Generally covariant Relativistic Quantum Theory: "Renormalization".

Johan Noldus*

September 13, 2016

Abstract

In a previous paper of this author [1] building upon insights reached in [2], we constructed the free theory on a rather general curved spacetime for spin-0, $\frac{1}{2}, 1$ particles and we wrote down the most general interaction vertices for the latter leading to the principle of local gauge invariance. In this paper, we further define the interacting theory and study the behavior of modified particle propagators, leading to a finite theory.

1 Introduction.

The reason for constructing a generally covariant realist quantum theory is multifold; first of all one wishes to get rid of the crucial role played by the observer which results in an operational instead of realistic framework in the old fashioned formulations of Schrodinger and Heisenberg. Second, one wishes quantum theory to speak the same language as does general relativity so that the two approach one and another and consistent ideas about nature on all scales may arise. As argued in a philosophical paper of this author [3], the application of the superposition principle to spacetime will require novel ideas beyond those explained in this series of papers and moreover, it is not clear at all that the superposition principle should be applied to spacetime in the first place. It might just be that the gravitational field is determined by the classical degrees of freedom in the universe and that individual particles will only influence gravitation once they acquire classical properties (during an act of measurement for example). After all, the world is not purely quantum alone and different rules emerge in different regimes. Third, finding a realist theory of processes for the quantum world should clarify the position of the measurement postulate; indeed, in our previous construction, the latter got a rather natural place on pair with the "Schrodinger" equation by means of processes which do not travel into the relativistic past but are possibly superluminal. So, we already gained some novel insights and the intention of this paper is to put what we know already on more solid grounds. That is, we rely upon the construction of the interacting theory in [2] and then move on to the issue of "renormalization" as well as the structure of gravitationally modified two point functions on a Friedmann universe. Hence, we investigate technical issues related to our novel conceptual

*email: johan.noldus@gmail.com, Relativity group, departement of mathematical analysis, University of Gent, Belgium.

approach in order to deepen our understanding of it; a next conceptual step forwards would consist in further understanding the pieceful coexistence between the quantum microworld and classical macroworld. The problem of quantum gravity is just at the end of this road and must be postponed for the future. To give an idea why this is so, quantum theory formulated in the way done in this series of papers relies crucially upon the spacetime metric, therefore one would suspect a quantum theory of the spacetime metric to depend upon a universal metric on the space of metrics. This was the topic of this author's PhD thesis and requires a rather abstract construction of the Gromov type: at this point, it is unclear how these considerations may lead to a well defined generalization of quantum mechanics.

Although the real essential parts of the references [1, 3] are explained in this paper, the reader is advised to read carefully through those references before embarking upon this paper. There are plenty of ideas in those papers which should be absorbed and digested prior to acquire a proper understanding of the construction below. This paper is written with a fairly high mathematical rigour and no sloppy arguments have a place in it unlike what is the case for almost all papers on renormalization. Actually, I really do not treat the issue of renormalization of the coupling constants but rather straightforwardly adress the finiteness of every Feynman diagram and the whole theory in particular.

2 A modified propagator for a free relativistic particle on a general space-time.

As argued in [1], the correct two point function for a spin-0 particle in a general curved background spacetime is given by

$$W(x,y) = \int_{T\mathcal{M}_x} \frac{d^4k}{(2\pi)^3} \delta(k^2 - m^2)\theta(k^0)\phi(x, k^a, y)$$

where

$$\phi(x, k^a, y) = \sum_{w^a \in T\mathcal{M}_x : \exp_x(w) = y} e^{ik_a w^a}$$

where the exponential map is defined as usual. In Minkowski spacetime, this expression is given by

$$W(x,y) = \int_{T\mathcal{M}_x} \frac{d^3k}{2(2\pi)^3\sqrt{\vec{k}^2 + m^2}} e^{ik_a(y^a - x^a)}$$

which may be computed further by making a distinction between the spacelike, null, and timelike case. For spacelike $y^a - x^a$, one may choose the Lorentz frame such that $y^a - x^a = \sqrt{(y-x)^2}e_3$ resulting in

$$W(x,y) = \int_{T\mathcal{M}_x} \frac{d^3k}{2(2\pi)^3\sqrt{\vec{k}^2 + m^2}} e^{ik^3\sqrt{(y-x)^2}} = \frac{1}{8\pi^2}\int_0^{\infty} r\,dr \int_{-\infty}^{+\infty} dk \frac{1}{\sqrt{k^2 + r^2 + m^2}} e^{ik\sqrt{(y-x)^2}}$$

$$= \frac{\infty}{4\pi}\delta(\sqrt{(y-x)^2})$$

where we performed the r integration prior to the k integration which does not only give the wrong answer but also shows that the original "integral" cannot be computed by appealing to Fubini's theorem in this coordinate system and therefore, the Lebesgue integral does not exist. Indeed, *no* momentum integral in standard field theory exists in the sense of Lebesgue as one considers integration of widely fluctuating functions which do not go sufficiently fast to zero at infinity so that the positive and negative, real and imaginary parts of the integrand do not give finite integrals by themselves. In fact, there does not exist a straightforward way how to define this expression. It does exist as a bi-distribution however:

$$W(f,g) = \int_{\mathcal{M}} dx \int_{T\mathcal{M}_x} \frac{d^3k}{2(2\pi)^3\sqrt{\vec{k}^2+m^2}} \int_{\mathcal{M}} dy\, e^{ik_a(y^a - x^a)} f(x)g(y)$$

or

$$W(f,g) = \int_{\mathbb{R}^3} \frac{d^3k}{2(2\pi)^3\sqrt{\vec{k}^2+m^2}} \int_{\mathcal{M}\times\mathcal{M}} dx\, dy\, e^{ik_a(y^a - x^a)} f(x)g(y)$$

since all tangent spaces are isomorphic and both definitions agree for smooth test functions f, g of compact support, where the integrals are taken in the order indicated in the above expressions. In the literature $W(x,y)$ is often presented as a smooth function $\tilde{W}(x,y)$ with a delta distribution on the light-cone; this *representation* however holds only when contractions with Schwartz functions f, g are made, in either

$$W(f,g) = \int_{\mathcal{M}\times\mathcal{M}} f(x)g(y)\tilde{W}(x,y)$$

and the reader may easily find out that $\tilde{W}(x,y)$ is given by special Bessel functions. Indeed, for x, y spacelike, we have that

$$\tilde{W}(x,y) := \frac{m}{\sqrt{(x-y)^2}4\pi^2} \int_0^\infty \frac{dk}{\sqrt{k^2+1}}\, k\, \sin(km\sqrt{(x-y)^2}) e^{-\epsilon k^2} = \frac{m}{\sqrt{(x-y)^2}4\pi^2} K_1(m\sqrt{(x-y)^2})$$

as a formal expression. Indeed, it is fairly easy to check by means of partial integration that $K_1(z)$ satisfies Bessels equation

$$z^2\ddot{K}_1(z) + z\dot{K}_1(z) - (z^2+1)K_1(z) = 0$$

with appropriate boundary conditions. However, $\tilde{W}(x,y)$ is not absolutely integrable given that it does not vanish at infinity (it remains constant on spacelike hyperbolae). Therefore, one cannot extend the definition of $\tilde{W}(x,y)$ from Schwartz functions to smooth L^2 functions of non-compact support as one would expect of realistic wave packages. However, it is worthwhile to mention that $K_1(z)$ diverges as $\frac{1}{z}$ at $z = 0$ and goes to zero as e^{-z} at $z = +\infty$. Indeed, coming back to the formal integral representation of $K_1(z)$ one may consider the effect of smoothening out with a Schwarz function of compact support as cutting off the integral at high momenta so that only the lower momenta count; this cutoff can be computed by means of a square contour in the complex plane which goes form 0 to R to $R + i\frac{\pi}{2}$ to $i\frac{\pi}{2}$ to 0 in the variable α where $k = \sinh(\alpha)$. The large vertical integral oscillates in a bounded way for large R but becomes irrelevant in the limit for R to infinity when smeared out with test functions while the vertical integral from 0 to $\frac{\pi}{2}$ is irrelevant. In this way, it can be shown that the Schwartz kernel $K_1(z)$ corresponds to the integral

$$K_1(z) = \int_0^\infty \cosh(t) e^{-\cosh(t) z}$$

and it is easy to see that this expression diverges as $\frac{1}{z}$ if z approaches zero. Hence, $K_1(z)$ is not uniformly bounded and therefore the best kind of duality one may set up is one of L^1_{loc} which are the absolutely integrable functions of compact support (disjoint from the lightcone). To construct interactions, we need to calculate the Feynman propagator, which is defined as $\Delta_F(x,y) = W(x,y)$ if $y \notin J^-(x)$ and $W(y,x)$ otherwise, and has a formal integral representation as

$$\Delta_F(x,y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik_a(y^a - x^a)}}{k^2 - m^2 + i\epsilon}$$

where ϵ is a positive infinitesimal which may be taken to zero after all computations have been performed. Hence, integrals of the kind

$$\int dx\, dv\, dy\, dz \Delta_F(v,x)\Delta_F(w,x)\Delta_F(y,x)\Delta_F(z,x) f(v,w,y,z)$$

are well defined since all *logical* orders[1] of integration give the same result for entire complex analytic f with exponential falloff on the real section towards infinity. I am not aware if such special functions are really needed to obtain this result but it certainly allows one to appeal to the residue theorem for complex analytic functions in order to compute the result[2]. Even stronger, the above integral exists in a distributional sense for ordinary multidimensional plane waves as the reader may easily compute. Loops, however, are not well digested since one cannot give direct meaning to

$$\int_{\mathcal{M}\times\mathcal{M}} dx\, dy\, \Delta_F(x,y)^2 f(x,y)$$

with $f(x,y)$ an absolutely integrable function, not necessarily of compact support[3]. Alternatively, one might suggest that the correct expression to compute is given by

$$\int_{\mathcal{M}\times\mathcal{M}} dx\, dy\, \tilde{\Delta}_F(x,y)^2 f(x,y)$$

[1] By logical, we mean any order which gives a well defined, finite, result.

[2] It would be interesting to have a result regarding the existence of the above integral if f were merely a Schwartz function.

[3] This follows easily from

$$\int d^4k d^4l \frac{\widehat{f}(-k-l, k+l)}{(k^2 - m^2 + i\epsilon)(l^2 - m^2 + i\epsilon)} = \int d^4r\, d^4k \frac{\widehat{f}(-r,r)}{(k^2 - m^2 + i\epsilon)((r-k)^2 - m^2 + i\epsilon)}$$

and for S-matrix elements $f(x,y) = e^{i(p+q).x} e^{i(r+s).y}$ where p,q are the on-shell incoming momenta and r,s the on-shell outgoing momenta so that

$$\widehat{f}(k,l) = \delta^4(k+p+q)\delta^4(l+r+s)$$

so that $\widehat{f}(-k-l, k+l) = \delta^4(k+l+r+s)\delta^4(p+q+r+s)$. It is easy to see that for generic absolutely integrable and differentiable $\widehat{f}$, the above integral is ill defined as

$$\int \frac{d^4k}{(k^2 - m^2 + i\epsilon)((r-k)^2 - m^2 + i\epsilon)}$$

where $\tilde{\Delta}_F$ is the smooth distribution constructed before. Taking for $f(x,y) = e^{i(p+q)x+i(k+l)y}$, one notices that for $x \sim y$ the integral reduces to

$$\delta^4(k+l+p+q)\int_{(y-x)\,\text{spacelike}} d(y-x)\,\frac{m^2K_1^2(y-x)}{16\pi^4(y-x)^2}e^{i(k+l)(y-x)}$$

and by an appropriate change of variables

$$\begin{aligned} t &= r\sinh\alpha \\ x &= r\cosh\alpha\,\sin\theta\sin\psi \\ y &= r\cosh\alpha\sin\theta\cos\psi \\ z &= r\cosh\alpha\cos\theta \end{aligned}$$

which reduces the metric to

$$ds^2 = -dr^2 + r^2 d\alpha^2 - r^2\cosh^2\alpha d\theta^2 - r^2\cosh^2\alpha\sin^2\theta d\psi^2$$

and the volume form to

$$r^3\cosh^2\alpha\sin\theta d\alpha d\theta d\psi dr$$

one obtains that the latter integral reduces to

$$I(k+l) = \int dr\,d\alpha\,\cosh\alpha\,\frac{m^2K_1^2(r)}{4\pi^3 t_0\sinh\alpha_0}e^{it_0\cosh\alpha_0\,r\sinh\alpha}\sin(t_0\sinh\alpha_0 r\cosh\alpha)$$

where $k+l = t_0(\cosh\alpha_0, 0, 0, \sinh\alpha_0)$. It is clear, again, that this integral does not exist in the Lebesgue sense but one might wish to regard it as a distribution in $k+l$ where k,l are on-shell. As before, we may extract a kernel $\tilde{I}(k+l)$ in the dual sense and equate the integral to that expression. However, in general, one superposes wave packages of such on-shell plane waves which do not have compact support in momentum space and therefore, even this method will fall short in the end although it can be consistenly applied on a much higher level than is usually argued for in standard QFT textbooks. The lightcone will give trouble since there we do have a $\delta((y-x)^2)$ distribution in the formula for $\tilde{\Delta}_F(x,y)$ and the square of that is of course ill defined; one might, however, wish to ignore these contributions and effectively “cut out” the null cone. However, such procedure seems to be hard to motivate from a physical point of view and we will proceed in a way which makes the propagator well defined in the Lebesgue sense so that W and $\tilde{W}$ coincide and are smooth distributions. In a general renormalization procedure, one takes “particular sums” of such nonsensical integrals, performs an associated ad-hoc analytic continuation, and makes the result finite by means of a redefinition of the bare parameters with an infinite amount. This happens, for example, in ϕ^4 field theory regarding corrections to the bare propagator; apart from the fact that this procedure is entirely arbitrary

---

is for generic r. This is most easily seen by application of the residue theorem and noticing that one is left with integrals of the kind

$$\int \frac{d^3k}{|k|^2+m^2}$$

which are linearly divergent.

