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k-Difference cordial labeling of graphs

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Abstract: In this paper we introduce new graph labeling called k-difference cordial labeling. Let G be a (p,q) graph and k be an integer, $2 \leq k \leq |V(G)|$. Let $f : V(G) \rightarrow \{1, 2, \cdots, k\}$ be a map. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denote the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we investigate k-difference cordial labeling behavior of star, m copies of star and we prove that every graph is a subgraph of a connected k-difference cordial graph. Also we investigate 3-difference cordial labeling behavior of some graphs.

Key Words: Path, complete graph, complete bipartite graph, star, k-difference cordial labeling, Smarandachely k-difference cordial labeling.

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§1. Introduction

All graphs in this paper are finite and simple. The graph labeling is applied in several areas of sciences and few of them are coding theory, astronomy, circuit design etc. For more details refer Gallian [2]. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge uv by a path uwv. The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. In [1], Cahit introduced the concept of cordial labeling of graphs. Recently Ponraj et al. [4], introduced difference cordial labeling of graphs. In this way we introduce k-difference cordial labeling of graphs. Also in this paper we investigate the k-difference cordial labeling behavior of star, m copies of star etc. $\lfloor x \rfloor$ denote the smallest integer less than or equal to x. Terms and results not here follows from Harary [3].

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§2. k-Difference Cordial Labeling

Definition 2.1 *Let G be a (p,q) graph and k be an integer $2 \leq k \leq |V(G)|$. Let $f : V(G) \to \{1, 2, \cdots, k\}$ be a function. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, and Smarandachely k-difference cordial labeling if $|v_f(i) - v_f(j)| > 1$ or $|e_f(0) - e_f(1)| > 1$, where $v_f(x)$ denote the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling or Smarandachely k-difference cordial labeling is called a k-difference cordial graph or Smarandachely k-difference cordial graph, respectively.*

Remark 2.2 (1) p-difference cordial labeling is simply a difference cordial labeling;

(2) 2-difference cordial labeling is a cordial labeling.

Theorem 2.3 *Every graph is a subgraph of a connected k-difference cordial graph.*

Proof Let G be (p,q) graph. Take k copies of graph K_p. Let G_i be the i^{th} copy of K_p. Take k copies of the $\overline{K}_{\binom{p}{2}}$ and the i^{th} copies of the $\overline{K}_{\binom{p}{2}}$ is denoted by G_i'. Let $V(G_i) = \{u_i^j : 1 \leq j \leq k, 1 \leq i \leq p\}$. Let $V(G_i') = \{v_i^j : 1 \leq j \leq k, 1 \leq i \leq p\}$. The vertex and edge set of super graph G^* of G is as follows:

Let $V(G^*) = \bigcup\limits_{i=1}^{k} V(G_i) \cup \bigcup\limits_{i=1}^{k} V(G_i') \cup \{w_i : 1 \leq i \leq k\} \cup \{w\}$.

$E(G^*) = \bigcup\limits_{i=1}^{k} E(G_i) \cup \{u_1^j v_i^j : 1 \leq i \leq \binom{p}{2}, 1 \leq j \leq k-1\} \cup \{u_1^k w, wv_i^k : 1 \leq i \leq \binom{p}{2}\} \cup \{u_p^j w_j : 1 \leq j \leq k\} \cup \{u_2^j u_2^{j+1} : 1 \leq j \leq k-1\} \cup \{w_1 w_2\}$.

