About The Geometry Of Cosmos and Beyond

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Abstract

The current paper presents an attempt to express the mass problem in a more concrete mathematical scheme where all the Physical quantities could come naturally. The Standard Model and its extensions where investigated where SU(4) has appeared as a conclusion of this attempt.

1 History

In Nature exist three elemental physical quantities LENGTH,TIME AND MASS.We could say that this statement could be almost axiomatic and we would like to figure how great pioneers in Physics treated those physical quantities historically.

1. ***Newton's idea***: Newton built a physical theory as follows,he used a three dimensional real space (R^3) which describes lengths and the pythagorian metric in order to measure this lengths plus a universal clock(scalar) that describes "times", plus a parameter m(scalar) that describes the property of mass.Schematically we could write about Newton's description about Cosmos:

$$R^3 \times R^+ \text{ plus mass}$$

2. ***Einstein-Minkowski's idea***:They thought to unify geometrically "lenghts" with "times" which gave us our familiar four dimensional real spacetime R^4 .The vector of R^4 is $\overrightarrow{r} = (ct, x_1, x_2, x_3)$ and the measure $ds^2 = c^2dt^2 - dx_1^2 - dx_2^2 - dx_3^2$ .But by this move (the unification of lengths and times) the form and the definition of mass was altered automatically.From Newton's absolute masses we moved to relativistic ones.Schematically we could write that their Cosmos is described:

$$R^4 \text{ plus mass}$$

3. ***What is next***?: Two of the three elemental physical quantities are already unified geometrically(lengths and times) so it is a logical scheme to unify all the three ones.But how?Someone could proceed just as Minkowski did.Minkowski altered the Newton's universal clock to a coordinate of a certain mathematical space (R^4) so someone could try the same logical step to alter the scales(weighting machine) to coordinate.We have to admit that such an idea looks so irrational to our mind or senses but is extremely logical as a scientific or mathematical one .As irrational was Minkowski's idea so irrational is this new one.In science everybody looks for new ideas and in Science's History someone can always find something inspiring.Definitely if an idea is promising will be judged by its results otherwise someone just lost time and effort.But trying is exactly what human beings can do:try and try and maybe something good could come.

2 If Minkowski was still alive

If someone would try and expand Minkowski' idea ,he could write a vector $\overrightarrow{k}$ in the form

$$\overrightarrow{k} = (x_1, x_2, x_3, ct, \tfrac{G}{c^2}m)$$

or if we set $G = c = 1$ for our convenience

$$\overrightarrow{k} = (x_1, x_2, x_3, t, m)$$

This way the mathematical description of such a hypothesis would be a five dimensional real space R^5.But many reasons led us to hypothesise that R^5 is not the right mathematical space.Some of these reasons are:

1. We wanted a space that could have real and complex representation meaning a mathematical space of dimension 2n
2. We wanted somehow to represent the time coordinate with a complex coordinate(Landau and Lifshitz)
3. We wanted a whole structure for mass
4. We wanted Cartan's theorem about triality to show the way,the right dimension [1] [2] [3] [4] [5] [6] [8].

Those reasons led us to suppose that we need an 8-dimensional real or a 4-dimensional complex space.We need to remark that we will built a physical theory by only one hypothesis:the existence of $R^8 \equiv C^4$.Afterwards we will let differential geometry to do the work and if we will end with a valid physical theory we will be very satisfied.A new theory or just an new idea would have a chance to be valid if and only if it can represent the current undeniable theories and something more.So our intend is, if we can represent by this idea or even better prove mathematically the standard model(SM),the Higg's mechanism(Hagen, Englert, Guralnik, Higgs, Brout, and Kibble mechanism to be more accurate) and hopefully something new.So let suppose a vector space $K = R^8 \equiv C^4$ and $\overrightarrow{k} \in K$

$$\overrightarrow{k} = (x_1, x_2, x_3, T, Bm_1, Bm_2, Bm_3, ct)$$

where $B = \frac{G}{c^2}$ and if we set once again $G = c = 1$ and write it in more compactified form:

$$\overrightarrow{k} = (\overrightarrow{r}, T, \overrightarrow{m}, t)$$

$$\overrightarrow{k} = (\overrightarrow{r}, T) + i(\overrightarrow{m}, t)$$

The first one is in the 8-d real representation while the second is in the 4-d complex one.Moreover $\overrightarrow{r}$ is our usual length vector with units in meters(length,width,height),$\overrightarrow{m}$ is a mass vector with units in kilograms(mass length,mass width,mass height),t our usual time with units in seconds and T a new dynamical parameter which behaves like a second "time" with units in meters.So we have two "clocks" or two "times" or just two "dynamical parameters" but in the complex representation we have one unified "time" $T + it$.One of the most peculiar element in this description is the meaning and the significance of T.Our initial aim was to give a symmetric form to our spacetime and at the same time to connect it with cosmological events.Our explanation in [8] was that T plays the role of "cosmic time" but in meters,the "true time" that COSMOS HIMSELF experiences.Such a hypothesis gave us the opportunity to give to COSMOS a new dynamical parameter,separated at the beginning by our usual time t.We will see and prove later that when we will formulate our usual space ,T and t, are fully connected.We are sure that some guesses have already crossed your minds about T!If we go back to our space $K = R^8 \equiv C^4$ we can write also(a spatial case easier to work at the beginning):

$$K = R^4 \times M^4 \equiv R^4 + iM^4$$

where R^4 is a 4-d real space which we like to call it the length space($(\overrightarrow{r}, T) \in R^4$)and M^4 is also a 4-d real space which we like to call it the mass space($(\overrightarrow{m}, t) \in M^4$).The reason why we put t in M^4 and Tin M^4 is fully explained in [8] and has to do with Cartan's theorem about triality.Till now we have a space and the form of its vectors and the next thing that we need is an elementary length or just a measure:

$$dk^2 = d\overrightarrow{r}^2 + dT^2 - d\overrightarrow{m}^2 - dt^2$$

giving us a signature $(4,4)$.Why we choose this signature?The answer is once again the Cartan's theorem about triality [1][2][3][4][5][6][8].We could start with $(8,0)$ or $(0,8)$ which are Pythagorean but as we can see in [1][2]the signatures $(4,4)$,$(8,0)$,$(0,8)$ are all correlated.Besides $(4,4)$ signature is easier connected to our belief about a Minkowskian spacetime.The most impressing information is that this way a certain belief that minus signs comes with "times" is not valid.Already without any argument we have the minus signs if we look the complex 4-d representation and as we mentioned R^8 is algebraically equivalent with C^4.Now that we have a space and a metric we can built a physical theory and let us hope that something good will show up or that this space has something to do with Cosmos.We can proceed as mechanics or classical differential geometry dictates.Which path someone will choose is just a matter of preference,both ways are equivalent.The only that matters is that we have a manifold,a tangent space,our maps,a tangent bundle,Poisson brackets etc.Or for someone that loves to work with fields,a vast variety of them,from the manifold our initial fields the vectors,from the tangent space the locally differential functions,Christoffel's symbols as fields ,the metric tensor and much more.Continuously ,we will separate our problem in three categories

1. ***Fully flat-pseudo-Euclidean space***
   If we symbolize as N_{IJ} or N_{ij} the flat symmetrical metric tensor of R^8 we have for $I, J = 1, 2...8$ or $i, j = 1, 2, 3, 4$

   $$dk^2 = N_{IJ} dK^I dk^J$$
   $$dk^2 = N_{ij}(dr^i, dm^i)(dr^j, dm^j)$$
   $$dk^2 = d\overrightarrow{r}^2 + dT^2 - d\overrightarrow{m}^2 - dt^2$$

   where the signature of N_{IJ} or N_{ij} is $(4,4)$

2. ***Semi-curved K***
   In this case we have a curved R^4, a curved M^4 but flat between R^4 and M^4.It seems like a Lorentz form between R^4 and M^4.If we define as G_{IJ} or G_{ij} semi-curved symmetrical metric tensor,it will appear with 4×4 sub-matrices in the diagonal and $\bigcirc_{4\times4}$ in the anti-diagonal.We have not terms of the kind $dr^i dm^j$

3. ***Fully-curved K***
   In this case all K is curved and interactions between R^4 and M^4 are allowed

   $$dk^2 = G_{IJ} dK^I dk^J$$
   $$dk^2 = G_{ij}(dr^i, dm^i)(dr^j, dm^j)$$

   or if we like to represent in C^4 instead, the metric tensor is a 4×4 complex Hermitian matrix G_{ij} $i, j = 1, 2, 3, 4$ with $k = (r, T) + i(m, t)$

Someone can now imagine the following situation,in the first case if we separate times we can form a Newtonian-Hamiltonian TYPE theory in $R^8 \equiv C^4$.If we will not separate times we can form a special relativity TYPE theory in $R^8 \equiv C^4$.In the second case we have a semi-special-general relativity TYPE theory while in the third case we have a general relativity TYPE theory in $R^8 \equiv C^4$.When we say the word TYPE it means that we can work in the same way as Newton or Einstein worked but in more dimensions.The most impressive and obvious element is that the second and third case will always give locally the first case and the first case hides magic, a famous "field" lies there(it is to soon to meet it).
OK someone could say that all these are mathematical beautiful but where is our usual spacetime R^4 and are there any clues in our existing theories about such a crazy consideration?Surprisingly our usual spacetime is already there and our existing theories are full of evidence that we have refused to considerate,,we were distracted.So let us examine some facts that comes from our existing theories

1. We could describe mathematically our Cosmos with 8 dimensions but human beings live,move and observe the usual 4 dimensional space due to our energy scale but the source of our existence is the mass space.The source of the existence of Cosmos is the mass space but is "small" enough to observe its dimensionality.The mass space generates amd its products move and live in R^4 where T makes R^4 expand.M^4 is extremelly "small" compared to R^4 a fact that can be seen in every physical theory we have.A wonderful picture about this fact comes from General Relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi\frac{G}{c^4}R_{\mu\nu} - \Lambda g_{\mu\nu}$$

   The left side of this equation is pure geometry described by R^4 and it has no physical constants.On the other hand the right side has the energy-momentum tensor plus the cosmological term Λ.First of all we could imagine by the sight of this equation,our hypothesis about a mass space where $T_{\mu\nu}$ could geometrised and look like an equivalent Ricci tensor in M^4.But maybe this could be premature.Moreover Einstein in order to balance the two sides of the equation used $\frac{G}{c^4} = \frac{1}{F_p}$ an extremely big constant(the Planck power has appeared) and Λ which is so so so small.