(but motivated by "physical intuition"), distinct "regularizations" might give different answers and this should not be the case for a physical theory unless there is a very good physical reason to prefer a particular regularization scheme over another. Moreover, this procedure splits theories into two categories: those to which *some* procedure of this kind can be applied, called the renormalizable theories, and those to which it cannot, the nonrenormalizable ones. The shear arbitrareness of the *infinite* renormalization procedure as well as the lack of a deep physical motivation behind it resulted in my thesis that interacting QFT on Minkowski does not exist and that gravitation had to play a fundamental role in making each Feynman diagram finite to the dismay of many field theorists I know of.

The reader notices that we had to twist ourselves into many small corners in order to give meaning to the two point function, the Feynman propagator and *some* "interaction" integrals. The results in the literature regarding renormalizability are alas much weaker than the kind of results we alluded to above; there, it is only shown that S-matrix elements in the distributional basis of plane Fourier waves can be given a distributional, perturbative, meaning due to renormalization. Nothing is said about physical, more general wave packages and not a single non-perturbative result is achieved. By this, I do not want to say that the results of 't Hooft and Veltman in the 1970's are virtually meaningless; they constituted a big step forwards in a time where everybody was concentrated upon Minkowski spacetime and the scattering matrix orginated by Wheeler. From a modern point of view, they do however fall short by many margins and better mathematicians such as Connes and Marcolli have tried to dig deeper in the mathematics behind renormalization. However, they seem to suggest that such a thing would only work for some noncommutative geometry, something I deeply disagree with and, moreover, might be in conflict with nature. We will now argue now that all these "dual" points of view are rather nonsensical from a physical point of view and that the propagator has to exist in a stronger sense than the dual one, that is the usual Lebesgue sense. It is here that gravitation by means of some positive energy condition becomes of primordial importance. Our fundamental formula for the two point function in a general curved spacetime has rather the same shortcomings than the standard Minkowski one; in [1] we therefore suggested to gravitationally deform it so that the resulting integrals become well defined in the standard Lebesgue sense. The particular proposal made in that reference however is not entirely complete and we shall discuss a better one in the remainder of this section. We want to keep the definition of $\phi(x, k^a, y)$ as a sum over geodesic(s) but we will provide every exponential $e^{ik_a w^a}$ with an exponential surpression factor which is *local* at x and y; these factors may be interpreted as a kind of resistance spacetime offers to the sending and receiving of geodesic signals. If w^a is causal, then this surpression factor might be defined by

$$\alpha(x, k^a, w^b) = R_{\alpha\beta}(x) k^\alpha k^\beta + R_{\alpha'\beta'}(y) k^{\alpha'}_{\star\, w^b} k^{\beta'}_{\star\, w^b} + \gamma (k_a w^a)^2$$

where $R_{\alpha\beta}$ is the Ricci tensor, $\star\, w^b : T\mathcal{M}_x \to T\mathcal{M}_y : k^a e^\mu_a(x) \to k^{a'}_{\star w^b} e^{\mu'}_{a'}(y)$ denotes parallel transport along the geodesic defined by $w^b e^\alpha_b(x)$; the latter induces an orthochronous Lorentz transformation and (un)primed indices do refer to y (x). Here, we require the weak energy condition that $R_{\alpha\beta} V^\alpha V^\beta > 0$

for all timelike vectors V^α. This certainly does the job for a timelike w^a, however for a null w^a this formula may be insufficient to get convergence. In case w^b is spacelike, then denote by $R(w^b)^\alpha_\beta$ the reflection around w^b: the latter is an idempotent isometry on the future pointing causal vectors. One could now define

$$\alpha(x, k^a, w^b) = R_{\alpha\beta}(x)k^\alpha k^\beta + R_{\alpha'\beta'}(y)k^{\alpha'}_{\star\, w^b} k^{\beta'}_{\star\, w^b} +$$

$$R_{\alpha\beta}(x)R(w^b)^\alpha_\kappa k^\kappa R(w^b)^\beta_\gamma k^\gamma + R_{\alpha'\beta'}(y)R(w^{b'}_{\star\, w^b})^{\alpha'}_{\kappa'} k^{\kappa'}_{\star\, w^b} R(w^{b'}_{\star\, w^b})^{\beta'}_{\gamma'} k^{\gamma'}_{\star\, w^b} + \gamma(k_a w^a)^2$$

and by using that $R(\lambda w^b)^\alpha_\beta$ is independent of λ for $\lambda \neq 0$ (a reflection is defined by an axis, not an orientation), we have that

$$\alpha(x, k^a, w^b) = \alpha(y, k^{a'}_{\star\, w^b}, -w^{b'}_{\star\, w^b})$$

and

$$\alpha(x, k^a, w^b) = \alpha(x, R(w^b)^a_b k^b, w^c).$$

The distinction between the spacelike and causal case is obvious since null w^a do not canonically define a reflection and the reflection around timelike vectors swaps the future and past lightcones. We define now

$$\phi_\mu(x, k^a, y) = \sum_{w^a \in T\mathcal{M}_x : \exp_x(w) = y} e^{ik_a w^a} e^{-\mu\alpha(x, k^a, w^b)}$$

and as before

$$W_\mu(x, y) = \int_{T\mathcal{M}_x} \frac{d^4k}{(2\pi)^3} \delta(k^2 - m^2)\theta(k^0)\phi_\mu(x, k^a, y).$$

From the above properties and similar reasoning as in [1] we obtain that

$$\overline{W_\mu(x, y)} = W_\mu(y, x)$$

and

$$W_\mu(x, y) = W_\mu(y, x)$$

for $x \sim y$. It is kind of obvious that this propagator on a de-Sitter spacetime is *not* finite for $\mu, \lambda > 0$ given that the Ricci tensor is proportional to the metric and therefore all curvature terms are constant. More precisely, for timelike w^a we do have exponential surpression due to the $(k_a w^a)^2$ term, but the latter does not do a proper job in case w^a is spacelike. Thus, in a maximally symmetric spacetime, where the Riemann tensor is fully equivalent to the metric itself, there is no way to get a theory out satisfying our finiteness criteria unless one simply ignores spatial propagation which would endanger the spin-statistics theorem. One can easily save the day by relying on geometries which do locally define a *preferred* timelike unit vectorfield V^μ; as is well known, such geometries are *generic* and may even be algebraically special; Wylleman has recently given an explicit construction hereof. Hence, one could simply replace the $(k_a w^a)^2$ term by a $(k_\mu V^\mu)^2$ or $(k_\mu V^\mu)^2 + (R(w)^\mu_\nu k^\nu V_\mu)^2$ term, in case w is spacelike, which would provide one with the necessary falloff and symmetry properties independent of w^a. The physical message here is plain and simple, in the non relativistic theory, one had that the two point function is well defined[4] and finite unlike in the Minkowski case; to restore these salient properties, we need

[4]The propagator is certainly well defined when applying a momentum cutoff and sending the cutoff towards infinity; I did not check if it exists in the Lebesgue sense.

a physical arrow of time which is realized by generic matter distributions. All maximally symmetric spacetimes are pathological in the sense that no realistic matter propagates on them; now, people would argue that such timelike vectorfield is not observed in nature as it might suggest a violation of "Lorentz invariance" although everything is formally locally Lorentz covariant. Such attitude is of course rather nonsensical given that we have not specified yet how the two point functions relate to observable quantities and moreover, the surpression terms in the amplitude are *local* and therefore do not influence the propagation part of the definition which resulted in the Fourier basis functions. All our surpression terms do is to incoorporate a kind of resistance of the spacetime fabric to the creation and annihilation of a signal of a particular type just like a liquid offers resistance to the creation and annihilation by pointlike motion of waves but little if nothing to the propagation theirof. The gravitational field is such an eather and Minkowski's idealization is just fictituous; I have no idea wether it is sensible to say that these surpression terms have to be small in some sense as, again, they seem not pertain to the propagation aspect of the signal but just to the creation and annihilation thereof. It is still possible to work in a spatially homogeneous and isotropic cosmology, such as the one given by the usual Friedmann universes: therefore, in a later section, we will compute the quantum theory on such a big bang type of universe. It will turn out however that a little friction on the propagation of the signal is also required in order to tame the divergencies of the lightcone and we shall adress that issue in a while.

3 No cosmological constant problem.

One might at this point reflect if one can still couple geometry semiclasically to our novel definition of a quantum theory and whether it is meaningful to do so. The main point of our discussion so far [1, 2, 3] turned around the two point function: any reference regarding quantum fields has been omitted so, the point of view of an energy momentum tensor is not natural anymore. More in particular, the creation and annihilation processes at events x and y respectively came with a local energy momentum dependent "viscosity" so that the total process is not of Hamiltonian nature anymore and therefore no conserved currents can be constructed, something which is badly needed if one might want to look for a source for gravitation. Also, we did not speak about virtual processes here, all processes in the computation of the relevant amplitudes [3] are real and the limit of instantaneous creation and annihilation is just unphysical; these processes do not happen since we cannot measure them and moreover, they should come with a viscous surpression. This last fact alone implies that our framework does not contain a natural energy momentum tensor anymore. Indeed, the only natural definition in our framework would be the following quadratic expression

$$\langle 0|T_{\mu\nu}(x)|0\rangle = \lim_{y\to x}\left(\partial_\mu\partial_{\nu'}W(x,y) - \frac{1}{2}g_{\mu\nu'}(x,y)\left(g^{\alpha\beta'}(x,y)\partial_\alpha\partial_{\beta'}W(x,y) - m^2W(x,y)\right)\right)$$

which does not exist because the limit differs when y approaches x from the spacelike or timelike side. The fundamental reason herefore is to be found in the "reflection symmetry" in the surpression terms for spacelike geodesics, something which only depends upon an axis and not a magnitude nor a specific orientation. We recall that this symmetry was needed to obtain Bose statistics which crucially determined the definition of the Feynman propagator. Now, it may very well be that Bose statistics is something which does not survive in a curved spacetime, but then the Feynman propagator would depend upon a frame of reference as there is no canonical way to define it. This is an avenue which we shall not take here; the reader, moreover, notices that the limit taken for y in the future lightcone of x gives an expression which is not covariantly conserved at all. This can be easily seen by noticing that for $y \in I^{\pm}(x)$ sufficiently close to x one has that

$$W_{\mu}(x,y) = \int \frac{d^4k}{(2\pi)^3}\delta(k^2-m^2)\theta(k^0)e^{-ik^a\sigma_a(x,y)}e^{-\mu\left(K_{ab}(x)k^ak^b+K_{a'b'}(y)k^{a'}_{\star\,-\sigma^c(x,y)}k^{b'}_{\star\,-\sigma^c(x,y)}\right)}$$

where $\sigma(x,y)$ denotes as usual Synge's world function and the index a refers to the operation $e^{\mu}_{a}(x)\partial_{\mu}$ applied to it. The quadratic form $K_{ab}k^ak^b$ satisfies the property that it blows up quadratically in any Lorentz frame towards infinity if k^0 goes to infinity. In this limit $y \to x$, $W_{\mu}(x,x)$ becomes a smooth function of $K_{ab}(x)$ only since $\sigma_a(x,x) = 0$. The latter, however, does not satisfy a conservation law since generically $K_{ab;\nu}(x) \neq 0$ and the same reasoning applies to the whole energy momentum tensor where second covariant derivatives of $K_{ab}(x)$ come ito play and the expression becomes much more complicated. More abstract and from first principles, there is a priori no good reason why the coincidence limit of derivatives applied to an amplitude for particle propagation should have something to do with a *vacuum* expectation value of some energy momentum tensor. Our above reasoning shows that this is not so and that therefore, no candidate for a conserved vacuum energy momentum tensor exists which makes the cosmological constant problem obsolete. There is another way the quantum influences the geometry than by means of simple propagation; indeed, particles do not propagate in quantum theory and, as we have seen here, wave functions also don't in the naive sense of second order hyperbolic PDE's. Indeed, the way geometry is influenced by particles must be encoded in a new theory which requires a super metric, a universal, and therefore background independent, metric on the space of all Lorentzian geometries (and matter configurations thereupon). This author has written ideas regarding this super-metric up in his Phd thesis [4].

