Is it possible to write down SU(2) electrodynamics?

By Victor Christianto,[1] email: victorchristianto@gmail.com

Introduction

This is a question that I asked in RG forum a few weeks ago.[2] The responses were quite interesting and insightful, so I guess it would be useful for others too. Hopefully you will find this article interesting.

Description:

It is known that Yang-Mills theory is part of classical field theory. Therefore it seems possible to write down SU(2) electrodynamics. What do you think? Your comments are welcome.

Answers:

1. **Daniel Baldomir**

   Dear Victor,

   the field equations that you have are not Maxwell's equations at all, which are macroscopic experimental laws that work everyday with fantastic accuracy. Notice that your Gauss gives more than an electric charge conservation and no linearization between fields-charge. The same happens for the div of B and Faraday´s law. You have written Ampere's law instead of Ampere-Maxwell with the displacement currents, if you write it for obtaining the coupling among all the equations as a wave equation, notice that the outcome is not at a simple wave equation.

   In fact you have for SU(2) three infenitesimal generators which give you three kind of electric charges instead of the usual electric charge. How do you make these compatible with the usual Electrodynamics? What could be the experimental result to purpose for showing this macroscopic classical Electrodynamics?

[1] URL: www.sci4God.com, http://researchgate.net/profile/VIctor_Christianto
[2] https://www.researchgate.net/post/Is_it_possible_to_write_down_SU2_electrodynamics

2. **Daniel Baldomir**

Dear Erik,

There are very different forms to represent Electrodynamics. One is through Maxwell equations, where the conserved quantity is the electric charge and the energy-momentum, which are Noether currents of the abelian U(1) group and the SO(3,1) Lorentz group. And the another is the Dirac equation where the spinors are introduced by the SU(2) group or in fact SU(2)xSU(2) group, as subgroups of the Lorentz group. Notice that in this case, the Dirac equation generalize Schrodinger equation to the relavistic form and Electrodynamics is introduced just by the electromagnetic potentials, but no Noether's currents are linked with the isospin group, as it happens for the Yang-Mills equations when the quark conservation is reached. In the Standard Model the SU(2) group plays a very different role that in Electrodynamics in the two previous discussed previously.

3. **Erik Baxter**

Ah I'm terribly sorry, I rethought my answer and deleted it; it's a bad habit. Here is the revised one: feel free to critique it if you want, I will not edit it again:

I agree with all of the points in the above analysis - your system is interesting as a toy model but seems very unphysical; as Daniel says, you would have violation of charge conservation and possible magnetic monopoles, which violates the basic features of Maxwell's theory. The validity of this theory in terms of experimental results confirming it cannot possibly be overstated - you'd need a very good motivation for this to "mean something" to physics. (e.g. you mention a 'non-zero charge density in the vacuum' - Wouldn't this again naively mean a single earthed wire would experience a current in a vacuum? So haven't you got a built-in violation of the 1st law of thermodynamics?)

Other points though (I accept these may be naive or plain wrong, but feel free to enlighten me if so!):

If you are defining the cross-product of two elements of SU(2) in the usual way, then like Daniel said, this would still give you 3 degrees of freedom in the field, leading to 3 charges (or similar) to 'fix' the system. After all your potential A is going to be SU(2)-valued. An element in SU(2) can be given by a 2x2 traceless anti-hermitian matrix containing one (independent) pair of complex numbers. Hence 3 independent degrees of freedom (When I think "three charges", I think of the quark model and the 'flavour' charge: but unless I am mistaken that's SU(3) and well understood!)

Also in your conclusions, on first glance:

a) I'm not so sure: doesn't it in fact just fatally violate Noether's theorem in some way?

c) How are you defining those derivatives in the original four Maxwell-like equations? If we're dealing with quantities in SU(2), surely those derivatives will need to be covariant, not ordinary? If not, then it seems like all you've got is SU(2) YM theory but without the gauge invariance…

d) What meaning is there in light speed being a vector instead of a scalar? This seems like an enormously onerous thing to come out of a theory. Have you considered how this affects basic Lorenz invariance, and hence everything afterwards (both SR and GR)? I haven't done the analysis but I hope someone has!

e) I wonder if you can give me details about the Higgs boson mass calculation - this is usually represented by a scalar field, not an SU(2) field, so a quick summary would be amazing

4. **Stam Nicolis**

Electrodynamics is defined by a U(1) gauge symmetry, not an SU(2) gauge symmetry. So the term ``SU(2) electrodymanics'' is meaningless. It's known, since the work of Yang and Mills how to write theories, invariant under any compact Lie group of local transformations and, since the work of 't Hooft and Veltman, how to describe their consistent quantization.

if the Lie group is non-compact, however, then it isn't known how to describe its consistent quantization. The typical example is gravity, where the gauge group is the group of diffeomorphisms.

Isospin is a global, not gauged, symmetry (of the strong interactions); and the electroweak interactions have gauge group $SU(2)_L$ x $U(1)_Y$ where $U(1)_Y$ is weak hypercharge. This gauge group gets broken, by the Brout-Englert-Higgs mechanism, to $U(1)_{em}$ i.e. the gauge group of electromagnetism.

5. **Victor Christianto**

Dear Dr. Stam Nicolis,

Thank you for your remarks, I really appreciate your correction. We admit that the term SU(2) electrodynamics may not be well-known. But let me cite three papers discussing electrodynamics different from U(1) context:

a. Asim Barut et al's paper in Found. Physics, 1982. They use the term "Electrodynamics in terms of functions over the group of SU(2)"

b. A.D. Boozer in his Am.J. Phys. vol. 79 (2011) paper, he uses the term: "color electrodynamics".

c. Other has used term such as "Gauss Law" in Yang Mills Theory.

