_2 = R = 54cm$ $\mu y = (\mu_v + \mu_R)/2 = 1.54$ (given)

for convex lens $1/f_c = (1.54-1)[(1/54)-(1/-54)$

$1/f_c = (1.54-1)[(1/54)+(1/54)]$ gives $f_{Crown} = 50$ cm (1)

now as combination of two thin lenses in contact $1/f_c + 1/f_F = 1/f$

$1/50 + 1/f_F = 1/150$ gives $f_{lint} = -75cm$ (2)

As the system is achromatic $\omega_f/f_F + \omega_c/f_c = 0$

$\omega_c = (\mu_v - \mu_R) / \mu_{y-1} = 1.55-1.53/1.54-1 = 1/27$ (3)

Substituting the values of f_c , f_f & ω_c in $\omega_f/f_F + \omega_c/f_c = 0$

$\omega_f = -\omega_c (f_f/f_c) = -(1/27)(-75/50) = \mathbf{1/18 = 0.055}$

Problem 7: Two lenses, one made of crown glass and the other of flint glass, are to be combined so that the combination is achromatic for the blue & red light and acts as a convex lens of focal length 35 cm.

Calculate the focal length of the components of it.

For crown glass $\qquad \mu_y = 1.5175$ $\mu_\beta - \mu_R = 0.00856$

flint glass $\qquad \mu_y = 1.6214$ $\mu_\beta - \mu_R = 0.01722$

Solution: According to given problem $(1/f_c) + (1/f_f) = 1/35$ (1)

the system will be achromatic if $(\omega_c/f_c) + (\omega_f/f_f) = 0$ i.e. $(f_c/f_f) = -(\omega_c/\omega_f)$ (2)

Now as by definition $\omega = d\mu/\mu-1 = (\mu_\beta - \mu_R)/(\mu_y-1)$

$\omega_c = 0.00856/(1.5175-1) = 0.01654$

$\omega_f = 0.01722/(1.6214-1) = 0.02771$

substituting these values of ω_c & ω_f in eq. (2)

$f_c / f_f = -[0.01654/0.02771]$ $f_c = -[1654/2771] f_f$ (3)

substituting f_c from eq.(3) in eq. (1)

$1/f_c + 1/f_f = 1/35$

$- [2771/1654f_f] + 1/1/f_f = 1/35$

$f_f = [-35*117]/1654 = -23.8cm$

$f_c = -[1654/2771]\ f_c = (-1654*-23.8)/2771 = 14.17\ cm$

$f_{flint} = -23.8$ & $f_{crown} = 14.17$ cm

Problem 8: (IIT-JEE)

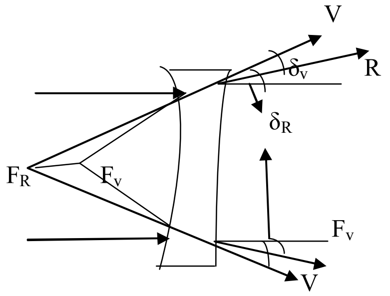
State whether the following statement is true or false giving reason in brief " A parallel light beam of white light is incident on a combination of a concave & convex lens both of same material. Their focal lengths are 15cm and 30cm respectively for the mean wavelength in white light on the other side of the lens system. One sees coloured patterns with violet colours at the outer edge''.

Solution:

The combination will behave as a single lens of focal length $1/f = 1/(-15) + 1/30$

$f = -30$ cm , As divergent lens of focal length 30cm.

Since for a lens f_v is lesser than f_R violet deviates more than red so on the other side we will see coloured pattern with violet on the outer side i.e. the given statement is true.



Monochromatic Aberration:

The size of the image as formed by a lens is not according to theoretical calculation even using monochromatic light the image formed will spread both along & perpendicular to the principle axis of the lens. Also the shape of the image is not according to the shape of object.

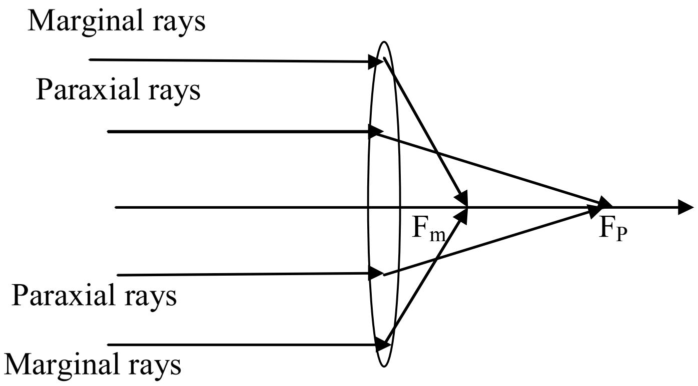
Monochromatic Aberration can be divided:

1.Spherical   2.Coma   3.Astigmatism   4.Curvature   5.Distortion

1. *Spherical Aberration:*

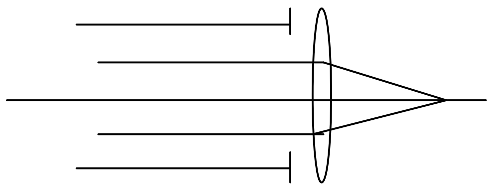
Figure shows the image formed by different parts of a lens of a point object.

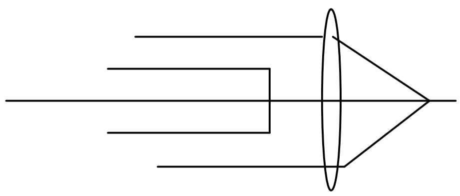
Paraxial rays of light form the image at longer distance from the lens than the marginal rays. The image is not separate any point on the axis the effect is called spherical aberration.



Minimization or removal of Spherical Aberration:

Spherical aberration can be minimized by





(1) Using stops or crossed lens.
(2) Using lens of large focal lengths
(3) Using Plano convex lens
(4) Using two thin lenses separated by a distance using crossed lens
(5) Using aplanatic lens
(6) Using parabolic reflectors

2. Coma: When object is situated off the axis, its image will spread obliquely perpendicular to the principle axis. It looks like comet & so called coma.

Removal of Coma:

(i) Coma may be reduced by placing a stop at a suitable distance from the lens.
(ii) Coma may be minimized by designing lenses of suitable shapes and materials.
For example, for an object at infinite distance, a lens with μ=1.5 and R_1/R_2= -1/9 forms an image sufficiently free from coma.

3. Astigmatism: When object is situated off the axis, the spread of image along the principle axis of the lens is known as astigmatism.
The object situated off the axis its image will spread along & perpendicular to the principle axis.

Removal of Astigmatism:

(i) For a single lens the astigmatic difference may be reduced by placing a stop in a suitable position so that only less oblique rays are permitted to form the image.
(ii) For a system of several lenses astigmatism may be eliminated by adjusting their relative positions. Such systems are widely used as photographic objectives on which narrow pencils are incident at large angles.

4. Curvature: The image of an object (extended plane object) formed by lens is not a flat but curved. This defect is called the curvature. This effect is even present it the aperture of the lens is reduced by a suitable stop.

Removal of Curvature:

(i) for a single lens, the curvature may be reduced by placing an aperture in a suitable position in front of the lens.
(ii) For a combination of lenses, the condition for absence of curvature is sum of $1/\mu f = 0$, where μ is refractive index & f is the focal length of a lens. For two lenses (whether in contact or separated by a distance) the condition reduces to

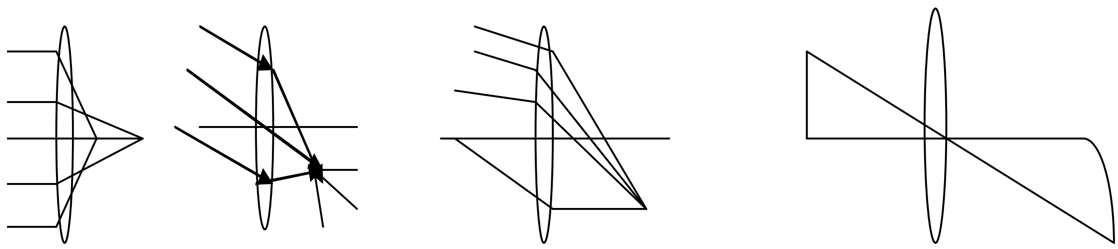
$$\mathbf{1/\mu_1 f_1 + 1/\mu_2 f_2 = 0 \quad \text{or} \quad \mu_1 f_1 + \mu_2 f_2 = 0 \qquad \text{(Petzval condition)}}$$

Since μ_1, μ_2 are positive, f_1 & f_2 should be of opposite signs. Hence by combining a convex lens of a certain material with a concave lens of the suitable material and focal length, a flat object is obtained.

5. Distortion: When a stop is used is used with a lens to reduce the various aberrations, the image of a plane square like objects placed perpendicular to the axis is not of the same shape as the object. This defect is called distortion.

Removal of Distortion:

A combination of two similar meniscus convex lenses with their concave surfaces facing each other and having an aperture stop in the middle is free from distortion, when the object & image are symmetrically placed.



(a) Object situated off axis

(b) Image like comet image spread perpendicular to principle axis

(c) Spread of the image along the principle axis

(d) Curved Image

Problem 9(AIEEE 2011): When monochromatic red light is used instead of blue light in a convex lens, its focal length will

(a) increase (b) decrease (c) remain same (a) does not depend on colours of light

Solution: $1/f = (\mu-1)[(1/R_1)-(1/R_2)]$

also, by Cauchy's formula $\mu = A+(B/\lambda^2)+(c/\lambda^4)+.......$

$\lambda_{blue} < \lambda_{red}$

$\mu_{blue} > \mu_{red}$

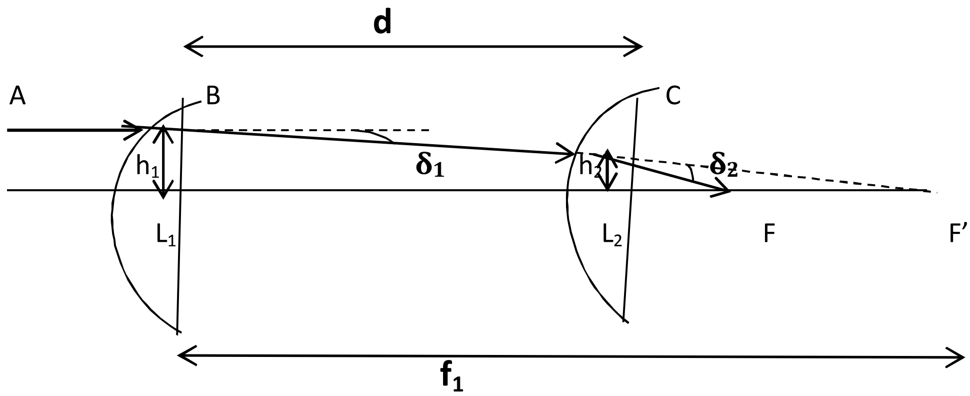
hence, $f_{red} > f_{blue}$

Condition for Minimum Spherical Aberration of two lenses separated by a distance d:

Problem 10: The condition for minimum spherical aberration for two thin lenses of focal length f_1&f_2 separated by distance d is given by

a) $d= f_1+f_2$ b) $d= f_1-f_2$ c) $d= f_1-f_2/2$ d) None

Solution: When a beam of light passes through a system of two thin lenses placed coaxially at a distance apart from each other, the refraction takes place at four surfaces. The spherical aberration is Minimum when there is an equal deviation at all a surface.



Let L_1&L_2 be two coaxial converging lenses of focal lengths f_1&f_2 and separated by a distance d. A ray parallel to the axis meets L_1 at a height h1and suffers deviation $\delta_1=h_1/f_1$ and is directed towards F', the second focal point of L_1. The reflected ray BC meets the lens L_2 at height h_2 and suffers deviation $\delta_2=h_2/f_2$. The final emergent ray meets the axis at F which is the second focal length of the combination.

For minimum spherical aberration, the total deviation should be equally shared by two lenses, that is

$\delta_1= \delta_2$

$h_1/f_1= h_2/f_2$, $h_1/ h_2= f_1/f_2$ (1)

Now from similar triangles BL_1F' & CL_2F', we have

$BL_1/CL_2=L_1F'/L_2F'=L_1F'/ (L_1F'-L_2F')$

Here $BL_1=+h_1$, $CL_2=+h_2$, $L_1F'=+ f_1$ & $L_1L_2=d$

$h_1/ h_2=f_1 /(f_1-d)$ (2)

comparing equation (1) &(2) we get

$f_1/f_2 = f_1 /(f_1-d)$, $f_2 =(f_1-d)$

$d =(f_1- f_2)$ This is required condition. (3)

So the two lenses should be placed with light incident on the lens of grater focal length and the distance between them should be equal to the difference between their focal lengths.

Note: An optical device satisfying Condition for Minimum Spherical Aberration of two lenses separated by a distance d:

The condition of minimum spherical aberration is satisfied in Hygen's eyepiece, which consists of two Plano convex lenses of focal lengths3f&f respectively and separated by a distance 2f.

Problem11: A convergent doublet of separated lenses, corrected for spherical aberration, has an equivalent focal length of 10 cm. the lenses of doublet are separated by 2 cm.

What are the focal lengths of its component lenses?

a) 20 cm , 18 cm
b) 20 cm , 15 cm
c) 15 cm , 18 cm
d) None

Solution:
Let f_1&f_2 be the focal length of two components.
Since the doublet is corrected for spherical aberration, it satisfies the following condition;
$d = (f_1 - f_2) = 2cm$
$f_1 -= f_2 + 2$ cm (1)
The equivalent focal length $1/F = 1/f_1 + 1/f_2 - d/(f_1 f_2)$
$F = (f_1 f_2) / (f_1 + f_2 - d) = 10$ cm (2)
Substituting the value of f_1 from equation (1) in equation (2) we get
$F = (f_2 + 2) f_2 / (f_2 + 2 + f_2 - 2) = 10$ cm
$(f_2 + 2) f_2 / (2f_2) = 10$
$(f_2 + 2) / 2 = 10$
$(f_2 + 2) = 20$
$f_2 = 18$ cm (3)
substituting value of f_1 in equation (1) $f_1 -= f_2 + 2$ cm
$f_1 = 18 + 2 = 20$ cm

Problem 12: From the condition for no chromatic aberration and minimum spherical aberration of a combination of two separated thin lenses, design a combination of equivalent focal length 5.0 cm.
To design the required condition,
a) Two Plano convex lenses of focal lengths 10 cm & 3.3 cm must be kept separated by a distance of 6.7 cm
b) Two Plano concave lenses of focal lengths 10 cm & 3.3 cm must be kept separated by a distance of 6.7 cm
c) A Plano convex lens of focal lengths 10 cm & A Plano concave lens of focal length 5cm must be kept separated by a distance of 6.7 cm
d) A Plano concave lens of focal lengths 10 cm & A Plano convex lens of focal length 5cm must be kept separated by a distance of 6.7 cm

Solution: Let f_1&f_2 be the focal length of two lenses of the same material and d the separation between them.
From the condition of the Achromatism, we have $d = (f_1 + f_2)/2$ (1)
From the condition of minimum spherical aberration, we have $d = (f_1 - f_2)$ (2)
Solving equation (1) & (2),
$d = (f_1 + f_2)/2 = (f_1 - f_2)$
$(f_1 + f_2) = 2(f_1 - f_2)$
$f_1 + f_2 = 2f_1 - 2f_2$

$3f_2 = f_1$ (3)

Putting the value from equation (3)in equation (2)

$d=(f_1-f_2)$

$d=3f_2 - f_2=2f_2$ (4)

The equivalent focal length of combination is given by $1/F = 1/f_1 + 1/f_2 - d/(f_1f_2)$

$F= (f_1f_2) /(f_1+ f_2-d)$ (5)

Here F=+5.0 cm (the combination is converging), $f_1= 3f_2$, $d=2f_2$

substituting these values in equation (5) $F= (f_1f_2) /(f_1+ f_2-d)$

$5= (3f_2\, f_2) /(3f_2+ f_2-2f_2)$

$5= (3f_2\, f_2) /(2f_2)$

$5= 3f_2/2$

$f_2=10/3= 3.33$ cm (6)

substituting the value of f_2 in equation (3) & (4), we get

$f_1=3f_2 =10$ cm

$d =2f_2 +6.7$ cm

Hence to design the required combination, two Plano convex lenses of focal lengths 10 cm & 3.3 cm must be kept separated by a distance of 6.7 cm.

Lecture-9

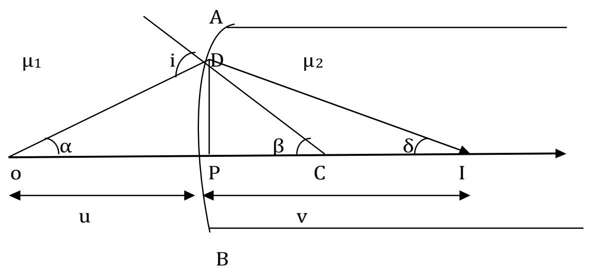
Geometrical Optics

Physics for IIT - JEE

Refraction from curved surfaces:

- *Refraction from single curved surface*
- *Lens makers formula*
- *Lense formula*
- *Displacement methods*

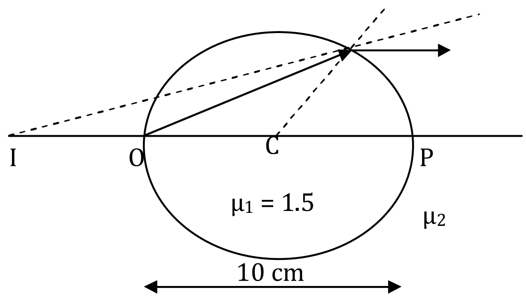
***Refraction from curved surfaces*:** If the boundary between two transparent media is not plane and an object O in medium of refractive index μ_1 forms an image I in medium of refractive index μ_2 as shown in figure.
Applying Snell's law at the boundary AB



$\mu_1 \sin i = \mu_2 \sin r$
for small angles
$\mu_1 \tan i = \mu_2 \tan r$
$\mathbf{\mu_1 i = \mu_2 r}$ (1)
$\mu_1 (\alpha + \beta) = \mu_2 (\beta - \delta)$
$\mu_1 \alpha + \mu_1 \beta = \mu_2 \beta - \mu_2\delta$
$\mu_1 \alpha + \mu_2\delta = (\mu_2 - \mu_1)\beta$
$\mu_1 \tan \alpha + \mu_2 \tan \delta = (\mu_2 - \mu_1) \tan \beta$
$\mu_1 (y/-u) + \mu_2 (y/v) = (\mu_2 - \mu_1) (y/R)$
$(\mu_1 /-u) + (\mu_2 /v) = ((\mu_2 - \mu_1)/R)$
$\mathbf{(\mu_2 /v) - (\mu_1 /u) = ((\mu_2 - \mu_1)/R)}$ (2)
This is desired result. If we compare it with mirror formula $(1/v) + (1/u) = (1/f)$
$u \rightarrow (-u/ \mu_1)$, $v \rightarrow (v/ \mu_2)$, $f \rightarrow (R/ (\mu_2 - \mu_1))$
so transverse magnification in this case
$m = I/O = [(-v/\mu_2) /(-u/\mu_1)] = (v/u)(\mu_1 / \mu_2)$
$\mathbf{m = (v/u)(\mu_1 / \mu_2)}$ (3)
While using eq. (1) and (3) keep in mind that
(1) these are valid for all single refractive surfaces, concave, convex or plane.
In case of plane refractive surface $f = \infty$, $R = \infty$
$(\mu_2 /v) - (\mu_1 /u) = 0 \Rightarrow u/v = \mu_1 / \mu_2$
$\mathbf{dA_c /dA_p = \mu_1 / \mu_2}$
(2) The rules for signs for single refracting surfaces are same as spherical mirrors.
(3) If object or image is itself present at refracting surface, refraction at that surface is not considered.

Problem 1(IIT.R -1997): If a mark of size 0.2 cm on the surface of a glass sphere of diameter 10 cm and $\mu = 1.5$ is viewed through diametrically opposite point, where will the image be seen and of what size.

(a) 20 cm towards object from point of observation & size is 0.6 cm



Solution: As mark is an surface, refraction will take place on the other surface only (which is curved)

$(\mu_2 /v) - (\mu_1 /u) = ((\mu_2 - \mu_1)/R)$

$(1/v) - (1.5/(-10)) = (1.5-1)/-5$

$1/v = (.5/-5) - (1.5/10)$

$1/v = (5/50) - (15/100)$

$1/v = (10/100) - (15/100)$

$1/v = (-5/100)$

v = -20 cm

The image is at a distance of 20 cm from P towards O as shown in figure.

Now as in case of refraction through curved surface

$m = I/O = (v/u)\ (\mu_1 / \mu_2) = (-20/-10)\ (15/1) = 3.0$ cm

Size of image: $m = I/O = 3.0$ cm

$I = 3.00 = 3*0.2 = 0.6$ cm

so image is erect, virtual & enlarged i.e. of size 0.6 cm

If refracting surface had been plane.