The reader might wonder whether, given the fact that there is a preferred timelike vectorfield, it would not be more convenient to work in an Euclidean theory by means of a Wick rotation. The answer is that it wouldn't do much: all Feynman diagrams give finite results as we will study in rather much detail in a short while. The only nontrivial question concerns the convergence of the total series defining the interacting theory [3], we adress that question in section six.

4 Computations on a Friedmann universe.

Before we proceed, some words of physical significance are in place, in a Schwarzschild and Kerr-Newman rotationally symmetric black hole solution we can speak of a null Killing horizon, which coincides with the union of black hole surfaces defined by Hawking [12], where our preferred timelike vectorfield, or gravitational arrow of time, becomes null and therefore quantum theory becomes ill defined again. It may be clear that generic perturbations in the initial data, even smooth ones of compact support, will destroy the Killing Horizon *and* most likely, also the strongly future asymptotically predictable character of the spacetime. Indeed, to my knowledge, the issue of stability regarding the very definition of an event horizon by means of the past of the boundary of the asymptotic future in some conformal spacetime has not been properly examined. I really do not care much about it, as I have always found this definition rather contrarian and "unphysical" to some extend (given that in quantum gravity the future is not known at all). What our thoughts above reveal is that Kerr-Newman spacetimes also cannot serve as a background for quantum theory as the Lebesgue well definedness of the propagator goes havoc on the horizon and also within. One might again want to resort to weaker, dual, interpretations as before but it could be that the old problems of Minkowski come back in some different jacket. With those words of caution, we now proceed to the definition of the two-point function on the $k = 0$ or spatially flat Friedmann universe in the way envisioned at the end of the previous section. The metric is given by

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$$

and the Einstein equations with cosmological constant $\Lambda' = 3\Lambda$ and homogeneous isotropic fluid reduce to

$$3\frac{\dot{a}^2}{a^2} = 8\pi\rho + 3\Lambda$$

and

$$\frac{3\ddot{a}}{a} = -4\pi(\rho + 3p) + 3\Lambda.$$

The energy momentum conservation law reads

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

while the geodesic equation equals

$$\frac{d^2t}{ds^2} + \dot{a}a\left|\frac{d\vec{x}}{ds}\right|^2 = 0, \;\; \frac{d^2\vec{x}}{ds^2} + 2\frac{\dot{a}}{a}\frac{dt}{ds}\frac{d\vec{x}}{ds} = 0.$$

In this section, we shall be interested in the cosmological vacuum defined by $\rho = p = 0$; in that case the scale factor reads

$$a(t) = \alpha e^{\sqrt{\Lambda}t}$$

with $\alpha > 0$ and the Ricci tensor is given by

$$R_{\alpha\beta} = -3\Lambda g_{\alpha\beta}$$

in other words, our cosmology is an Einstein space. Performing the coordinate transformation $\tilde{t} = \frac{e^{-\sqrt{\Lambda}t}}{\alpha\sqrt{\Lambda}}$ leads to the expression

$$ds^2 = \frac{1}{\tilde{t}^2\Lambda}(d\tilde{t}^2 - dx^2 - dy^2 - dz^2)$$

which shows that our Einstein space is conformally flat. It is also a space of constant negative sectional curvature as the Riemann tensor takes on the form

$$R_{\alpha\beta\mu\nu} = -\Lambda\left(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}\right)$$

a property which will be most convenient later on. It is nevertheless not a maximally symmetric spacetime such as is the case for a de Sitter spacetime. Taking $\tilde{t}$ as a time coordinate suggests a big crunch while the t coordinate hints to an exponentially expanding universe. They both determine the same unit norm timelike vectorfield up to a time orientation, which explains the qualitative difference, and in the sequel, we will keep on working in the t, x, y, z instead of in the $\tilde{t}, x, y, z$ system. Further specialization of the geodesic equation leads to

$$\frac{d^2t}{ds^2} + \sqrt{\Lambda}\alpha^2 e^{2\sqrt{\Lambda}t} \left|\frac{d\vec{x}}{ds}\right|^2 = 0$$

and

$$\frac{d^2\vec{x}}{ds^2} + 2\sqrt{\Lambda}\frac{dt}{ds}\frac{d\vec{x}}{ds} = 0$$

from which it can be deduced that

$$\left|\frac{d\vec{x}}{ds}\right| = \beta e^{-2\sqrt{\Lambda}t}$$

with $\beta \geq 0$. These equations show that the affine time derivative slows down so that one may wonder wether it is possible to get at $t = +\infty$ in the first place. As we will show, this is the case for future oriented timelike geodesics but *not* so for spacelike geodesics for which the $\frac{dt}{ds} > 0$ part of the solution has a finite future t and s extend. One obtains the Newtonian law

$$\frac{d^2t}{ds^2} + \sqrt{\Lambda}(\alpha\beta)^2 e^{-2\sqrt{\Lambda}t} = 0$$

which can be integrated to give

$$\frac{e^{-\sqrt{\Lambda}t}}{\sqrt{\frac{\delta}{\alpha^2\beta^2} + e^{-2\sqrt{\Lambda}t}} + \frac{\sqrt{\delta}}{\alpha\beta}} = e^{-\sqrt{\delta\Lambda}(s+\gamma)}$$

where $\alpha, \beta, \delta \geq 0$ and $\gamma \in \mathbb{R}$. This, again, leads to

$$t(s) = -\frac{1}{\sqrt{\Lambda}} \ln\left(\sqrt{\frac{4\delta}{\alpha^2\beta^2}} \frac{e^{-\sqrt{\delta\Lambda}(s+\gamma)}}{1 - e^{-2\sqrt{\delta\Lambda}(s+\gamma)}}\right)$$

and $\gamma > 0$. It is clear that for $s < -\gamma$ the space is past geodesically incomplete, unless $\gamma = +\infty$ which corresponds to $\vec{v} = 0$, while for s to plus infinity, we obtain again an approximate linear relation between t and s. The geodesic equation for the spatial part then becomes

$$\frac{d^2\vec{x}}{ds^2} + 2\sqrt{\delta\Lambda}\frac{1 + e^{-2\sqrt{\delta\Lambda}(s+\gamma)}}{1 - e^{-2\sqrt{\delta\Lambda}(s+\gamma)}}\frac{d\vec{x}}{ds} = 0$$

which leads to

$$\frac{d\vec{x}}{ds} = \vec{\beta}\frac{4\delta}{\alpha^2\beta^2}\frac{e^{-2\sqrt{\delta\Lambda}(s+\gamma)}}{(1 - e^{-2\sqrt{\delta\Lambda}(s+\gamma)})^2}$$

where $\left|\vec{\beta}\right|^2 = \beta^2$. This last formula may again be integrated to yield

$$\vec{x}(s) = \vec{r}_0 - 2\vec{\beta}\sqrt{\frac{\delta}{\Lambda}}\frac{1}{\alpha^2\beta^2}\frac{1}{1 - e^{-2\sqrt{\delta\Lambda}(s+\gamma)}}$$

where, in the limit for β to 0, $\vec{r}_0$ has to renormalize by an infinite constant too. As it turns out, we have only given a parametrization for future oriented causal geodesics; in terms of the initial values x and $v = (\frac{dx}{ds})_{s=0}$ the original parameters read

$$\begin{aligned}
\vec{\beta} &= \vec{v} e^{2\sqrt{\Lambda} t} \\
e^{-\sqrt{\delta\Lambda}\gamma} &= \frac{1}{\alpha e^{\sqrt{\Lambda} t} |\vec{v}|} \left(v - \sqrt{v^2 - \alpha^2 e^{2\sqrt{\Lambda} t} |\vec{v}|^2} \right) \\
\delta &= v^2 - \alpha^2 e^{2\sqrt{\Lambda} t} |\vec{v}|^2 \\
\vec{r}_0 &= \vec{x} + \frac{\vec{v}}{\sqrt{\Lambda}(v - \sqrt{v^2 - \alpha^2 e^{2\sqrt{\Lambda} t} |\vec{v}|^2})}
\end{aligned}$$

so in the limit of Λ to zero $\vec{r}_0$ renormalizes $\vec{x}_0$ by an infinite amount. One notices that δ has the geometric significance of the length squared of the tangent vector of the geodesic at x which we may put to one since we deal with timelike geodesics. This further simplifies our formulae to

$$\begin{aligned}
e^{-\sqrt{\delta\Lambda}\gamma} &= \sqrt{\frac{v-1}{v+1}} \\
\vec{r}_0 &= \vec{x} + \frac{\vec{v}}{\sqrt{\Lambda}(v-1)}
\end{aligned}$$

and with these reservations, we obtain that

$$\begin{aligned}
t(s) &= -\frac{1}{\sqrt{\Lambda}} \ln \left(\frac{2e^{-\sqrt{\Lambda}(t+s)}}{v + 1 - (v-1)e^{-2\sqrt{\Lambda} s}} \right) \\
\vec{x}(s) &= \vec{x} + \frac{\vec{v}}{\sqrt{\Lambda}(v-1)} - \frac{2\vec{v}}{\sqrt{\Lambda}(v-1)\left(v + 1 - (v-1)e^{-2\sqrt{\Lambda} s}\right)}.
\end{aligned}$$

From the first equation, one can solve v in function of $z = e^{-\sqrt{\Lambda} s}$; the formula is given by

$$v = \frac{2z e^{\sqrt{\Lambda}(t'-t)} - 1 - z^2}{1 - z^2}$$

with $z > e^{-\sqrt{\Lambda}(t'-t)}$. Insertion into the second equation fixes z by the polynomial

$$z^2 + 1 - \left(2\cosh(\sqrt{\Lambda}(t'-t)) - \Lambda \left| \vec{x'} - \vec{x} \right|^2 \alpha^2 e^{\sqrt{\Lambda}(t'+t)} \right) z = 0$$

where the evaluation holds for $(t', \vec{x}')$ future timelike related to $(t, \vec{x})$. Notice that we have an asymptotic region of radius $\frac{1}{\sqrt{\Lambda}\alpha e^{\sqrt{\Lambda} t}}$, so unlike Minkowski spacetime, in our vacuum cosmology, it is impossible for $\vec{x}'$ to become infinite and therefore any observer has a nontrivial horizon. It is easy to solve our equation to

$$s = -\frac{1}{\sqrt{\Lambda}} \ln \left(g(x, x'; \Lambda, \alpha) - \sqrt{g(x, x'; \Lambda, \alpha)^2 - 1} \right)$$

where

$$g(x, x'; \Lambda, \alpha) = \cosh(\sqrt{\Lambda}(t'-t)) - \Lambda \left| \vec{x'} - \vec{x} \right|^2 \frac{\alpha^2 e^{\sqrt{\Lambda}(t'+t)}}{2}.$$

In the limit for $\sqrt{\Lambda}$ to zero, this expression becomes

$$s_0^2 = \lim_{\sqrt{\Lambda}\to 0} \frac{\left((t'-t)\sinh(\sqrt{\Lambda}(t'-t)) - \sqrt{\Lambda}\left|\vec{x'}-\vec{x}\right|^2 \alpha^2 e^{\sqrt{\Lambda}(t'+t)} + O(\lambda)\right)^2}{g(x,x';\Lambda,\alpha)^2 - 1} = (t'-t)^2 - \alpha^2 \left|\vec{x}'-\vec{x}\right|^2$$

as it should be. This formula can be easily analytically continued to the region

$$-1 < g(x,x';\Lambda,\alpha) < 1$$

by

$$is' = -\frac{1}{\sqrt{\Lambda}} \ln\left(g(x,x';\Lambda,\alpha) - i\sqrt{1-g(x,x';\Lambda,\alpha)^2}\right)$$

where we have made the branch cut for the complex square root in the upper half plane at for example $\frac{\pi}{2}$. It is then easily computed that

$$-s'(x,x';\Lambda,\alpha)^2 = -\frac{1}{\Lambda}\left(\arccos\left(g(x,x';\Lambda,\alpha)\right)\right)^2$$

and one can again check that the $\sqrt{\Lambda}$ to zero limit is given by

$$-s_0'(x,x';\alpha)^2 = (t'-t)^2 - \left|\vec{x}'-\vec{x}\right|^2 \alpha^2$$

as is should, so our formula is entirely correct. One can easily see that this result comes by considering the case $\delta < 0$ which corresponds to spacelike geodesics which live a finite amount of time t in the future as well as a finite amount of affine parameter time s in the past and the future. This is again a distinction with Minkowski which is geodesically complete and where spacelike geodesics reach out to infinite values of time in the future. The relevant formulae are deduced by performing the analytic continuation to $\delta < 0$ and putting $\delta = -1$:

$$\begin{aligned} t(s) &= -\frac{1}{\sqrt{\Lambda}} \ln\left(\frac{\sqrt{-\delta}}{\alpha\beta\sin(\sqrt{-\delta\Lambda}(s+\gamma))}\right) \\ \vec{x}(s) &= \vec{r}_0 - \sqrt{\frac{-\delta}{\Lambda}} \frac{\vec{\beta}}{\alpha^2\beta^2\tan(\sqrt{-\delta\Lambda}(s+\gamma))}. \end{aligned}$$

