# Carotid-Kundalini Functions and Chaos


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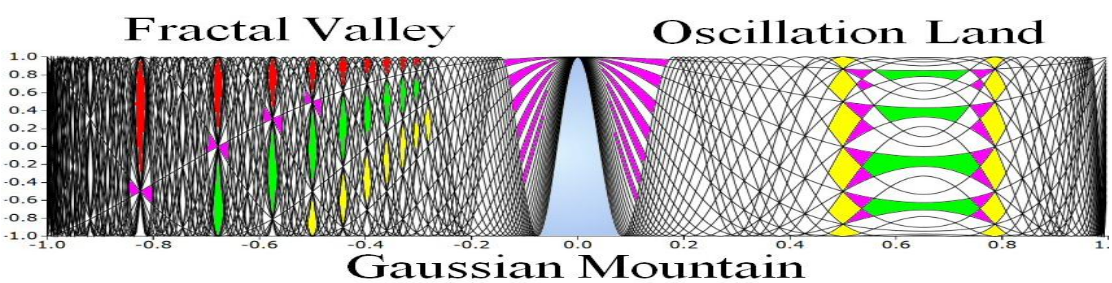
**This article explores a case of signal based chaos generation, using the Carotid-Kundalini function, shown in literature to possess fractal artifacts. Specifically, we set the input to a two tone signal, with the frequency ratio between the sinusoids acting as the control parameter. We explore the iterative map using the time derivatives, and upon plotting the bifurcation plot, observe the chaotic nature of the generated signal. Phase portraits are plotted for different orders, and presence of rich patterns are observed. True to the nonlinear nature, the frequency spectrum shows a horde of new frequency components generated at the output. Lyapunov Exponents also quantitatively confirm the presence of generated chaos in the Carotid-Kundalini signal.**


As the flagship of nonlinear dynamics, Chaos Theory is characterized by systems exhibiting determinism and an extremely sensitive dependence on initial conditions [1]. Research literature exists, illustrating various innovative methods of generating chaos, using signal frequency based control parameters, that determine the transition of the system from order to chaos, and some of these systems involve the use of special mathematical functions such as the Ramanujan Theta function and Bessel functions[2-7].

This article demonstrates how to generate such frequency dependent chaos using the Carotid-Kundalini function [8]. Discovered by Pickover and called so because some of the sinusoid based patterns in the function resemble the flow of the Kundalini energy in Indian Spirituality, chaotic patterns in these functions using the period doubling route have been proposed earlier [8,9]. The function is noteworthy, because, when one superposes plots of the Carotid Kundalini function for different orders, one ends up with a plot displaying three distinct regions: ‘oscillation land’, ‘Gaussian mountain’, and ‘fractal valley’, with the last one seen to possess fractal patterns [10].




In this article, we demonstrate signal based chaos generation using the Carotid Kundalini function. We start with the basic definition of the function, which for an input ‘x’ and order ‘n’ is given by,

$$C_n(x) = \cos(n x \cos^{-1}(x))$$

We set the input ‘x’ to a sum of sinusoids, given by $x = \sin(ft) + \sin(rft)$, and in accordance with the function definition, normalize ‘x’ to restrict its range to [-1,1]. The ratio between the frequencies, ‘r’ serves as the control parameter in our chaos generator.

To examine the chaotic nature, we use the bifurcation plots, obtainable through the iterative map. First, we start by substituting for ‘x’ and taking the time derivative to yield $C_n^{'}(t)$. Next, we replace ‘t’ by discrete samples ‘i’, and note that the time derivative is nothing but the difference between successive samples, i.e. $C_n^{'}(t) = C_n(i+1) - C_n(i)$. Using this, we write the equation for $C_n(i+1)$, which becomes our iterative map.

$$C_n(i+1) = C_n(i) - \left(A(i)B(i) - \frac{nC(i)A(i)}{D(i)}\right)\sin(C(i)B(i))$$