$(\mu_2 /v) - (\mu_1 /u) = ((\mu_2 - \mu_1)/R)$

$(\mu_2 /v) - (\mu_1 /u) = ((\mu_2 - \mu_1)/\infty) = 0$

$(\mu_2/\mu_1) = (v/u)$

$(1/ 1.5) = (v/-10) \Rightarrow v = (-10/1.5) = - 100/15 = -20/3$

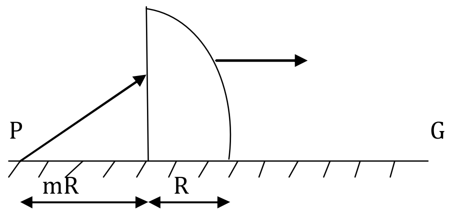
V= -20/3 cm

$m = (\mu_2/\mu_1)\ (v/u) = (1.5/1)\ [(-20/3)/-10] = (15/10)(20/3)(1/10) = 1$

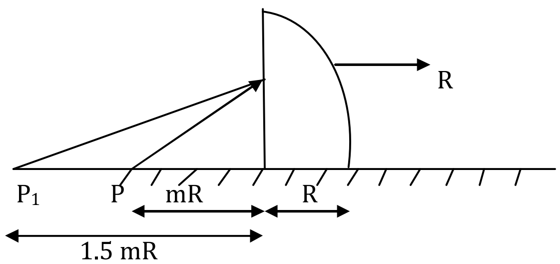
$m = I/O = 1 \Rightarrow$ **I = 0** i.e. image is at a distance of 20/3 cm from P towards O as shown in the figure.

Problem 2(IIT 1999): A quarter cylinders of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance of mR from it. Find the value of m for which a ray from P will emerge parallel to the table as shown in figure.

(a) m = 4/3 (b) m = 4 (c) m = 2 (d) m = 3/4



Solution:



From refraction at plane surface

$(\mu_2 /v) - (\mu_1 /u) = ((\mu_2 - \mu_1)/R)$

$\mu_2 = 1.5$, $\mu_1 = 1$, $R_1 = R = \infty$ (plane), $u = -mR$

$(1.5/v)- (1/-mR) = (1.5 -1)/\infty$

$(1.5/v) = (1/-mR)$

V = -1.5 mR So image will be formed at point P_1

for curved surface image of P at P_1 will act as an object, for this surface

$(\mu_2 /v) - (\mu_1 /u) = ((\mu_2 - \mu_1)/R)$

$\mu_2 =1$, $\mu_1 = 1.5$, $u = -(1.5mR +mR) = -R[1.5m + 1]$, $v = \infty$

$(1/\infty)-(1.5/-(1.5m+1)R) = (1-1.5)/(-R)$

$[1.5/(1.5m+1)R] = [.5/+R]$

$1.5/(1.5m+1) = +.5/1 \Rightarrow 1.5/(1.5m+1) = +.5$

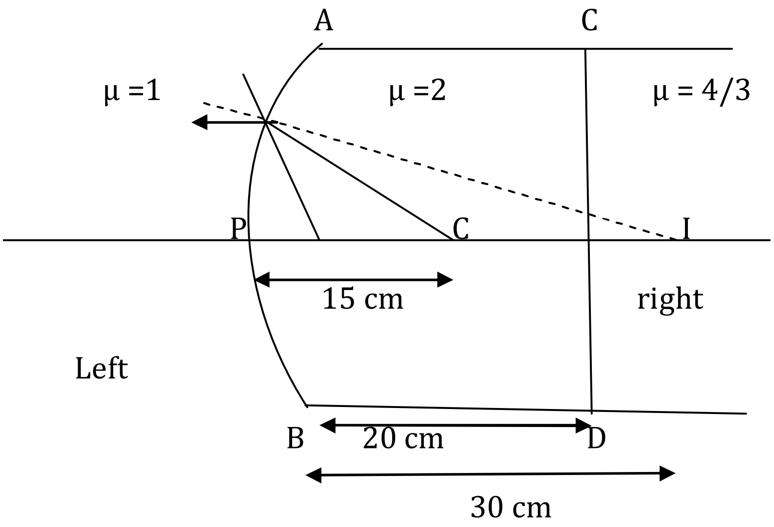
$3/(1.5m+1) = \Delta \Rightarrow 3 = 1.5m +1 \Rightarrow 2 = 1.5m$

$m = 2/1.5 = 20/15$

m= 4/3

Problem 3 (IIT 1991): The slab of material of refractive index 2 shown in figure has a curved surface APB of radius of curvature 10 cm and a plane surface ID on the left of APB is air and on the right of CD is water with refractive indices as given in the figure. An object O is placed at a distance of 15 cm from the pole P as shown. Which of the

following statements are correct
(a) Distance of final image of O from P as viewed from the left is 30 cm.
(b) Curved surface will form virtual image I at a distance of 30 cm from P
(c) There will be no refraction at the plane surface CD
(as the rays are not actually passing through the boundary)
(d) All statements are correct.



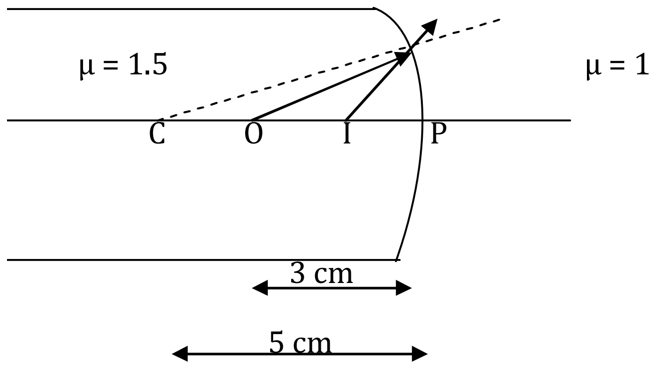
Solution:

In case of refraction from a curved surface, we have
$(\mu_2 /v) - (\mu_1 /u) = ((\mu_2 - \mu_1)/R)$
$\mu_1 = 2$, $\mu_2 = 1$, $R = -10$ cm, $u = -15$ cm
so $(1/v) - (2/-15) = (1-2)/ -10$
$\Rightarrow v = -30$ cm

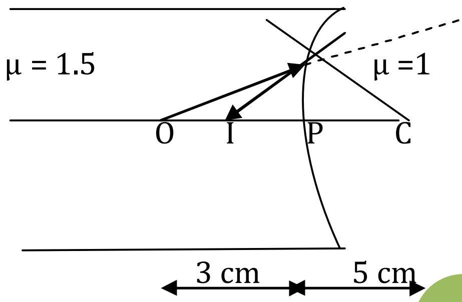
i.e. the curved surface will form virtual image I at a distance of 30 cm from P since the image is virtual there will be no refraction at the plane surface CD (as the rays are not actually passing through the boundary), the distance of final image I from P will remain 30 cm.

Problem 4: An air bubble in glass ($\mu = 1.5$) is situated at a distance 3 cm from a spherical surface of diameter 10 cm as shown in figure (A,B). At what distance from the surface will the bubble appear if the surface is (1) convex (2) concave (3) plane
(a) The bubble will appear at a distance 2.5 cm from the curved surface inside the glass.
(b) The bubble will appear at a distance 1.66 cm from the curved surface inside the glass.
(c) The bubble will appear at a distance .2 cm from the curved surface inside the glass
(d) none

Solution:



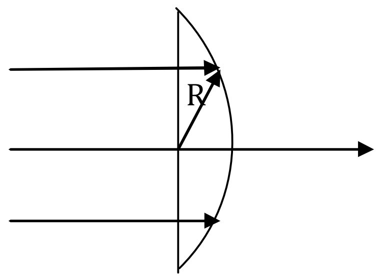
(A)



(B)

In case of refraction from curved surface $(\mu_2 / v) - (\mu_1 / u) = ((\mu_2 - \mu_1)/R)$
(a) In case convex surface, $\mu_1 = 1.5$, $\mu_2 = 1$, u = -3cm, R = -5cm
$(1/v) - (1.5/-3) = [(1-1.5)/-5] \Rightarrow v = -2.5$ cm
i.e. the bubble will appear at a distance 2.5 cm from the curved surface inside the glass.
(b) In case concave surface, $\mu_1 = 1.5$, $\mu_2 = 1$, u = -3cm, R = +5cm
$(1/v) - (1.5/-3) = [(1-1.5)/5] \Rightarrow v = -10/6 = -1.66$ cm
i.e. the bubble will appear at a distance 1.66 cm from the curved surface inside the glass.
(c) In case plane surface, $\mu_1 = 1.5$, $\mu_2 = 1$, u = -3cm, R = ∞
$(1/v) - (1.5/-3) = [(1-1.5)/\infty]=0$
$(1/v) = -[1.5/3] \Rightarrow v = .2$cm
i.e. the bubble will appear at a distance .2 cm from the curved surface inside the glass.

Problem 5: A parallel beam of light is incident normally on the flat surface of a hemisphere of radius R = 6 cm and refractive index 1.56. Assuming paraxial ray approximation,
(a) determine the point at which the beam is focussed as measured along the axis from the curved surface;
(b) determine the new focal length measured from the flat surface if the rays are incident at the curved surface

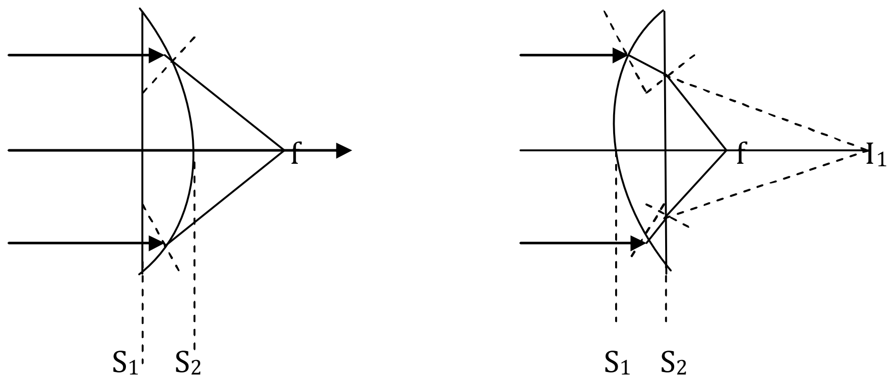


Solution: Rays pass without deviation at flat surfaces. From single surface refraction equation for single surface refraction equation for surface S_2, we have
$(1/v) - (n/\infty) = (1-n)/(-R)$
$v = R/(n-1) = 6/[1.56-1] = 6/0.56 = 10.7$ cm
This focal point as rays travel from left.

From single surface refraction equation at S_1, we have

$(n/v) - (1/\infty) = (n-1)/R$

$v = nR/(n-1)$



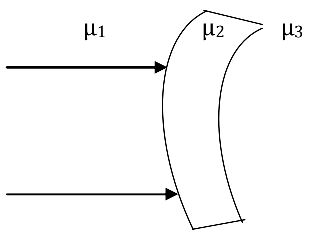
This is the first image, it acts as an object for refraction at the plane surface. Object distance for refraction at S_2, = $[R/(n-1)]-R = R/(n-1)$

so we have $(1/v')- [n/+R(n-1)] =0$

$v' = R/n(n-1) = 6/1.56(1.56-1) = 6.9$ cm

Hence focal point is at 6.9 cm from the plane surface.

Problem 6: In fig. Light is incident on a thin lens as shown. The radius of curvature of both surfaces is R. Determine the focal length of the system.



For refraction at first surface $(\mu_2/v_1)- (\mu_1/\infty) = (\mu_2-\mu_1)/R$ (1)

for 2nd surface: $u_2 =v_1$, $v_2 = f$

$(\mu_3/f) -(\mu_2 /v_1) =(\mu_3 -\mu_2)/R$ (2)

on adding (1) and (2)

$(\mu_2 /v_1) =(\mu_2 -\mu_1)/R$

$(\mu_3/f) -(\mu_2 /v_1) =(\mu_3 -\mu_2)/R$

$(\mu_3/f) = [\mu_2 - \mu_1 + \mu_3 -\mu_2]/R$

$(\mu_3/f) =(\mu_3 -\mu_1)/R$

$\mathbf{f = \mu_3R /(\mu_3 -\mu_1)}$

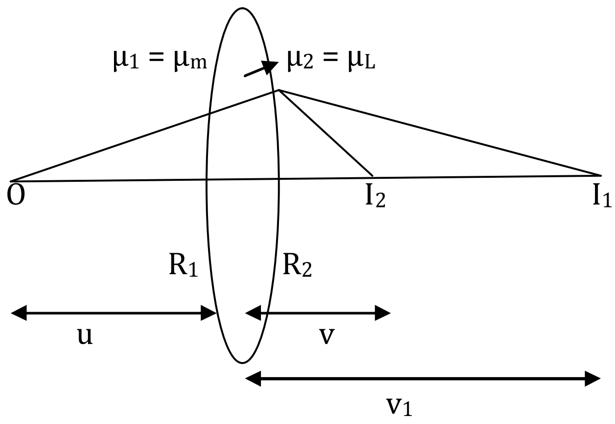
If we want nature as a convex then $f > 0$

$[\mu_3R /(\mu_3 -\mu_1)] > 0$

$\mathbf{\mu_3 >(\mu_3 -\mu_1)/R}$

Lens maker formula:

In case of a mage formula by lens the incident ray is reflected twice at first surface & second surface respectively. The image formed by the first surface acts as object for the second



Formula for refraction at curved surfaces

$(\mu_2 /v) - (\mu_1 /u) = ((\mu_2 - \mu_1)/R)$ (*)

for 1st surface $(\mu_2 /v_1) - (\mu_1 /u) = ((\mu_2 - \mu_1)/R_1)$ (1)

for 2nd surface $(\mu_1 /v) - (\mu_2 /v_1) = ((\mu_1 - \mu_2)/R_2)$ (2)

Adding (1) & *(2) we get

$(\mu_2 /v_1) - (\mu_2 /v_1) +(\mu_1 /v) - (\mu_1 /u) = ((\mu_2 - \mu_1)/R_1) +((\mu_1 - \mu_2)/R_2)$

$\mu_1[(1/v) - (1/u)] = (\mu_2 - \mu_1)[1/R_1 - 1/R_2]$

$[(1/v) - (1/u)] =[(\mu_2 / \mu_1)-1][1/R_1 - 1/R_2]$ $\{ \mu_2 / \mu_1 = \mu = \mu_2 / \mu_m\}$

$\mathbf{[(1/v) - (1/u)] = [\mu-1][1/R_1 - 1/R_2]}$ (*1)

If an object is placed at infinity $u = \infty$, $v = f$

$(1/f) - (1/\infty) = [\mu-1][1/R_1 - 1/R_2]$

$\mathbf{(1/f) = [\mu-1][1/R_1 - 1/R_2]}$ **→Lens makers formula** (*2)

comparing (*1) and (*2) we get

$\mathbf{1/f = [(1/v) - (1/u)]}$ → **Lens formula** (*3)

Note: Lens formula $\mathbf{1/v - 1/u = 1/f}$

(i) The above formula is valid for convex as well as concave lens for all positions of object and it is independent of nature of object (real or virtual).

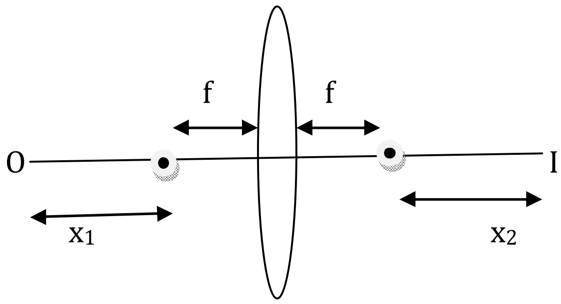
(ii) In numerical problems, it is convenient to use following expressions:

$\mathbf{v = uf/ (u +f), u = vf/(f-v), f = vu/(u-v)}$

Problem 7: In case of a thin lens of focal length f in f an object is placed at a distance x_1 from first focus and its image is formed at a distance x_2 from the second focus then show that $f^2 = x_1x_2$

(a) $f = \sqrt{x_1x_2}$ (b) $f = (x_1 + x_2)/2$ (c) $f = (2x_1x_2)/(x_1 + x_2)$ (d)none

Solution:



$u = x_2 + f$, $v = x_1 + f$

$(1/v) + (1/u) = (1/f)$

$[1/(x_1 + f)] + [1/(x_2 + f)] = 1/f$

$[(x_2 + f) + (x_1 + f)]/[(x_1 + f)(x_2 + f)] = 1/f$

$f(x_1 + x_2 + 2f) = (x_1 + f)(x_2 + f)$

$2f^2 + fx_1 + fx_2 = x_1x_2 + x_1f + x_2f + f^2$

$\mathbf{f^2 = x_1 x_2}$

$\mathbf{f = \sqrt{x_1x_2}}$ ***This is the Newton's formula.***

Problem 8: If the distance between the real objects a real image formed by a lens & f is the focal length of lens then L_{min} is

(a) f (b) 2f (c) 3f (d) 4f

Solution: $L = u + v$

$L = (\sqrt{u} - \sqrt{v})^2 + 2\sqrt{uv}$

$L_{min} \Rightarrow (\sqrt{u} - \sqrt{v})^2 = 0 \Rightarrow u = v$

$L_{min} = u + v = v + v = 2v$

$(1/v) + (1/u) = (1/f)$

$(1/v) + (1/v) = (1/f)$

$2/v = 1/f$

$\mathbf{V = 2f}$

$\mathbf{L_{min} = u + v = 2f + 2f = 4f}$

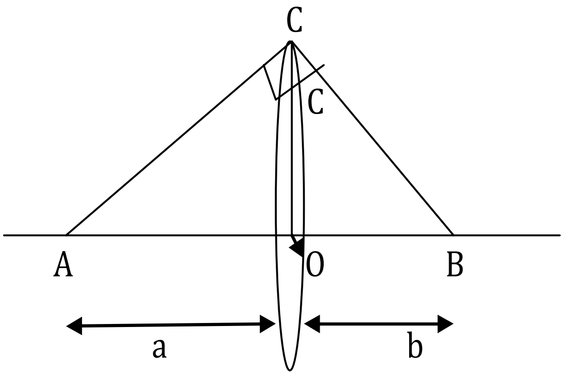
Problem 9: An object is placed at A (OA >f). Here f is the focal length of the lens. The image is formed at B. A $\perp_r$ erected at O & C is chooses such that $\angle BCA = 90^0$ let OA = a, OB = b & OC = c then the values of f is

(a) $(a + b)^3/c^2$

(b) $(a + b)c/(a+ c)$

(c) $c^2/(a + b)$

(d) $a^2/ (a +b +c)$



$[(1/v) - (1/u)]= 1/f$

$[(1/b) - (1/-a)]= 1/f$

$1/b + 1/a = 1/f$

$(1/f) = 1/a + 1/b$

$\mathbf{f = ab/ (a + b)}$ (1)

From Δ ABC

$Ac^2 + Bc^2 = AB^2$

$a^2 + c^2 + b^2 + c^2 = (a + b)^2$

$a^2 + b^2 +2c^2 = a^2 +b^2 + 2ab$

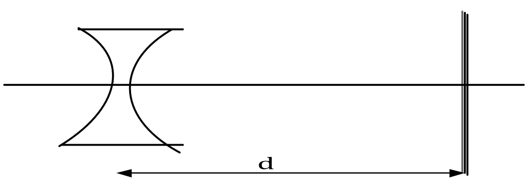
$\mathbf{c^2 = ab}$ (2)

substituting (3) in (2)

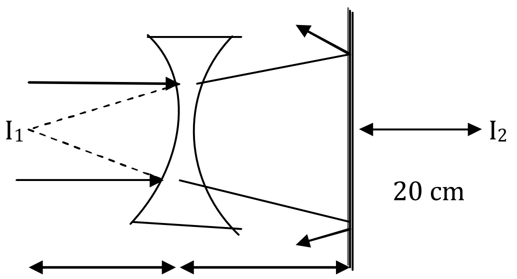
$f = ab/ (a + b)$

$\mathbf{f = c^2 / (a +b)}$

Problem 10: A diverging lens of focal length 10 cm is placed 10 cm in front of a plane mirror as shown in fig. Light from a very far away source falls on the lens what is the distance of the final image?



Solution:



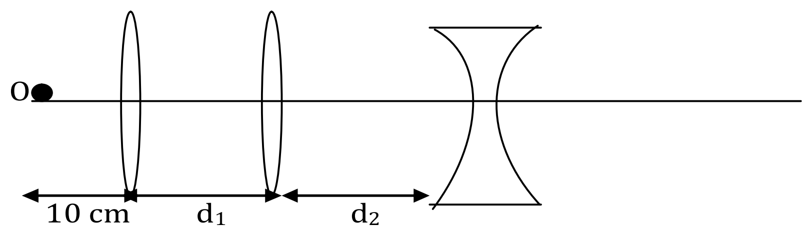
(1/v) – (1/u) = (1/f)

(1/v) – (1/-30) = + (1/-10)

(1/v) = (-1/10) – (1/30) = -(4/30) ⇒ v = -7.5

The final image distance = 2.5 cm in front of the mirror.

