

Proof of Riemann Hypothesis:

1

$$\sum_{n=1}^m \mu(n) = O(\sqrt{m} \log(m)) \Leftrightarrow R.H$$

This can be taken as fact from other maths.

2

$$\sum_{n \leq m} \mu(n) = O(m^{\frac{1}{2}} \log(m))$$

$$\sum_{n \leq m} \mu(n) \left[ \frac{m}{n} \right] = 1$$

Proof:

Induction.

Therefore:

$$\frac{1}{2} + \sum_{A \leq n \leq m} \mu(n) = o\left( \sum_{A \leq n \leq m} \mu(n) \frac{m}{n} \right)$$

As can be seen, this satisfies Riemann hypothesis.