Assign the label i to the vertices of G_i, $1 \leq i \leq k$. Then assign the label $i+1$ to the vertices of G_i', $1 \leq i \leq k-1$. Assign the label 1 to the vertices of G_k'. Then assign 2 to the vertex w. Finally assign the label i to the vertex w_i, $1 \leq i \leq k$. Clearly $v_f(i) = p + \binom{p}{2} + 1$, $i = 1, 3, \ldots, k$, $v_f(2) = p + \binom{p}{2} + 2$ and $e_f(1) = k\binom{p}{2} + k$, $e_f(0) = k\binom{p}{2} + k + 1$. Therefore G^* is a k-difference cordial graph. □

Theorem 2.4 *If k is even, then k-copies of star $K_{1,p}$ is k-difference cordial.*

Proof Let G_i be the i^{th} copy of the star $K_{1,p}$. Let $V(G_i) = \{u_j, v_i^j : 1 \leq j \leq k, 1 \leq i \leq p\}$ and $E(G_i) = \{u_j v_i^j : 1 \leq j \leq k, 1 \leq i \leq p\}$. Assign the label i to the vertex u_j, $1 \leq j \leq k$. Assign the label $i+1$ to the pendent vertices of G_i, $1 \leq i \leq \frac{k}{2}$. Assign the label $k - i + 1$ to the pendent vertices of $G_{\frac{k}{2}+i}$, $1 \leq i \leq \frac{k}{2} - 1$. Finally assign the label 1 to all the pendent vertices of the star G_k. Clearly, $v_f(i) = p + 1$, $1 \leq i \leq k$, $e_f(0) = e_f(1) = \dfrac{kp}{2}$. Therefore f is a k-difference cordial labeling of k-copies of the star $K_{1,p}$. □

Theorem 2.5 *If $n \equiv 0 \pmod{k}$ and $k \geq 6$, then the star $K_{1,n}$ is not k-difference cordial.*

Proof Let $n = kt$. Suppose f is a k-difference cordial labeling of $K_{1,n}$. Without loss of generality, we assume that the label of central vertex is r, $1 \leq r \leq k$. Clearly $v_f(i) = t$,

$1 \le i \le n$ and $i \ne r$, $v_f(r) = t+1$. Then $e_f(1) \le 2t$ and $e_f(0) \ge (k-2)t$. Now $e_f(0) \ge (k-2)t - 2t \ge (k-4)t \ge 2$, which is a contradiction. Thus f is not a k-difference cordial. □

Next we investigate 3-difference cordial behavior of some graph.

§3. 3-Difference Cordial Graphs

First we investigate the path.

Theorem 3.1 *Any path is* 3*-difference cordial.*

Proof Let $u_1 u_2 \ldots u_n$ be the path P_n. The proof is divided into cases following.

Case 1. $n \equiv 0 \pmod 6$.

Let $n = 6t$. Assign the label 1, 3, 2, 1, 3, 2 to the first consecutive 6 vertices of the path P_n. Then assign the label 2, 3, 1, 2, 3, 1 to the next 6 consecutive vertices. Then assign the label 1, 3, 2, 1, 3, 2 to the next six vertices and assign the label 2, 3, 1, 2, 3, 1 to the next six vertices. Then continue this process until we reach the vertex u_n.

Case 2. $n \equiv 1 \pmod 6$.

This implies $n-1 \equiv 0 \pmod 6$. Assign the label to the vertices of u_i, $1 \le i \le n-1$ as in case 1. If u_{n-1} receive the label 2, then assign the label 2 to the vertex u_n; if u_{n-1} receive the label 1, then assign the label 1 to the vertex u_n.

Case 3. $n \equiv 2 \pmod 6$.

Therefore $n-1 \equiv 1 \pmod 6$. As in case 2, assign the label to the vertices u_i, $1 \le i \le n-1$. Next assign the label 3 to u_n.

Case 4. $n \equiv 3 \pmod 6$.

This forces $n-1 \equiv 2 \pmod 6$. Assign the label to the vertices u_1, u_2, $\ldots u_{n-1}$ as in case 3. Assign the label 1 or 2 to u_n according as the vertex u_{n-2} receive the label 2 or 1.

Case 5. $n \equiv 4 \pmod 6$.

This implies $n-1 \equiv 3 \pmod 6$. As in case 4, assign the label to the vertices u_1, u_2, $\cdots$, u_{n-1}. Assign the label 2 or 1 to the vertex u_n according as the vertex u_{n-1} receive the label 1 to 2.

Case 6. $n \equiv 5 \pmod 6$.