As before

$$\begin{aligned} \vec{\beta} &= \vec{v}e^{2\sqrt{\Lambda}t} \\ e^{i\sqrt{\Lambda}\gamma} &= \frac{v+i}{\sqrt{v^2+1}} \\ \vec{x} &= \vec{r}_0 - \frac{v\vec{v}}{\sqrt{\Lambda}(v^2+1)}. \end{aligned}$$

This reshapes our solutions as

$$\begin{aligned} t(s) &= -\frac{1}{\sqrt{\Lambda}} \ln\left(\frac{e^{-\sqrt{\Lambda}t}}{\sin(\sqrt{\Lambda}s)v + \cos(\sqrt{\Lambda}s)}\right) \\ \vec{x}(s) &= \vec{x} + \frac{v\vec{v}}{\sqrt{\Lambda}(v^2+1)} - \frac{\vec{v}(v-\tan(\sqrt{\Lambda}s))}{\sqrt{\Lambda}(v^2+1)(1+v\tan(\sqrt{\Lambda}s))} \end{aligned}$$

and the reader notices that in the limit $\tan(\sqrt{\Lambda}s) = v$, our assumption $\frac{dt}{ds} \geq 0$ no longer holds. Nevertheless, this solution is past incomplete in the sense that for $s = \frac{1}{\sqrt{\Lambda}} \arctan(-\frac{1}{v})$ it diverges to $t = -\infty$ and $|\vec{x}| \to \infty$. This limit cannot be attained towards the future however and we notice that for $\tan(\sqrt{\Lambda}s) = v$ one has that $\frac{dt}{ds} = 0$ and for later times s, the geodesic evolves again towards lower $t(s)$ values. Our parameter domain reaches only up till $s = \frac{\pi}{2\sqrt{\Lambda}}$ at which point nothing special happens given that the limit of $\vec{x}$ as well as its derivatives are well defined if $\tan(\sqrt{\Lambda}s)$ blows up to infinity. Hence, we need to glue a new solution to the old one which makes the construction of Synge's function for spacelike geodesics rather complicated but we proceed first by determining the world function for the above parametrization. We again obtain the following formulae

$$\begin{aligned} v &= \frac{e^{\sqrt{\Lambda}(t'-t)} - \cos(\sqrt{\Lambda}s)}{\sin(\sqrt{\Lambda}s)} \\ \Lambda\alpha^2 \left|\vec{x}' - \vec{x}\right|^2 e^{2\sqrt{\Lambda}t} &= \frac{(v^2+1)\tan^2(\sqrt{\Lambda}s)}{(1 + v\tan(\sqrt{\Lambda}s))^2} \end{aligned}$$

which leads to

$$s'(x, x'; \Lambda, \alpha) = \frac{1}{\sqrt{\Lambda}} \arccos(g(x, x; \Lambda, \alpha))$$

a result which we obtained previously by means of analytic continuation; this formula covers the full spacelike region as the maximal length of a spacelike geodesic equals $\frac{\pi}{\sqrt{\Lambda}}$ which is precisely the range of that function. It is interesting to study the limit for $v \to +\infty$ of our solution; from any starting point in spacetime one arrives at $t = +\infty$ in a parameter time $s = \frac{\pi}{2\sqrt{\Lambda}}$ at which $\frac{dt}{ds} = 0$ and still the limit of the tangent vectors has unit norm. This means, in particular, that in any direction of space one can trace back these data for smaller t values providing one with a null hypersurface of events in spacetime demarcating, within the region of events which can be connected by means of a spacelike curve to the initial point, those events which can be reached by a spacelike *geodesic* starting at x. In particular, this horizon is given by

$$|\vec{x}' - \vec{x}| = \frac{1}{\alpha\sqrt{\Lambda}e^{\sqrt{\Lambda}t}} + \frac{1}{\alpha\sqrt{\Lambda}e^{\sqrt{\Lambda}t'}}$$

and it obviously lies fully in the region

$$-1 < g(x, x'; \Lambda, \alpha) < 1.$$

This leads us to the following definition: given a spacetime point x, the spacelike geodesic horizon $HS(x)$ is the boundary of the region which can be reached by means of a spacelike geodesic. Likewise, we define the future timelike horizon $HT(x)$ at x as the boundary of the region of spacetime which can be reached by means of timelike geodesics. $HS(x)$ is not necessarily a null hypersurface as it the case for our cosmology and neither does $HT(x)$ need to coincide with the boundary of $J^+(x)$. Note that $HS(x)$ coincides in our case with the boundary of $J^-(I^+(x))$ which is the standard horizon for timelike signals in a general cosmology. Hence, there is a region of spacetime which cannot be reached by any geodesic starting at x; this is a novel feature to be taken into account in the quantum theory which we shall do in the next section. We finish this section by making a comment upon the way the vectorfield e_0 is chosen from local physical considerations. The most obvious criterion is a *quasi*-local one which says that the Riemann curvature squared (or the Ricci curvature squared) of the Riemannian metric on the orthogonal spacelike hypersurface attains an absolute minimum 0. It may be that there exists some *ultra*-local criterium by looking for minima of some function in the spacetime Riemann tensor components evaluated in a tetrad with timelike vector given by ∂_t. The latter characterization would be preferred in my mind but we leave such fine points for the future.

5 The modified propagator on the cosmological vacuum.

Before we come to the calculation of the two point function, we need to calculate the parallel transporter $S^{\alpha'}_{\beta}(x,y)$ between two points; the latter is defined by means of the transport of a vector along the unique geodesic connecting x with y. Before we come to the explicit computations, let us try to guess the structure of the result based upon symmetry considerations. As is well known $-\sigma_\mu(x,y)$ gives the tangent co-vector at x to the geodesic connecting x with y of length equal to the geodesic length; that is

$$g^{\mu\nu}(x)\sigma_\mu(x,y)\sigma_\nu(x,y) = 2\sigma(x,y)$$

where we have surpressed Λ, α in the notation of Synge's function $\sigma(x,y)$. For future convenience, let us denote by $e_0 = \partial_t$, $e_i = \frac{e^{-\sqrt{\Lambda}t}}{\alpha}\partial_i$ the standard tetrad which is constant under parallel transport on timelike geodesics of constant $\vec{x}$. Hence, the transporter expressed with respect to this tetrad $S^{a'}_{b}(x,y)$ is the unit matrix if y has the same space coordinate than x. More in general, one would expect $S^{a'}_{b}(x,y)$ to be a Lorentz boost determined by the $e_0, e_a\sigma^a(x,y)$ plane with a magnitude proportional to $\sqrt{\sum_i \sigma_i(x,y)^2}, \sigma^0(x,y)$ where $\sigma^a(x,y) = e^{a\mu}(x)\sigma_\mu(x,y)$ and it has been understood that the a index has been raised with the flat Minkowski metric η^{ab}. Let us now make the explicit computations; the transport equation is given by

$$\begin{aligned}\frac{d}{ds}Z^0(s) + \alpha^2\sqrt{\Lambda}e^{2\sqrt{\Lambda}t}\vec{v}(s).\vec{Z}(s) &= 0\\ \frac{d}{ds}\vec{Z}(s) + \sqrt{\Lambda}\left(\vec{v}(s)Z^0(s) + \vec{Z}(s)v(s)\right) &= 0\end{aligned}$$

where $v^\alpha(s)$ is the unit tangent to the geodesic in affine parametrization. From our solutions for timelike and spacelike geodesics, it is easy to see that initial vectors Z perpendicular to e_0 and $\vec{v}$ remain so which confirms our claim that unit vectors perpendicular to e_0 and $e_a\sigma^a(x,y)$ are left invariant for as well spacelike as timelike geodesics[5]. Remains to figure out the boost parameter; here we study the transport of $Z = e_0$. The fact that parallel transport preserves the norm allows us to write

$$Z(s) = (\cosh(\gamma(s)), \sinh(\gamma(s))\frac{\vec{v}(s)}{\sqrt{v(s)^2 - 1}})$$

[5]Invariant in the sense that the components only undergo a rescaling as to preserve the local norm.

for timelike geodesics with $\gamma(0) = x$. Hence, we obtain that the first transport equation reduces to

$$\frac{d\gamma(s)}{ds} = -\sqrt{(v(s)^2 - 1)\Lambda}$$

and taking the explicit formula for

$$v(s) = \frac{v + 1 + (v-1)e^{-2\sqrt{\Lambda}s}}{v + 1 - (v-1)e^{-2\sqrt{\Lambda}s}}$$

results in

$$\gamma(s) = \left(\ln \left(\frac{1 + \sqrt{\frac{v-1}{v+1}} e^{-\sqrt{\Lambda}s}}{1 - \sqrt{\frac{v-1}{v+1}} e^{-\sqrt{\Lambda}s}} \right) - \ln \left(\frac{1 + \sqrt{\frac{v-1}{v+1}}}{1 - \sqrt{\frac{v-1}{v+1}}} \right) \right).$$

Upon substitution by the well known formulae for v in function of t, t', s and s in function of $g(x, x'; \Lambda, \alpha)$, we arrive after some algebra at

$$\sqrt{\frac{v-1}{v+1}} = \sqrt{\frac{e^{\sqrt{\Lambda}(t'-t)} - e^{\sqrt{\Lambda}s}}{e^{\sqrt{\Lambda}(t'-t)} - e^{-\sqrt{\Lambda}s}}}$$

and some rather complicated formula

$$\begin{aligned} \gamma(s) &= \ln \left(\frac{1 - z^2}{\left(\sqrt{1 - z e^{-\sqrt{\Lambda}(t'-t)}} - \sqrt{z^2 - z e^{-\sqrt{\Lambda}(t'-t)}} \right)^2} \right) \\ &\quad - \ln \left(\frac{2z e^{\sqrt{\Lambda}(t'-t)} - 1 - z^2}{2z e^{\sqrt{\Lambda}(t'-t)} - 1 - z^2 - 2\sqrt{\left(z^2 (e^{2\sqrt{\Lambda}(t'-t)} + 1) - z^3 e^{\sqrt{\Lambda}(t'-t)} - z e^{\sqrt{\Lambda}(t'-t)} \right)}} \right) \end{aligned}$$

where $z = g(x, x'; \Lambda, \alpha) - \sqrt{g(x, x'; \Lambda, \alpha)^2 - 1}$. A similar result holds for spacelike geodesics and the above calculations show already that exact calculations for the two point function will look rather messy. However, regarding the issue of convergence, we can make useful estimates and it is important to notice that

$$-\ln \left(\frac{1 + \sqrt{\frac{v-1}{v+1}}}{1 - \sqrt{\frac{v-1}{v+1}}} \right) \leq \gamma(s) \leq 0$$

meaning that in the limit for the affine parameter towards future infinity, the boost parameter converges to a finite negative value. Only in the limit for v towards infinity does $\gamma(s)$ converge to infinity too. For spacelike geodesics, one obtains a different qualitative result which is that in the limit for the affine time towards its finite negative and positive values (with a difference of $\frac{\pi}{\sqrt{\Lambda}}$), $\gamma(s)$ blows up towards minus infinity in the limit towards the positive value and to plus infinity in the limit towards the negative value.