where,

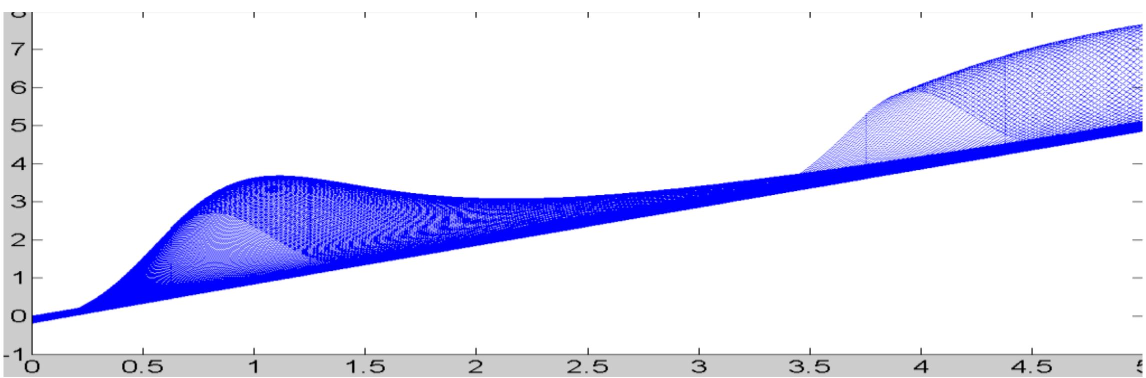
$$A(i) = n(frcos(fri) + fcos(fi))$$

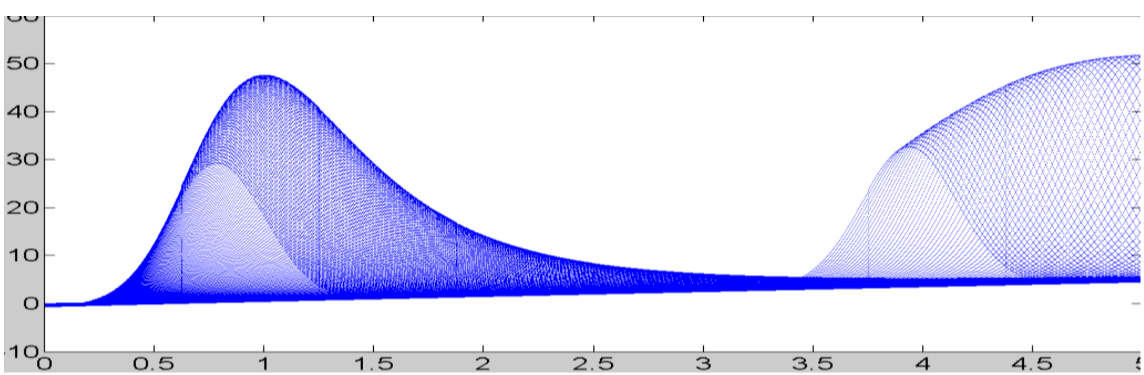
$$B(i) = cos^{-1}(\sin(fri) + \sin(fi))$$

$$C(i) = n(\sin(fri) + \sin(fi))$$

$$D(i) = \sqrt{1 - \sin(fri) + sin^2(fi)}$$

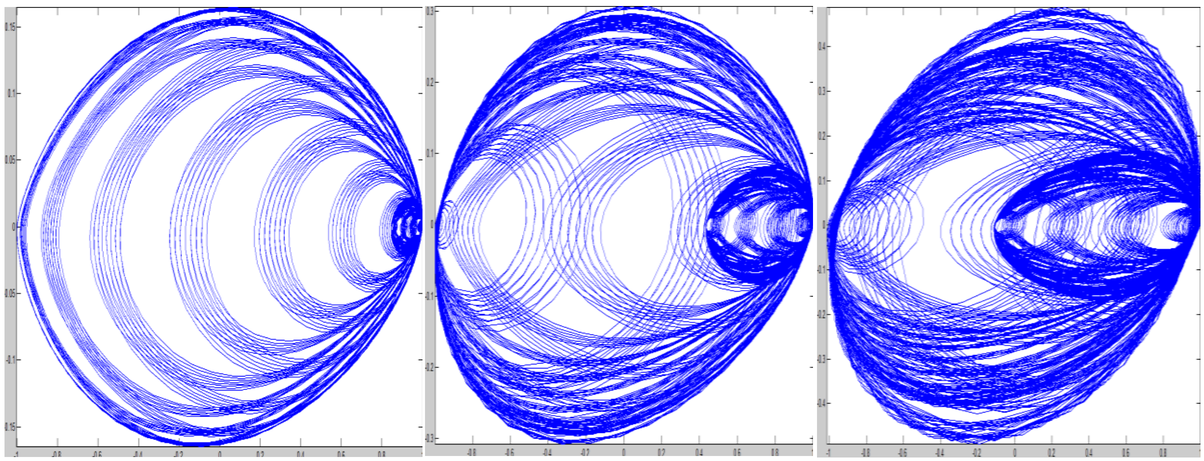
Plotting $C_n(i+1)$ as a function of ‘r’ gives the bifurcation plot as follows, for orders n = 1and 2.