This implies $n-1 \equiv 4 \pmod 6$. Assign the label to the vertices u_1, u_2, $\cdots$, u_{n-1} as in Case 5. Next assign the label 3 to u_n. □

Example 3.2 A 3-difference cordial labeling of the path P_9 is given in Figure 1.

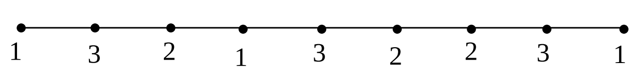

Figure 1

Corollary 3.3 *If $n \equiv 0, 3 \pmod 4$, then the cycle C_n is 3-difference cordial.*

Proof The vertex labeling of the path given in Theorem 3.1 is also a 3-difference cordial labeling of the cycle C_n. □

Theorem 3.4 *The star $K_{1,n}$ is 3-difference cordial iff $n \in \{1, 2, 3, 4, 5, 6, 7, 9\}$.*

Proof Let $V(K_{1,n}) = \{u, u_i : 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{uu_i : 1 \leq i \leq n\}$. Our proof is divided into cases following.

Case 1. $n \in \{1, 2, 3, 4, 5, 6, 7, 9\}$.

Assign the label 1 to u. The label of u_i is given in Table 1.

$n \backslash u_i$	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9
1	2								
2	2	3							
3	2	3	1						
4	2	3	1	2					
5	2	3	1	2	3				
6	2	3	1	2	3	2			
7	2	3	1	2	3	2	3		
9	2	3	1	2	3	2	3	1	2

Table 1

Case 2. $n \notin \{1, 2, 3, 4, 5, 6, 7, 9\}$.

Let $f(u) = x$ where $x \in \{1, 2, 3\}$. To get the edge label 1, the pendent vertices receive the label either $x - 1$ or $x + 1$.

Subcase 1. $n = 3t$.

Subcase 1a. $x = 1$ or $x = 3$.

When $x = 1$, $e_f(1) = t$ or $t + 1$ according as the pendent vertices receives t's 2 or (t+1)'s 2. Therefore $e_f(0) = 2t$ or $2t - 1$. Thus $e_f(0) - e_f(1) = t - 2 > 1$, $t > 4$ a contradiction. When $x = 3$, $e_f(1) = t$ or $t + 1$ according as the pendent vertices receives t's 2 or (t+1)'s 2. Therefore $e_f(0) = 2t$ or $2t - 1$. Thus $e_f(0) - e_f(1) = t$ or $t - 2$. Therefore, $e_f(0) - e_f(1) > 1$, a contradiction.

Subcase 1b. $x = 2$.

In this case, $e_f(1) = 2t$ or $2t + 1$ according as pendent vertices receives t's 2 or (t-2)'s 2. Therefore $e_f(0) = t$ or $t - 1$. $e_f(1) - e_f(0) = t$ or $t + 2$ as $t > 3$. Therefore, $e_f(0) - e_f(1) > 1$, a contradiction.

Subcase 2. $n = 3t + 1$.

Subcase 2a. $x = 1$ or 3.

Then $e_f(1) = t$ or $t+1$ according as pendent vertices receives t's 2 or (t+1)'s 2. Therefore $e_f(0) = 2t+1$ or $2t$. $e_f(0) - e_f(1) = t+1$ or $t-1$ as $t > 3$. Therefore, $e_f(0) - e_f(1) > 3$, a contradiction.

Subcase 2b. $x = 2$.

In this case $e_f(1) = 2t$ or $2t+1$ according as pendent vertices receives t's 1 and t's 3 and t's 1 and (t+3)'s 3. Therefore $e_f(0) = t+1$ or t. $e_f(1) - e_f(0) = t-1$ or t as $t > 3$. Therefore, $e_f(0) - e_f(1) > 1$, a contradiction.

Subcase 3. $n = 3t+2$.

Subcase 3a. $x = 1$ or 3.

This implies $e_f(1) = t+1$ and $e_f(0) = 2t+1$. $e_f(0) - e_f(1) = t$ as $t > 3$. Therefore, $e_f(0) - e_f(1) > 1$, a contradiction.