We now come to the determination of the two point function and will denote the relevant formula in terms of first derivatives of Synge's function $\sigma_a(x, x'; \Lambda, \alpha)$ and the boost parameter

$$\gamma(x, x'; \Lambda, \alpha).$$

There is no need to use their explicit expressions to arrive at the desired results and if the reader wants to, he or she can manipulate the final expressions by substituting for the above obtained formulae. The two point function we shall study is given by

$$W_\mu(x, x'; \Lambda, \alpha) = \int \frac{d^4k}{(2\pi)^3} \delta(k^2 - m^2)\theta(k^0) e^{-ik^a \sigma_a(x,x';\Lambda,\alpha)} e^{-\mu(k^0)^2 - \mu(S_a^{0'}(x,x';\Lambda,\alpha)k^a)^2}$$

where x' is causally related to x, since otherwise we would have to include reflection symmetric terms, and $S_a^{0'}(x, x'; \Lambda, \alpha)$ is given by

$$S_a^{0'}(x, x'; \Lambda, \alpha)k^a = \cosh(\gamma(x, x'; \Lambda, \alpha))k^0 + \sinh(\gamma(x, x'; \Lambda, \alpha)) \frac{\vec{k}.(\vec{x}' - \vec{x})}{|\vec{x}' - \vec{x}|}$$

and x' is supposed to lie within the total geodesic horizon of x (here the total geodesic horizon is defined as the boundary of the set of events which can be reached from x by means of a geodesic). Prior to making any further computation, let us study this integral in Minkowski spacetime where ∂_t has to be associated to the timelike vectorfield defined by some physical observer making the quantum particle feel an eather due to the him or herself and see if our integral has all desired properties. As is evident from the previous discussion, the only problem with the two point function really resides near the null cone and for this purpose it is sufficient to take the massless limit $m \to 0$. With these reservations

$$W_\mu(x, x') = \frac{1}{2(2\pi)^3} \int \frac{d^3\vec{k}}{\left|\vec{k}\right|} e^{i\left(|\vec{k}|(t'-t) + \vec{k}.(\vec{x}'-\vec{x})\right)} e^{-2\mu\left|\vec{k}\right|^2}.$$

The reader may further calculate this expression to be

$$\begin{aligned} W_\mu(x, x') &= \frac{1}{(2\pi)^3 |\vec{x}' - \vec{x}|} \int_0^\infty dk \sin(k |\vec{x}' - \vec{x}|) e^{ik(t'-t) - 2\mu k^2} \\ &= \frac{1}{2i(2\pi)^3 \sqrt{2\mu} |\vec{x}' - \vec{x}|} e^{-\frac{(t'-t+|\vec{x}'-\vec{x}|)^2}{2\mu}} \int_0^\infty dk e^{-\left(k - i\frac{(t'-t+|\vec{x}'-\vec{x}|)}{\sqrt{2\mu}}\right)^2} - \\ &\quad \frac{1}{2i(2\pi)^3 \sqrt{2\mu} |\vec{x}' - \vec{x}|} e^{-\frac{(t'-t-|\vec{x}'-\vec{x}|)^2}{2\mu}} \int_0^\infty dk e^{-\left(k - i\frac{(t'-t-|\vec{x}'-\vec{x}|)}{\sqrt{2\mu}}\right)^2} \end{aligned}$$

and to study the limit $\mu \to 0$ is a rather subtle issue since, albeit the real part of both integrals equals $\frac{\sqrt{\pi}}{2}$ independent of the arguments $t' - t \pm \left|\vec{x'} - \vec{x}\right|$, the complex part is diverging and cannot be computed exactly. More precisely, we note that both integrals are of the form

$$I(c) = \int_0^\infty dk e^{-(k - ic)^2}$$

and the integrand is complex analytic in k and c. For real c, we may compute the integral by considering the limit of a contour in the complex plane from 0 to R to $R+ic$ to ic and finally back to 0. As usual, the integral over the large vertical part vanishes in the limit for R to infinity while the remainder gives

$$I(c) = \int_0^{\infty} dke^{-k^2} + i\int_0^c dke^{k^2}.$$

This shows that the imaginary part of $W_\mu(x,x')$ equals

$$\frac{\sqrt{\pi}}{4(2\pi)^3\sqrt{2\mu}\,|\vec{x}'-\vec{x}|}\left(e^{-\frac{(t'-t-|\vec{x}'-\vec{x}|)^2}{2\mu}} - e^{-\frac{(t'-t+|\vec{x}'-\vec{x}|)^2}{2\mu}}\right)$$

which converges in the limit for μ to zero to the usual delta functions on the lightcone. The real part however is given by

$$\frac{1}{2(2\pi)^3\sqrt{2\mu}\,|\vec{x}'-\vec{x}|}\left(e^{-\frac{(t'-t+|\vec{x}'-\vec{x}|)^2}{2\mu}}\int_0^{\frac{t'-t+|\vec{x}'-\vec{x}|}{\sqrt{2\mu}}} dke^{k^2} - e^{-\frac{(t'-t-|\vec{x}'-\vec{x}|)^2}{2\mu}}\int_0^{\frac{t'-t-|\vec{x}'-\vec{x}|}{\sqrt{2\mu}}} dke^{k^2}\right)$$

and the task remains to get insight into the large c behavior of

$$\int_0^c dke^{k^2}.$$

A crude estimate

$$\frac{\sqrt{\pi}}{2}e^{\frac{c^2}{2}} \leq \int_0^c e^{k^2}dk \leq \frac{\sqrt{\pi}}{2}e^{c^2}$$

may be shown immediately by means of

$$\left(\int_0^c e^{k^2}dk\right)^2 \leq \frac{\pi}{2}\int_0^{\sqrt{2}c} dr\, re^{r^2} = \frac{\pi}{4}(e^{2c^2}-1)$$

and likewise for the lower bound. However, this is not good enough and for $c>0$ one can, by means of analytic methods, obtain that

$$\int_0^c e^{k^2}dk = \frac{1}{g(c)c}\left(e^{c^2}-1\right)$$

where $1 \leq g(c) \leq 2$ and $g(0)=1$ and $g(+\infty)=2$ which is precisely what we need. Hence, the real part of the two point function behaves as

$$\begin{aligned}\mathcal{R}e\, W_\mu(x,x') &= \frac{1}{2(2\pi)^3\sqrt{2\mu}\,|\vec{x}'-\vec{x}|}\frac{1}{c_+(x,x',\mu)g(c_+(x,x',\mu))}\left(1-e^{-c_+(x,x',\mu)^2}\right)\\ &\quad -\frac{1}{2(2\pi)^3\sqrt{2\mu}\,|\vec{x}'-\vec{x}|}\frac{1}{c_-(x,x',\mu)g(c_-(x,x',\mu))}\left(1-e^{-c_-(x,x',\mu)^2}\right)\end{aligned}$$

and

$$c_\pm(x,x',\mu)) = \frac{t'-t\pm|\vec{x}'-\vec{x}|}{\sqrt{2\mu}}.$$

It is easy to see that for x' in the lightcone of x, one has that the limit of μ to zero of $\mathcal{R}e\, W_\mu(x,x')$ vanishes and the same holds when x' is null related. The convergence of the right hand side towards spacetime infinity for x' causally related to x is only slow since, along a branch of $t' - t - |\vec{x}' - \vec{x}| = c$, it goes proportional to

$$\frac{1}{|\vec{x}' - \vec{x}|}$$

which is not quadratically integrable in $\vec{x}'$. Similar results hold when x' is spacelike related to x albeit the computation is somewhat more difficult there due to the reflection symmetry. It is obviously so that in Minkowski spacetime, it will never be possible to get the integral

$$\int |\Delta_{F,\mu}(x,y)|^2 \, dxdy = \int |W_\mu(x,y)|^2 \, dxdy$$

finite due to the translation symmetry. However, this is not something we should be ambitious of as such integrals have nothing to do with real physics. We shall examine now wether this weak asymptotic behavior is sufficient to get finite loop diagrams by studying some cases which usually give infinite results. Before we proceed, let us notice that, under the agreement that the coincidence limit is defined by the causal presciption, we have

$$W_\mu(x,x) = \frac{1}{4\pi^2} \int_0^\infty dk \, k e^{-2\mu k^2} = \frac{1}{8\pi^2 \mu}$$

which is a finite number usually much larger than one since μ is taken to be small. Therefore, the simpelest one vertex correction to the propagator from x to y reads

$$O_\mu(x,z) = \frac{-i\zeta}{8\pi^2 \mu} \int_{\mathcal{M}} \Delta_{F,\mu}(x,y) \Delta_{F,\mu}(y,z) dy$$

where $\zeta > 0$ is the coupling constant of the theory. We will now isolate a, fairly special, subintegral which diverges to infinity: consider the geometrical situation where z is in the future of x and y in the future of z. These three points determine a plane and consider now the set of spacetime point y' such that

$$c_-(x, y', \mu) = c_-(x, y, \mu)$$

and

$$c_-(z, y', \mu) = c_-(z, y, \mu).$$

The set of y' constitutes a two dimensional manifold as it is the intersection of two three dimensional manifolds and the product $\Delta_{F,\mu}(x, y')\Delta_{F,\mu}(y', z)$ behaves as

$$\sim \frac{1}{8(2\pi)^6 \mu} \frac{1}{|\vec{y}' - \vec{x}| \, |\vec{y}' - \vec{z}|} \left(- \frac{1}{c_-(x,y,\mu) g(c_-(x,y,\mu))} \left(1 - e^{-c_-(x,y,\mu)^2}\right) + i \frac{\sqrt{\pi}}{2} e^{-c_-(x,y,\mu)^2} \right)$$
$$\left(- \frac{1}{c_-(z,y,\mu) g(c_-(z,y,\mu))} \left(1 - e^{-c_-(z,y,\mu)^2}\right) + i \frac{\sqrt{\pi}}{2} e^{-c_-(z,y,\mu)^2} \right)$$

for sufficiently large $|\vec{y}'|$. It is clear that the integration of this expression over the one dimensional manifold defined as the intersection of the previous two dimensional manifold with the plane formed by x, y, z diverges linearly (since we have to take into account a $|\vec{y}'|^2$ coming from the measure). One can “smell” that this pathological behavior of well chosen subintegrals is going to cause general trouble which brings us back to a suggestion made at the end of section two. That is, we have to include friction on the propagation of the signal as well; however, this friction should be momentum independent as is dictated by Lorentz invariance. Obviously, we might just have excluded loop diagrams of this type since no propagation from x to x should ever happen but one might envision problems with other diagrams containing two interaction vertices with one loop and four externel legs. Naively, a logarithmic divergency might occur there.

In this short intermezzo, we find the physical principles, and appropriate formula, behind a friction term imposed upon propagation. This delivers the ultimate death blow to unitarity, something which has for long been envisioned by this author [2]. The reader should understand very well that our formalism, so far, *is* Lorentz covariant in spite of the local momentum dependent supression terms on the amplitudes $\phi_\mu(x, k^a, y)$. The propagation aspect does not depend upon any local frame of reference[6] and this is *all* experiments reveal so far. Therefore, we have to preserve this salient feature and we now look for the maximal extension of the physical law behind the determination of $\phi(x, k^a, w^b)$ obeying this principle. In [1], we came to the prescription for $\phi(x, k^a, w^b)$ by means of

$$\phi(x, k^a, w^b) = \phi(x, k^a, w^b, 1)$$

where

$$\frac{d}{ds}\phi(x, k^a, w^b, s) = iw^\mu(s)k_\mu(s)\phi(x, k^a, w^b, s)$$

with $k^\mu(s)$ parallely transported over the geodesic in affine parametrization s with tangent vector $w^\mu(s)$ fixed by initial conditions at $s = 0$ given by 1, k^a and w^b respectively. The unique solution clearly is given by the exponential function

$$\phi(x, k^a, w^b) = e^{ik^a w_a}.$$

The reader notices that the equation in time s is reparametrization invariant with respect to general, orientation preserving diffeomorphisms of the real line. We now ask ourselves the question what kind of "energy" term could be added which respects Lorentz covariance. As mentioned already, we assume that our geometry provides for a unit timelike vectorfield V^μ causing friction in the creation and annihilation of particles at definite spacetime points: as is well known, a unit timelike vectorfield determines a unique Riemannian metric tensor $h_{\mu\nu}(x)$ as

$$h_{\mu\nu} = 2V_\mu V_\nu - g_{\mu\nu}$$

given our signature convention $(+---)$. The reader should keep in mind that all indices are raised and lowered with the Lorentzian metric and associated vierbein; so $h_{ab} = e^\mu_a e^\nu_b h_{\mu\nu}$ with the standard vielbein e^μ_a. With these lessons in mind, we can now write down another covariant energy term given by

$$\sqrt{h_{ab}(x_{w^c}(s))w^a(s)w^b(s)}$$

where $w^\mu(s) = \frac{dx^\mu(s)}{ds}$. So, our differential equation becomes

$$\frac{d}{ds}\phi_\kappa(x, k^a, w^b, s) = \left(iw^\mu(s)k_\mu(s) - \kappa\sqrt{h_{\mu\nu}(x_{w^b}(s))w^\mu(s)w^\nu(s)}\right)\phi_\kappa(x, k^a, w^b, s)$$

[6] By this, I mean that the momentum only couples to the zero'th and first derivatives of the vielbein.

giving rise to the solution

$$\phi_\kappa(x, k^a, w^b) = e^{ik^a w_a} e^{-\kappa \int_0^1 \sqrt{h_{\mu\nu}(x_{w^b}(s)) w^\mu(s) w^\nu(s)} ds}.$$