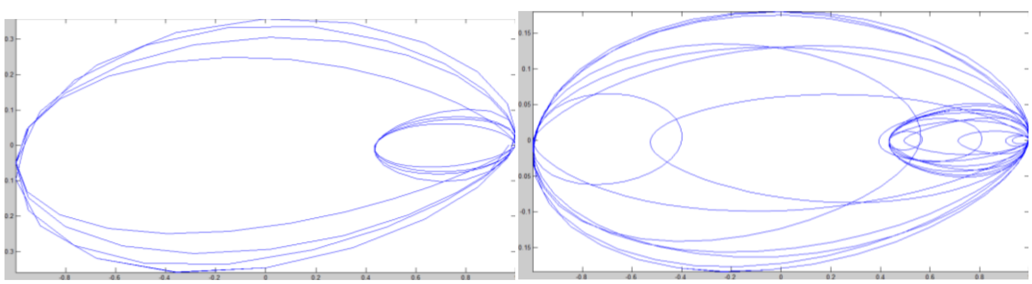

Interesting texture formations are seen in the bifurcation plots, with varying sparse and dense regions.

We select an irrational number, pi, as our value of ‘r’, since this value yields one of the denser regions for both orders. For this value of r, and for the first three orders, n=1,2,3, we plot the time derivative $C_n^{'}(t)$ as a function of $C_n(t)$. This gives us the phase portraits as follows:

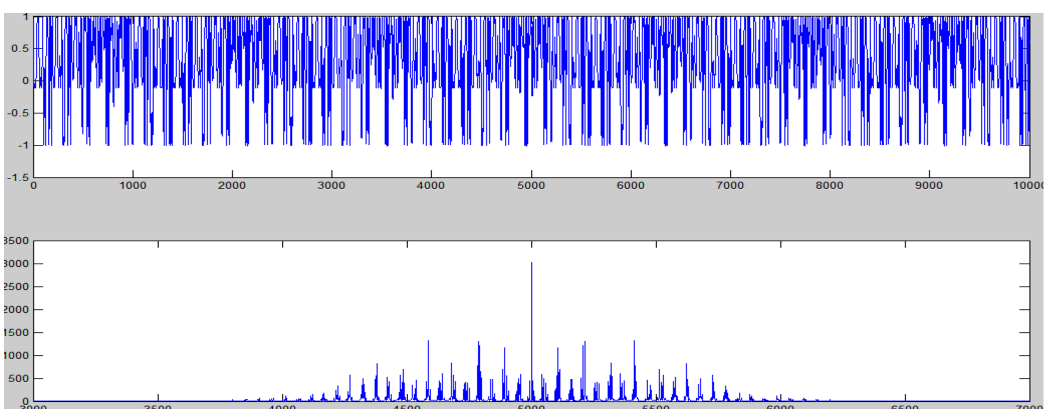


One can see that for the same values of ‘r’ and ‘f’, the phase portraits show increasing degrees of intricate richness, as the order ‘n’ increases. One can also see a particular region of high density in the far right portion of each phase portrait, which on expansion reveals fractal patterns, in accordance with the original discoveries of the Carotid-Kundalini ‘fractal valley’ [8].

It is also seen that for integer values, such as r=3, or rational values, such as r=1.6, the phase portraits, shown below, exhibit considerable lack of richness, and hence are not as chaotic as r=pi.



We now plot the time series and spectrum of the chaotic signal for n=2 and for r=pi.



We note that the waveform shows a considerable lack of periodicity, one of the necessary properties of chaos. Though our starting input was a two-tone signal, with frequencies ‘f’ and ‘rf’, we note from the frequency spectrum that $C_n(t)$ consists of a number of frequency components, harmonics, sidebands and sub-harmonics. This shows that our system for chaos generation is highly nonlinear, introducing several new frequency components into the output.

Quantitatively, we also note that the signal $C_n(t)$ is shown to exhibit a largest Lyapunov Exponent value of 9.37, the positive value of the measure asserting the presence of chaos [11].

Thus, in summary, the article explores a case of signal based chaos generation, using the Carotid-Kundalini function, shown in literature to possess fractal artifacts. Specifically, we set the input to a two tone signal, with the frequency ratio between the sinusoids acting as the control parameter. We explore the iterative map using the time derivatives, and upon plotting the bifurcation plot, observe the chaotic nature of the generated signal. Phase portraits are plotted for different orders, and presence of rich patterns are observed. True to the nonlinear nature, the frequency spectrum shows a horde of new frequency components generated at the output. Lyapunov Exponents also quantitatively confirm the presence of generated chaos in the Carotid-Kundalini signal.

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