See below.

So perhaps this discussion over the classical interpretation of Yang-Mills theory may be not merely useless.

Best wishes,

6. **Victor Christianto**

Dear Dr. Erik Baxter:

Thank you for your comments, and sorry for this late reply.

You wrote "you'd need a very good motivation for this to "mean something" to physics". Allow me to give you a few reasoning for the above paper discussing SU(2) electrodynamics:

a. It is known that the Lagrangian of Yang-Mills is just the Lagrangian of Maxwell plus some nonlinear terms. It seems to suggest that it is possible to discuss Yang-Mills theory in terms of classical electromagnetic.

b. There is a book written by Boris Kosyakov with title: An Introduction to Classical Particle and Field. He discuss Yang-Mills theory as part of classical field theory. Unfortunately, he does not discuss the electromagnetic interpretation of Yang-Mills Theory, unlike A.D. Boozer's paper (2011).

c. It can be shown that Yang-Mills theory can be derived from quaternion algebra. In the meantime, it is known that Maxwell himself once used quaternion to express his equations. Therefore it seems possible to discuss Yang-Mills theory in terms of electromagnetic fields. See our paper below from 2007.

d. We need some new approaches if we wish to go beyond the Standard Model in a meaningful way. And one way to do that is to elucidate the electromagnetic interpretation of Yang-Mills theory.

The above reasoning are just a few which motivate us to explore this approach. Plus one story which I experienced myself: Back in Moscow around April or May 2009, after listening a long lecture by an old professor concerning Yang-Mills theory, I decided to move forward and ask the professor: "Professor, we know that the Lagrangian of Yang-Mills theory is just Lagrangian of Maxwell plus some nonlinear term, so do you think it is possible to discuss Yang-Mills theory in terms of classical electromagnetic fields?" He starred at me for a moment, and then replied with a mysterious smile: "Of course, of course you can." (The exact actual conversation may be somewhat different, but that was the question that I asked him.) After you read this story, I hope you begin to see our motivation.

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Now, pertaining to your question on calculation of Higgs boson mass. That problem is beyond me, but I can mention a paper by Prof. Bo Lehnert which discusses how to derive a Higgs-like particle mass from his Revised Quantum Electrodynamics. No, I do not say that he derives Higgs boson mass. Instead, he suggests that what CERN observed in their particle detector is not actually Higgs boson, but a Higgs-like particle. He also discuss in another paper about strong interaction. See his papers below.

Hopefully the above reply is sufficient.

Thank you for your kind attention and best wishes

7. Stam Nicolis

The Lagrangian of Yang-Mills is definitely not the Lagrangian of electrodynamics along with some non-linear terms-that statement is, simply, wrong. It's pointless to make such wrong statements, when the correct statements are easily available, in any textbook on quantum field theory.

The equations of motion, obtained by varying the Yang-Mills action, are non-linear PDEs and describe classical vacuum configurations of the quantum theory, as is, always, the case.

If the gauge group is SU(2) then the algebra is that of quaternions-if not, it's not.

So far for the mathematics. Regarding the physics, once more, electromagnetism is a U(1) gauge theory and is unified with the weak interactions in a SU(2)_L x $U(1)_Y$ gauge theory, that's broken to $U(1)_{em}$ by the Brout-Englert-Higgs mechanism in a way that's described in all courses on the subject. The two U(1) groups have totally different meaning.

Gauss' law in electrodynamics is a constraint, since it doesn't contain time derivatives. There's a similar constraint among the equations of motion in any Yang-Mills theory and that constraint carries the same name-but it doesn't have anything to do with electromagnetism.

While Boozer uses the *words* ``color electrodynamics'', what he *means* is a gauge theory with a group other than U(1). It's misleading terminology, tthat's all. He's solving the classical equations of motion for Yang-Mills fields coupled to corresponding charges.

8. **Dmitri Martila**

Victor: "It can be shown that Yang-Mills theory can be derived from quaternion algebra."

How far are you from the Millennium Prize of 1 000 000 USD?

9. **Stam Nicolis**

That the SU(2) algebra is the algebra of quaternions doesn't imply anything about whether the quantum Yang-Mills theory, with that gauge group, has a mass gap (which is a special case of the corresponding Millenium problem). However it should be recalled that it's known, since the early 1980s, how the Yang-Mills theory, based on the SU(2) gauge group, http://journals.aps.org/prd/abstract/10.1103/PhysRevD.21.2308 , behaves quantitatively. And similar studies have become standard for any compact Lie group.

(The Millenium problem involves mathematics, not physics. It involves developing the mathematical tools for proving the existence of the mass gap. Computer simulations indicate that quantum Yang-Mills theory does have a mass gap-but it's not possible to prove that the extrapolation from the finite lattice and finite sample size data to the continuum and infinite sample size is controlled.)

In fact, as Polyakov stressed in the 1970s, any compact gauge group has a confining phase, at strong coupling-the non-trivial statement is to prove whether, as the coupling is varied, there is a phase transition to a deconfining phase.

10. **Stoil Donev**

Dear Victor!

My viewpoint may be found in the book "Geometric view on Photon-Like Objects", ArXiv, math-phys: 1210.8323, authors: Stoil Donev, Maria Tashkova.

Good reading!

Stoil.

Version: 1.0, date: July 28th, 2016.
VC