Subcase 3b. $x = 2$.

This implies $e_f(1) = 2t+2$ and $e_f(0) = t$. $e_f(1) - e_f(0) = t+2$ as $t > 1$. Therefore, $e_f(1) - e_f(0) > 1$, a contradiction. Thus $K_{1,n}$ is 3-difference cordial iff $n \in \{1,2,3,4,5,6,7,9\}$. □

Next, we research the complete graph.

Theorem 3.5 *The complete graph K_n is 3-difference cordial if and only if $n \in \{1,2,3,4,6,7,9,10\}$.*

Proof Let u_i, $1 \leq i \leq n$ be the vertices of K_n. The 3-difference cordial labeling of K_n, $n \in \{1,2,3,4,6,7,9,10\}$ is given in Table 2.

$n\backslash u_i$	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
1	1									
2	1	2								
3	1	2	3							
4	1	1	2	3						
6	1	1	2	2	3	3				
7	1	1	1	2	2	3	3			
9	1	1	1	2	2	2	3	3	3	
10	2	2	2	2	1	1	1	3	3	3

Table 2

Assume $n \notin \{1,2,3,4,6,7,9,10\}$. Suppose f is a 3-difference cordial labeling of K_n.

Case 1. $n \equiv 0 \pmod 3$.

Let $n = 3t$, $t > 3$. Then $v_f(0) = v_f(1) = v_f(2) = t$. This implies $e_f(0) = \binom{t}{2} + \binom{t}{2} + \binom{t}{2} +$

$t^2 = \frac{5t^2-3t}{2}$. Therefore $e_f(1) = t^2 + t^2 = 2t^2$. $e_f(0) - e_f(1) = \frac{5t^2-3t}{2} - 2t^2 > 1$ as $t > 3$, a contradiction.

Case 2. $n \equiv 1 \pmod 3$.

Let $n = 3t+1$, $t > 3$.

Subcase 1. $v_f(1) = t+1$.

Therefore $v_f(2) = v_f(3) = t$. This forces $e_f(0) = \binom{t+1}{2} + \binom{t}{2} + \binom{t}{2} + t(t+1) = \frac{1}{2}(5t^2+t)$. $e_f(1) = t(t+1) + t^2 = 2t^2 + t$. Then $e_f(0) - e_f(1) = \frac{1}{2}(5t^2+t) - (2t^2+t) > 1$ as $t > 3$, a contradiction.

Subcase 2. $v_f(3) = t+1$.

Similar to Subcase 1.

Subcase 3. $v_f(2) = t+1$.

Therefore $v_f(1) = v_f(3) = t$. In this case $e_f(0) = \frac{5t^2+t}{2}$ and $e_f(1) = t(t+1) + t(t+1) = 2t^2 + 2t$. This implies $e_f(0) - e_f(1) = \frac{5t^2+t}{2} - (2t^2+2t) > 1$ as $t > 3$, a contradiction.

Case 3. $n \equiv 2 \pmod 3$.

Let $n = 3t+2$, $t \geq 1$.

Subcase 1. $v_f(1) = t$.

Therefore $v_f(2) = v_f(3) = t+1$. This gives $e_f(0) = \binom{t}{2} + \binom{t+1}{2} + \binom{t+1}{2} + t(t+1) = \frac{5t^2+3t}{2}$ and $e_f(1) = t(t+1) + (t+1)^2 = 2t^2+3t+1$. This implies $e_f(0) - e_f(1) = \frac{5t^2+3t}{2} - (2t^2+3t+1) > 1$ as $t \geq 1$, a contradiction.

Subcase 2. $v_f(3) = t$.

Similar to Subcase 1.

Subcase 3. $v_f(2) = t$.