2. The Standard Model(SM) is maybe the biggest element.Let us begin with the "covariant" derivative of SM for electromagnetism for simplicity

$$D_\mu = \partial_\mu - iqA_\mu$$

   We have to admit that this might looks like a covariant derivative but we rather prefer to consider it as a connection.Some scientists say that $iqA_{\mu\nu}$ looks like a Christoffel symbol,there is a resemblance.But in which space $iqA_{\mu\nu}$ could **really be** a Christoffel symbol?This is a good question with a simple answer.In this connection already exist the R^4 partial derivative meaning that the Minkowski metric tensor is already there.Moreover we cannot use our usual Christoffel symbols because they are connected with gravity.An affine connection is totally out because a manifold equipped with a metric tensor already exists,R^4 is a metric space(the Minkowski space is a pro-Hausdorff pseudo-metric space).So the most simple answer is that we have to expand our space to a bigger one, which must be at least pro-Hausdorff.Then these $iqA_{\mu\nu}$ could be Christoffel symbols in the expanded part of the space.If we continue ,we can read in any book about SM:

• All elementary particles have an internal local gauge symmetry $U(1)$(Electromagnetism)
• All elementary particles have a second internal local gauge symmetry $SU(2)$(Weak nuclear field)
• All particles have a third internal symmetry $SU(3)$(Strong nuclear field)
With every local gauge symmetry ,exists a corresponding gauge field B_μ, W_μ, G_μ.The key word is **INTERNAL**!!!What does it mean internal?In Physics we mean this mysterious symmetries that come with **MASS** fields that come with elementary particles and describes their properties.The external symmetries come with the properties of our usual spacetime.Truly we feel little uncomfortable.In Mathematics there is no such a seperation between internal or external symmetries,just symmetries.For instance in $SO(8)$ there are rotations many different kinds of rotations and many symmetries,we do not separate them into internal or external symmetries or rotations.So why we have to talk about internal symmetries or internal space?.The most logical step is to give to mass a metric space where all the properties will come out of this space.This way we do not have internal or external symmetries byt just symmetries in R^4 or M^4 or anywhere else in R^8.So the fields B_μ, W_μ, G_μ that look so much like Christoffel symbols could be **linked** with the Christoffel symbols of M^4

3. In CKM and PMNS matrices in bibliography you can always find the phrase "mass eigenvalues and mass eigenvectors".It is very common in quantum physics to talk about eigenvalues and eigenvectors ,but before you have to define an operator and by solving the eigenvalue equation you find the eigenvalues and eigenvectors of the associated operator.The operator comes first.Arthur Jaffe and Edward Witten in their famous paper "Quantum Yang-Mills Theory" [7] write "The mass operator in natural units is $M = \sqrt{H^2 - P^2} \geq 0$,and the vacuum vector Ω is a null vector $M\Omega = 0$.Massive single particle states are eigenvectors of an eigenvalue $m \geq$. If m is an isolated eigenvalue of M then one infers from the Wightman axioms ...". So the question that arises is: where is that "mass" operator?If you have a manifold and its fibre bundle you can find such an operator.By inserting a mass space you can find easily a pure "mass" operator.
4. The Higg's mechanism.This wonderful and notorious mechanism introduces us a Lagrangian of the form:
$$L = (D_\mu\varphi)^+(D^\mu\varphi) - V$$
where V is a potential.This potential V is the key of this mechanism,where V is:
$$V = \mu^2\varphi^+\varphi + \lambda(\varphi^+\varphi)^2$$
For $\mu^2 > 0$ it gives nothing particular or new,but for $\mu^2 < 0$ Nature spoke to us.A big secret was revealed which was nicely hidden.But why $\mu^2 < 0$?Why we do not write ?:
$$L = (D_\mu\varphi)^+(D^\mu\varphi) + \mu^2\varphi^+\varphi - \lambda(\varphi^+\varphi)^2$$
Although it looks nice enough, physicists want to keep this precious minus sign.And then ,suddenly from this classical term we jump into quantum physics and the minus sign is hidden under the carpet of our physics.In the Standard Model, the Higgs field is a scalar tachyonic field which is something annoying but we can find a way to overcome.But is it so?Maybe there are many more secrets in the closet of physics concerning this mechanism.But we have always to remember that if you open a closet there is a whole SPACE inside.

As a conculion of this chapter ,there is so much information which is linked to mass or just elementary particles.Let us make the most logical step to introduce a metric space where all this information is inside.Afterwards let us add this new metric space next to our usual space of lengths.Finally we will have a space to work

3 The role of T

Let us go back to our flat case of $R^8 \equiv C^4$ and look once again the elementary length(all physical constants equal to one)

$$dk^2 = d\overrightarrow{r}^2 + dT^2 - d\overrightarrow{m}^2 - dt^2$$

• if we "hide" the dm and dT terms we have:

$$dk^2 = d\overrightarrow{r}^2 - dt^2$$

which is of course our usual Minkowski's spacetime
• if we "hide" the dm and dt terms we have:

$$dk^2 = d\overrightarrow{r}^2 + dT^2$$

and looks like a new Pythagorean-Galilean spacetime
• if we"hide" only the dm term we have :

$$dk^2 = d\overrightarrow{r}^2 + dT^2 - dt^2 = dl^2 - dt^2$$

a five dimensional space that looks like our familiar five dimensional De-Sitter space.On the other hand our basic cosmological metric is the Friedman-Robertson-Walker one:

$$ds^2 = dt^2 - R(t)^2(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + sin^2\theta d\varphi^2))$$

where R(t) is the time-depended term of cosmic scale with units in meters.The coordinate r is dimensionless and the "vector" (r, ϑ, ϕ) are the coo moving coordinates.This "vector" is constant in time for observers that move under the cosmic expansion.We can see that we have two pictures in our current physics.The first picture is when someone decides to describe a motion in spacetime and the second one is when someone decides to describe spacetime.It seems to us that we have two different dynamics that both are connected to t our usual time.So we though to insert two "times" one for R^3(meaning T) and one for M^3(meaning t).This way our space $R^8 \equiv C^4$ has a "time" of the form (T, t) or $T + it$.It is true that we like the fact to put this "it"(imaginary) coordinate.We like to see that "T time" as the "time" that Cosmos himself experiences,it give us his "growth",his expanse,his "age".At this part T and t are two different parameters that are connected just as a total time in $R^8 \equiv C^4$.So we faced with two pictures,one with T for events in R^4 and one with t for events concerned M^4.Of course if someone chooses to live and observe in $R^8 \equiv C^4$ he will have the one total "time" $(T, t) \equiv T + it$.Now it is easy to observe that this wonderful parameter T resemles or coincides with the time-depended term of cosmic scale R(t).Moreover we would like to remember that Hubble's constant H is $H = \frac{R^{\bullet}}{R}$.The correct path now is to **show** that T is R(t),not just hypothesize it because of resemblance.But we will do something stronger,we will start wit T and show that its classical limit is R(t) and as a conclusion Quantum Cosmology will appear.Before we proceed,it is time to make a big statement:

"All the geometrical objects or physical phenomena in M^4 will eventually appear in our usual spacetime R^4 as mean values or eigenvalues of certain defined operators in M^4"

All our missing information as we see it in our usual spacetime comes from M^4 meaning mass,charge,spin isospin,fields etc.Now we can proceed as follows:

1. In $R^8 \equiv C^4$ we define momentums p_r in R^4 and p_m in M^4.Our usual momentum will appear only if we pass from R^8 to our usual spacetime (S)
2. We can define two different "energies" E_T in R^4 and E_t in M^4.
3. We will start with the flat metric:
$$dk^2 = d\overrightarrow{r}^2 + dT^2 - d\overrightarrow{m}^2 - dt^2$$
4. A special relativity TYPE in R^8 can be formed with an energy equation
   - $\mid E_T \pm E_t \mid^2 = \mid p_r \pm p_m \mid^2$

   if R^8 is semi-curved and
   - $\mid E_T^2 \pm E_t^2 \mid = \mid p_r^2 \pm p_m^2 \mid$

   if R^8 is flat.Those equations can be derived from the elementary length in R^8 if we introduce two "speeds" $v = \frac{dr}{dT}$ and $U = \frac{dm}{dt}$.Afterwards we define the associated Lagrangian and by taking the Legendre transformation we can define the Hamiltonian.The full details are presented in [8] but it is obvious that we form a special relativity in R^8.
5. In [8] we have introduced the generalised or canonical "momentums" and "energies" so our next step is to introduce the associated operators.In bibliography if someone wants to connect the tangent vectors of our usual spacetime (S) with the usual operators of quantum mechanics can find the above:
$$\partial_\mu = (\partial_0, \nabla)$$
and if $g_{\mu\nu}$ is the Minkowski's metric with signature (1,-1,-1,-1) we can raise indices so:
$$\partial^\mu = g^{\mu\nu}\partial_\nu = (\partial^0, -\nabla)$$
and if we multiply ∂^μ by (ih) we have our usual operators of quantum mechanics $\widehat{p}$ and $\widehat{E}$.So we could proceed in the same spirit with the tangent vectors of R^8 (and please forgive that we put "times" in the end)
$$\partial_\mu = (\nabla_r, \partial_T, \nabla_m, \partial_t)$$
and if we use the flat metric tensor $G_{\mu\nu}$ with signature (4,-4) to raise indices:
$$\partial^\mu = G^{\mu\nu}\partial^\nu = (\nabla^r, \partial^T, -\nabla^m, -\partial^t)$$
and if we multiply by (-i) we can have the operators for all our physical quantities $\widehat{p_r}, \widehat{p_m}, \widehat{E_T}, \widehat{E_t}$ as:

$$\widehat{E_T} = -i\hbar c\frac{\partial}{\partial T}$$
$$\widehat{E_t} = i\hbar\frac{\partial}{\partial t}$$
$$\widehat{p_r} = -i\hbar\nabla_r$$
$$\widehat{p_m} = i\hbar c\nabla_m$$

6. We have a new type of "energy" which appears and is connected with "time" T.It is very attempting and pleasant to interpretate it as Dark Energy.This interpretation and the $\widehat{E_T}$ operator could form the quantum dark energy .
7. The next step is to "quantize" .We have the energy-momentum equation,we have a Lagrangian and we have a Hamiltonian,so we can proceed and replace the operators $\widehat{p_r}, \widehat{p_m}, \widehat{E_T}, \widehat{E_t}$.As a result we will have a quantum wave equation,a TYPE Klein-Gordon wave equation in R^8.This wave equation is a second rank differential equation with partial derivatives and the parameters r,m,t,T can be separated.The exact process and solution is presented in [**?**]At this chapter we are interesting only in the "time" part.The solution that comes from [8] for T is:
$$< T >_t = \frac{\sqrt{2}}{2}\frac{c}{H}e^{Ht}$$
where $< T >$ is the mean value of T and H is the Hubble 's parameter.(It is important to mention that in order to solve the Klein-Gordon type wave equation in R^8 we moved into spherical coordinate system and infinite spherical well as boundary conditions.)As a result ,we begun with a strictly quantum procedure(a Klein-Gordon type equation) and we end with a mean value of T.It seems like we fulfilled our promise.The mean value of T is exactly the R(t) solution as it is presented in a vacuum Cosmos by De-Sitter.While T,t are two different parameters in $R^8 \equiv C^4$,when we will transform the theory in our usual spacetime(S),T will become by mean value as R(t).This way we begun with Quantum Cosmology and we ended up with Classical Cosmology.The cause of a Cosmos that expands is cleared out

"T produces the mechanism of Cosmos' expansion'

Notes:

1. To be honest we do not know if the definition of the operators $\widehat{p_r}, \widehat{p_m}, \widehat{E_T}, \widehat{E_t}$ is correct.Maybe some signs or (i) (imaginary unit) could be different and that is the main reason that we put $\pm$ in the energy-momentum equation.But the results are similar no matter what signs we will choose.Of course the picture is not exact but the spirit is the same.The signs at this part are just a matter of fine-tuning of the whole theory.The most interesting element is the interpretation that someone could give.It looks like we rotate the tangent space and by this rotation we can define the operator's space.
2. It is clear that we do not need T if we want to describe the motion of a mass-body or a particle.T is needed only in the case of the dynamic description of whole Cosmos
3. A new catholic invariance principle is presented.In particular,someone could begin with an invariance principle and afterwards he could present a metric which is invariant

under this principle.The new invariance principle is $\frac{G}{c^2}$ with units $\frac{kgr}{m}$ and is concerned for the space R^8.We do not oppose to Einstein's invariant principle which is valid in our usual spacetime(S).Our invariance principle will be transformed to Einstein's one under the transformation from R^8 to (S).