Problem 11:



(a)

The values of d_1 and d_2 for final rays to be parallel to the principle axis are: (focal length of the lenses are written on the lenses)

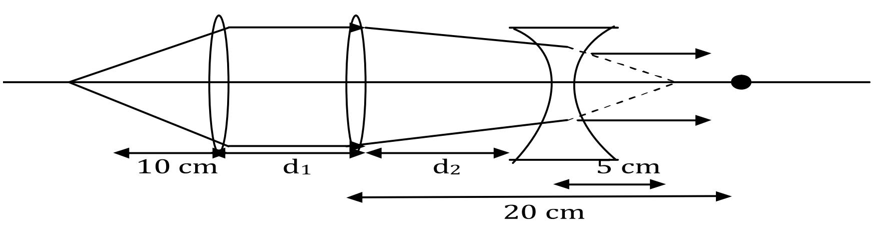
(a) d_1 = 10 cm, d_2 = 15 cm

(b) d_1 = 20 cm, d_2 = 15 cm

(c) d_1 = 30 cm, d_2 = 15 cm

(d) none of these

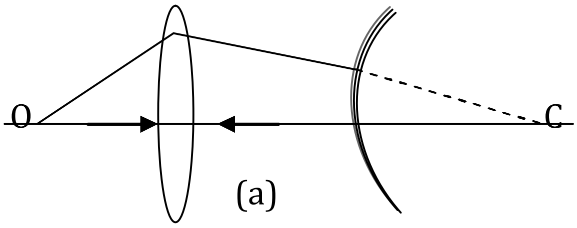
Solution:



(b)

$d_2 = 20 - 5 = 15$ cm
d_1 can take any value.
Hence option (a), (b), (c) all are correct

Problem 12:



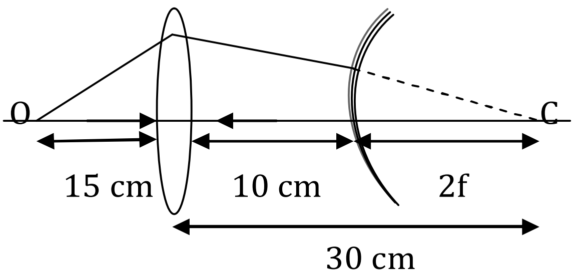
(a)

An object is placed at a distance 0f 15 cm from a convex lens of focal length 10 cm on the other side of the lens, a convex mirror is placed at its focus such that the image formed by combination coincides with the object itself. Find the focal length of concave mirror.

Solution:

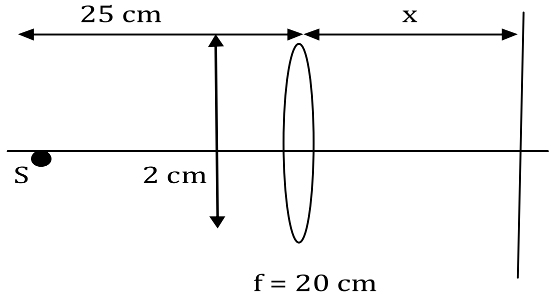
For retracting of ray; ray must fall normally on mirror i.e., towards the centre of curvature.

$(1/v) - (1/u) = (1/f)$ $\quad$ $u = -15$
$(1/v) + (1/15) = (1/10)$ $\quad$ $f = 10$ cm, $v = 30$ cm
For mirror $(1/v) + (1/u) = (1/f)$ $\quad$ $u = 2f$
$(1/v) + (1/2f) = (1/f)$ $\quad$ $v = 2f$
Hence from the ray diagram
$2f = 30 - 10$
$f = 10$ cm



(b)

Problem 13:



(a)

A point source of light S is placed on the axis of a lens of focal length 20 cm as shown in the figure. A screen is placed normal to the axis of lens at a distance x from it. Treat all rays as paraxial.

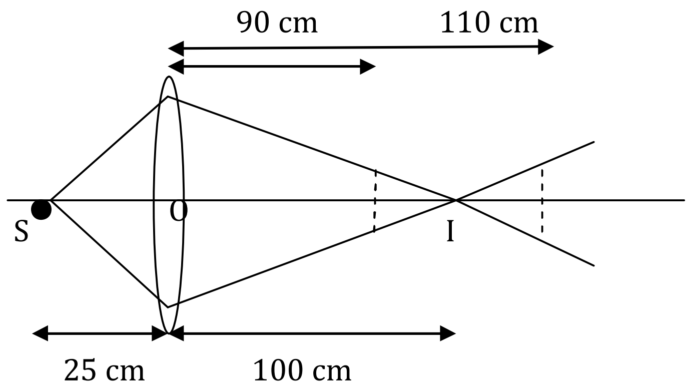
(a) As x is increased from zero intensity continuously decreases

(b) As x is increased from zero intensity first increases then decreases

(c) Intensity at centre of screen for x =90cm and x = 110 cm is same

(d) Radius of bright circle obtained on screen is equal to 1 cm for x = 200 cm

Solution: (b, c, d)



$(1/v) - (1/u) = (1/f) \Rightarrow (1/v) - [1/(-25)] = (1/20)$

$\Rightarrow (1/v) = (1/20) - (1/25) = (1/100)$

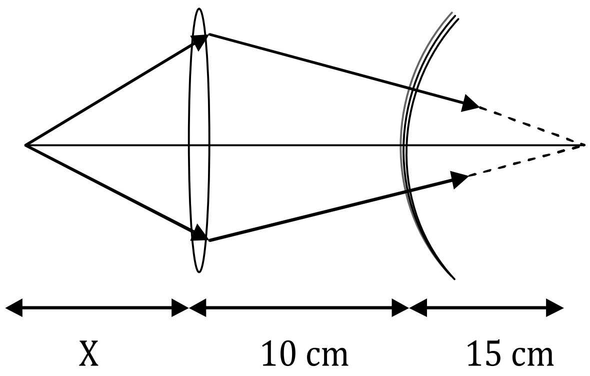
$\Rightarrow$ v = 100 cm

From O to I intensity increases and then decreases at x = 90 cm and 110 cm intensity is same.

(d) Radius at x = 200 cm is equal to radius of lens.

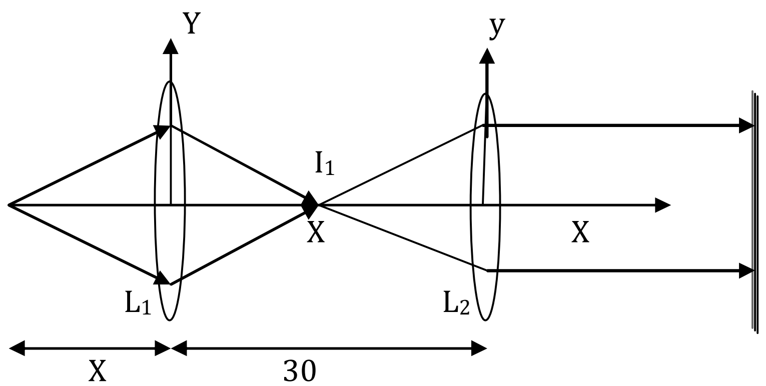
Problem 14: A convex lens of focal length 20 cm is placed 10 cm in front of a convex mirror of radius of curvature 15 cm. Where should a point object be placed in front of the lens so that it images on to itself?

Solution:



The convex lens and the convex mirror are shown in fig. The combination behaves like a concave mirror.
Let the distance of the object from the lens be x.
For the ray to retrace is path it should be incident normally on the convex mirror, or in other words the rays should pass through the centre of curvature of the mirror.
From the diagram we see that for the lens
u = -x, f = +20 cm, v = +10+15 = +25 cm
From the lens equation we get
(1/v) – (1/u) = (1/f)
or (1/25) – (1/-x) = (1/20)
or x = 100 cm

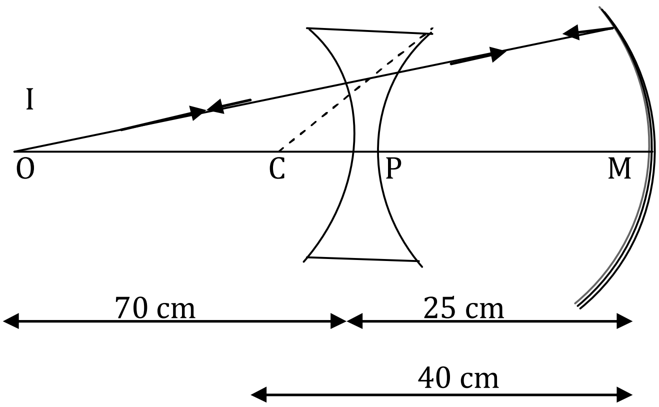
Problem 15:



A convex lens of focal length 10 cm is placed 30 cm in front of a second convex lens also of the same focal length. A plane mirror is placed after the two lenses. Where should a point object be placed in front of the first lens so that it images on to itself?

Solution: The convex lenses and the plane mirror are shown in figure. The combination behaves like a concave mirror.
Let the distance of the object from the first lens be x.
For the ray to retrace its path it should be incident normally on the plane mirror.
From the diagram we see that for lens L_2,
From the lens equation we get
(1/v) – (1/u) = (1/f)
or u = -10 cm
From the diagram we see that for lens L_1,
v = 30-10 = 20 cm, f = +10cm, u =-x
From the lens equation we get (1/v) – (1/u) = (1/f)
or (1/20) – (1/-x) = (1/10)
or x = 20 cm

Problem 16:



A diverging lens, focal length f_1 = 20 cm, is separated by 5 cm from a converging mirror, focal length f_2 = 10 cm. Where an object should be placed so that a real image is formed at the object itself?

Solution: Let the object be placed at a distance x to the left of the lens.
From the lens equation,
(1/v) = (1/-20) + (1/-x)
v = - [20x/(x+20)]
A virtual image is formed due to first refraction at the lens. This image is an object for reflection from the concave mirror. Object distance is
– [5+ {20x/ (20+x)}] = - [(25x +100)/(x+20)]
From mirror equation, (1/v') + [-(x+20)/(25x+100)] = (1/-10)
(1/v') = (-1/10) +[(x+20)/(25x+100)]= [(10x +200-25x-100)/250(x+4)]

$v' = -[50(x+4)/(3x-20)]$

This image is formed to the left of the mirror. Object distance for second refraction through concave lens, $u'' = -[5-\{50(x+4)/(3x-20)\}]$

we assumed that second image lies between lens and mirror.

The final image is produced at the object itself; hence $v'' = +x$

From lens equation, $(1/x) - 1/[5-\{(50x+4)/(3x-20)\}] = 1/-20$

on solving for x, we get

$25x^2 - 1400x - 6000 = 0$

$x^2 - 56x - 240 = 0$

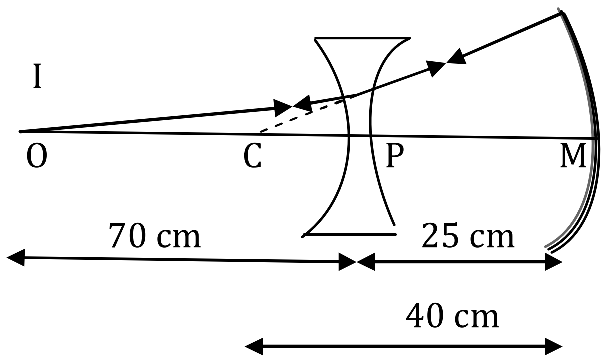
$(x-60)(x+4) = 0$

Hence x = 60 cm

The object must be placed at 60 cm to the left of the diverging lens.

Problem 17: A diverging lens (f_1=20 cm) is separated by 25 cm from a concave mirror (f_2 =20 cm). An object is placed 70 cm to the left of the lens. Find the focal length of the lens if the image coincides with the object.

Solution: The light refracts through the lens, reflects at the mirror and finally passes once again through the lens. The final image will coincide with the object if it retraces its path after reflection from the mirror, i.e. the ray strikes the mirror normally. The normal rays at the mirror after being extended must pass through the centre of curvature of the mirror. Thus, the object for second refraction at the lens is at C and its image is at I



From figure PC = (40-25) = 15 cm

from lens equation, $(1/-15) - (1/-70) = (1/f)$

f = -19.1 cm

Displacement method:

Problem 18 (A):

(i) A thin converging lens of focal length f is placed between an object and a screen fixed at a distance D apart, there are two positions of the lens at which a sharp image of the object is formed on the screen if

(a) D = 4f (b) D > 4f (c) D < 4f (d) None

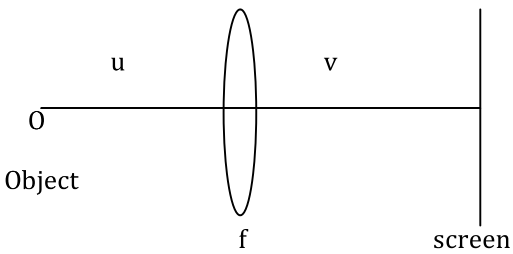
(ii) In above question if distance between the two positions of the lens is x then the focus length of the convergent lens is

(a) $f = (D^2 - x^2)/4D$ (b) $f = x^2 / 4D$ (c) $(D^2 + x^2)/4D$ (d) None

(iii) If m_1 & m_2 is the magnification in the above case then focal length f is

(a) $f = x/ m_1-m_2$ (b) $f = m_1-m_2/x$ (c) $f = x/ m_1+m_2$ (d) None

Solution:



If the object is at distance D from the lens, the distance of image from the lens is

$u + v = D$, $v = D - u$

so from lens formula $(1/v) - (1/u) = (1/f)$ for convergent lens $u \Rightarrow -u$

$[1/(D-u)] + (1/u) = 1/f$

$f [u + (D-u)] = u(D-u)$

$fu + fD - fu = Du - u^2$

$\mathbf{u^2 - Du + f D = 0}$

$u = \pm [\sqrt{D^2 - 4(1)fD} + D]/2$

$u = [D \pm \sqrt{D^2 - 4fD}]/2 = [D \pm \sqrt{D(D-4f)}]/2$

Now there are 3 possibilities:

(1) if $D - 4f > 0$ then two roots of the equation will be real i.e.

$u_1 = [D + \sqrt{D(D-4f)}]/2$, $u_2 = [D - \sqrt{D(D-4f)}]/2$

so there are two positions of lens at u_1 & u_2 from the object for which real image is formed.

(2) If $D - 4f = 0$ then only one roots of the equation which is real, $\mathbf{u = D/2}$,

$v = D-u = [D - (D/2)] = D/2$, $v = D/2 = 2f$

so only one position of image is possible.

(3) If $D - 4f < 0$, then roots are imaginary, so physically no position of lens is possible.

Note: *this method is called* ***displacement method*** *& is used in lab to determine the focal length of convergent lens.*

In case of displacement method:

(1) If the distance between the two positions of the lens is x

$x = u_2 - u_1 = ½[D +\sqrt{D (D-4f)}] -1/2 [D-\sqrt{D (D-4f)}] = \sqrt{D (D-4f)}$

$x^2 = D (D -4f)$

$\mathbf{f = (D^2 - x^2)/4D}$

(2) The image distances corresponding to two positions of lens will be

$v_1 = D - u_1 = D -(1/2)[D+\sqrt{D(D-4f)}] = (1/2)[D-\sqrt{D(D-4f)}] = u_2$

$v_2 = D - u_2 = D -(1/2)[D-\sqrt{D(D-4f)}] = (1/2)[D+\sqrt{D(D-4f)}] = u_1$

i.e. for two positions of the lens object & image distances are interchangeable.

(3) As $x = u_2 - u_1$ & $D = u_1 + v_1 = u_1 + u_2$ as $v_1 = u_2$

$x = u_2 - u_1$, $D = u_2 + u_1 \Rightarrow D + x = 2u_2 \Rightarrow u_2 = (D + x)/2$

$D - x = 2u_1 \Rightarrow u_1 = (D - x)/2$

so the magnification for the positions of the lens will be

$m_1 = u_1/v_1= u_1/u_2 = [(D-x)/2]/[(D +x)/2] = (D-x)/(D +x)$

$m_2= u_2/v_2 = u_2/u_1 = [(D +x)/2]/[(D-x)/2] = (D+ x)/(D -x)$

(a) $m_1m_2 = [(D-x)/(D +x)][(D+ x)/(D -x)] = 1\Rightarrow (I_1/O_1)(I_2/O_2) =1\Rightarrow \mathbf{O = \sqrt{I_1I_2}}$

(b) $m_1/m_2 = [(D-x)/(D +x)]/[(D+ x)/(D -x)]= [(D-x)/(D +x)]^2$

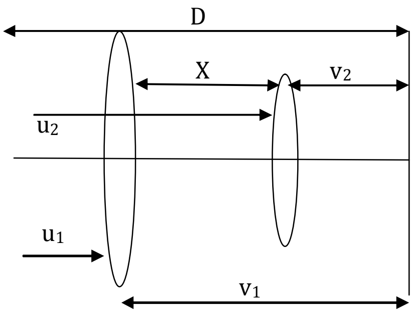
(c) $m_1-m_2 = [(D-x)/(D +x)]-[(D+ x)/(D -x)]= [(D-x)^2 - (D+ x)^2]/(D^2 - X^2) = 4Dx/(D^2 - X^2)$

$m_1-m_2 = x[4D/(D^2 - X^2)]$

as we know $f =(D^2 - X^2)/4D$

$m_1-m_2 = x/f$

$\mathbf{f = x / m_1-m_2}$



Problem 18(B): A luminous object and a screen are at a fixed distance D apart.

(a) show that a converging lens of focal length f, placed between object and screen will form a real image on the screen for two lens positions that are separated by a distance $d = \sqrt{D(D-4f)}$

(b) Show that $[(D-d)/(D+d)]^2$ gives the ratio of the two image sizes for these two positions of the lens.

(c) If the heights of two images are h_1 and h_2 respectively, the height of the object is $h = \sqrt{h_1 h_2}$

(d) If the distance between two positions of the lens is d,

$f = (D^2-d^2)/4D$ and $f = d/(m_1-m_2)$

where m_1 and m_2 are magnifications in the two positions of the lens.

Solution:

(a) Let the object distance be x; then the image distance is D-x.

From lens equation, $(1/x) + (1/(D-x)) = (1/f)$

On algebraic rearrangement we get $x^2 - Dx + Df = 0$

On solving for x, we get $x_1 = [D-\sqrt{D(D-4f)}]/2$

$x_2 = [D+\sqrt{D(D-4f)}]/2$

The distance between the two object positions is $d = x_2 - x_1 = \sqrt{D(D-4f)}$

(b) If the object is at $u = x_1$

$m_1 = I_1/O = (D-x_1)/x_1$

Now, $x_1 = (1/2)(D-d)$, where $d = \sqrt{D(D-f)}$

so, $m_1 = [D-(D-d)/2]/[(D+d)/2] = (D+d)/(D-d)$

Similarly, when the object is at x_2, the magnification is $x_2 = (1/2)(D+d)$

$m_2 = I_2/O = (D-x_2)/x_2 = [D-(D+d)/2]/[(D+d)/2] = (D-d)/(D+d)$

The ratio of magnification is $(m_2/m_1) = [(D-d)/(D+d)]/[(D+d)/(D-d)] = [(D-d)/(D+d)]^2$

(c) As $m_1 = I_1/O = v_1/u_1$

and $m_2 = I_2/O = v_2/u_2$

$m_1 * m_2 = I_1 I_2/O^2 = (v_1/u_1)(v_2/u_2) = 1$

Hence $O = \sqrt{I_1 I_2}$

(d) As $d = \sqrt{D(D-4f)}$

$d^2 = D^2 - 4Df$

$f = (D^2-d^2)/4D$

Also $m_1 - m_2 = [(D+d)/(D-d)] - [(D-d)/(D+d)] = 4Dd/(D^2-d^2)$

Hence $m_1 - m_2 = d/f$

or $f = d/(m_1-m_2)$

Formula $\mathbf{(1/f) = (\mu-1)\,[(1/R_1)-(1/R_2)]}$ for different shapes of thin lenses:

For Convex lens: $(1/f) = (\mu-1)\,[(1/R_1)-(1/R_2)] = (\mu-1)\,[(1/R_1)-(1/-R_2)]$

$\Rightarrow \mathbf{(1/f) = (\mu-1)\,[(1/R_1) + (1/R_2)]}$

For Concave lens: $(1/f) = (\mu-1)\,[(1/R_1)-(1/R_2)] = (\mu-1)\,[(1/-R_1)-(1/+R_2)]$

$\Rightarrow \mathbf{(1/f) = (\mu-1)\,[(1/R_1) + (1/R_2)]}$

For Plano Convex lens: $(1/f) = (\mu-1)\,[(1/R_1)-(1/R_2)] = (\mu-1)\,[(1/\infty)-(1/-R)]$

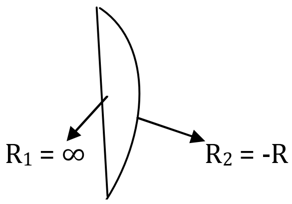
$\Rightarrow \mathbf{(1/f) = (\mu-1)\,[1/R]}$ **OR**

$(1/f) = (\mu-1)\,[(1/R_1)-(1/R_2)] = (\mu-1)\,[(1/R)-(1/-\infty)]$
$\Rightarrow$ **$1/f = (\mu-1)/R$**

For Plano Concave lens: $(1/f) = (\mu-1)\,[(1/R_1)-(1/R_2)] = (\mu-1)\,[(1/-R)-(1/-\infty)]$
$\Rightarrow$ **$(1/f) = -[(\mu-1)/R]$ OR**
$(1/f) = (\mu-1)\,[(1/R_1)-(1/R_2)] = (\mu-1)\,[(1/\infty)-(1/R)]$
$\Rightarrow$ **$(1/f) = -[(\mu-1)/R]$**

Note: **(1)** If surface is double convex or double concave then $R_1 = R_2$
for equiconvex $(1/f) = 2[(\mu-1)/R]$ for equiconcave $(1/f) = -2[(\mu-1)/R]$
(2) Sun goggles: In case of sum goggles radii of curvatures $R_1 = R_2 = R$
$(1/f) = (\mu-1)\,[(1/R_1)-(1/R_2)] = (\mu-1)\,[(1/R)-(1/R)] = 0$
$P=0,\ f = \infty$ This is why sun goggles have no power or infinite focal length.
Glass slab: same is true for glass slab (transparent sheet) but with difference
$R_1 = \infty,\ R_2 = \infty$
$(1/f) = (\mu-1)\,[(1/R_1)-(1/R_2)] = (\mu-1)\,[(1/\infty)-(1/\infty)] = 0$, **$P=0,\ f = \infty$**

Problem 19: What is the refractive index of material of a Plano convex lens? If the radius of curvature of convex surface is 10 cm & focal length of the lens is 30 cm.
Solution:

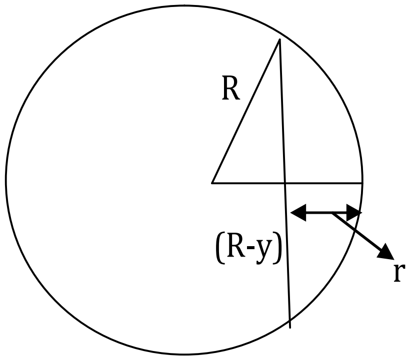


Plano convex $(1/f) = (\mu-1)\,[(1/R_1)-(1/R_2)] = (\mu-1)\,[(1/\infty)-(1/-R)] = (\mu-1)/R$
$f = [R/(\mu-1)] \Rightarrow (\mu-1) = R/f \Rightarrow \mu = [1+(R/f)] = [1+(10/30)]$
$\mu = 1+(1/3) = 4/3$

Problem 20: Diameter of a Plano convex lens is 6 cm and its thickness at the centre is 3mm. What is the focal length of the lens? If the speed of light in the material of lens is 2×10^8 m/s

(a) 30cm (b) 20 cm (c) -30 cm (d) none

Solution:



According to lens maker's formula

$(1/f) = (\mu-1) [(1/R_1)-(1/R_2)]$

As the lens is Plano convex $R_1=R$, $R_2=\infty$ [or $R_1 = \infty$, $R_2 = -R$]

$f = R/(\mu-1)$ (1)

Now as speed of light in vacuum = $3*10^8$ m/s

Now as speed of light in medium = $2*10^8$ m/s

$\mu = c/v = [(3*10^8 \text{ m/s})/(2*10^8 \text{ m/s})] = 3/2 = 1.5$ (2)

If r is the radius & y is the thickness of lens (at the centre) the radius of curvature R of its curved surface in accordance to fig. Will be given by

$R^2 = r^2 +(R -y)^2$

$R^2 = r^2 +R^2 +y^2-2Ry$

$[r^2 +y^2]/2y = R$

$r = 6/2$, $y = 3$mm

$R = [3^2 + 3^2(1/100)]/[2(3/10)]$

$R=3^2[1+(1/100)]/(6/10)$

$R = 3^2[101/100][10/6]$

$R= (3/2)10 = 15$ cm

so substituting the values μ & R in eq.(1)

$f = R/(\mu-1)$

$f= 15/(1.5-1) = 15/.5 = 150/5$

f= 30 cm

Problem 21: *Lens immersed in water:*

A glass convex lens of refractive index 3/2 has got a focal length of 0.3m. If it is immersed in water of refractive index 4/3, then Find the focal length of the lens.