Therefore $v_f(1) = v_f(3) = t+1$. In this case $e_f(0) = \binom{t+1}{2} + \binom{t+1}{2} + \binom{t}{2} + (t+1)(t+1) = \frac{5t^2+5t+2}{2}$ and $e_f(1) = t(t+1) + t(t+1) = 2t^2 + 2t$. This implies $e_f(0) - e_f(1) = \frac{5t^2+5t+2}{2} - (2t^2+2t) > 1$ as $t \geq 1$, a contradiction. $\square$

Theorem 3.6 *If m is even, the complete bipartite graph $K_{m,n}$ $(m \leq n)$ is 3-difference cordial.*

Proof Let $V(K_{m,n}) = \{u_i, v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(K_{m,n}) = \{u_iv_j : 1 \leq i \leq$

$m, 1 \leq j \leq n\}$. Define a map $f : V(K_{m,n}) \to \{1, 2, 3\}$ by

$$\begin{array}{rlll}
f(u_i) & = 1, & 1 \leq i \leq \frac{m}{2} & \\
f(u_{\frac{m}{2}+i}) & = 2, & 1 \leq i \leq \frac{m}{2} & \\
f(v_i) & = 3, & 1 \leq i \leq \left\lceil \frac{m+n}{3} \right\rceil & \\
f(v_{\lceil \frac{m+n}{3} \rceil + i}) & = 1, & 1 \leq i \leq \left\lceil \frac{m+n}{3} \right\rceil - \frac{m}{2} - 1 & if \quad m+n \equiv 1, 2 \pmod 3 \\
 & & 1 \leq i \leq \left\lceil \frac{m+n}{3} \right\rceil - \frac{m}{2} & if \quad m+n \equiv 0 \pmod 3 \\
f(v_{2\lceil \frac{m+n}{3} \rceil - \frac{m}{2} - 1 + i}) & = 2, & 1 \leq i \leq n - 2\left\lceil \frac{m+n}{3} \right\rceil + \frac{m}{2} + 1 & if \quad m+n \equiv 1, 2 \pmod 3 \\
f(v_{2\lceil \frac{m+n}{3} \rceil - \frac{m}{2} + i}) & = 2, & 1 \leq i \leq n - 2\left\lceil \frac{m+n}{3} \right\rceil + \frac{m}{2} & if \quad m+n \equiv 0 \pmod 3
\end{array}$$

Since $e_f(0) = e_f(1) = \frac{mn}{2}$, f is a 3-difference cordial labeling of $K_{m,n}$. □

Example 3.7 A 3-difference cordial labeling of $K_{5,8}$ is given in Figure 2.

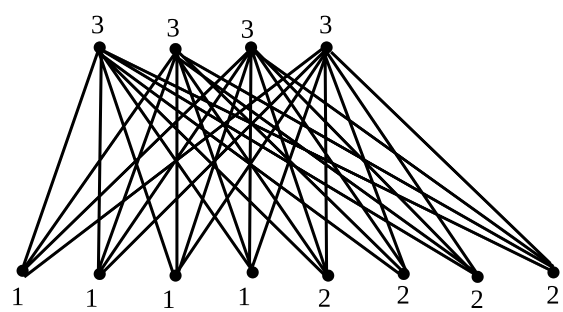

Figure 2

Next, we research some corona of graphs.

Theorem 3.8 *The comb $P_n \odot K_1$ is 3-difference cordial.*

Proof Let P_n be the path $u_1u_2 \ldots u_n$. Let $V(P_n \odot K_1) = V(P_n) \cup \{v_i : 1 \leq i \leq n\}$ and $E(P_n \odot K_1) = E(P_n) \cup \{u_iv_i : 1 \leq i \leq n\}$.

Case 1. $n \equiv 0 \pmod 6$.

Define a map $f : V(G) \to \{1, 2, 3\}$ by

$$\begin{array}{rclcll}
f(u_{6i-5}) & = & f(u_{6i}) & = & 1, & 1 \leq i \leq \frac{n}{6} \\
f(u_{6i-4}) & = & f(u_{6i-1}) & = & 3, & 1 \leq i \leq \frac{n}{6} \\
f(u_{6i-3}) & = & f(u_{6i-2}) & = & 2, & 1 \leq i \leq \frac{n}{6}.
\end{array}$$

In this case, $e_f(0) = n - 1$ and $e_f(1) = n$.