4 The mass problem.A final solution?

Historically ,the mass problem was successfully investigated by Higg's mechanism(SM).The mechanism starts with a field $\phi \in C^2$:

$$M = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_2 \end{pmatrix}$$

It is very common in Physics to start with a field but as we referred previously we want to start with a more geometrical picture.The establishment of a manifold will lead us normally to the necessary field and its properties.Moreover SM is still an ad-hoc mechanism,we still do not know what mathematics are hidden behind or even better what mathematical structures are behind.We are lucky that the pioneers of SM thought this mechanism back in the decade of 1960 but we owe as a generation to fully explain and understand it.So let us go back again to our Klein-Gordon type wave equation in $R^8 \equiv C^4$.We have already referred and presented what is happening with times T and t and now we can proceed with M^3 part(the mass part as we like to call it).The solutions of the wave equation can be separated to r,m,t,T parts so it is right to focus only at m which is nevertheless the new element.To be honest,it is as we treat the r coordinate in standard quantum mechanics.Instead of x_1, x_2, x_3 we have m_1, m_2, m_3 but with different constants and interpretation.We solve the problem as follows:
• We move to spherical coordinate system so from m_1, m_2, m_3 we have two angles ϑ, ϕ in mass space(ϑ, ϕ are mass-angles) and a radius m so :

$$(m_1, m_2, m_3) \longrightarrow (m, \vartheta, \phi)$$

• We treat the problem in a spherical infinite well
• The solutions are presented in any bibliography on this subject and in [8] .The solution has the form

$$\xi(m, \vartheta, \varphi) = \Upsilon^l_m(\vartheta, \varphi) R(m)$$

where Υ^l_m are our familiar spherical harmonics and l,m the related quantum numbers in M^3 and R(m) the radial part.The angular part and its interpretation is presented in the next chapter "spin".The radial part is described by the spherical Bessel functions.The analysis about the radial part is presented in [8].So we can proceed and use the solution .The ***eigenvalue m*** is :

$$m = k\pi A m_P$$

this solution comes for $l = 0$.The interesting fact is that $k \in N$ but $k \neq 0$.The ground state or vacuum of this solution is ***not zero***.So for $k = 1$, which is the ground state we have

$$m = \pi A m_P$$

Our next step is to define this constant A which is:

$$A = \frac{1}{6}\sqrt{\frac{2}{3}}\sqrt{\frac{G}{G_F}\frac{h}{c}} = \frac{\sqrt{2}}{2}\sqrt{\frac{1}{n-1}}\sqrt{\frac{G}{G_F}\frac{h}{c}} = 3,26297x10^{-18}$$

Notes on A

1. This constant A is necessary in order to form a special relativity in $R^8 \equiv C^4$.We choose to insert it at this part but from now and in the future we must include it in the mass operator.So the mass operator will be :
$$\widehat{p_m} = i\hbar c A \nabla_m$$
or if we want in pure mass form:
$$\widehat{m} = i m_P^2 A \nabla_m$$
2. We could leave $\frac{1}{6}\sqrt{\frac{2}{3}}$ out of A and incude it in the covariant derivative .It was a move for simplicity to include it in A.
3. n is the number of generators of the group that leaves our metric tensor invariant.The group as it is presented analytically in [8] is $SO(8)$.So in our theory $n = 28$.
4. The constant A is dimensionless.This is the most suprising and exciting property of the constant A .All our basic constants united in one elegant form.We could easily set :
$$\sqrt{\frac{G}{G_F}\frac{h}{c}} = 1$$
and we could have immediate information about the initial conditions of our Cosmos.Somehow this is the key behind the values of our physical constants.
5. The eigenvalue m,for $k = 1$ and $l = 0$ is :
$$m = 125,17394(5) Gev/c^2$$
6. In Standard Model the Higg's boson mass value is:
$$m = \sqrt{2\lambda}\upsilon$$
where $\upsilon = \frac{G_F}{\sqrt{2}}, h = c = 1$.If we combine it with our result λ will be:
$$\lambda = \frac{1}{2(n-1)}$$
So we could say that λ is a number that comes straightforward from the symmetry of our problem.Such is a result would be fascinating for anyone in the world.
7. Surprisingly the eigenvalue m is not depended by G ,the gravitational Newton's constant.
8. In 5) we thought as our eigenvalue m is the Higg's boson mass value.Is such a consideration valid?Yes definatelly it is .This is Higg's boson and we shall prove it analytically.Our knowledge about Higg's boson is that it comes from a scalar field,it has spin 0,a value around $125 Gec/c^2$ and gives mass to other elementary particles.So we have to prove that our "thing" has all those properties and even better more.Before we proceed with these properties we have need to focus in our new weapon.Finally we have the mass mo-

mentum operator(or the mass operator),from which we can find mass eigenvalues.This is the new weapon and totally agrees with our current picture and belief in Quantum Physics which is:

> "if we want to compute the values of a physical quantity we need an operator.The eigenvalues of this operator are the values that we will take from our experiments.So we defined a new operator in order to measure mass values.The most logical and consistent move.This operator will reflect to us in our usual spacetime S as mc^2,where m is the corresponding eigenvalue or the mean value."