$(1/f) = (\mu-1) [(1/R_1)-(1/R_2)]$

$\mu = \mu_L/\mu_m$

$(1/f_a) = [\mu_g/\mu_a)-1] [(1/R_1)-(1/R_2)]$ (1)

$(1/f_\omega) = [\mu_g/\mu_\omega)-1] [(1/R_1)-(1/R_2)]$ (2)

Dividing eq. (1) and (2)
$[(1/f_a)/ (1/f_\omega)] = \{[\mu_g/\mu_a)-1][(1/R_1)-(1/R_2)]\}/\{ [\mu_g/\mu_\omega)-1] [(1/R_1)-(1/R_2)]\}$
$f_\omega /f_a =[\mu_g/\mu_a)-1]/[\mu_g/\mu_\omega]$

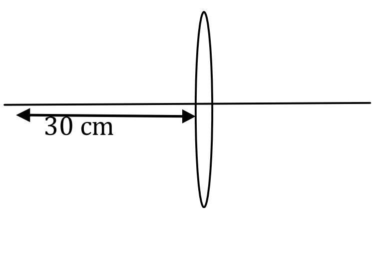
$\Rightarrow$ ***$f_\omega = [[\mu_g/\mu_a)-1]/[\mu_g/\mu_\omega]]f_a$***

In this problem $f_a = 0.3$, $\mu_a=1$, $\mu_g=3/2$
$f_\omega = [[\mu_g/\mu_a)-1]/[\mu_g/\mu_\omega]]f_a$
$f_\omega = (0.3) [\{[(3/2)-1]/1\}/\{[(3/2)/(4/3)]-1\}]$
$f_\omega = [(0.3)(1/2)]/[(9/8)-1]$
$f_\omega = [(0.3)(1/2)]/(1/8) = (0.3*1*8)/2$
$\mathbf{f_\omega = 1.2m}$

Problem 22(AIEEE 2005): A thin glass (refractive index 1.5) lens has optical power in a liquid medium with refractive index 1.6 will be
(a) 1D (b) -1D (c) 25D (d) -25D

Solution:
$(1/f_o) = (\mu-1)[(1/R_1)-(1/R_2)]$
$(1/f_o) =(1.5-1) [(1/R_1)-(1/R_2)]$ (1)
$1/f_m =[(\mu_g - \mu_m)/ \mu_m] [(1/R_1)-(1/R_2)]$
$1/f_m = [(1.5/1.6)-1] [(1/R_1)-(1/R_2)]$ (2)
Dividing eq. (1) and (2)
$f_m/f_0 =(1.5-1) / [(1.5/1.6)-1] = -8$
$(\because f_0 = 1/p = (-1/5)m$
$f_m = -8* f_0 = -8*(-1/5) =1.6$ m
$P_m= \mu/ f_m$
$P_m = 1.6/1.6 = 1D$

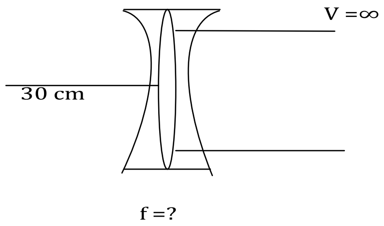
Problem 23: A point object is places at 30 cm from a convex glass lens ($\mu_g = 3/2$) of focal length 20 cm. The final image will be formed at infinity if
(a) The whole system is immersed in liquid of refractive index 4/3.
(b) The whole system is immersed in liquid of refractive index 9/8.
(c) Another concave lens of focal length 60 cm is placed in contact with its previous lens.
(d) Another concave lens of focal length 60 cm is placed at a distance of 30 cm from the first lens.



Solution:

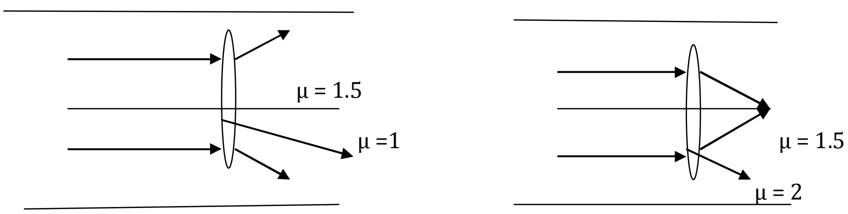
$\mu_g = 3/2$, $u = -30$cm, $f = 20$cm, $v = \infty$ (final image)
after contact of lens $(1/v) - (1/u) = (1/f)$

$(1/\infty) - (1/-30) = 1/f$
$f = +30$cm this is combined focal length



$1/f = 1/f_1 + 1/f_2$, $1/+30 = 1/20 + 1/f_2$
$1/f_2 = -1/20 + 1/30 = (-3+2)/60 = -1/60$ **$f_2 = -60$**
Hence another concave lens of focal length 60 cm is placed with contact in first lens.
$1/f = (\mu-1)[(1/R_1)-(1/R_2)] \Rightarrow 1/20 = [(\mu_g/\mu_a)-1][(1/R_1)-(1/R_2)]$ (1)
If whole system is immersed in a liquid of refractive index μ & focal length becomes 30 cm.
$1/30 = [(\mu_g/\mu_\omega)-1][(1/R_1)-(1/R_2)]$ (2)
dividing eq.(1) and (2) we get
$(3/2) = [(3/2)-1]/[\{(3/2)/\mu\}-1] = (1/2)/[(3-2\mu)/2\mu] = (1/2)[2\mu/(3-2\mu)] \Rightarrow 3/2 = \mu/(3-2\mu) = 9-6\mu = 2\mu \Rightarrow$ **$\mu = 9/8$**

Problem 24: As shown in fig. a spherical lens of radii of curvature$R_1=R_2=10$ cm is cut in a glass cylinder. Determine the focal length and nature of air lens. If a liquid of refractive index 2 is filled in the lens what will happen to its focal length and nature.

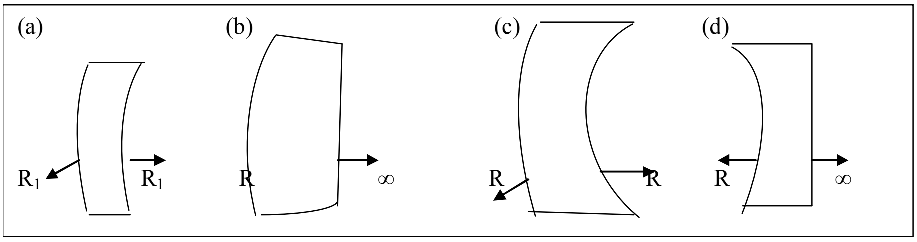


Solution:

According to lens maker's formula
$(1/f) = (\mu-1)[(1/R_1)-(1/R_2)]$ with $\mu = \mu_L/\mu_m$
$\mu = \mu_A/\mu_g = 1/(3/2) = 2/3$, $R_1 = +10$, $R_2 = -10$ cm
$(1/f) = [(2/3)-1][(1/10)-(1/10)] = -2/30 \Rightarrow f = -15$cm
i.e. the air lens in glass behaves as divergent lens of focal length 15 cm.

When the liquid $\mu = 2$ is filled in the air cavity $\mu = \mu_L/\mu_m = 2/1.5 = 4/3$
so that $(1/f') = [(4/3)-1][(1/10)-(1/-10)] = 2/30$
$f' = +15$ cm
i.e. the liquid lens in glass behave as a convergent lens of focal length 15 cm.

Problem 25: Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surface of the lenses are as given in the diagrams.



Solution: $1/f = (\mu-1)[(1/R_1) - (1/R_2)]$
$d(1/f) = d(\mu-1)[(1/R_1) - (1/R_2)]$
for no dispersion $d(1/f) = 0$,
$d\mu[(1/R_1) - (1/R_2)] = 0$
$d\mu \neq 0$ then $[(1/R_1) - (1/R_2)] = 0$ $\mathbf{R_1 = R_2}$
Hence optics c is correct.

Lecture-10

Geometrical Optics

Physics for IIT - JEE

Refraction from curved surfaces:

- *Magnification problems Lenses*
- *Positions, nature and size of the image for different positions of object*
- *Analysis of graphs*
- *Measurement of refractive index of liquid by a convex lens*:
- *Formula related to combinations of thin lenses*

Magnification: This is defined as the ratio of size of image to the size of object
magnification (m) = Size of image (I)/Size of object (O)

(a) Transverse or lateral magnification (m_T): It is defined as the ratio of any transverse dimension of the image to the corresponding transverse dimension of the object. Let the size of the object and image be denoted by 'O' and 'T' respectively

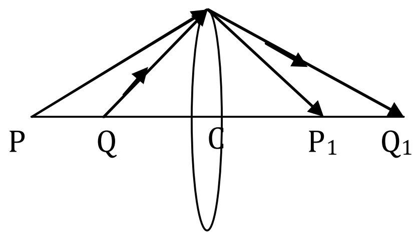
$$m_T = (I/O) = v/u$$

Note:

(i) The above formula is valid for convex as well as concave lens for all positions of object and it is independent of nature of object (real or virtual)
(ii) linear magnification for a lens can also be expressed as

$$m = I/O = v/u = (f-v)/f = f/(f+u)$$

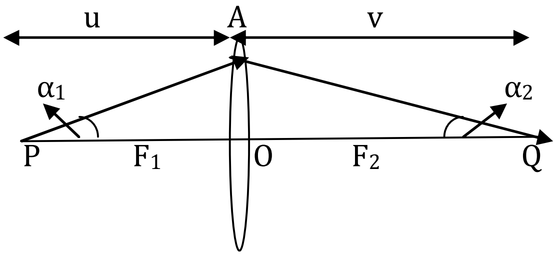
(b) Longitudinal or axial magnification (m_L):



It is defined as the ratio of the length of the image to the corresponding length of the object ,
$m_L = (v_P - v_Q)/(u_p - u_Q)$
For small object, $m_L = dv/du$
differentiating lens equation, we get $-[dv/v^2]-[-du/u^2] = 0$
or $[dv/v^2] - [du/u^2] = 0$
and therefore $m_L = dv/du = (v^2/u^2) = (v/u)^2 = m^2$
Therefore, for small object longitudinal magnification is square of transverse magnification.

(c) Angular magnification: It is defined as the ratio $\tan\alpha_2/\tan\alpha_1$, where α_1 and α_2 are the slope angles of the object ray and the image ray respectively as shown in the figure it is denoted by γ.
$\therefore$ $\gamma = \tan\alpha_2/\tan\alpha_1$

By Lagrange-Helmholtz equation
$\mu_1 h_1 \sin\alpha_1 = \mu_2 h_2 \sin\alpha_2$
$\mu_1 h_1 \tan\alpha_1 = \mu_2 h_2 \tan\alpha_2$ [$\because \alpha_1$ and α_2 are small]



since $\mu_1 = \mu_2$

$h_1 \tan \alpha_1 = h_2 \tan \alpha_2$

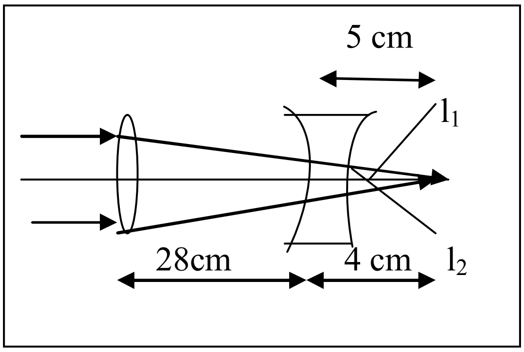
∴ Angular magnification $\gamma = \tan \alpha_2/\tan \alpha_1 = h_1/h_2 = 1/m$

That is, the angular magnification is reciprocal of the lateral magnification produced by a lens.

Problem 1(IIT – JEE 2003)*:* The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image.

(a) 1.25 cm (b) 2.5 cm (c) 1.05 cm (d) 2cm

Solution:



$(1/v) - (1/u) = 1/f$ or $(1/v) - ¼ = 1/ -20$ or $v = 5$ cm

magnification for concave lens $m = v/u = 5/4 = 1.25$

As size of the image at I_1 is 2 cm. Therefore, size of image at I_2 will be 2* 1.25 = 2.5 cm

Problem 2: When an object is at distance x &y from lens, a real image and a virtual image is formed having same magnification. The focal length of the lens is

(a) $(x + y)/2$ (b) $x - y$ (c) $\sqrt{xy}$ (d) $x + y$

Solution: The given lens is a convex lens. Let the magnification be m.

$(1/v) + (1/u) = (1/f)$

convex lens $(1/v) + (1/u) = (1/f)$

$m = v/u \Rightarrow$ real image $m = +ve \Rightarrow v = mu$

$m = -v/u \Rightarrow$ virtual image $m = -ve \Rightarrow v = -mu$

$u_{real} \rightarrow x$ & $v_{virtual} \rightarrow y$

$(1/v) + (1/u) = (1/f) \Rightarrow v = mx$ for real image

$(1/mx) + (1/x) = (1/f)$ (1)

$(1/v) + (1/u) = (1/f) \Rightarrow v = -my$ for virtual image

$(1/-my) + (1/y) = (1/f)$ (2)

Now equating eq.(1) & (2) we get

$(1/mx)+ (1/x) = (1/-my) + (1/y) = (1/f)$

$(1/mx) +(1/my) = (1/y) – (1/x)$

$(1/m)[(1/x)+(1/y)] = [(1/y)- (1/x)]$

$(1/m) [(x + y)/(x- y)]=[(x -y)/xy]$

$m = [(x + y)/(x- y)]$ (3)

substituting values of m in eq. (1) $(1/mx) + (1/x) = (1/f)$

$(1/x)[(1/m)+1]= 1/f$

$(1/x) [(x - y)/(x+ y)]+1=1/f$

$(1/x)[(x –y +x + y)/ (x +y)]= 1/f$

$2/(x +y) = 1/f$

f= (x +y)/2 Hence option (a) is correct

Problem 3(IIT-JEE 2010): The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50} The ratio m_{25} / m_{50} is

(a) 6 (b) 7 (c)8 (d) 9

Solution:

$(1/v)-(1/u) = 1/f$

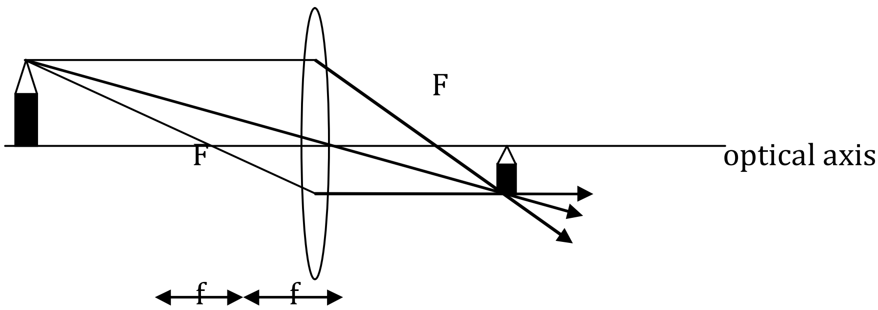
$(u/v)-1=u/f$

$(u/v) = (u+f)/f$

$m=v/u=f/ (u+f)$

$m_{25}/m_{50} = [20/ (-25+20)]/ [20/ (-50+20)] =6$

Problem 4: A pencil of height 1 cm is placed 30 cm from an equiconvex lens, refractive index n =3/2, radius of curvature for both the surfaces, $R_1 = R_2 = R = 10$ cm. Find the location of the image and describe its characteristics.



Solution: The ray of light diverges from the object; it is a real object. Focal length of a convex lens is positive.

$(1/f) = [(n_2-n_1)/n_1](2/R) = [(3/2) -1]/1(2/10) = 1/10$

The given parameters are

u = -30 cm

f= +10 cm

substituting these values in thin lens equation, we have

$(1/v) – (1/u) = (1/f)$

$(1/v) – (1/-30) = (1/10)$

which on solving gives v = +15 cm.

The image distance is positive, which implies that the image is real. The lateral magnification is

m = v/u = 15/(-30) = -1/2

The image is half the size of the object; size of the image is 0.50 cm. The negative sign with magnification shows inverted image.

Problem 5: If the pencil in the previous problem is kept at 6 cm from the lens, locate and characterize the image.

Solution: from lens equation, $(1/v) = (1/f) + (1/u)$

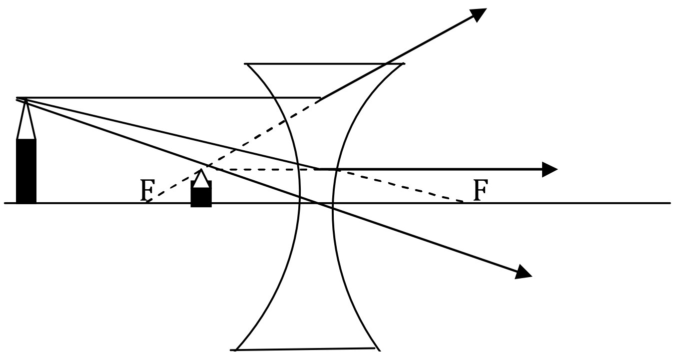
$= (1/10) + (1/-6) = -(1/15)$

v = - 15 cm

Lateral magnification, m = v/u = -15/-6 = 2.50

the minus sign with distances shows that the image is located on the side of the object. The image is upright, because magnification is positive.

Problem 6*:*



If in the previous example we use a diverging lens with a focal length 10 cm to form an image of the pencil kept 15 cm in front of the lens, locate and characterise the image.