Case 2. $n \equiv 1 \pmod 6$.

Assign the label to the vertices u_i, v_i $(1 \leq i \leq n-1)$ as in case 1. Then assign the labels 1, 2 to the vertices u_n, v_n respectively. In this case, $e_f(0) = n - 1$, $e_f(1) = n$.

Case 3. $n \equiv 2 \pmod 6$.

As in Case 2, assign the label to the vertices u_i, v_i $(1 \le i \le n-1)$. Then assign the labels 3, 3 to the vertices u_n, v_n respectively. In this case, $e_f(0) = n$, $e_f(1) = n-1$.

Case 4. $n \equiv 3 \pmod 6$.

Assign the label to the vertices u_i, v_i $(1 \le i \le n-1)$ as in case 3. Then assign the labels 2, 1 to the vertices u_n, v_n respectively. In this case, $e_f(0) = n-1$, $e_f(1) = n$.

Case 5. $n \equiv 4 \pmod 6$.

As in Case 4, assign the label to the vertices u_i, v_i $(1 \le i \le n-1)$. Then assign the labels 2, 3 to the vertices u_n, v_n respectively. In this case, $e_f(0) = n-1$, $e_f(1) = n$.

Case 6. $n \equiv 5 \pmod 6$.

Assign the label to the vertices u_i, v_i $(1 \le i \le n-1)$ as in case 5. Then assign the labels 3, 1 to the vertices u_n, v_n respectively. In this case, $e_f(0) = n-1$, $e_f(1) = n$. Therefore $P_n \odot K_1$ is 3-difference cordial. □

Theorem 3.9 *$P_n \odot 2K_1$ is 3-difference cordial.*

Proof Let P_n be the path $u_1u_2\cdots u_n$. Let $V(P_n \odot 2K_1) = V(P_n) \cup \{v_i, w_i : 1 \le i \le n\}$ and $E(P_n \odot 2K_1) = E(P_n) \cup \{u_iv_i, u_iw_i : 1 \le i \le n\}$.

Case 1. n is even.

Define a map $f : V(P_n \odot 2K_1) \to \{1, 2, 3\}$ as follows:

$$\begin{aligned} f(u_{2i-1}) &= 1, \quad 1 \le i \le \tfrac{n}{2} \\ f(u_{2i}) &= 2, \quad 1 \le i \le \tfrac{n}{2} \\ f(v_{2i-1}) &= 1, \quad 1 \le i \le \tfrac{n}{2} \\ f(v_{2i}) &= 2, \quad 1 \le i \le \tfrac{n}{2} \\ f(w_i) &= 3, \quad 1 \le i \le \tfrac{n}{2}. \end{aligned}$$

In this case, $v_f(1) = v_f(2) = v_f(3) = n$, $e_f(0) = \frac{3n}{2}$ and $e_f(1) = \frac{3n}{2} - 1$.

Case 2. n is odd.

Define a map $f : V(P_n \odot 2K_1) \to \{1, 2, 3\}$ by $f(u_1) = 1$, $f(u_2) = 2$, $f(u_3) = 3$, $f(v_1) = f(v_3) = 1$, $f(w_1) = f(w_2) = 3$, $f(v_2) = f(w_3) = 2$,

$$\begin{aligned} f(u_{2i+2}) &= 2, \quad 1 \le i \le \tfrac{n-3}{2} \\ f(u_{2i+3}) &= 1, \quad 1 \le i \le \tfrac{n-3}{2} \\ f(v_{2i+2}) &= 2, \quad 1 \le i \le \tfrac{n-3}{2} \\ f(v_{2i+3}) &= 1, \quad 1 \le i \le \tfrac{n-3}{2} \\ f(w_{i+3}) &= 3, \quad 1 \le i \le n-3. \end{aligned}$$

Clearly, $v_f(1) = v_f(2) = v_f(3) = n$, $e_f(0) = e_f(1) = \frac{3n-1}{2}$. □

Next we research on quadrilateral snakes.