9. A new uncertainty principle will appear where the "quantum" will be $\frac{h}{c}$ because of the structure of $R^8 \equiv C^4$ (Cauchy-Swartz inequality) so that:

$$(\Delta r)(\Delta m) \leq \frac{h}{c}$$

the "quantum" $\frac{h}{c}$ shall be treated as a whole unified constant,rather than two separated constant h,c.

5 The nature of bosons and mass gap

It is time to examine if the solution we presented in the previous chapter is the Higg's boson.In order to fullfill our scope we have to prove accordingly to our present theories the followings:

1. Higg's boson is presented by a scalar field
2. Higg's boson has spin 0
3. Higgs boson has mass value around $125Gev/c^2$ which was shown in chapter 4
4. It is responsible for the masses of other elementary particles

So let us proceed with the proof:
Proof

1. Our approach in order to examine 1,2,4 at this part is mostly geometrical.We will not start with a scalar field.On the contrary we will see that Geometry dictates a scalar field.In the previous chapter we had a Klein-Gordon type wave equation in flat R^8 and by solving the mass part we concluded with an eigenvalue m.The key is that we examined the flat case geometry,meaning that we have a flat metric tensor ,let us call in N_{ij}, with signature $(4, 4)$ .Our solution is strictly connected with N_{ij}.But what exactly means a flat metric tensor in Differential Geometry?Let us remember some simple meanings from Differential Geometry(the solution is always in the basics):
Definition:If X is a Euclidean space then the Cartesian coordinate system has segment coordinate lines meaning that they are geodesic lines.At this system the metric tensor g_{ij} is constant at every point:
$$\frac{\partial g_{ij}}{\partial x^k} = 0$$
Moreover the Christoffel's symbols of the first and second kind are equal to zero and the covariant derivative coincided with the usual partial derivative.
Definition:If X is Riemannian there are not exist Cartesian or geodesic systems.On

the contrary we can find a variety of systems where $\dfrac{\partial g_{ij}}{\partial x^k} = 0$ holds ***locally***.Those systems are called locally geodesic.

Theorem:Let us suppose X is a Riemannian space and x^i, x_0^i coordinates of a point P of the space X.If $\xi^l = \left(\dfrac{dx^l}{ds}\right)_0$ and we set $\xi^l = X^l$ then we can have the transformation

$$X^i = x^i - x_0^i + \frac{1}{2}(\Gamma^i_{jk})_{(0)}(x^j - x_0^j)(x^k - x^k_{(0)} + ...$$

where the indice (0) means that we are referred to the origin of the coordinates of P and we will call it the pole of the system.Then we can prove:

(a) The Cristoffel symbols are equal to zero

(b) If we set $x^i = x_0^i$ then $X^i = 0$ that means that in this new coordinate system X^i the pole is at coordinate origin.

So we have a pseudo-Euclidean,Riemannian space R^8 equipped with a quadratic form.We start with a metric tensor N_{ij} and we move to spherical coordinate system and so N_{ij} will be transformed to $(N_{ij})_{sph}$.Then locally we can always find coordinate system such that

$$\frac{\partial (N_{ij})_{sph}}{\partial x^k} = 0$$

and if $OP = x^l$ a vector in R^8 where O is the origin then OP locally degenerates to the point O.So from a vector x^l we ended locally with a point(***scalar***).That is exactly what Higg's mechanism and Higg's boson is.The Higg's boson is associated with the metric tensor $(N_{ij})_{sph}$ which locally is constant and can be represented with a vector field φ as we begin in the Higg's mechanism and ends to be scalar because of locality.The most impressive element is that you ***can not escape from the existence of Higg's boson***.You will always find it because of the locality of the Geometry.No matter what metric tensor G_{ij} we choose to equip our problem locally we will find the Higg's boson or just the vacuum as we usually refer which as we saw in ***not zero***.As a conclusion there is always a mass gap and can be calculated from the Klein-Gordon type equation we presented in [8] .This K-G type equation comes naturally with our space R^8 if the metric tensor G_{ij} is flat or just locally flat.

2. As concerned why Higg's boson has spin 0 at this point let us say that is logical because as we saw is associated with a scalar field.But we have to remember that this value came for $l = 0$.The full explanation will be presented in next chapter where we will define the nature of spin.We can say for now that spin is associated with the angular momentum in M^4(a most natural assumption).

3. Let us suppose for the shake of simplicity that we start not with R^8 but with C^4.We have the right for such a consideration because R^8 and C^4 are algebraically isomorphic.With C^4 our results will be exactly as we picture them in our usually theories.In C^4 the metric tensor G is not symmetric (as it is in R^8) but Hermitian.The general Hermitian metric tensor G is invariant under transformations of the group $GL(4, C)$.Because $G \in GL(4, C)$ we can expand G at the Dirac-Gell-Mann basis and our favourite Gell-Mann matrices will appear [9] .For simplicity let us skip the μ, ν indices of $G_{\mu\nu}$ .If $G \in GL(4, C)$ there are $a_n \in C$ and λ^n Gell-Mann matrices such that :

$$G = \sum_{n=0}^{15} \lambda_n a^n = I + \sum_{n=1}^{15} \lambda_n a^n = I + a_n \lambda^n = I + a^n \lambda_n$$