In our case of Minkowski spacetime, and some vielbein with $e_0 = \partial_t$, $h_{ab} = \delta_{ab}$ and

$$\phi_\kappa(x, k^a, y) = e^{ik^a(y_a - x_a)} e^{-\kappa|y-x|}.$$

For sake of convergence, it is assumed that the real part of κ is greater than zero. It turns out that this surpression mechanism is sufficient as integrals of the kind

$$\int \Delta_{F,\mu,\kappa}(x,y)\Delta_{F,\mu,\kappa}(y,z)$$

are in the same "function class" as $\Delta_{F,\mu,\kappa}$ meaning they have similar fallof properties towards infinity so that the proof of perturbative renormalizability becomes self evident[7]. Roughly speaking, all cases are covered if integrals of the kind

$$\int dy e^{-\kappa|x-y|-\rho|y-z|}$$

where $\kappa, \rho > 0$ belong to the same function class as $e^{-\zeta|x-z|}$ for some other $\zeta > 0$. From a simple triangle inequality estimate, one obtains that

$$\frac{1}{2}|x-z| + \left| y - \frac{x+z}{2} \right| \leq |x-y| + |z-y|$$

for $\left|y - \frac{x+z}{2}\right| \geq |x-z|$. This splits the integral into two parts as follows

$$e^{-\frac{1}{2}\min\{\kappa,\rho\}|x-z|} \int_{\left|y-\frac{x+z}{2}\right| \geq |x-z|} e^{-\min\{\kappa,\rho\}\left|y-\frac{x+z}{2}\right|} dy + e^{-\min\{\kappa,\rho\}|x-z|} \int_{\left|y-\frac{x+z}{2}\right| \leq |x-z|} dy$$

and this may further be bounded by

$$2\pi^2 \left(\frac{6}{(\min\{\kappa,\rho\})^4} e^{-\frac{1}{2}\min\{\kappa,\rho\}|x-z|} + \frac{1}{4} e^{-\min\{\kappa,\rho\}|x-z|} |x-z|^4 \right)$$

where $2\pi^2$ equals the volume of the three dimensional unit sphere with radius one. These functions obiously belong to the same class as $x^n e^{-\kappa x} \leq a e^{-\zeta x}$ for some $a > 0$ and $0 < \zeta < \kappa$ for all n. The same technique can be applied to an arbitrary number of points $x, z, \ldots$ in the integral as the reader may easily verify for himself. The bound above is slightly inconvenient because of the division of $\min\{\kappa, \rho\}$ by a factor of two in the exponential; this can however be repaired by noticing that

$$|x-z| + \left| y - \frac{x+z}{2} \right| \leq |x-y| + |z-y|$$

for $\left|y - \frac{x+z}{2}\right| \geq \frac{3}{2}|x-z|$ which would only change the coefficients in the polynomial. Hence, we are completely set for perturbative finiteness and hopefully also for nonperturbative finiteness.

We proceed now with the computation of the two point function $W_{\mu,\kappa}(x, x'; \Lambda, \alpha)$

[7] Although the proof of convergence of the series is more involved as we will see later on.

in the cosmological vacuum for x' in the causal future of x where we have included both momentum dependent friction on the creation and annihilation of particles as well as momentum independent friction on the propagation of information. Denoting by $\vec{\sigma}(x, x'; \Lambda, \alpha) = (\sigma_i(x, x'; \Lambda, \alpha))$, where the i index refers to the spatial part of the vierbein and not to the space components of σ_μ, and correspondingly

$$|\vec{\sigma}(x, x'; \Lambda, \alpha)| = \sqrt{\sum_i \sigma_i(x, x'; \Lambda, \alpha)^2}$$

we arrive, after some algebra, to

$$\begin{aligned} W_\mu(x, x'; \Lambda, \alpha) &= \frac{1}{8\pi^2} \int_0^\infty dk \frac{k}{\sqrt{k^2 + m^2}} \int_{-k}^{k} dz e^{-i\sqrt{k^2+m^2}\sigma_0 - \mu(1+\cosh^2(\gamma))(k^2+m^2)} \\ & e^{-\mu \sinh^2(\gamma)\left(z + \left(\frac{\cosh(\gamma)}{\sinh(\gamma)}\sqrt{k^2+m^2} + i\frac{|\vec{\sigma}|}{2\mu \sinh^2(\gamma)}\right)\right)^2} e^{\mu \sinh^2(\gamma)\left(\frac{\cosh(\gamma)}{\sinh(\gamma)}\sqrt{k^2+m^2} + i\frac{|\vec{\sigma}|}{2\mu \sinh^2(\gamma)}\right)^2} \end{aligned}$$

where we have surpressed all dependencies upon x, x', Λ, α in the right hand side and, as mentioned previously, this expression only holds for x' in the causal future of x. At this point, it is instructive to give some comment about the general structure of the integral. The μ surpression terms we included are sufficient for our purposes just as it is the case for Minkowki. This property is rather independent of the behavior of γ which we have shown to converge to an asymptotic, finite negative value in the limit of the parameter time towards plus infinity for future timelike related events. It may be better to replace the $(V_a k^a)^2$ surpression term by a $h_{ab}k^a k^b$ surpression where h_{ab} is, as before, the Riemannian metric determined by the timelike vectorfield. The relevant estimates for W_μ will be thighter for this choice and therefore a proof of convergence for our choice for cosmological spacetimes should automatically result in a general theorem about spacetimes with a canonical Wick transform defined on it. It is immediately seen that the absolute value of $W_\mu(x, x'; \Lambda, \alpha)$ is bounded by a universal constant proportional to $\frac{1}{\mu}$, which is actually sufficient for our proof of finiteness since we have to take into account the Riemannian surpression term due to κ. However, we are interested in more detailed properties of this function and carry on.

Coming back to the calculation of $W_\mu(x, x'; \Lambda, \alpha)$, the integral over z is a Gaussian one which cannot be exactly done, but to which we can find a useful upper bound. In particular, we estimate integrals of the type

$$F(k, c) = \int_{a(k)}^{b(k)} dz e^{-a(z+ic)^2}$$

for $c \geq 0$. Taking the differential of $F(k, c)$ with regards to c results in

$$\frac{d}{dc}F(k, c) = i \int_{a(k)}^{b(k)} \frac{d}{dz} e^{-a(z+ic)^2} = i\left(e^{-a(b(k)+ic)^2} - e^{-a(a(k)+ic)^2}\right).$$

Therefore we obtain that

$$|F(k, c)| \leq \int_0^c dz e^{az^2} \left(e^{-a\, b(k)^2} + e^{-a\, a(k)^2}\right) + \frac{\sqrt{\pi}}{\sqrt{a}}$$

and upon using our previous results, the latter expression reduces to

$$|F(k,c)| \leq \frac{1}{acg(\sqrt{a}c)} \left(e^{ac^2} - 1\right)\left(e^{-a\,b(k)^2} + e^{-a\,a(k)^2}\right) + \frac{\sqrt{\pi}}{\sqrt{a}}.$$

For the purpose of asymptotic analysis, we may clearly ignore the constant on the right hand side, since the resulting expressions converge exponentially fast in the limit for $|\vec{\sigma}|$ towards infinity, and we obtain that

$$\begin{aligned}|W_\mu(x,x';\Lambda,\alpha)| \quad &\sim \quad \frac{1}{4\pi^2\,|\vec{\sigma}|}\int_0^\infty dk \frac{k}{\sqrt{k^2+m^2}} \\ &e^{-\mu(k^2+m^2)}\left(e^{-\mu\sinh^2(\gamma)\left(k+\frac{\cosh(\gamma)}{\sinh(\gamma)}\sqrt{k^2+m^2}\right)^2} + e^{-\mu\sinh^2(\gamma)\left(k-\frac{\cosh(\gamma)}{\sinh(\gamma)}\sqrt{k^2+m^2}\right)^2}\right).\end{aligned}$$

which shows that $W_\mu(x,x';\Lambda,\alpha)$ converges to zero in the limit for $|\vec{\sigma}|$ to infinity for x' future causally related to x. It is much harder to obtain an estimate in case $|\vec{\sigma}|$ remains finite but σ_0 blows up to plus infinity. The only result I am able to obtain is that of convergence in σ_0 along $|\vec{x}' - \vec{x}| = 0 = \vec{\sigma}$ and $\gamma = 0$ as $\frac{1}{\sigma_0}$.

We now turn our head towards the study of the impact of κ on $W_{\mu,\kappa}(x,x';\Lambda,\alpha)$. Denote by

$$E(x,x';\Lambda,\alpha,\kappa) = e^{-\kappa\int_0^{\tilde{s}}\sqrt{h_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}}}$$

the exponentiated energy along the timelike geodesic connecting x with x', then

$$W_{\mu,\kappa}(x,x';\Lambda,\alpha) = E(x,x';\Lambda,\alpha,\kappa)W_\mu(x,x';\Lambda,\alpha)$$

and, in case $|\vec{x}' - \vec{x}| = 0$, then one has

$$E(x,x';\Lambda,\alpha,\kappa) = e^{-\kappa|t'-t|}.$$

In order for every subintegral of

$$\alpha^3 \int dx' e^{3\sqrt{\Lambda}t'}\,|\Delta_{F,\mu,\kappa}(x,x';\lambda,\alpha)|^n$$

to be finite, it is therefore necessary that $\kappa > 3\sqrt{\Lambda}$, a condition which did not appear in Minkowski spacetime. Regarding the proof of perturbative finiteness, we will require some other bound to which we will come back to in a short while. Actually, without any further computation, the reader should realize that our cosmology behaves very different from ordinary Minkowski; on one side, one has the existence of all horizons and on the other, one notices that Minkowski can be conformally compactified while the Friedmann cosmology can't. The latter feature causes scattering processes in the future to occur with a higher amplitude which might ultimately not be surpressed anymore by our geodesic energy terms $E(x,x';\Lambda,\alpha,\kappa)$. To jump a bit ahead in our terminology, this would forbid Type III quantum theories but not Type II or Type I; in Minkowski spacetime, there is no such distinction between the past and the future and therefore, such behavior is not to be expected. Coming back to our computation, one immediately sees that

$$\int ds\sqrt{h_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}} = \int_0^{\sqrt{2\sigma(x,y)}} ds\sqrt{2\left(\frac{dt}{ds}\right)^2 - 1}$$

where

$$\frac{dt}{ds} = \frac{v+1+(v-1)e^{-2\sqrt{\Lambda}s}}{v+1-(v-1)e^{-2\sqrt{\Lambda}s}}$$

an expression wich decreases from v to 1 at $s=\infty$. In Minkowski $\Lambda = 0, \alpha = 1$ and this expression equals $\sqrt{2(\sigma^0(x,y))^2 - 2\sigma(x,y)} = |x-y|$; for a cosmological spacetime this is very different. In general, we have that,

$$\int_0^{\sqrt{2\sigma}} ds \sqrt{h_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}} \geq \sqrt{2\sigma} \sim |t'-t|$$

for $|t'-t|$ large and $|\vec{x}' - \vec{x}| < \frac{e^{-\sqrt{\Lambda}t}}{\alpha\sqrt{\Lambda}}$ fixed. Moreover, the inequalities and similarities become equalities in the limit for σ to infinity. Note that σ is infinite within the lightcone and zero on the lightcone in the limit for t' towards ∞, but the pathology on the lightcone needs to be studied further. Actually, one obtains that the energy increases from the symmetrical point $|\vec{x}' - \vec{x}| = 0$ towards the boundary of the lightcone along the "hyperbola" of constant σ which is contained within a domain of compact $\vec{x}'$. We need a finer estimate in order to obtain conclusive results on convergence; some algebra shows that

$$\begin{aligned}\int_0^{\sqrt{2\sigma}} ds \sqrt{h_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}} \quad &\geq \quad \frac{1}{\sqrt{\Lambda}}\sqrt{\frac{v+1}{v-1}} \\ &\left(\ln\left(\frac{1+\sqrt{\frac{v-1}{v+1}}}{1+\sqrt{\frac{v-1}{v+1}}e^{-2\sqrt{2\Lambda\sigma}}}\right) + \ln\left(\frac{1-\sqrt{\frac{v-1}{v+1}}e^{-2\sqrt{2\Lambda\sigma}}}{1-\sqrt{\frac{v-1}{v+1}}}\right)\right)\end{aligned}$$

upon substitution of v by

$$v = \frac{2e^{-\sqrt{\Lambda}(\sqrt{2\sigma}-(t'-t))} - 1 - e^{-2\sqrt{2\Lambda\sigma}}}{1-e^{-2\sqrt{2\Lambda\sigma}}}.$$

In order to study the σ to zero limit, we only need to take into account the second term; this one reduces in leading order to

$$\frac{1}{\sqrt{\Lambda}}\ln\left(\frac{3 + \frac{1}{1-e^{-\sqrt{\Lambda}(t'-t)}} + \frac{1}{e^{\sqrt{\Lambda}(t'-t)}-1}}{\frac{1}{1-e^{-\sqrt{\Lambda}(t'-t)}} + \frac{1}{e^{\sqrt{\Lambda}(t'-t)}-1} - 1}\right)$$

meaning that for large $|t'-t|$ this expression behaves approximately as $|t'-t| + \frac{\ln(4)}{\sqrt{\Lambda}}$ which is all we need. Actually, due to the nature of the Riemannian metric, we immediately have a lower bound of $|t'-t|$ on the (Lorentzian) energy and an upper bound on the *Riemannian* distance of $|t'-t| + \frac{1}{\sqrt{\Lambda}}$; the constant of $\frac{\ln(4)}{\sqrt{\Lambda}}$ is the only nontrivial thing in the above formula and the reader can easily see that this estimate is very accurate. This means that in the limit for σ equal to zero and $|t'-t|$ towards infinity, the exponentiated energy goes as

$$E(x,x';\Lambda,\alpha,\kappa) = \frac{1}{(\sigma^0)^{\frac{\kappa}{\sqrt{\Lambda}}}}$$

something which falls quicker off than $\frac{1}{(\sigma^0)^3}$ given our previous bound on κ.