Theorem 3.10 *The quadrilateral snakes Q_n is 3-difference cordial.*

Proof Let P_n be the path $u_1u_2\cdots u_n$. Let $V(Q_n) = V(P_n) \cup \{v_i, w_i : 1 \le i \le n-1\}$ and $E(Q_n) = E(P_n) \cup \{u_iv_i, v_iw_i, w_iu_{i+1} : 1 \le i \le n-1\}$. Note that $|V(Q_n)| = 3n-2$ and $|E(Q_n)| = 4n-4$. Assign the label 1 to the path vertices u_i, $1 \le i \le n$. Then assign the labels 2, 3 to the vertices v_i, w_i $1 \le i \le n-1$ respectively. Since $v_f(1) = n$, $v_f(2) = v_f(3) = n-1$, $e_f(0) = e_f(1) = 2n-2$, f is a 3-difference cordial labeling. □

The next investigation is about graphs $B_{n,n}$, $S(K_{1,n})$, $S(B_{n,n})$.

Theorem 3.11 *The bistar $B_{n,n}$ is 3-difference cordial.*

Proof Let $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \le i \le n\}$ and $E(B_{n,n}) = \{uv, uu_i, vv_i : 1 \le i \le n\}$. Clearly $B_{n,n}$ has $2n+2$ vertices and $2n+1$ edges.

Case 1. $n \equiv 0 \pmod 3$.

Assign the label 1, 2 to the vertices u and v respectively. Then assign the label 1 to the vertices u_i, v_i $(1 \le i \le \frac{n}{3})$. Assign the label 2 to the vertices $u_{\frac{n}{3}+i}$, $v_{\frac{n}{3}+i}$ $(1 \le i \le \frac{n}{3})$. Finally assign the label 3 to the vertices $u_{\frac{2n}{3}+i}$, $v_{\frac{2n}{3}+i}$ $(1 \le i \le \frac{n}{3})$. In this case $e_f(1) = n+1$ and $e_f(0) = n$.

Case 2. $n \equiv 1 \pmod 3$.

Assign the labels to the vertices u, v, u_i, v_i $(1 \le i \le n-1)$ as in Case 1. Then assign the label 3, 2 to the vertices u_n, v_n respectively. In this case $e_f(1) = n$ and $e_f(0) = n+1$.

Case 3. $n \equiv 2 \pmod 3$.

As in Case 2, assign the label to the vertices u, v, u_i, v_i $(1 \le i \le n-1)$. Finally assign 1, 3 to the vertices u_n, v_n respectively. In this case $e_f(1) = n$ and $e_f(0) = n+1$. Hence the star $B_{n,n}$ is 3-difference cordial. □

Theorem 3.12 *The graph $S(K_{1,n})$ is 3-difference cordial.*

Proof Let $V(S(K_{1,n})) = \{u, u_i, v_i : 1 \le i \le n\}$ and $E(S(K_{1,n})) = \{uu_i, u_iv_i : 1 \le i \le n\}$. Clearly $S(K_{1,n})$ has $2n+1$ vertices and $2n$ edges.

Case 1. $n \equiv 0 \pmod 3$.

Define a map $f : V(S(K_{1,n})) \to \{1, 2, 3\}$ as follows: $f(u) = 2$,

$$\begin{aligned} f(u_i) &= 1, \quad 1 \le i \le t \\ f(u_{t+i}) &= 2, \quad 1 \le i \le 2t \\ f(v_i) &= 3, \quad 1 \le i \le 2t \\ f(v_{2t+i}) &= 1, \quad 1 \le i \le t. \end{aligned}$$

Case 2. $n \equiv 1 \pmod 3$.

As in Case 1, assign the label to the vertices u, u_i, v_i $(1 \leq i \leq n-1)$. Then assign the label 1, 3 to the vertices u_n, v_n respectively.

Case 3. $n \equiv 2 \pmod 3$.