for different a_n,G can be written also in $U(4)$.Moreover $GL(4, C)$ can be fallen naturally (locally) to $SU(4)$.But as we can see in [1][2][8] we have to expand $GL(4, C)$ to $SO(8) \equiv SO(4, 4)$.This group $SO(8)$ came out as a conclusion because of Cartan's theorem about triality.Specifically the signature $(4, 4)$ is associated with the group $SO(8)$ [1][2] .The group $SO(8)$ has $\frac{7X8}{2} = 28$ generators and we can find as algebra which would be isomorphic to $SO(8)$ in order to stay in C^4.The algenbras of $GL(4, C)$ and $U(4)$ are isomorphical and we can expand those algebras under the chain $U(1) \subset SU(2) \subset SU(3)$....The product

$$U(1) \times SU(2) \times SU(3) \times U(4) = Z$$

creates an algebra of exactly 28 generators.Moreover both Z and $SO(8)$ have rank 7.As a conclusion we can work with the algebra of Z instead of $SO(8)$ for now.We choose to work with the algebra of Z because we are used to work with the groups $U(1), SU(2), SU(3)$ in the Standard Model and so wanted to stay in the same notation.So G can be written as:

$$G = a_0 I + a_1 \lambda^1 + a_2 \lambda^2 + a_3 \lambda^3 + a_4 \tau^1 + ... a_{11} \tau^8 + a_{12} \omega^1 + ... + a_{27} \omega^{16}$$

I generator of $U(1)$

$\lambda_1, \lambda_2, \lambda_3$ generators of $SU(2)$

$\tau_1, \tau_2, ..., \tau_8$ generators of $SU(3)$

$\omega_1, \omega_2, ..., \omega_{16}$ generators of $SU(4) \times U(1)$

or in more simple notation

$$G = a_n \lambda^n, n = 1, ..., 28$$

We have managed to express the metric tensor G through our familiar Gell-Mann matrices.The next step is the covariant derivative in $K = C^4$ which can be written through Cauchy derivative(we skip the $\frac{1}{2}$ in Cauchy derivative for simplicity).

$$(D_\mu)_K = (\partial_\mu)_K - \Gamma^2_K$$

once again we skipped indices for simplicity and $(\partial_\mu)_K$ is Cauchy derivative in $K = C^4$ space and Γ^2 the Cristoffel symbols of second type in K.But we can split K in R^4 and M^4 spaces because $K = R^4 + iM^4$. So $(D_\mu)_K$ can be written as

$$(D_\mu)_K = (\partial_\mu)_R - i(\partial_\mu)_M - (\Gamma^2_R - i\Gamma^2_M)$$

$$(D_\mu) = \partial_\mu - i\eth_\mu - (\Gamma^2 - i\Delta^2)$$

where

$\partial_\mu = \partial_R$ the partial derivative in R^4

$\eth_\mu = \partial_M$ the partial derivative in M^4

Γ^2 the Christoffel symbol of second type R^4

Δ^2 the Christoffel symbol of second type M^4

or

$$D_\mu = (\partial_\mu - \Gamma^2) - (i\eth_\mu - i\Delta^2)$$

but we can also write $\Gamma^2 = G\Gamma^1$ where G is the metric tensor and Γ^1 the Cristoffel symbols of first type so :

$$D_\mu = (\partial_\mu - G\Gamma^1) - (i\eth_\mu - iG\Delta^1)$$

and if we put for now $\Gamma^1 = 0$ and skip $i\eth_\mu$

$$D_\mu = (\partial_\mu - iG\Delta^1)$$

and if we remember that we wrote G as linear combination of Gell-Mann matrices ,this covariant derivative looks like our usual covariant derivative of Standard Model with two new surprising elements.The first element is that the fields A_μ, W_μ, G_μ are associated with the Christoffel symbols of the first type in the mass space. Every physicist assumed that the fields A_μ, G_μ, W_μ look like Christoffel's symbols but we could not see in what space or just where(the notorious internal space).Now we can see that we have a space,a simple one.The second element is that we have not only the $U(1) \times SU(2) \times SU(3)$ part that appears in Standard Model but $U(4) \equiv SU(4) \times U(1)$ as well.Some scientists believe that we could represent dark matter with $SU(4)$.We have the same opinion because it seems logical and natural.If this interpretation is valid ,then dark matter consists of 15+1 dark bosons.More elements on this subject are presented in [8] .The one boson which comes from the $U(1)$ could be a dark photon or just the Higg's boson.Let us go back at the part we skipped the $\eth_\mu$ and we set $\Gamma^1 = 0$.We set $\Gamma^1 = 0$ because we did not wanted for now to have troubles with gravity.Christoffel's symbols in R^4 ,even if we do not have our usual time t but T(but we already saw that classically there is connection)would bring gravity in the game.We will deal with gravity later.As concerned $\eth_\mu$ it represents the operator from which we can find mass eigenvalues,as we saw.The part $\eth_\mu \eth^\mu$ is associated with the $\mu^2(\phi\phi^*)$ as we can find it in the Higg's mechanism.But we have to be careful because in this theory we do not need the part $\mu^2(\phi\phi^*)$ any more ,instead $\eth_\mu \eth^\mu$ will do our job.As a conclusion if we want tp present the right and full covariant in K:

$$D_\mu = (\partial_\mu - G\Gamma^1) - (i\eth_\mu - iG\Delta^1)$$

The most fascinating element is that through G we appeared our usual bosons of spin 1.Once again the metric tensor seems to carry all the information needed for a physical theory.Moreover the most general form of the metric tensor G(in curved space) will always be locally flat and as we proved locality means Higg's boson or just vacuum state.So through the metric tensor G we have already presented Higg's boson (spin 0) and our usual bosons (spin 1) plus something new.But in a curved space we can always find a system where the Christoffel symbol's can be vanished.We could say that all bosons of spin 1 at the pole (locally) are just Higg's bosons.As they move from the pole they became spin 1 bosons.As a conclusion ,we think that the right picture is not that bosons of spin 1 interact with Higg's boson but all bosons of spin 1 are locally Higg's boson.So ,all bosons of spin 1 will get the property of mass and their masses' values are expressed through the Higg's boson mass value or vacuum.The only thing that has to be investigated is if the gluons "interact "directly with Higg's field or the whole hadronic-mesonic structure interacts with the Higg's filed.The same will happen if $SU(4)$ represents dark matter(for further information [8]).