Solution: In accordance with Cartesian sign convention the given parameters are

f = -10 cm, u = -15 cm

From lens equation, we have $(1/v) = (1/f) + (1/u)$

$= [1/(-10)] + [1/(-15)] = -(1/6)$

v = -6 cm

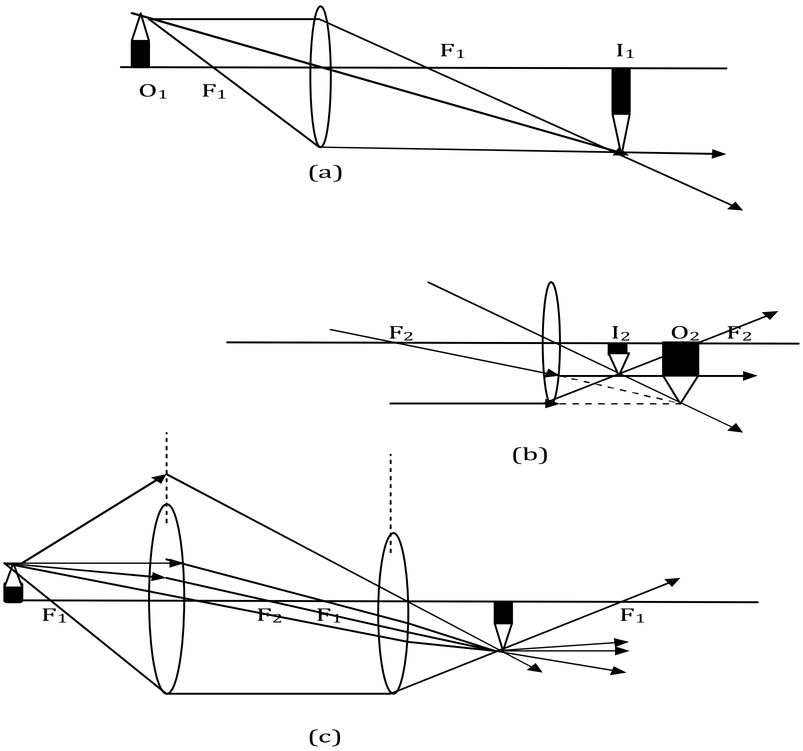
Lateral magnification, $m = (v/u) = -6/-15 = 0.40$

The minus sign with image shows that the image is located on the side of the object. The magnification is positive and m <1, which shows that the image is upright and diminished.

Problem 7: A pencil with a height of 5 mm is placed 45 cm to the left of a converging lens of focal length 20 cm. A diverging lens of focal length 15 cm is at a distance 10 cm from the first lens. What is (a) the image distance v_1 and (b) the height I_1 of the image produced by the first lens? (c) What is the object distance for the second lens? Find (d) the image distance v_2 and (e) the height I_2 of the image formed by the second lens.

Solution: The image formed by a converging lens can be real or virtual. It depends on where the object is located relative to the focal point. If the object is to the left of the focal point as in this case, the image is real and formed on the right side of the lens. If on the other hand the object is located between the focal point and the lens, the image is virtual, located on the left of the lens.

The image produced by the first lens acts as an object for the second lens; this image is called the first image. The first image can lie between the lenses or to the right of the second lens. In the first case it will be a real object and in the second case it will be a virtual object for the second lens.



(a) For the image formed by the first lens,

$(1/v_1) = (1/f_1) + (1/u_1) = (1/20) + (1/-45) = (1/36)$

$v_1 = +36$ cm

We thus find that the first lens is attempting to forms a real image, 36 cm behind it. However, before this image can be formed the rays are intercepted by the second lens located 10 cm behind it, see fig. (a) The point of convergence of these rays is a **virtual object** for the second lens.

(b) The height of the image, $I_1 = O_1 (v_1/u_1) = (5*10^{-1}) (36/-45) = -4mm$

The minus sign indicates that the image is inverted.

(c) The object distance for the second lens = (36-10) cm = 26 cm

to the right of the second lens; hence $v_2 = +26$ cm

(d) From lens equation for the second lens,

$(1/v_2) = (1/f_2) + (1/u_2) = (1/-15) + (1/+26) = -(1/35.5)$

$v_2 = -35.5$ cm

The negative sign shows that it is on the left of the diverging lens (virtual image)

(e) The image height for the first lens is object height for the second lens. Height of the image formed by the second lens

$I_2 = O_2 (v_2/u_2) = (-4*10^{-1}) [-35.5/+26] = 5.46mm$

The positive sign for I_2 means that the final image has the same orientation as the first image.

Problem 8: An equiconvex lens, $f_1 = 10$ cm, is placed 40 cm in front of a concave mirror, $f_2 = 7.50$ cm, as shown in fig. An object 2 cm high is placed 20 cm to the left of lens. Find the location of three images:

(a) the image formed by lens as rays travel to the right,

(b) the image formed after rays reflect from mirror and

(c) the final image after leftward travelling rays once again pass through lens.

Complete the ray diagram and characterise the image.

Solution: (a) from lens equation, $(1/v) – [1/(-20)] = (1/10)$, v = +20cm magnification,

$m_1 = v/u = [+20/-20] = -1$

image is real and inverted, same size as object.

(b) The first image acts as object for concave mirror.

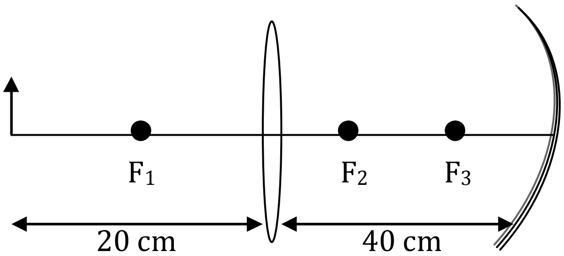
Object distance for mirror is (40-20)cm

From mirror equation, $(1/v') + (1/-20) = (1/-7.5)$

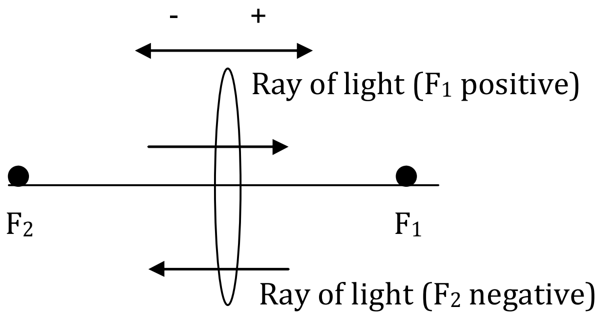
$v' = -12$ cm

magnification, $m_2 = -(u'/v') = -[12/-20] = -0.6$

The second image acts as objects for the lens. The object distance for second refraction at the lens,

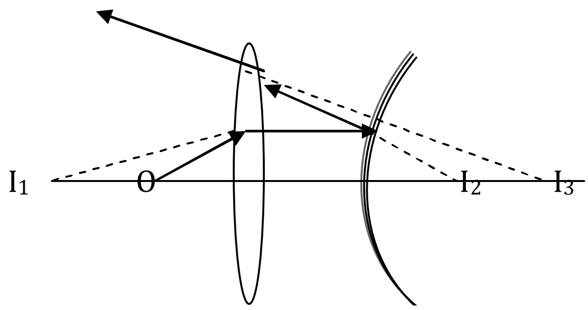


u'' = +28 cm
From lens equation, (1/v'') - (1/+28) = 1/(-10), v'' =-15.6 cm
Note the sign convention for f and u
magnification,m_3 = v''/u'' = (-15.6)/(+28) = -0.556



The final image is real, inverted and lies 15.6 cm to the left of the lens. Overall magnification, m = m_1*m_2*m_3 = (-1) (-0.6) (-0.556) = -0.333

Problem 9: A biconvex lens, f_1 = 20 cm, is placed 5 cm infront of a convex mirror, f_2 = 15 cm, an object of length 2 cm is placed at a distance 10 cm from the lens. Find the location of three images: (a) the image formed by the lens as the rays travel to the right, (b) the image formed after the rays reflect from the mirror and (c) the final image after the leftward travelling rays once again pass through the lens.



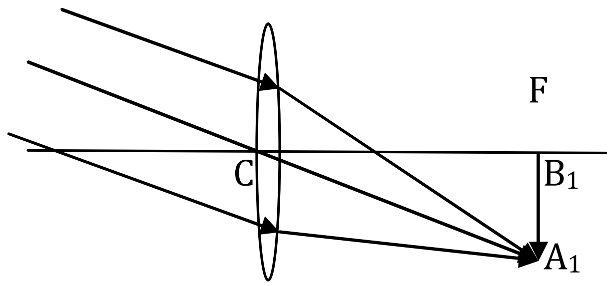
Solution: (a) From lens equation, (1/v) – (1/-10) = -(1/20) , v = -20 cm
magnification m_1 = (-20)/(-10) = +2
image is virtual, erect and magnified.
(b) the first image acts as an object for the convex mirror. Object distance for the mirror, u' = (20+5) = 25 cm.
From mirror equation, (1/v') + (1/-25) = (1/15)

$v' = +75/8$ cm
magnification, $m_2 = (+75/8)/(-25) = 3/8$
image is virtual (to the right of the mirror), erect and diminished.
(c) the object distance for second refraction at the lens = $(75/8) + 5 = 115/8$
From lens equation, $(1/v'') - 1/(+115/8) = 1/-20$
$v'' = 460/9 = +51.1$ cm
magnification $m_3 = (+460/9)/(115/8) = 32/9$
overall magnification $m = m_1 * m_2 * m_3 = 2(3/8)(32/9) = 8/3$
hence size of image=(8/3)2 cm= 5.33 cm
Final image is to the right of the lens at a distance 51.1 cm from the lens, real, erect and magnified.

Positions, nature and size of the image for different positions of object:

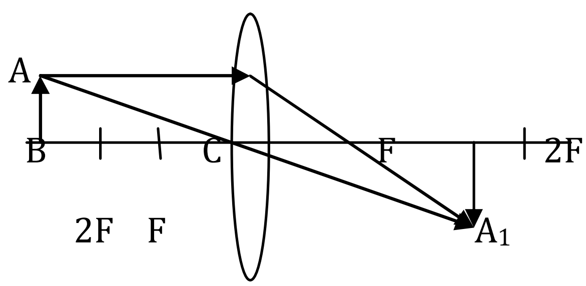
Convex lens:

(i) Object at infinity:



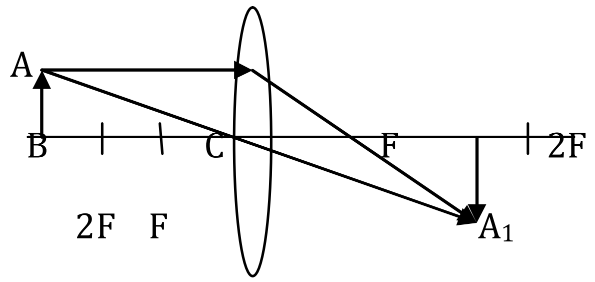
In this case, $u \to \infty$
$(1/f) = (1/v) - (1/\infty)$ or $(1/f) = (1/v)$
or $v = f$
Hence
Position: At F
Nature: Real and inverted
Size: Diminished (very small)

(ii) Object lying beyond 2F:



From lens equation, we have (1/v) = (1/f) + (1/u)
If u > 2f, 2f > v > f, i.e., the image lies between f and 2f
Also, m =v/u always is less than one
Position: Between f and 2f
Nature: Real and inverted
Size: Small

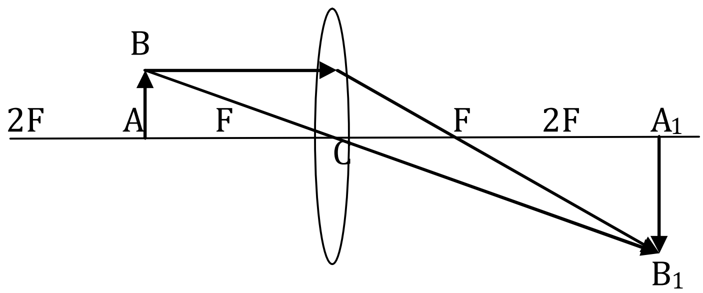
(iii) Object at 2F:



Here, u = -2f
From lens equation, we have v = uf/(u + f) =[(-2f)(f)]/[-2f + f] = 2f
Also, m = v/u = 2f/-2f = -1

Position: At 2f
Nature: Real and inverted
Size: Same as that of object

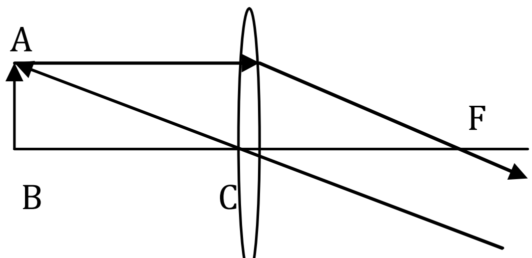
(iv) Object lying between F and 2F:



If 2f > u > f, it can be seen from lens equation that, v > 2f
Also, m = v/u is greater than one

Position: Beyond 2F
Nature: Real and inverted
Size: Enlarged

(v) Object at F:



Object at F

Here, u = -f

From lens equation, we have v = uf/(u+ f) = [(-f)(-f)]/[-f+ f] = ∞

Also, m = v/u → ∞

Position: At infinity

Nature: Real and inverted

Size: Highly magnified

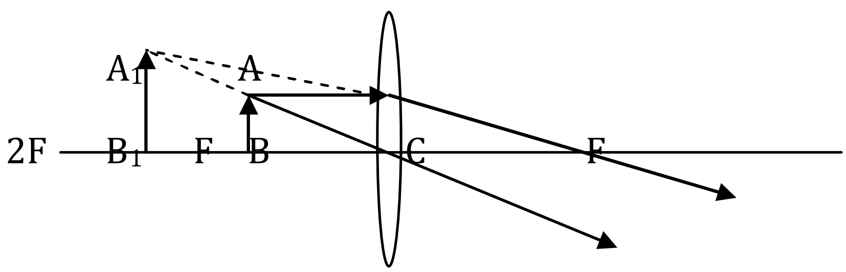
(iv) Object lying between 'F' and optical centre 'C':

Here, u = -f

From lens equation, we have

(1/f) = (1/v) – (1/-f) = (1/v) + (1/f)

or (1/v) = (1/f) – (1/f) =0



Object between F and optical centre

At C, u = 0

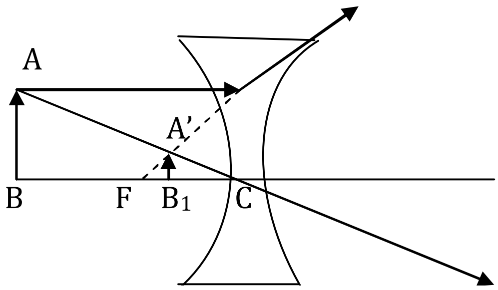
(1/v) = (1/f) + (1/0) = ∞ or v = 0

Position: Same side of lens

Nature: Virtual and erect

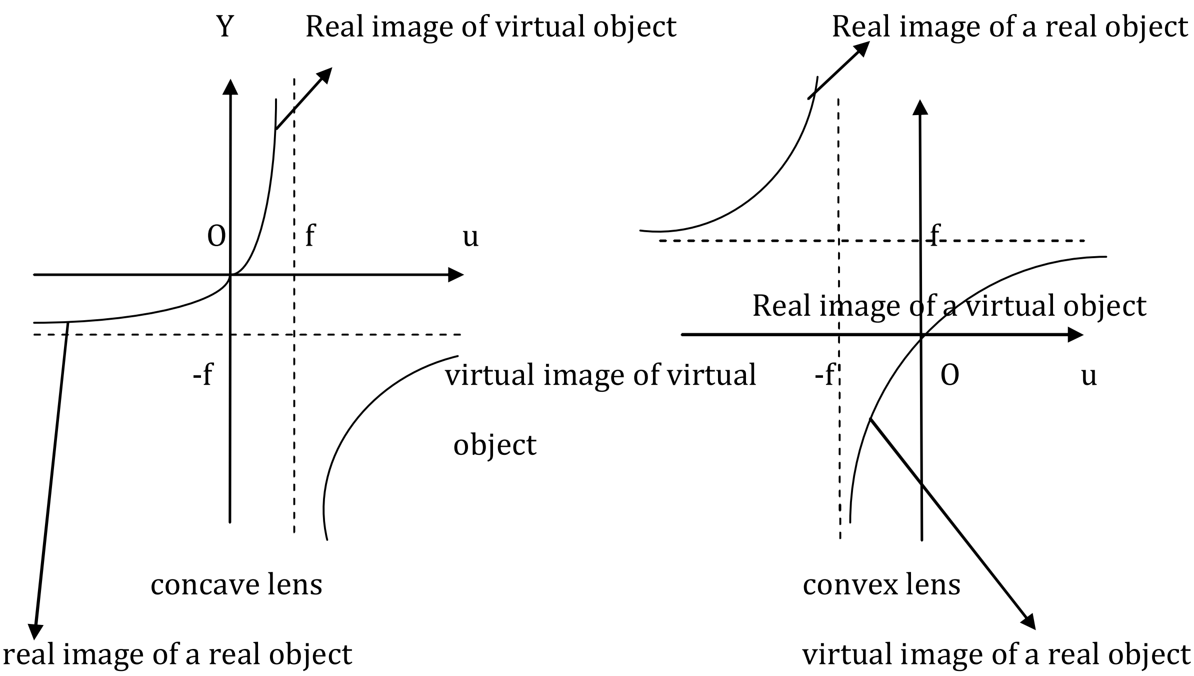
Size: Magnified

Concave lens:

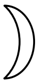
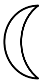
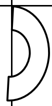
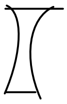
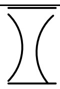


In concave lens whatever be the position of object, image formed is always virtual, erect and small in size. If object is virtual concave lens may form real or virtual image depending on the position of object. If u < f, image is real and if u > f, then image is virtual.

Note:



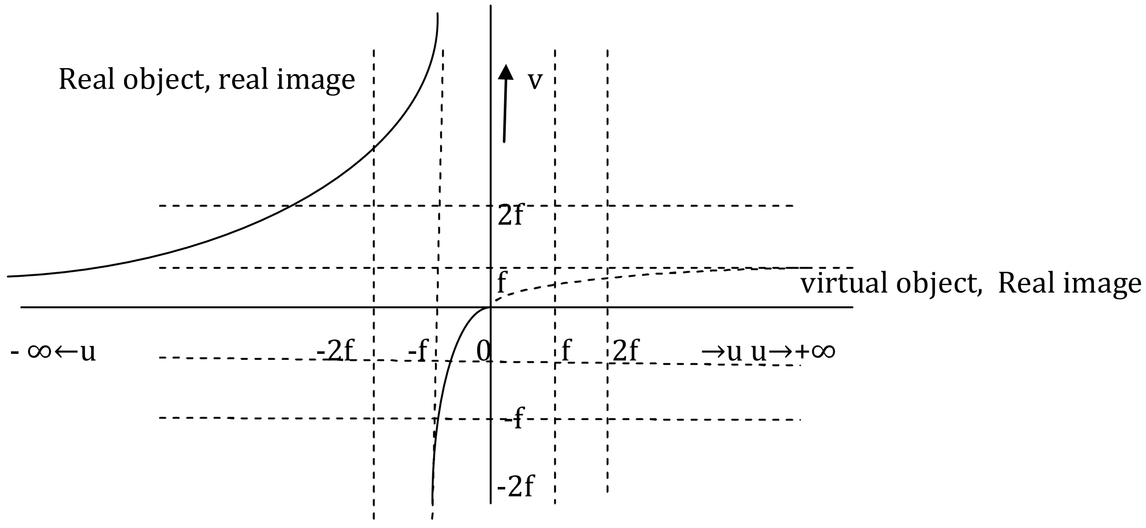
In general, all situations in lenses can be summarised in u-v graph as shown in fig.
While interpreting these graphs remember following points:
(i) u is negative for real object and u is positive for virtual object
(ii) v is positive for real image and v is negative for virtual image

SNo.	Type of lens	Shape of lens	The factor $[(1/R_1)-(1/R_2)]$	Reciprocal of the focal length in air or vacuum	Nature of lens
(1)	Concavo - convex	 or 	$[(1/x)-(1/y)]$	$(\mu-1)\,[(1/x)-(1/y)]$	Converging
(2)	Convexo-concave	 or 	$-[(1/x)-(1/y)]$	$-(\mu-1)\,[(1/x)-(1/y)]$	Diverging
(3)	Biconvex		$(1/x)+(1/y)$	$(\mu-1)\,[(1/x)+(1/y)]$	Converging
(4)	Biconcave		$-[(1/x)+(1/y)]$	$-(\mu-1)\,[(1/x)+(1/y)]$	Diverging
(5)	Plano convex	 or 	$1/R$	$(\mu-1)/R$	Converging
(6)	Plano concave	 or 	$-1/R$	$-(\mu-1)/R$	Diverging
(7)	Symmetrical biconvex		$2/R$	$(\mu-1)/(2/R)$	Converging
(8)	Symmetrical biconcave		$-2/R$	$-(\mu-1)(2/R)$	Diverging

Graph of u vs. v for a lens:

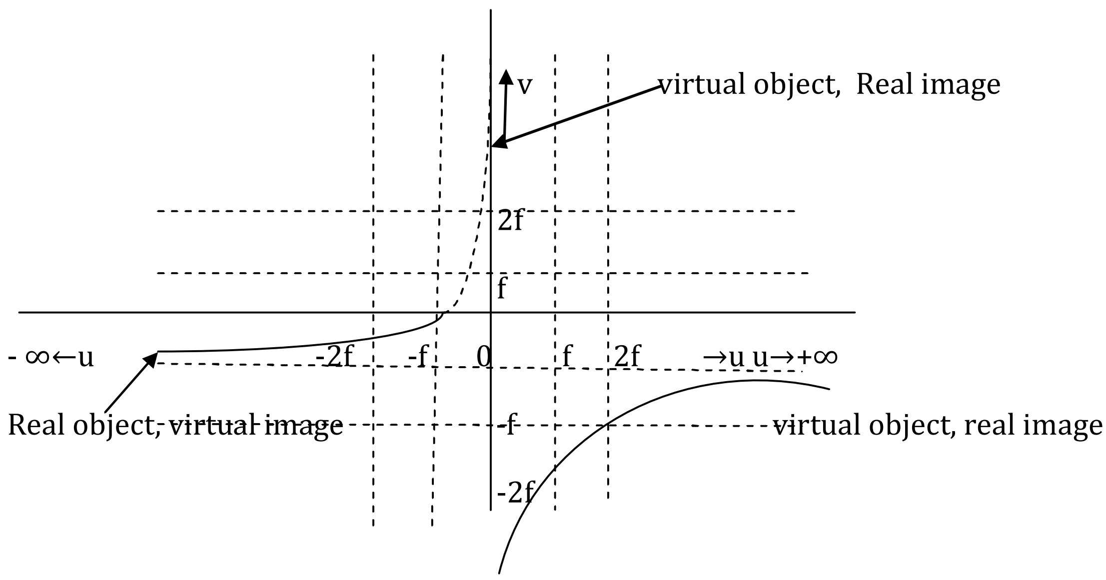
According to lens equation, it is hyperbola, as shown in fig.