As in Case 2, assign the label to the vertices u, u_i, v_i $(1 \leq i \leq n-1)$. Then assign the label 2, 1 to the vertices u_n, v_n respectively. f is a 3-difference cordial labeling follows from the following Table 3.

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$e_f(0)$	$e_f(1)$
$n = 3t$	$2t$	$2t+1$	$2t$	$3t$	$3t$
$n = 3t+1$	$2t+1$	$2t+1$	$2t+1$	$3t+1$	$3t+1$
$n = 3t+2$	$2t+2$	$2t+2$	$2t+1$	$3t+2$	$3t+2$

Table 3

Theorem 3.13 *$S(B_{n,n})$ is 3-difference cordial.*

Proof Let $V(S(B_{n,n})) = \{u, w, v, u_i, w_i, v_i, z_i : 1 \leq i \leq n\}$ and $E(S(B_{n,n})) = \{uw, wv, uu_i, u_iw_i, vv_i, v_iz_i : 1 \leq i \leq n\}$. Clearly $S(B_{n,n})$ has $4n+3$ vertices and $4n+2$ edges.

Case 1. $n \equiv 0 \pmod 3$.

Define a map $f : V(S(B_{n,n})) \to \{1, 2, 3\}$ by $f(u) = 1$, $f(w) = 3$, $f(v) = 2$,

$$\begin{aligned}
f(w_i) &= 2, \quad 1 \leq i \leq n \\
f(v_i) &= 1, \quad 1 \leq i \leq n \\
f(z_i) &= 3, \quad 1 \leq i \leq n \\
f(u_i) &= 1, \quad 1 \leq i \leq \tfrac{n}{3} \\
f(u_{\frac{n}{3}+i}) &= 2, \quad 1 \leq i \leq \tfrac{n}{3} \\
f(u_{\frac{2n}{3}+i}) &= 3, \quad 1 \leq i \leq \tfrac{n}{3}.
\end{aligned}$$

Case 2. $n \equiv 1 \pmod 3$.

As in Case 1, assign the label to the vertices u, w, v, u_i, v_i, w_i, z_i $(1 \leq i \leq n-1)$. Then assign the label 1, 2, 1, 3 to the vertices u_n, w_n, v_n, z_n respectively.

Case 3. $n \equiv 2 \pmod 3$.

As in Case 2, assign the label to the vertices u, w, v, u_i, v_i, w_i, z_i $(1 \leq i \leq n-1)$. Then assign the label 2, 2, 1, 3 to the vertices u_n, w_n, v_n, z_n respectively. f is a 3-difference cordial labeling follows from the following Table 4.

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 3$	$\frac{4n+3}{3}$	$\frac{4n+3}{3}$	$\frac{4n+3}{3}$	$\frac{4n+2}{2}$	$\frac{4n+2}{2}$
$n \equiv 1 \pmod 3$	$\frac{4n+5}{3}$	$\frac{4n+2}{3}$	$\frac{4n+2}{3}$	$\frac{4n+2}{2}$	$\frac{4n+2}{2}$
$n \equiv 2 \pmod 3$	$\frac{4n+4}{3}$	$\frac{4n+4}{3}$	$\frac{4n+1}{3}$	$\frac{4n+2}{2}$	$\frac{4n+2}{2}$

Table 4

Finally we investigate cycles $C_4^{(t)}$.

Theorem 3.14 *$C_4^{(t)}$ is 3-difference cordial.*

Proof Let u be the vertices of $C_4^{(t)}$ and i^{th} cycle of $C_4^{(t)}$ be $uu_1^iu_2^iu_3^iu$. Define a map f from the vertex set of $C_4^{(t)}$ to the set $\{1,2,3\}$ by $f(u)=1$, $f(u_2^i)=3$, $1 \le i \le t$, $f(u_1^i)=1$, $1 \le i \le t$, $f(u_3^i)=2$, $1 \le i \le t$. Clearly $v_f(1)=t+1$, $v_f(2)=v_f(3)=t$ and $e_f(0)=e_f(1)=2t$. Hence f is 3-difference cordial. $\square$

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