We think that we may managed to convince some people that we described the mass problem and that the boson we found is Higg's boson.Moreover it seems that in Cosmos there exist foul elementary fields the electromagnetic, the weak nuclear,the strong nuclear and the dark one plus one to rule them all, the gravity.With those new informations it seems that the line between a classical and a quantum theory is thinner than ever.A quantum theory seems to be our effort to represent M^4 in our usual spacetime.We are not sure yet if we need a quantum theory in $R^8 \equiv C^4$ or just a classic one in $R^8 \equiv C^4$.The gravity is a total different field than the other four.As we examined the metric tensor G in $R^8 \equiv C^4$ we splitted it into Gell-Mann matrices which we interpreted as bosons of spin 1.Moreover we set the Christoffel symbols of R^4 equal to zero.It seems that gravity was all the time there.In general relativity we use to think that the field is the metric tensor g_{ij}.In our new perspective we have a new metric tensor G in $R^8 \equiv C^4$ which can be linked directly to the graviton because all our description till now seems like quantum-gravity.Further information on this subject can be found in [8].

6 Spin and mass' eigenvalues corrections

Usually ,some physicist used to refer to spin as self-rotation of the particle,just as Earth rotates around itself.This picture of course is wrong because spin is a clearly quantum property of a particle,without any classical example or analogue.Historically ,spin was introduced ad-hoc by Pauli and later Dirac introduced it naturally in his famous equation.The spin operator share almost the same eigenvalues with the angular-momentum operator $\widehat{L}$ but is seems that the spin eigenvalues are short of "standardised" amounts at values $0, \frac{1}{2}, 1$ .For instance we can write for fermions:

$$S^2 = s(s+1)|_{\frac{1}{2}}$$

Trully,spin looks like a type of angular-momentum.Some scientists believe that spin could be angular momentum in the internal space that characterise particles. Let us start from the beginning.If $a = (a_i)$,$b = (b_i)$ are two n-dimensional vectors then the exterior product $a \times b = \tau_{ij}$ is a second rank antisymmetric tensor with dimension 6.Then we can write this tensor as:

$$\tau_{ij} = a_i b_j - a_j b_i$$

$$\tau_{ij} = 0$$

$$\tau_{ij} = -\tau_{ji}$$

In our space $K = R^8 \equiv C^4$ OR $K = R^4 + iM^4$ the vectors have the form :

$$k = (\overrightarrow{r}, T, \overrightarrow{m}, t) \equiv \overrightarrow{r} + i\overrightarrow{m} + T + it = (\overrightarrow{r} + T) + i(\overrightarrow{m} + t)$$

If we keep only the "length-mass" part then we can define the total angular-momentum in K as

$$L = \overrightarrow{k} \times \overrightarrow{p}_k$$

This tensor $L = (L_{ij})$ has $\frac{n(n+1)}{2} = \frac{6x5}{2} = 15$ components and can be written as a matrix:

$$L_{ij} = \begin{pmatrix} 0 & l_{12} & l_{13} & l_{14} & l_{15} & l_{16} \\ -l_{12} & 0 & l_{23} & l_{24} & l_{25} & l_{26} \\ -l_{13} & -l_{23} & 0 & l_{34} & l_{35} & l_{36} \\ -l_{14} & -l_{24} & -l_{34} & 0 & l_{45} & l_{46} \\ -l_{15} & -l_{25} & -l_{35} & -l_{45} & 0 & l_{56} \\ -l_{16} & -l_{26} & -l_{36} & -l_{46} & -l_{56} & 0 \end{pmatrix}$$

or

$$L_{ij} = \begin{pmatrix} L_R & L_{RM} \\ -L_{RM}^T & L_M \end{pmatrix}$$

where L_R is our usual angular-momentum tensor in R^3 ,the L_M is the angular-momentum in M^3 and the L_{RM} is the mixture between them.We could interpretate as spin the physical quantity that comes from the angular-momentum in the mass space M^3.This way the interaction angular-momentum of the spaces can be the interaction known as S-L(angular momentum-spin).But if someone tries to investigate the solution of the Klein-Gordon type equation in R^8 he will see that the radial part R(m) is depended of the angular momentum in mass space .As a conclusion all the mass eigenvalues are depended of the quantum number l in mass space.The Higg's mass value came for $l = 0$ and gave us $125, 173945 Gev/c^2$ which is slightly bigger because we have not include the analogous Darwin term which comes for $l = 0$.But where this Darwin term comes from?If we investigate the mass solution for $l = 0$ the Bessel 's functions are $\frac{sinx}{x} \simeq 1$ and we have missed to include the constant term of the power series $\frac{sinx}{x}$ which is the so called Darwin term in the mass space(let us call it m_d).For particles that the mass comes for spin 1 we will have corrections that comes from the interaction L_{RM} or without any confusion S-L.

7 Conclusion

The main quantities of our Cosmos are length,time and mass.A.Einstein managed to unify geometrically length with time, where the scalar quantity t(Newton's clock) became a coordinate.The most natural and logical step is to unify geometrically all the quantities by substituting the scalar quantity of mass with coordinates and only then,the mysteries of our Cosmos will be releaved to us ,the observers of R^4.The heart of existence, the main character is mass.Our Cosmos is just a problem of initial conditions

8 References

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