(a) **Convex lens**:



u =	-∞	-2f	-f	-(f/2)	-(f/4)	0	+f	+2f	+∞
V =	+f	+2f	+∞	-f	-(f/3)	0	+f/2	+2f/3	+f

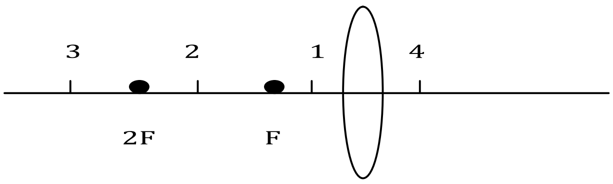
(b) **Concave lens**:

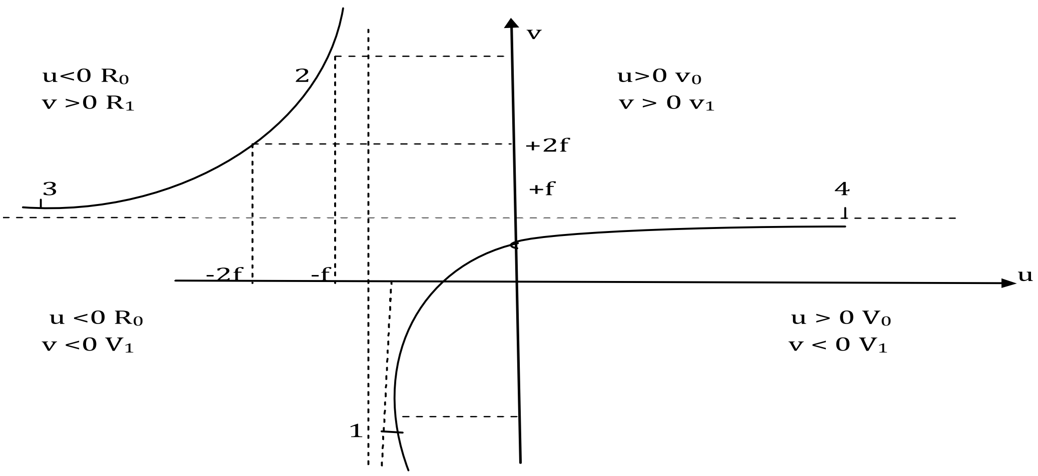


u =	-∞	-2f	-f	-(f/2)	0	+f/2	+f	+f	+2f	+∞
V =	-f	-2f/3	-f/2	-f/3	0	+f	+∞	-∞	-2f	-f

Analysis of graphs:

Convex lens:





Region 1: object lies between optical centre and focus real object, virtual image |v| > |u|.
m = v/u thus |m| > 1 image is enlarged is u < 0: v < 0 thus m >0 image is erect. As object moves from focus towards optical centre magnification goes on decreasing.

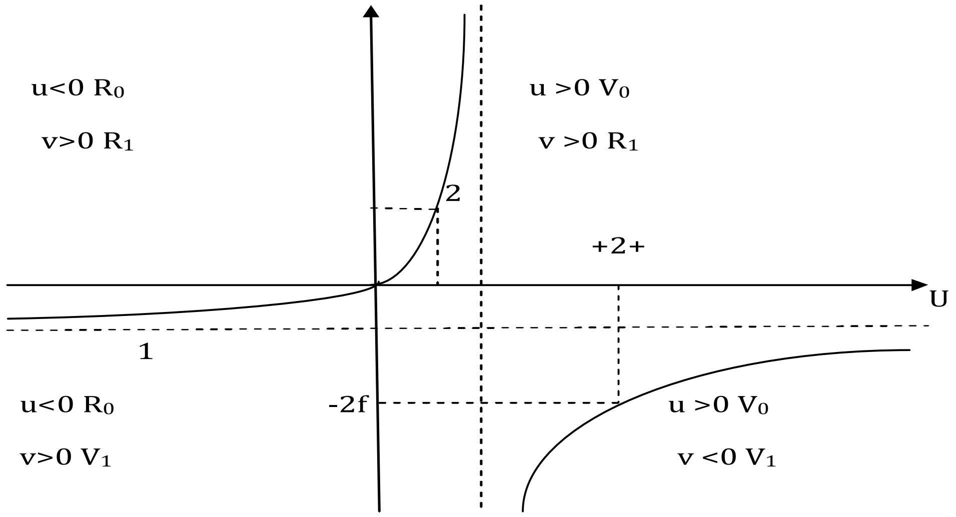
Region 2: object lies between F and 2F object is real image is also real |v| > |u|.
|m| < 1 image is enlarged v> 0; u<0, m < 0 image is inverted as object moves from F to 2F magnification decreases and equal 1 at 2F.

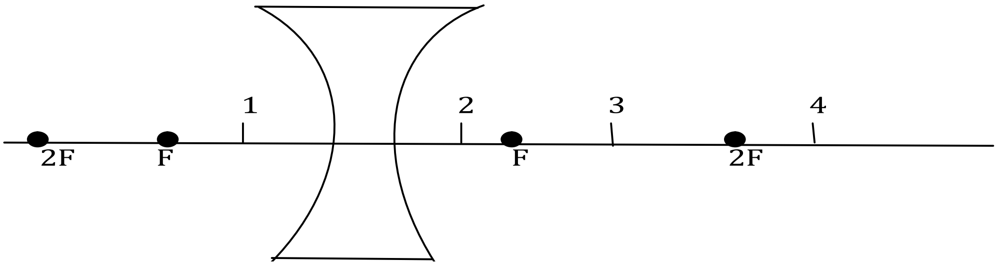
Region 3: object lies between 2F and infinity object is real and image is also real. |v| < |u| |m| < 1 image is also real. |v| > |u|
|m| > 1 image is smaller in size v > 0;u < 0, m <0 image is inverted as object moves farther away magnification goes on decreasing.

Region 4: object is virtual and it lies between optical centre and infinity. Object is virtual image is real |v| > |u|.
|m| < 1 image is smaller in size v > 0, u > 0, m>0 image is erect as object moves away magnification goes on decreasing.

Analysis of graphs:

Concave lens:





Region 1: For all the positions of a real object in front of a lens. Object is real image is virtual |v| > |u|.
|m| < 1 image is smaller in size v<0, u<0 m>0 image is erect. As object is moved away from lens image size i.e. magnification goes on decreasing

Region 2: Virtual object that lies between optical centre and focus. Object is virtual and image is real |v| > |u|; |m| > 1 image is enlarged v>0, u>0, m>0 image is erect. As object is moved towards F image size goes on increasing i.e., m increases.

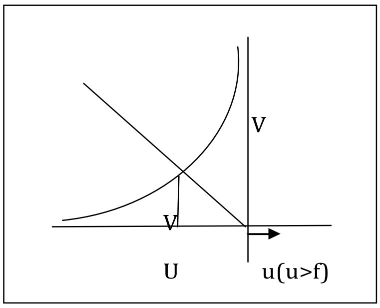
Region 3: virtual object lies between +F and +2F object is virtual and image is virtual |v| > |u|; |m| > 1 image is inverted. As object moves from +F to+2F magnification goes on decreasing.

Region 4: virtual object lies between 2F and infinity object is virtual and image is virtual |v| < |u|, |m| < 1 image is smaller in size u >0; v<0; m<0 image is inverted.

Problem 10(AIEEE 2009): In an optics experiments, with the position of the object fixed, a student varies the position of a convex lens and for each position; the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45^0 with the x-axis meets the experimental curve at p. The coordinates of p will be

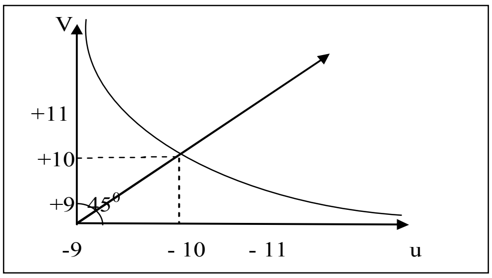
(a) (2f, 2f) (b) ((f/2),(f/2)) (c) (f, f) (d) (4f, 4f)

Solution:



It is possible when object kept at centre of curvature u=v,
u= 2f, v=2f.

Problem 11(IIT- JEE 2006): The graph between object distance u and image



Distance v for lens is given. The focal length of the lens is

(a) 5 ± 0.1 (b) 5 ± 0.05 (c) 0.5 ± 0.1 (d) 0.5 ± 0.05

Solution: from the lens formula: $1/f = (1/v) - (1/u)$ we have

$1/f = (1/10) - (1/-10)$ or $f = +5$

further $\Delta u = 0.1$ and $\Delta v = 0.1$ (from the graph)

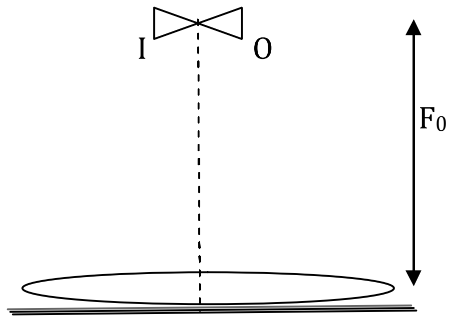
now differentiating the lens formula we have,

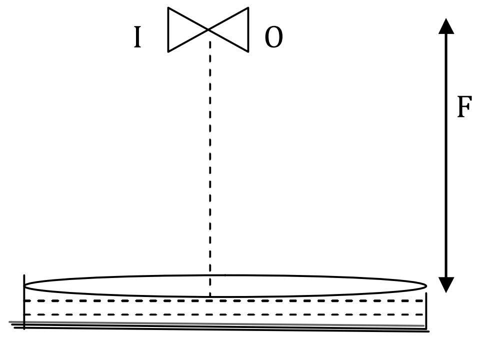
$(\Delta f/f^2) = (\Delta v/v^2)+(\Delta u/u^2)$ or $\Delta f = [(\Delta v/v^2)+(\Delta u/u^2)]f^2$

substituting the value we have $\Delta f = [(0.1/10^2)+(0.1/10^2)]\,(5)^2 = 0.05$

so $f \pm \Delta f = 5 \pm 0.05$

Measurement of refractive index of liquid by a convex lens:





(a) (b)

Fg. Shows an equiconvex lens placed on a plane mirror. An object pin is moved up and down. When the pin lies at the focus of the lens, there is no parallax between the object and the image. When a liquid whose refractive index is to be obtained is placed between a plane mirror and a convex lens, the object pin O has to be shifted downward so that no parallax exists between it and its image. The position of object O from the lens is now equal to the combined focal length of 'lens and liquid' combination. If f is the focal length of the liquid lens, then the combined focal length is given by

$(1/F) = (1/f_0) + (1/f)$

The liquid lens is Plano -concave type as its lower surface is the plane surface of the mirror and the upper surface is the curved surface of the convex lens, then

$(1/f) = (\mu-1)\ [(1/-R) - (1/\infty)]$

or $(1/f) = -[(\mu-1)/R]$

Thus, $\mu = 1+[R/f]$

Formula related to combinations of thin lenses:

If lenses are placed in contact

Net Deviation: $\boldsymbol{\delta_{net} = \delta_1 + \delta_2 + \delta_3 + ...}$

Net Power: $\mathbf{P_{net} = P_1 + P_2 + P_3}$

Net Magnification: $\mathbf{m_{net} = m_1\ m_2\ m_3 ...}$ If many steps are involved

Magnification = $(I_1/O)(I_2/I_1)(I_3/I_2)...(I/)\ = I/O$

If two lenses are placed at a separation d apart

$\mathbf{P_{net} = P_1 + P_2 - dP_1P_2}$

$\mathbf{1/f_{eq} = 1/F_1 + 1/F_2 - d\ (1/F_1F_2)}$

Problem 12(AIEEE 2007): Two lenses of Power - 15D and + 5D are in contact with each other. The focal length of the combination is

(a) -20 cm (b) -10 cm (c) + 20 cm (d) + 10 cm

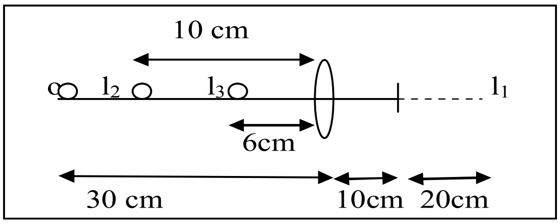
Solution: Power of a lens is reciprocal of its focal length. Power of combined lens is
$p = p_1 + p_2 = -15 + 5 = -10$ D so $f = 1/p = 100/(-10)$ cm
$f = -10$ cm

Problem 13(IIT – JEE 2005): A convex lens is in contact with concave lens. The magnitude of the ratio of their focal length is 2/3. Their equivalent focal length is 30 cm. What are their individual focal lengths?
(a) -75, 50 (b) -10, 15 (c) 75, 50 (d) -15, 10

Solution: Let focal length of convex lens is +f then focal length of concave lens would be (-3/2) f.
$1/F_{net} = 1/F_1 + 1/F_2$
from the given condition.,
$1/30 = (1/f) - (2/3f) = 1/3f$
$f = 10$ cm
Therefore, focal length of convex lens=+10 cm and that of concave lens = -15 cm.

Problem 14(IIT- JEE 2010): A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10cm. A small object is kept at a distance of 30 cm from the lens. The final image is
(a) virtual and at a distance of 16 cm from the mirror
(b) real and at a distance of 16 cm from the mirror
(c)virtual and at a distance of 20 cm from the mirror
(d) none of the above

Solution:



Object is placed at distance 2f from the lens. So first image I will be formed at distance 2f on other side. This image I_1 will behave like a virtual object for mirror. The second image I_2 will be formed at distance 20 cm in front of the mirror.

Problem 15(IIT-JEE 2006): A biconvex lens of focal length f forms a circular image of radius r of sun in focal plane. Then which option is correct?
(a) $\pi r^2 = f$
(b) $\pi r^2 = f^2$
(c) if lower half part is covered by black sheet, then area of the image is equal to $\pi r^2/2$
(d) if f is doubled, intensity will increase

Solution:

r = f tan θ

or r=f

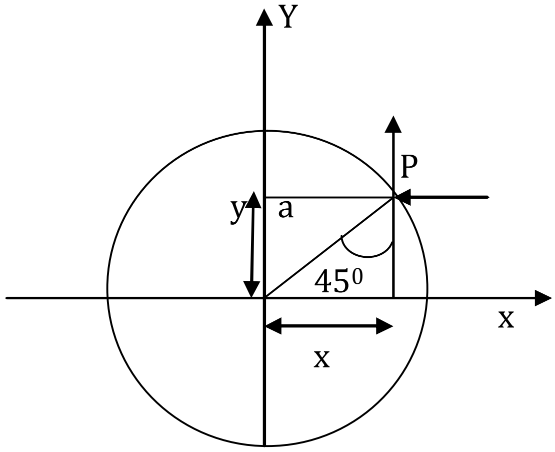
so $\pi r^2 = f^2$

Problem 16: A reflecting surface is represented by the equation $x^2 + y^2 = a^2$. A ray travelling in negative x direction is directed towards positive y direction after refraction from the surface at point P. Then the coordinates of point P are

(a) (0.8a, 0.6a) (b) (0.6a, 0.8a) (c) [(a/√2), (a/√2)]

(d) none (e) [(a/2), (a/2)]

Solution:



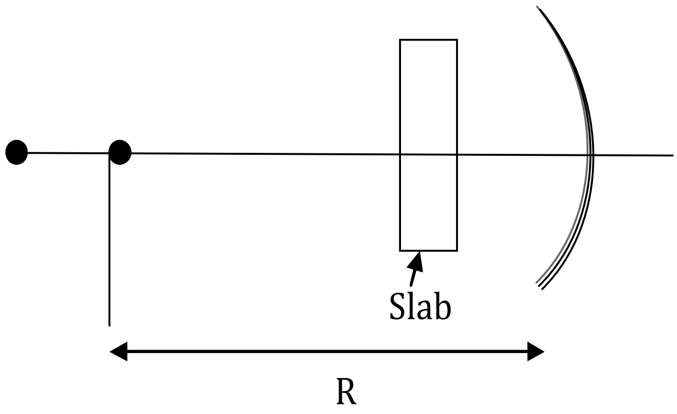
X = a/√2, y = a/√2

x = a cos 45^0 ⇒ a/√2 = x

y = a cos 45^0 ⇒ a/√2 = y

Hence coordinates [(a/√2), (a/√2)]

Problem 17: A glass slab of thickness 3 cm and refractive index 1.5 is placed in front of a concave mirror of focal length 20 cm. Where should a point object be placed if it is to image on to itself? The glass slab and the concave mirror are shown in fig.



Solution:

Let the distance of the object from the mirror be x.

We know that the slab simply shifts the object.

The shift being equal to

$s = t[1-(1/\mu)] = 1$ cm

The direction of shift is towards the concave mirror.

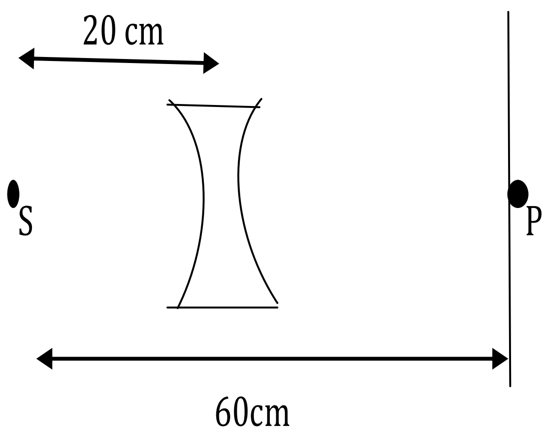
∴ The apparent distance of the object from the mirror is (x-1)

If the rays are to retrace their paths, the object should appear to be at the centre of curvature of the mirror.

∴ (x-1) = 2f = 40 cm

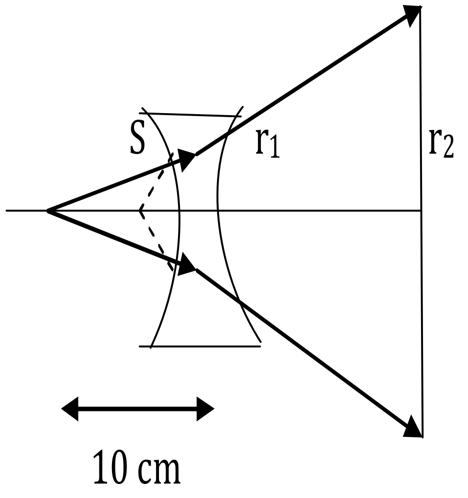
or x = 41 cm from the mirror

Problem 18:



A point source of light is placed 60 cm away from screen. Intensity detected at point P is I. Now a diverging lens of focal length 20 cm is placed 20 cm away from S between S and P. The lens transmits 75% of light incident on it. Find the new value of intensity at P.

Solution:



$u = -20$, $f = -20$,

gives $v = -10$

Let P = power of source

$I = P/4\pi(60)^2$

Energy received by lens $E_2 = [P/4\pi(20)^2]A_1$

$\therefore I_2 = 0.75E_2/A_2$

From similar triangles $A_2/A_1 = 25$

$\therefore I_2 = 0.271$

Lecture-11

Geometrical Optics

Physics for IIT - JEE

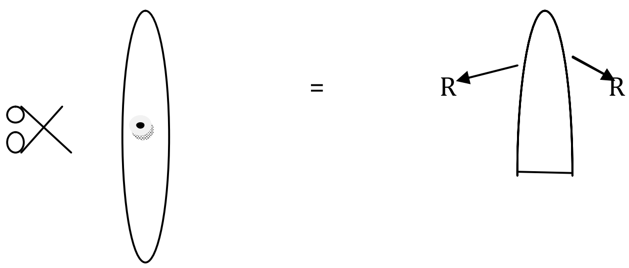
Refraction from curved surfaces:

- *Cut lense*
- *Silvered lenses*
- *Combination of lenses & mirrors*

Cut lens:

- Depending upon how a lens is cut its behaviour gets changes in such a way that either its power or brightness of image or both get changed.
- Generally a lens is cut by a plane surface (It can be cut by curved surface as well) in two possible ways.

(A) ***By a plane parallel to principle axis***:



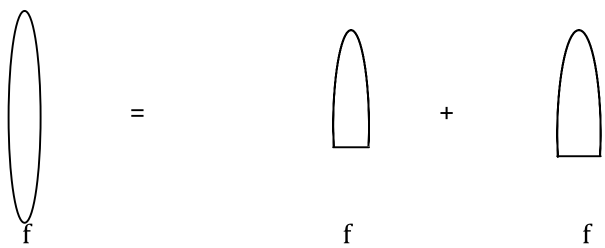
Power, focal length no change

Intensity, brightness change

In this situation power and focal length of the lens do not change (As radius of curvature of both refracting surface & refractive index do not change) but amount of light energy passing through lens decreases that is why brightness or intensity of image gets reduced.