Towards the past, we have that the local energy is an increasing quantity and

$$\infty > \sqrt{h_{\alpha\beta}\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds}} \geq \sqrt{2v^2 - 1}$$

which means that the energy is larger than

$$\sqrt{2(\sigma^0)^2 - 2\sigma}.$$

Akin to the future timelike case, this lower bound is actually insufficient as in the limit for $t'(s)$ to minus infinity, one obtains that

$$\sigma^0 = \sqrt{2\sigma}\frac{1 + e^{2\sqrt{2\Lambda\sigma}}}{e^{2\sqrt{2\Lambda\sigma}} - 1}$$

which converges to $\frac{1}{\sqrt{\Lambda}}$ in the limit for σ to zero. Just like in the previous case, one could perform the full integration,

$$\int_{-\sqrt{2\sigma}}^{0} ds\sqrt{h_{\alpha\beta}\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds}} \geq \frac{1}{\sqrt{\Lambda}}\sqrt{\frac{v+1}{v-1}} \left(\ln\left(\frac{1 + \sqrt{\frac{v-1}{v+1}}e^{2\sqrt{2\Lambda\sigma}}}{1 + \sqrt{\frac{v-1}{v+1}}}\right) + \ln\left(\frac{1 - \sqrt{\frac{v-1}{v+1}}}{1 - \sqrt{\frac{v-1}{v+1}}e^{2\sqrt{2\Lambda\sigma}}}\right)\right)$$

where

$$v = \frac{1 + e^{2\sqrt{2\Lambda\sigma}} - 2e^{\sqrt{\Lambda}(t'-t+\sqrt{2\sigma})}}{e^{2\sqrt{2\Lambda\sigma}} - 1}$$

or simply remark that the energy is always greater or equal to $|t' - t|$, which is all we actually need.

Similar convergence properties apply for spacelike geodesics, as the reader may want to verify for himself which finishes the discussion of this section. The only important conclusion is that the energy is always larger that the t' distance travelled which is sufficient to obtain convergent integrals. There remains something to be said about the Riemannian metric $h_{\alpha\beta}$ associated to our cosmological spacetime: it is a metric of constant negative sectional curvature $-\Lambda$ and therefore, balls in this metric have a volume which blows up at most exponentially fast in the radius due to a well known theorem in Riemannian geometry. Our Riemannian space has constant sectional curvature but is again not maximally symmetric; this behavior of balls in the Riemannian metric poses however no problem for our Type II quantum theory as the volume of the past lightcone blows up linearly in $-t'$ for t' towards minus infinity in opposition to the volume of the future lightcone which blows up exponentially in $\left|\tilde{t} - t\right|$ and the $t < t' < \tilde{t}$ slice of the lightcone contains the intersection of the future lightcone with the $\left|\tilde{t} - t\right|$ ball which reaches above the $\tilde{t} - t - \frac{1}{\sqrt{\Lambda}}$ slice and therefore has a volume scaling as $e^{3\sqrt{\Lambda}(t'-t)}$ which shows indeed exponential scaling of the balls for late times t'.

6 Finiteness of Feynman diagrams and the interaction series.

Prior to embarking upon the easy proof that Feynman diagrams are finite and the somewhat more involved argument that the series in terms of the coupling constant is analytic, let us repeat some of the crucial definitions for a spin zero bosonic theory made in [3]. There, we defined quantities similar to the r-point functions in quantum field theory as follows. Consider n points x_i in spacetime which are spacelike separated to one and another[8] and likewise m points y_j which are not to the relativistic past of x_i. The x_i form an IN state $|\mathrm{IN}\, x_i, i = 1 \ldots n\rangle$ while the y_j constitute an OUT state $|\mathrm{OUT}\, y_j, j = 1 \ldots m\rangle$ and the only quantity we are interested in computing is the potentiality $\langle \mathrm{OUT}\, y_j, j = 1 \ldots m | \mathrm{IN}\, x_i, i = 1 \ldots n\rangle$ for the IN state to evolve into the OUT state. In [3], we came up with rougly speaking three definitions which we labelled as Type I, II and III. The distinction is a deep philosophical one and stems from the fact that, in a general evolving cosmology, it is impossible to define an instantaneous vacuum state as has been explained in [2]. In our formalism, we do not speak about states, operators and so on so we look at things from a different vantage point of view, which opens new possibilities and lines of thought. Let me explain these differences again, the proofs in this section we will provide for do not depend upon them and are universal amongst all types. Apart from a local physical arrow of time, every quantum theory needs a notion of instantaneous existence, by which I mean the following: I exist now, what kind of other stuff in the universe exists at this point? From the point of relativity, this question seems to be abundant since everything is deterministic and the future exists in a sense we exist right now. In quantum theory, this is not the case, however, given that the future of spacetime and matter is not fixed but uncertain and therefore, it is impossible to speak about it in a sense which would imply that it exists right now. Hence, we have to complement our situation exlained so far with an initial S_I and final S_F spatial hypersurface associated to the IN state and OUT state, meaning that they contain x_i and y_j respectively and are disjoint. Associated to two hypersurfaces, one can define the sandwished region $R(S_I, S_F)$ as the set of events x such that every curve emanating from x either remains within $R(S_I, S_F)$ or leaves it by crossing $S_I \cup S_F$; hereby, it is assumed that any inextendible past oriented causal curve leaves $R(S_I, S_f)$ at S_I and any inextendible future oriented causal curve leaves $R(S_I, S_F)$ at S_F. Note that this definition is framed as such that closed timelike curves are allowed for given that we did not demand the hypersurfaces to be achronal; moreover, S_I, S_F are chosen such that $R(S_I, S_F)$ is nonempty. In a *classical* theory of the universe, one can speak about the realized past as a classical spacetime to the past of S_I; this is *not* so in a quantum theory where the past consists out of measurements made and those do not constitute a classical spacetime at all since spacetime is rather unknown when no measurement occurs. In that regard, for classical spacetime theories, we defined a quantum theory to be of type II when all events past to S_F have to be taken into account in the computation of the transition amplitude $\langle \mathrm{OUT}\, y_j, j = 1 \ldots m | \mathrm{IN}\, x_i, i = 1 \ldots n\rangle$. In a sense, this would mean that the recorded spacetime history plays a role in the behavior of elementary particles when evolving to the future: this is not a silly idea but one remniscent

[8] The definition we make can be extended to the case that they are not spacelike separated.

of Einstein causality. Type I is the most logical one in the sense that elementary particles do not care about the future nor about the past and all computations have to occur within $R(S_I, S_F)$. Type III is the opposite of Type II meaning that the potential (deterministic) future of S_I beyond S_F plays a role in the determination of the relevant amplitudes; the computations in quantum field theory are of Type II and III in the sense that the entire spacetime is taken into account. My personal guess is that nature works according to Type I principle but in all proofs below we shall make absolute norm estimations of scattering amplitudes meaning that we only have to show absolute convergence in that sense for the mixed Type II and III, in either the standard situation. From a philosophical point of view, Type I is the cleanest principle, while Type II and III are somewhat more far fetched but possible nevertheless; only nature knows the true answer in this regard. For this mixed type, the transition amplitude in ϕ^4 theory is given by

$$\langle \mathrm{OUT}\, y_j, j = 1 \ldots m | \mathrm{IN}\, x_i, i = 1 \ldots n \rangle = \sum_D \frac{(-i\lambda)^V}{s(D)} \left(\prod_{j=1}^{V} \int_{\mathcal{M}} dz_j \sqrt{g(z_j)} \right) \prod_E \Delta_{F,\mu,\alpha}(E)$$

where D denotes a diagram with V internal vertices all connected to some external point, E denotes an edge of the diagram and $\Delta_F(E)$ the Feynman propagator attached to the edge. Finally $s(D)$ is a symmetry factor of the diagram which equals the number of graph isomorphisms leaving the external points fixed[9].

We now investigate wether, on the backgrounds considered, every Feynman diagram gives a finite result. Here, it will be important to obtain thight bounds in order to be able to investigate analycity of the perturbation series in a suitable range of our constants κ, Λ, μ. As mentioned previously, we will show perturbative finiteness for Minkowski concerning the most general mixed Type (II and III) and for the Friedmann universe for Type II; here, we will argue that Type III is potentially troublesome regarding diagrams with many vertices. In general, our local Wick rotation preserves the spacetime volume and therefore, a desired property would be that for any x, we have that

$$\int_{\mathcal{M}} P(d(x,y)) e^{-\kappa d(x,y)} \sqrt{h(y)} dy < R(P, \kappa)$$

for any $\kappa > 0$, polynomial P and some $R(P, \kappa) > 0$. Here $R(P, \kappa)$ is supposed to go to zero in the limit for κ to plus infinity. Such Riemannian spaces are called exponentially finite; Euclidean spacetime, the Wick rotation of Minkowski, is exponentially finite but the Wick rotated Friedmann cosmology is *not* so when considering the entire asymptotic future. It is however exponentially finite towards the geodesic past of every point x restricted to a region $t \leq \tilde{t}$ and it is this result we will use in our exposition. In other words, the exponential blow up in the radius r for balls $B(x, r)$ poses no problem when considering the intersection with the region contained within the Lorentzian horizon of x restricted to that slice, since for large r the intersection of the ball with that region blows up *linearly* in r as opposed to the short scale r^4 behavior. Hence, from now on,

[9] A graph isomorphism is a permutation of internal vertices and edges leaving all relations fixed.

we switch over to Riemannian geometry, the behavior of

$$W_{\mu}(x, x'; \Lambda, \alpha)$$

only being important by means of its upper bound of

$$c(m, \mu) = \frac{e^{-\mu m^2}}{4\mu\pi^2}$$

for as well Minkowski as the Friedmann cosmology. Actually, this constant is universal as it does not depend upon the details of the geometry but only on μ and m; therefore, everything we say holds for spacetimes with an exponentially finite Wick rotated Riemannian geometry. Let us start by mentioning an obvious equation which is that

$$V - I = C - L$$

where V is the total number of *internal* vertices of a Feynman diagram, I its number of internal lines, hereby excluding the legs towards the external points, and L is the number of loops. Finally, C is the number of components of a graph; for a ϕ^4 theory and for connected diagrams C is bounded by

$$C \leq \frac{n+m}{2}$$

where n, m are the number of IN and OUT vertices respectively. We will also assume that our Riemannian manifold satisfies a volume bound for the ball of radius r around x by

$$\mathrm{Vol}_4(B(x, r)) \leq K r^4$$

for some metric dependent constant K. Likewise, we will uphold a bound of this kind on the spheres of radius r; that is

$$\mathrm{Vol}_3(S(x, r)) \leq M r^3.$$

Minkowski and our cosmological spacetimes, for the restrictions of the balls to the region within the geodesic region past to the slice of constant $\tilde{t}$, satisfy this property. With these conventions, we have that the absolute value of every Feynman diagram is bounded by

$$c(m, \mu)^{I + \frac{n+m+n'+m'}{2}} \int dz_1 \sqrt{h(z_1)} \ldots \int dz_V \sqrt{h(z_V)} \prod_{\text{internal lines}\,(z_i, z_j)} e^{-\kappa d(z_i, z_j)} \prod_{i=1}^{n} e^{-\kappa d(x_i, z(x_i))} \prod_{j=1}^{m} e^{-\kappa d(y_j, z($$