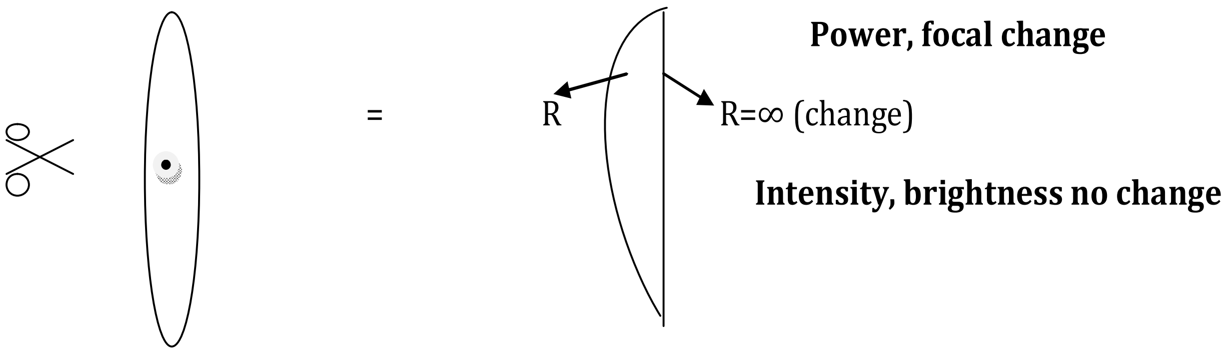
Problem 1: A convex lens of focal length f is cut by a plane parallel to principle axis. The pieces are then placed side by side in such a way that light passes through both the pieces. Find the power of such a combination.

Solution: As a focal length of both the pieces remain same as a whole



$P_{eff} = 1/f_{eff} = (1/f) + (1/f) = (2/f)$

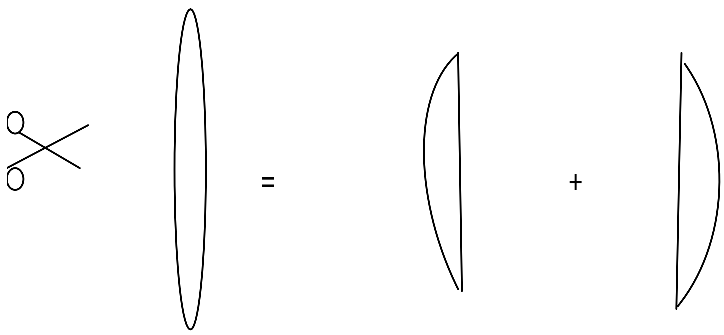
(B) ***By a plane perpendicular to its principle axis***:



- In this situation power & focal length of the lens gets change (As radius of curvature of one of the refracting surface changed)
- Position nature & size of image may get changed.
- But amount of light energy passing through remains unchanged and that is why brightness of image remains same.

Problem 2: A convex lens of focal length f is cut by a plane perpendicular to principle axis. The pieces are then placed side by side. In such a way that light passes through both the pieces. Find the power of such a combination.

Solution:



As power of each half becomes half or the focal length is twice so
$P_{eff} = 1/f_{eff} = [(1/2f) + (1/2f)] = 1/f$

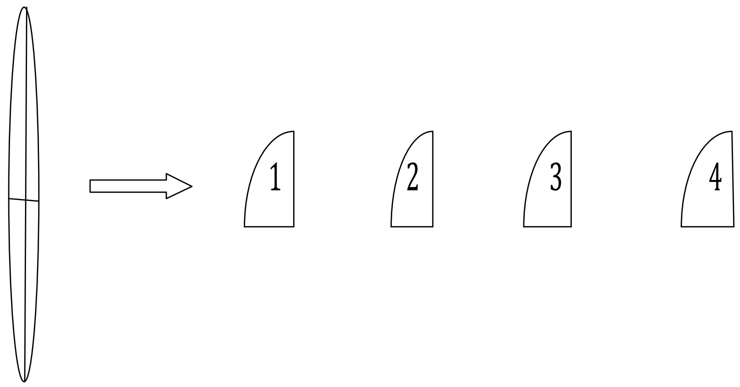
Problem 3: A lens of focal length f is cut in to four pieces and placed as shown. Find the power of combination.

a) $\mathbf{P_{eff} = 1/f}$

b) $\mathbf{P_{eff} = 2/f}$

c) $\mathbf{P_{eff} = 3/f}$

d) $\mathbf{P_{eff} = 5/f}$

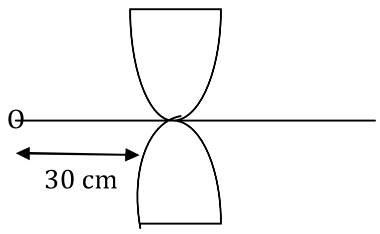


Solution:

$(1/f) = (\mu-1)[(1/R_1) - (1/R_2)]$ for convex
$(1/f) = (\mu-1)[(1/R_1) + (1/R_2)]$
$(1/f_1) = (\mu-1)[(1/R_1) - (1/\infty)]$
$(1/f_1) = (\mu-1)/R_1 = 1/f_3$ (1&3 are at $\perp$ to principle axis radii of)
$(1/f_2) = (\mu-1)[(1/R_1) + (1/R_2)]$
$(1/f_2) = (\mu-1)[(1/\infty) - (1/-R_2)]$
$(1/f_2) = (\mu-1)/R_2 = 1/f_4$
$P_{eff} = 1/f_{eff} = (1/f_1)+(1/f_2)+(1/f_3)+(1/f_4)$
$P_{eff} = [(2/f_1) + (2/f_2)]$
$P_{eff} = 2[(1/f_1) + (1/f_2)]$
$P_{eff} = 2[[(\mu-1)/R_1]+[(\mu-1)/R_2]]$
$P_{eff} = 2\ (\mu-1)\ [(1/R_1) + (1/R_2)]$
$\mathbf{P_{eff} = 2/f}$

Problem 4: A point object is placed at a distance of 30 cm along the principle axis of a convex lens of focal length 20 cm & aperture 4 cm. It is cut by a plane parallel to its principle axis and pieces are placed inverted as shown. Find distance between the images formed

(a) 4 cm (b) 8 cm (c) 12 cm (d) none



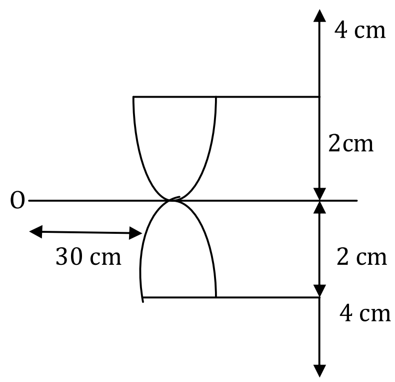
f = 20 cm

R = 4 cm

u = 30 cm

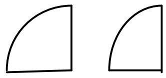
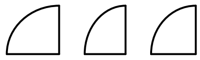
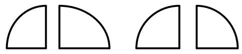
Solution:

(1/f) = (1/v) + (1/u)   Convex
(1/20)= (1/v) + (1/30) ⇒ (1/20) - (1/30) = (1/v)
(1/v) = (1/10)[(3-2)/6]= 1/60
V = 60 cm
m = v/u = 60/-30 = -2



so images are formed by each half, with twice magnification and as they are real, they are formed 2*2 = 4 cm away from their respective principle axis.
m = m_1, m_2, m_3, ------ magnification multiplied.
Thus distance between the image is 4 cm + 4 cm + 4 cm ⇒ **d = 12 cm**

Problem 5: An equiconvex lens of focal length f is cut into four pieces by two planes as shown and the pieces are places as shown and the pieces are placed as shown in column I. Then match them with column II

Column I	Column II
(a) 	p) f/2
(b) 	q) (2/3)f
(c) 	r) f
(d) 	s) None

Solution: focal length of each part becomes 2f

(a) $1/f_{eq} = 1/2f + 1/2f = 2/2f = 1/f$

$\Rightarrow f_{eq} = f$ **a → r**

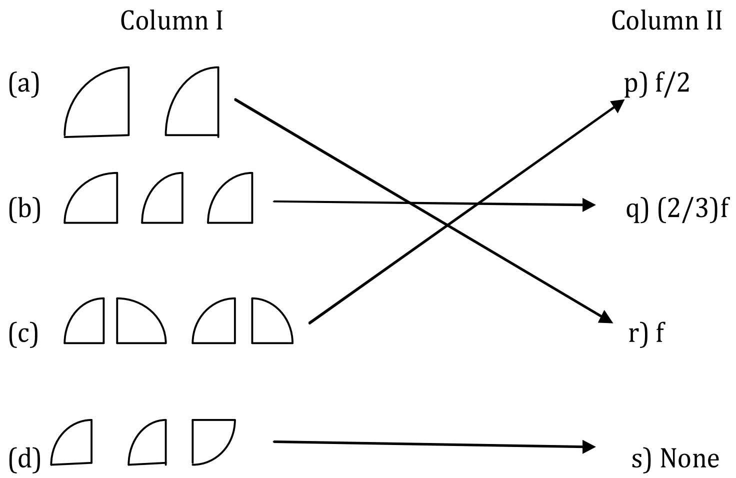
(b) $1/f_{eq} = 1/2f + 1/2f + 1/2f = 3/2f$

$\Rightarrow f_{eq} = (2/3)f$ **b → q**

(c) $1/f_{eq} = 1/2f + 1/2f + 1/2f + 1/2f = 4/2f = (2/f)$

$\Rightarrow f_{eq} = f/2$ **c → p**

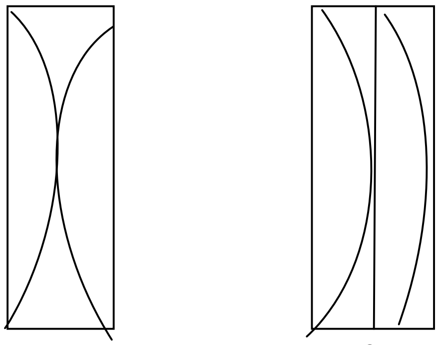
(d) As a paraxial ray for one lens is merging for other so focal length of such a combination cannot be calculated. **d → s**



Combination of cut lenses problems:

Problem 6: A liquid of $\mu_e = 1.62$ is placed between two Plano convex identical lenses of $\mu_2 = 1.54$. Two possible arrangements P & Q are shown. The system is

(a) divergent in P
(b) convergent in P
(c) divergent in Q
(d) convergent in Q



Solution:

For lens L:

$1/F_L = (1.54-1)[(1/R) - (1/-R)] = 1.08/R$

$F_L = R/1.08$

For P:

$(1/F_l) = (1.62-1)][(1/-R)-[1/+R]] = -1.24/R$

$f_l = -R/1.24$

$1/f_p = (1/f_l) + (1/F_L) = [(1.08/R)- (1.24/R) = -(0.16/R)$

$\mathbf{f_p = -R/0.16}$

For Q:

$1/f_l' = (1.62-1) [(-1/R)- 1/\infty] = -(0.62/R)$

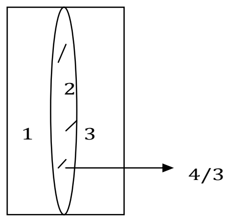
$1/f_Q = 1/f_l' + 1/F_L = (-0.62/R) + (1.08/R) = 0.46/R$

$\mathbf{f_Q = R/0.46}$

As focal length in P is negative & in Q is positive, so system is divergent in P and convergent in Q.

Problem 7: Two Plano concave lenses of glass of refractive index 1.5 have radii of curvature of 20 & 30 cm. They are placed in contact with curved surfaces towards each other & the space between them is filled with a liquid of refractive index 4/3. Find the focal length of the system

(a) divergent lens of focal length 72cm (b) divergent lens of focal length 60cm
(c) convergent lens of focal length 72cm (d) convergent lens of focal length 60cm



Solution:

The system is equivalent to the combination of three thin lenses.

$(1/f) = (1/f_1) + (1/f_2) + (1/f_3)$

but by lens maker's formula $1/f = (\mu -1)[(1/R_1) - (1/R_2)]$

$1/f_1 = (1.5 -1)[(1/\infty) - (1/+20)] = -1/40$

$1/f_2 = (4/3 -1)[(1/20) + (1/30)] = 5/180$

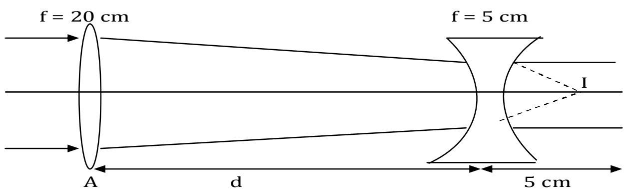
$1/f_3 = (1.5 -1)[(1/-\infty) - (1/30)] = -1/60$

$(1/f) = (-1/40) + (5/180) -(1/60) \Rightarrow (1/f) = [(-9 +10-6)/360]$

f = -72 cm i.e. the system will behave as a divergent lens of focal length 72 cm.

Problem 8(IIT 1985)(MNR 1992): A convex lens A of focal length 20 cm & a concave lens B of focal length 5 cm are kept along the same axis with a distance d between them. What is the values of d if a parallel beam of light incident on A leaves B as a parallel beam

(a) 15 cm (b) 30 cm (c) 10 cm (d) none



Solution:

As the incident beam is parallel in absence of concave lens it will form an image at a distance v from it such that $(1/v) - (1/-\infty) = (1/20) \Rightarrow v = 20$ cm = f

Now since d is the distance between convex & concave lens. The distance of image I from concave lens will be (20-d). Since the image I will act as an object for concave lens which forms its image at ∞, so $(1/v)- (1/u) = (1/f)$

$(1/\infty) - [1/(20-d)] = 1/(-5)$, $20-d =5 \Rightarrow$ **d = 15 cm**

Silvered lenses:

When one side of a lens is silvered such that it effectively behaves like a mirror, then it is called a silvered lens.

Silvered lens

⇒Combination of two lenses & mirror

⇒Behaves like a mirror

A light ray entering into a silvered lens goes through

(1) Refraction
(2) Reflection
(3) Refraction

That means as bending ray takes place three times deviation

$$\boldsymbol{\delta_{eff} = \delta_1 + \delta_2 + \delta_3}$$

$$P_{eff} = P_1 + P_2 + P_3 = P_L + P_M + P_L = 2P_L + P_M$$

$$-1/F_{eff} = (2/F_L) - (1/F_M)$$

Effective behaviour mirror,

$$P_M = -1/f,\ P_L = 1/f$$

$$\mathbf{1/F_{eff} = -(2/F_L) + (1/F_M)}$$

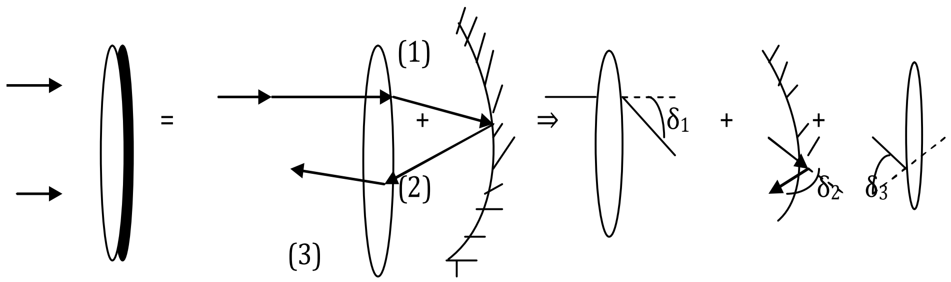


Fig: silvered lens

Working procedure for problems based on silvered lenses:

Step 1st:

Formula $1/F_{eff} = (-2/F_L) + (1/F_M)$

Step 2nd:

$F_M = R/2$

(please take proper sign)

concave mirror $\rightarrow$ -R, convex mirror $\rightarrow$ +R, plane mirror $\rightarrow \infty$

$\mathbf{1/F_M = 2/R}$

Step 3rd:

$1/F_L = (\mu-1)[(1/R_1)-(1/R_2)]$

for convex $R_1 \rightarrow$ +ive, $R_2 \rightarrow$ -ive

for concave $R_1 \rightarrow$ -ive, $R_2 \rightarrow$ +ive

Step 4th:

Use $1/F_L$ and $1/F_M$ in eq.

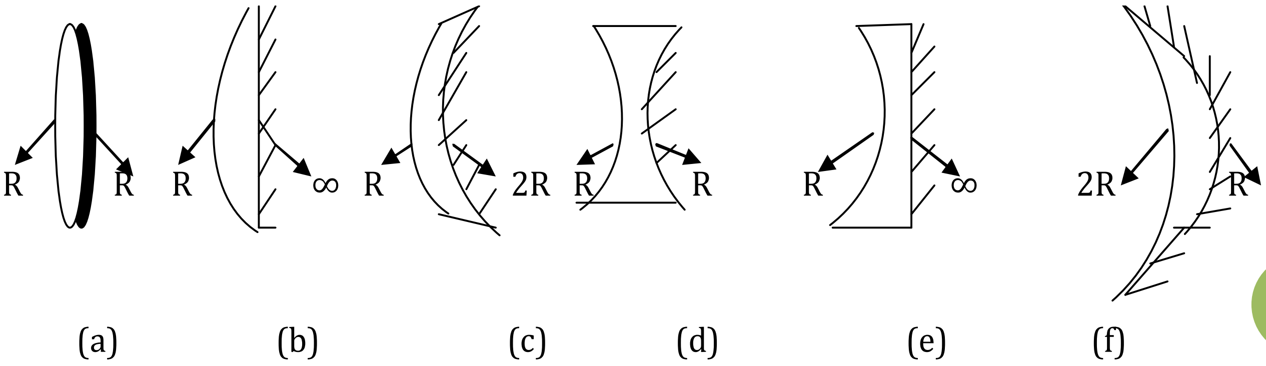
$1/F_{eff} = (1/F_M) - (2/F_L)$

then find $F_{eff} \rightarrow$ this is mirror

if $F_{eff} \rightarrow$ +ive $\rightarrow$ convex mirror diverging

if $F_{eff} \rightarrow$ -ive $\rightarrow$ convex mirror converging

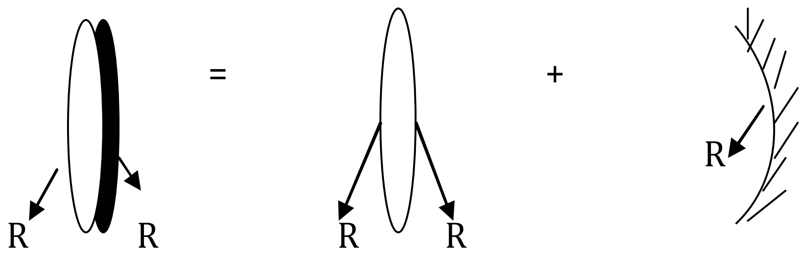
Problem 9: Find focal length & overall nature of the following silvered lenses. In all cases refractive index of material of lens is greater than that of surroundings.



Answer: (a) $f_{eff} = -[R/2(2\mu-1)]$ (b) $f_{eff} = -[R/2(\mu-1)]$ (c) $f_{eff} = [R/(2-\mu)]$ (d) $f_{eff} = [R/2(2\mu-1)]$ (e) $f_{eff} = [R/2(\mu-1)]$ (f) $f_{eff} = -[R/(\mu+1)]$

Solutions:

(a)



$1/F_{eff} = (1/F_M) - (2/F_L)$, $F_M = R/2$, $F_L = (\mu-1)[(1/R_1) - (1/R_2)]$

$F_M = -2/R$

$1/F_L = (\mu-1)[(1/R_1) - (1/R_2)] = (\mu-1)[(1/R) - (1/-R)] = (\mu-1)(2/R)$

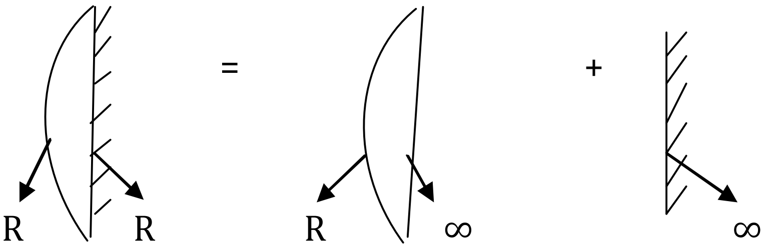
$1/F_{eff} = (1/F_M) - (2/F_L) = (-2/R)-2[2(\mu-1)/R] = -2/R[1+2(\mu-1)]$

$1/F_{eff} = -2/R[2\mu-1]$

$\mathbf{F_{eff} = R/-2[2\mu-1]}$

$F_{eff} \rightarrow$-ive silvered lens behaves like concave mirror & nature is converging.

(b)



$1/F_L = (\mu-1)[(1/R_1) - (1/R_2)] = (\mu-1)[(1/R) - (1/-\infty)] = (\mu-1)/R$

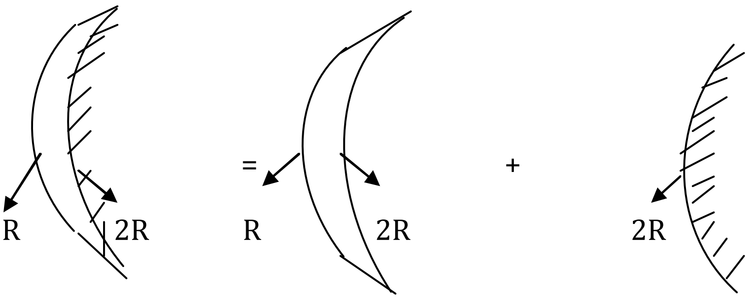
$F_M = R/L = \infty$, $1/F_M = 0$

$1/F_{eff} = (1/F_M) - (2/F_L) = 0 - 2[(\mu-1)/R] = -2(\mu-1)/R$

$\mathbf{F_{eff} = -R/[2\mu-1]}$

Hence silvered lens behaves like concave mirror & nature is diverging

(c)



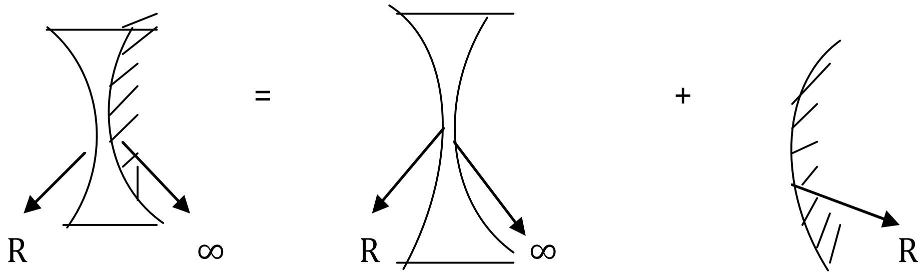
$F_M = R/L = 2R/L \Rightarrow 1/F_M = 1/R$

$1/F_L = (\mu-1)[(1/R_1) - (1/R_2)] = (\mu-1)[(1/+R) - (1/2R)] = (\mu-1)/2R$

$1/F_{eff} = (1/F_M) - (2/F_L) = (1/R) - 2[(\mu-1)/2R] = (1/R)[1-2(\mu-1)/2] = (1/R)[1-\mu+1]$

$= (2-\mu)/R \Rightarrow \mathbf{F_{eff} = [R/(2-\mu)]}$

(d)



$F_M = R/L = +R/L$, $1/F_M = 2/R$

$1/F_L = (\mu-1)[(1/R_1) - (1/R_2)] = (\mu-1)[(1/-R) - (1/R)] = -2(\mu-1)/R$

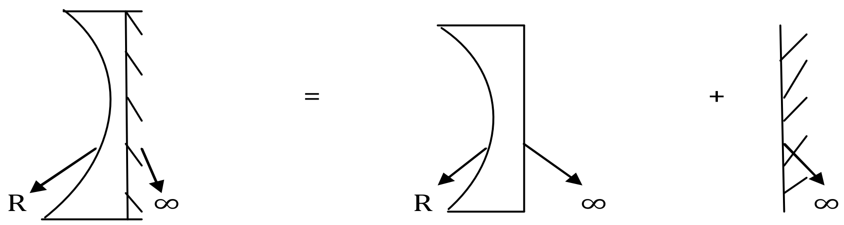
$1/F_{eff} = (1/F_M) - (2/F_L) = 2/R - 2[-2(\mu-1)/R] = 2/R[1+2(\mu-1)] = (2/R)[1+2\mu-2]$

$= (2/R)[2\mu-1]$

F_{eff} = [R/2(2μ-1)]

focal length of silvered lens is +ive hence its behaves like a convex mirror or its nature will be diverging.

(e)



$F_M = R/L = \infty/2 = \infty$, $1/F_M = 0$

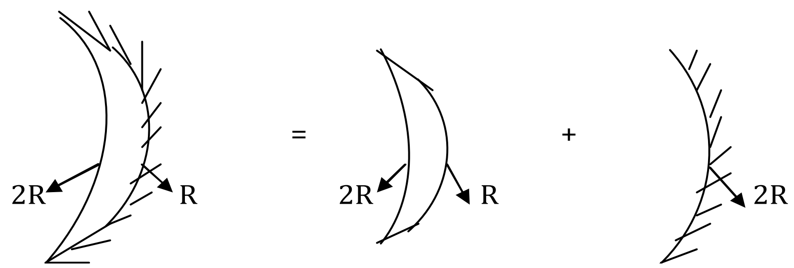
$1/F_L = (\mu-1)[(1/R_1) - (1/R_2)] = (\mu-1)[(1/-R) - (1/\infty)] = -(\mu-1)/R$

$1/F_{eff} = (1/F_M) - (2/F_L) = 0 - 2[-(\mu-1)/R] = 2(\mu-1)/R$

F_{eff} = [R/2(μ-1)]

focal = +ive, hence silvered lens behaves like convex mirror or its nature will be diverging.

(f)



$F_M = R/2 = -R/2 \Rightarrow 1/F_M = -2/R$

$1/F_L = (\mu-1)[(1/R_1) - (1/R_2)] = (\mu-1)[(1/-2R) - (1/-R)] = [(\mu-1)/R][(-1/2)+1]$
$= (\mu-1)/2R$

$1/F_{eff} = (1/F_M) - (2/F_L) = -2/R - 2[(\mu-1)/2R] = -2/R[1+(\mu-1)/2]$

F_{eff} = [-R/(μ-1)]

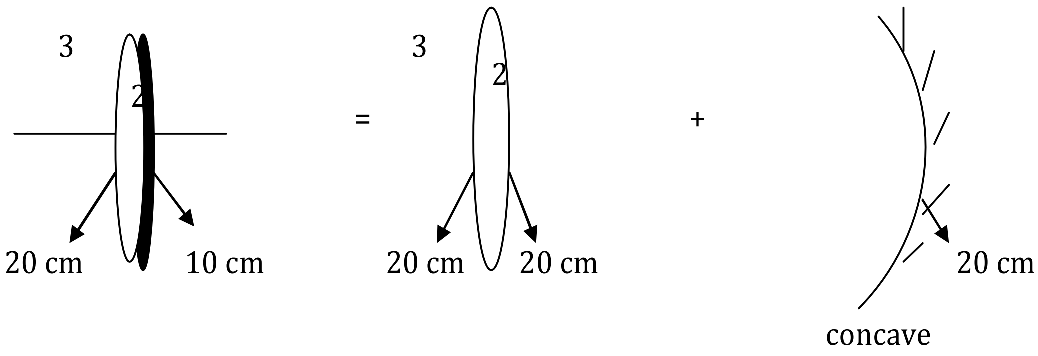
hence the focal length of the silvered lens is –ive it behaves like concave mirror or converging

F_{eq} = -R/(μ+1)

Problem 10: Find focal length and nature of a silvered equiconvex lens of radius of curvature 20 cm & refractive index 2 placed in a medium of refractive index 3.

(a) +30 cm, silvered lens will be like a concave mirror or converging
(b) +30 cm, silvered lens will be like a convex mirror or diverging
(c) +20 cm, silvered lens will be like a concave mirror or converging
(d) +20 cm, silvered lens will be like a convex mirror or diverging

Solution:



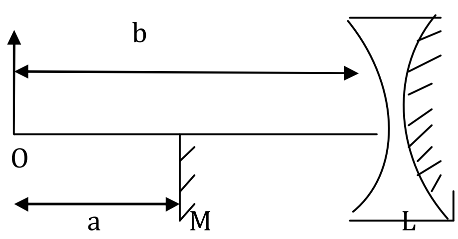
$F_M = R/2 = -20/2 = -10cm$
$1/F_L = (\mu-1)[(1/R_1) - (1/R_2)]$
$1/F_L = [(\mu_L/\mu_M)-1][(1/R_1) - (1/R_2)]$
$1/F_L = [(2/3)-1][(1/20) - (1/-20)]$
$1/F_L = (-1/3)(1/10) = -1/30$

$1/F_{eff} = (1/F_M) - (2/F_L)$
$1/F_{eff} = (-1/10) - 2(-1/30)$
$\mathbf{F_{eff} = -30cm}$

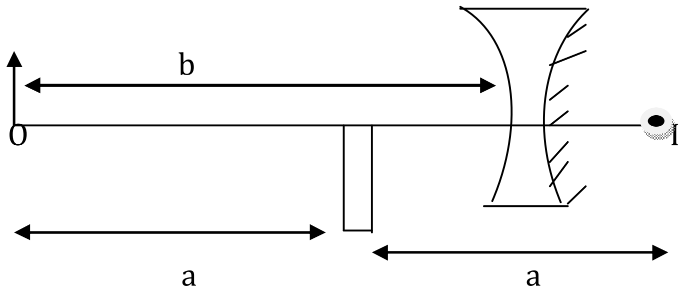
i.e. silvered lens will be like a concave mirror of focal length 30 cm i.e. nature will be converging.

Problem 11: Half part of a convex glass lens L ($\mu = 3/2$) of radius of curvature 40 cm is silvered at right side. A plane mirror M and an object O is placed $\perp$ to the principle axis, as shown at the indicated positions. Image formed by silvered lens and plane mirror has no parallel x but size of image formed by silvered is one third of that formed by plane mirror. Find the distance of plane mirror a & silvered lens b from the object.

(a) a = 40/3 cm & b = 20 cm
(b) a = 20 cm & b = 40/3 cm
(c) a = 20/3 cm & b = 20 cm
(d) a = 40/3 cm & b = 10 cm



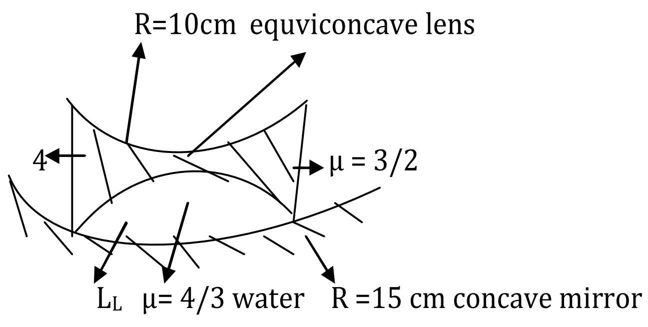
Solution:



From the problem 9(d) $F_L = R/2(2\mu-1) = 40/2[2(3/2)-1] = 10$ cm (+convex)
It is given in problem image formed by silvered lens & plane mirror has no parallel but size of image formed by silvered is (1/3) of that formed by plane mirror. So image formed by both mirrors coincide.
So for convex mirror $u = -b$ & $v = (2a-b)$
$M_S = -(v/u) = -(2a-b)/-b = (2a-b)/b$
so for plane mirror $M_P = 1$
$M_S/M_P = [I_s/O]/[I_p/O] = I_S/I_p = (I_p/3)/I_p = 1/3$
$M_S = (1/3)\ M_P = (1/3)1$
$M_S = (1/3) = (2a-b)/b \Rightarrow (1/3) = (2a-b)/b \Rightarrow b = 6a-3b \Rightarrow 6a = 4b$
$\mathbf{a = (2/3)b}$
$(1/f) = (1/v) + (1/u) = 1/(2a-b) + 1/(-b) = 1/10 \Rightarrow 1/[2(2/3)b-b]-(1/b)] = 1/10$
$\Rightarrow b = 20$ cm, $a = (2/3)b = \mathbf{40/3\ cm}$

Problem 12: A thin equiconvex lens (μ=3/2) of radius of curvature 10 cm is placed in contact with a concave mirror of radius of curvature 15 cm and the space between them is filled with water. Find the focal length and overall combination of nature.

(a)Concave mirror of focal length 22.5 cm
(b) Convex mirror of focal length 22.5 cm
(c) Concave mirror of focal length 18 cm
(d) Concave mirror of focal length 7.5 cm



Solution:

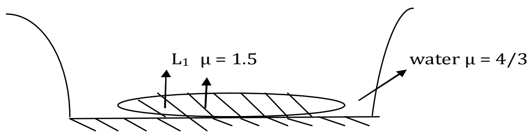
$1/F_L = (\mu-1)[(1/R_1) - (1/R_2)]$
$1/F_L = [(3/2)-1][(1/-10)-(1/10)]$
$1/F_L = (1/2)(-2/10) = -1/10$ cm
$\mathbf{F_L = -10cm}$

$1/F_L = [(4/3)-1][(1/10)-(1/-15)]$
$1/F_L = (1/3)[(3+2)/30] = (1/3)(5/30) = 1/18$
$\mathbf{F_L = 18\ cm}$

$F_M = -R/2$ concave mirror
$F_M = -15/2$
$1/F_{eff} = (1/F_M) - (2/F_{L1}) - (2/F_{L2})$
$1/F_{eff} = -(2/15) - 2(-1/10) - 2(1/18)$
$1/F_{eff} = -2[(1/15)-(1/10)+(1/18)]$
$1/F_{eff} = -2[(6-9+5)/90] = (-2*2)/90 = -4/90$
$F_{eff} = -90/4 = -22.5$ cm
$F_{eff} = -22.5$ cm,
i.e. overall combination behaves as a concave mirror or convergent nature.

Problem 13: A thin equiconvex glass lens (μ = 1.5) is placed on a plane mirror. When the space between lens and mirror filled with water μ = 4/3, then image is found to coincide with object at 15 cm above the lens on its principle axis. When water is replaced by another liquid, It is found that image coincides with object at 25 cm above the lens. Find the refractive index of the other liquid.
(a) μ = 1.6 (b) μ = 1.5 (c) μ = 1.3 (d) μ = 9/8

Solution:



For equiconvex glass lens $(1/f_{L1}) = (\mu-1)[(1/R_1) - (1/R_2)]$
$(1/f_{L1}) = (1.5-1)[(1/R) - (1/-R)] = (1/2)(2/R) = 1/R$
$\mathbf{f_{L1} = R}$
For Plano concave water lens:
$(1/f_{L2}) = (\mu-1)[(1/R_1) - (1/R_2)] = [(4/3)-1][(1/-R) - (1/\infty)] = -1/3R$
$\mathbf{f_{L2} = -3R}$
$F_M = R/2 = \infty$, $\mathbf{F_M = R/2 = \infty}$
$1/F_{eff} = (1/F_M) - (2/F_{L1}) - (2/F_{L2}) = (1/\infty) - (2/R) - (2/-3R) = (2/3R) - (2/R)$
$= (2/R)[(1/3)-1] = -4/3R$
$\mathbf{F_{eff} = -3R/4}$ i.e. a concave mirror
In case of concave mirror, object & its image coincide at the centre of curvature.
-15 cm = radius of curvature
$f_e = +R/2 \Rightarrow$ Radius of curvature = -2 focal

$-15 = -2\, f_{eff} = +2[-3R/a]$

$-15 = -(3/2)R \Rightarrow$ **$R = 10$ cm** for combination

Now for Plano concave liquid lens

$1/F'_{L2} = (\mu-1)[(1/R_1) - (1/R_2)]$

$1/F'_{L2} = (\mu-1)[(-1/R) - (1/\infty)] = -(\mu-1)/R$

Now $1/F'_{eff} = (1/F_M) - (2/F_{L1}) - (2/F'_{L2}) = (1/\infty) - (2/R) - 2[-(\mu-1)/R]$

$= (-2/R) + 2[(\mu-1)/R] = 2/R[-1+\mu-1] = 2(\mu-2)/R = (2\mu-4)/R$

$F'_{eff} = R/(2\mu-4) = -R/(4-2\mu)$

Focal length = Radius of curvature /2

$+R/(2\mu-4) = -25/2$ (now image coincides object at a distance 25 cm)

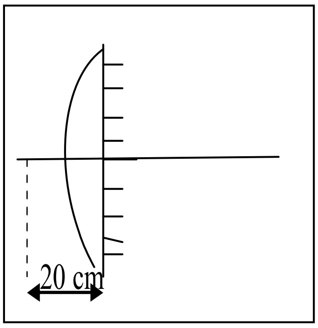
$10/(2\mu-4) = -25/2$

$-20 = 50\mu - 100, -20 + 100 = 50\mu$

$80/50 = \mu$

$\mu = 1.6$

Problem 14 (IIT – JEE 2006):



A point object is placed at distance of 20 cm from a thin Plano convex lens of focal length 15 cm. The plane surface of the lens is now silvered.
The image created by the system is at

(a) 60 cm to the left of the system
(b) 60 cm to the right of the system
(c) 12 cm to the left of the system
(d) 12 cm to the right of the system

Solution:

Refraction from lens :

$(1/v)-(1/u) = 1/f$

$(1/v_1)-(1/-20) = 1/15$

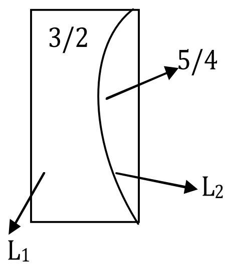
$v_1 = 60$ cm +ive direction

i.e. first image is formed at 60 cm to the right of lens system.

Reflection from mirror:
After reflection from the mirror, the second image will be formed at a distance of v_2 =60 cm to the left of lens system.

Refraction from lens:
(1/v)-(1/u) = 1/f
$(1/v_3) – (1/60) = (1/15)$ +ive direction
$v_3 = 12$ cm
Therefore, the final image is formed at 12 cm to the left of the lens system.

Problem 15 :(Based on silvered lens): A thin Plano convex lens (μ = 5/4) fits exactly into a Plano concave lens μ = 3/2. The radius of curvature of the curved interface is 30 cm.



(1) If plane surface of Plano convex lens is silvered, then the
(a) System behaves like a concave mirror
(b) System behaves like a convex mirror
(c) Focal length of the system is 45 cm
(d) Focal length of the system is 60 cm

(2) An object of height 5 cm is placed at a distance of 15 cm from equivalent mirror of the previous problem. Then transverse magnification produced by the system is.

(a) -4/5 (b) + 4/5 (c) -5/4 (d) +5/4

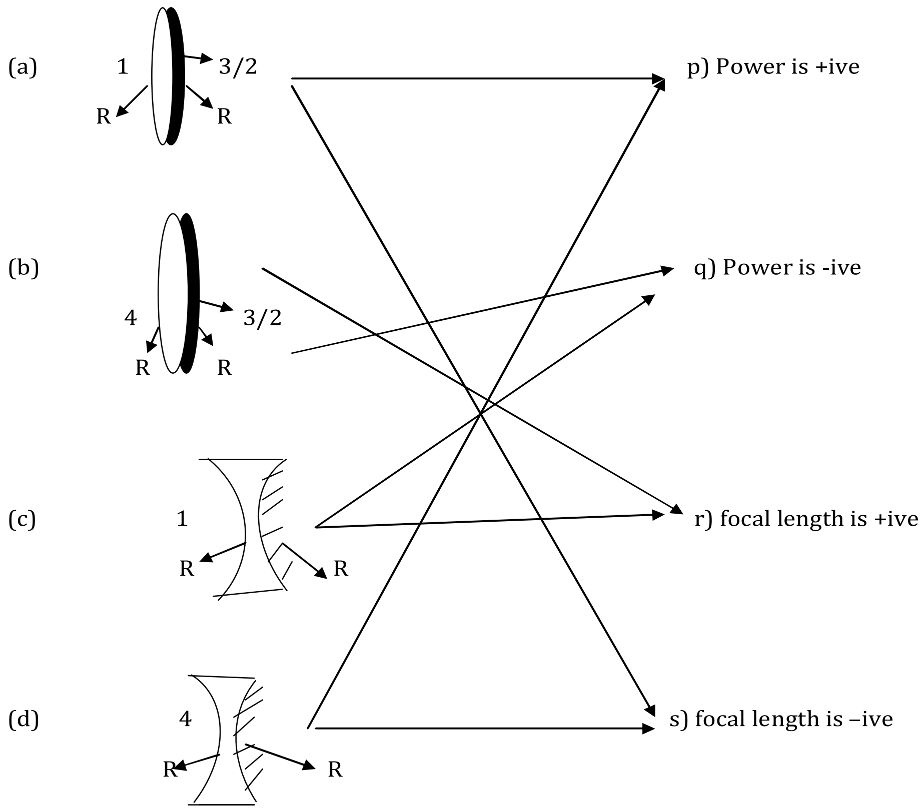
Solution:

$1/f_1 = [(3/2)-1][(1/\infty)- (1/30)] = -1/60$
$1/f_2 = [(5/4)-1][(1/30)- (1/\infty)] = 1/120$
$f_M = \infty$
$1/f_{eq} = (1/F_M) – (2/F_1)-(2/F_2)= (1/\infty) –(2/-60)-(2/120) = (1/30)- (1/60) = 1/60$
f_{eq} = +60cm
So it behaves like a convex mirror

(1/v) +(1/u) = (1/f)
(u/v) +(u/u) = (u/f)
(-1/m) +1 = (u/f)

$(1/m) = 1-(u/f)$
$(1/m) = (f-u)/f$
$m = f/(f-u) = -f/(u-f)$
$m = -(v/u) =-f/(u-f)$
$m =-60/(-15-60) =$
$m =--60/-75 =$ **+4/5**

Problem 16: In following four situations, silvered lenses of refractive index 3/2 are placed in medium of refractive index 1 or 4. Then match the column.



S***olution***:

For (a) $F_{eq} = -R/2(2\mu-1) = -R/[2(2(3/2)/1)-1]= -R/4$
a→(p, s) F →-ive, p → +ive

For (b)) $F_{eq} = -R/2(2\mu-1) = -R/[2(2(3/2)/4)-1]= 2R$
b→(q, r) F →+ive, p → -ive

For (c)) $F_{eq} = R/2(2\mu-1) = R/[2(2(3/2)-1)]= R/4$
c→(q, r) F →+ive, p → -ive

For (d)) $F_{eq} = R/2(2\mu-1) = R/[2(2(3/2)/4)-1]= -2R$
d→(p, s) F →-ive, p → +ive