Moreover, for ϕ^4 theory, one has that

$$I + \frac{n'+m'}{2} = 2V$$

where $0 \leq n' \leq n$ and $0 \leq m' \leq m$ so that the prefactor may be exactly written as

$$c(m, \mu)^{2V + \frac{n+m}{2}}$$

and

$$2V - \frac{n'+m'}{2} \geq L = V + C - \frac{n'+m'}{2} \geq 0$$

so that $V \geq \frac{n'+m'}{4}$. Here n', m' denote the number of IN or OUT vertices which are connected to an internal vertex. Before we proceed, let us mention some easy to see fact about the Friedmann cosmology; if z is within the geodesic horizon of x and y, then it is in the geodesic horizon of the midpoint of x and y in the Riemannian metric[10]. This observation is most convenient in the following estimates which constitute a straightforward generalization of our previous inequalities. Consider n points z_i and take the integral

$$\int_{\mathcal{M}} dz \sqrt{h(z)}\, e^{-\kappa(\sum_{i=1}^{n} d(z_i,z))}$$

then, as previous, this may be bouded by

$$e^{-\frac{\kappa}{n-1}\sum_{i<j} d(z_i,z_j)}$$

$$\left(\int_{y;\, \exists z_i,z_j : d(y, \frac{z_i+z_j}{2}) < \frac{3}{2} d(z_i,z_j)} \sqrt{h(y)} dy \, + \int_{y;\, \forall i,j \; d(y, \frac{z_i+z_j}{2}) \geq \frac{3}{2} d(z_i,z_j)} dy\, e^{-\frac{\kappa}{2(n-1)}\left(\sum_{i<j} d(y, \frac{z_i+z_j}{2})\right)} \sqrt{h(y)} \right).$$

Note here the factor of 2 in the denominator of the exponential in second integral; this originates from the fact that in a general Riemannian space

$$d(x,y) + d(y,z) \geq d(x,z) + \frac{1}{2} d(y, \frac{x+z}{2})$$

for $d(y, \frac{x+z}{2}) \geq \frac{3}{2} d(x,z)$ whereas in Euclidean space this factor $\frac{1}{2}$ is not present. The latter formula can again be estimated by

$$e^{-\frac{\kappa}{n-1}\sum_{i<j} d(z_i,z_j)} \left(\left(\frac{3}{2}\right)^4 K \sum_{i<j} d(z_i,z_j)^4 + R\left(1, \frac{\kappa}{2(n-1)}\right) \right)$$

and the only thing the reader should notice is the division of κ through $n-1$ which lowers convergence for diagrams with multiple internal vertices. We will not apply the above estimate consistently but look for a finer estimate which will provide one with better convergence properties. Actually, we will be set with a Kirchoff diagram where the flow is given by some rational proportion of $\kappa d(x_i, z_j)$ or $\kappa d(y_j, z_k)$; at any instant of the computation, these proportions add up to one. The optimal way of spreading around is by ensuring that the you do not subdivide into smaller portions; in that way, the surpression factor at the vertex remains constant κ. Homogeneous fractalizing is the worst that can happen since it lowers κ substantially after a few vertices have been run through. Loops make no difference whatsoever, in case we have a loop and there are three external vertices, two with current κ and one with current 2κ then we obtain that κ does not get renormalized, nor at the vertex nor at the legs. Also, in case we have a loop with only two external points each with current κ, there is no lowering of κ neither at the vertex nor at the legs.

Let us reason why homogeneous spreading is a bad idea; in case any of the currents associated to a leg consists out of several pieces, then a lowering of κ will occur, but such lowering will always be less than is the case for a vertex

[10]This follows most easily from the convexity of the horizon of z in the Riemannian metric d which the reader may prove as an exercise.

with four external currents associated to four distinct graph points. We will now determine the maximal contribution of homogeneous fractalizing: start at any vertex z_i, then the most severe contribution regarding the integral comes when no loop is present and likewise, this situation divides κ through the largest number three. Pick now any neighboring vertex, then again, the largest division occurs again when there are three other external legs, dividing the $\frac{1}{3}$ leg into 3 times $\frac{1}{9}$ and the remaining $\frac{2}{3}$ per other leg by two which gives $\frac{1}{3}$ and yields the surpression factor of $\frac{\kappa}{6}$ on the second vertex. In the third step, the worst that can happen is that a leg of the first and second vertex meet since that would cause maximal diversification. The leg from the first vertex contains two factors $\frac{1}{3}$ and 3 factors $\frac{1}{9}$ and the same for the leg coming from the second vertex. Therefore, diversification would lead to 4 times $\frac{1}{6}$ and 6 times $\frac{1}{18}$ on the other two legs, giving a surpression of $\frac{\kappa}{12}$ at the third vertex. Clearly, this reasoning is catastrophic and we now turn our head towards no fractalizing.

This case is easy and one can partition the set $S = \{x_i, y_j\}$ into pairs $(\alpha_{2i-1}, \alpha_{2i})$; with these reservations, the quantitative result reads

$$c(m,\mu)^{2V+\frac{n+m}{2}} P(d(\alpha_{2i-1}, \alpha_{2i})\,; i = 1 \ldots \frac{n'+m'}{2})e^{-\kappa \sum_{i=1}^{\frac{n+m}{2}} d(\alpha_{2i-1},\alpha_{2i})}$$

where P is a polynomial of degree $4V$ and the highest order coefficient is bounded by

$$\left(\frac{3}{2}\right)^{4V} K^V \left(2^4\right)^{\frac{V(V-1)}{2}} .$$

It is the behavior of this last coefficient which makes our bound on the series nonanalytic. The above formula is always true for any diagram as the reader may wish to show by induction on the number of internal vertices, by integrating out a vertex without altering the connectivity properties[11], and does not hinge upon special features of the diagram such as the property that there exists a partition of the edges into paths, connecting the exterior points, and loops such that no internal vertex belongs to two loops. It is always possible to cover a graph by means of curves connecting the exterior points and loops but sometimes it is the case that two loops always intersect[12]. The reader might wonder wether the above estimate is not too crude given that we do not rely upon the details of $W_\mu(x, x')$ at all. Also, we replaced the Lorentzian geodesic energy by the inferior Riemannian distance, wich is an approximation as well. My answer is a resounding no: these approximations will not significantly influence the result for the following reasons. Regarding W_μ, only very slight falloff behavior towards infinity can be shown which effectively can be minorized by means of a slight renormalization of κ (increasing its value a bit). Concerning the replacement of

[11]Such a vertex always exists as the following reasoning shows: start at an exterior vertex and go in the diagram. On the first vertex one meets, there is another edge which can be connected to a different exterior vertex given that every vertex is connected to at least two different exterior vertices. Suppose now that the remaining two edges are part of a loop, then one can integrate this vertex out and connect the remaining vertices without changing the connectivity properties of the diagram. Suppose, otherwise, that the remaining edges cannot be joined by a loop; then any vertex connected by exactly one the two remaining edges must have another edge which is connected to an exterior point. Therefore, we can integrate out this vertex and connect the remaining vertices in such way that the connectivity properties are preserved.

[12]The reader may easily find an example of such diagram.

the energy term by the Riemannian distance; not much is to be expected here since they coincide in Minkowski given that the geodesics of both metrics are the same. Therefore, in a general analysis, these details should not matter.

As always, one has to be careful about what I mean with the fact that no better estimate would be possible: what I meant is that it would not be possible to obtain a better bound on the *asymptotic* behavior of the amplitude when some particles are far away from one and another. In general, it is possible to obtain a bound which does not depend at all on the details of the interaction vertices as well as on the distances between the exterior vertices. It is simply given by

$$c(m,\mu)^{2V+\frac{n+m}{2}}R(1,\kappa)^V$$

which is most easily proved by induction on the number of internal vertices V. If $V=0$, then the bound is easily seen to hold since $e^{-\kappa d(\alpha,\beta)} \leq 1$ for every leg joining two external vertices. Suppose now the bound is true for $V \geq 0$, we will prove it for $V+1$. Take any internal vertex connected by one edge to an exterior vertex α and remove it; the effect is that we obtain a diagram with four extra external vertices (we copied four times the internal vertex) but with one internal vertex less. Remove the edge to α from the diagram, then the remaining part is bounded by

$$R(1,\kappa)^{V-1}.$$

Now there remains to identify the four vertices again and perform the remaining integration over this vertex; the latter gives an extra factor of $R(1,\kappa)$ because we still have the leg to α which proves the result. This shows that the diagram blows up in a suitable way, but there remains of course the "entropy" factor associated to all Feynman diagrams with V internal vertices and n IN and m OUT vertices. The latter remains to be investigated but it is very well possible that unitarity may have to be given up to make the series analytic. What lesson to learn from this? I know of many field theorists who would say that our series determines a C^{∞} function, but such line of reasoning is quite silly since it would only determine that function in an infinitesimal neighborhood of zero in the coupling constant λ which is desastrous. Our theory would be virtually empty if I were to believe that; only with some slight handsight could one say that the first few terms give a good indication for the behavior of the true function in some unknown neighborhood of $\lambda = 0$. I think such line of reasoning is simply demented: a theory is a theory and then it is well formulated or it is none. To my liking, it indicates that for large V we might need heavier surpression terms in the series: coming back to our original definition, there was indeed no good reason *why* the coefficient should be

$$\frac{(-i\lambda)^V}{s(D)}.$$

Every physicist knows of course it comes from unitarity, but we already came to the conclusion we had to dispell unitarity in the free theory when defining the two point function. I would see the eventual failure of perturbative finiteness, or analycity, as another failure of unitarity, even on Minkowski.

7 Conclusions.

In this paper, we have proved perturbative finiteness of our recently proposed quantum theory [1, 3] for a general type of backgrounds; a result which has no precedent in the physics literature. No strings or extended objects were needed for this result and no distinction between renormalizable and non-renormalizable theories exists. We have shown that our result is actually generic within the class of exponentially finite Riemann spaces and therefore, our line of thought was directly amenable to general analysis. Of course, the sceptic may argue that we did not include particles of nonzero spin in the picture but this is not going to change anything to our results as only two properties are crucial: the exponential falloff in the Riemannian metric of the two point function and the removal of the singular structure. I must emphasize that I have not "merely" introduced any regularization scheme of any kind: all our parameters have a physical significance and there is no violation of Lorentz invariance whatsoever. It is so that our notion of Lorentz invariance makes a lot of more sense than the overly rigid, and rather unphysical, notion in standard particle theory. Much remains to be done about this construction and we postpone further generalizations towards higher spin and an even wider variety of backgrounds towards the future. I cannot, but emphasize again, that a few lessons had to be learned here: (a) unitarity is dead, burried once and for all, it does not make any sense when combined with a generally covariant spacetime language (b) the idea of partial differential equations is a dead one too, as turned out, the features associated to "viscous friction" were the most important ones to make the idea work out. Therefore, pure wave propagation does not hold and is only part of the story.

It would be desirable, if necessary, to find novel physical principles to guide us in how to modify the defining series for the interacting theory such that it becomes analytic in all physical parameters λ, κ, μ. We postpone such quest for the future and content ourselves with the very idea that it can be done in a way which is faithful to experimental results. Our analysis also pointed in the direction that Type III quantum theories should not be taken into account, at least not in the way it is done now. In a forthcoming publication we shall point out that a Type III theory is nevertheless possible for our vacuum cosmology but that in that case only the

$$c(m, \mu)^{2V + \frac{n+m}{2}} R(1, \kappa)^V$$

bound can be shown for general diagrams. To me, Type I remains preferrable and we will say more about this in forthcoming publications.

References

[1] J. Noldus, General Covariance: a new Paradigm for Relativistic Quantum Theory, Vixra

[2] J. Noldus, On the Foundations of Physics, Vixra.

[3] J. Noldus, Generally Covariant Quantum Theory: Examples, Vixra.

[4] J. Noldus, Lorentzian Gromov Hausdorff theory as a tool for quantum gravity kinematics, arXiv.

[5] C.J. Fewster and R. Verch, Dynamical locality, what makes a physical theory the same in all spacetimes? arXiv;1106.4785

[6] C.J. Fewster and R. Verch, On a recent construction of vacuum like quantum field states in a curved spacetime, arXiv:1206.1562

[7] C.J. Fewster and R. Verch, Algebraic quantum field theory in curved spacetimes, arXiv:1504.00586

[8] M. Brum and K. Fredenhagen, "Vacuum-like" Hadamard states for quantum fields on a curved spacetime, arXiv:1307.0482

[9] Steven Johnston, Particle propagators on discrete spacetime, Classical and Quantum gravity 25:202001, 2008 and arXiv:0806.3083

[10] Steven Johnston, Quantum fields on causal sets, PhD thesis, Imperial College London, September 20120, arXiv:1010.5514

[11] Steven Weinberg, The quantum theory of fields, foundations, volume one, Cambridge university press.

[12] S.W. Hawking and G.F.R. Ellis, The large scale structure of space-time, Cambridge monographs on mathematical physics.