Neutrosophic Crisp Probability Theory & Decision Making Process

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Abstract

Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent. For that purpose, it is natural to adopt the value from the selected set with highest degree of truth-membership, indeterminacy membership and least degree of falsity-membership on the decision set. These factors indicate that a decision making process takes place in neutrosophic environment. In this paper, we introduce and study the probability of neutrosophic crisp sets. After giving the fundamental definitions and operations, we obtain several properties and discuss the relationship between them. These notions can help researchers and make great use in the future in making algorithms to solve problems and manage between these notions to produce a new application or new algorithm of solving decision support problems. Possible applications to mathematical computer sciences are touched upon.

Keyword

Neutrosophic set, Neutrosophic probability, Neutrosophic crisp set, Intuitionistic neutrosophic set.

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 42] such as the neutrosophic set theory. The fundamental concepts of neutrosophic set, introduced by Smarandache in [37, 38, 39, 40], and Salama et al. in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21], provides a natural foundation for treating mathematically the neutrosophic phenomena which pervasively exist in our real world and for building new branches of neutrosophic mathematics.

In this paper, we introduce and study the probability of neutrosophic crisp sets. After giving the fundamental definitions and operations, we obtain several properties, and discuss the relationship between neutrosophic crisp sets and others.

2 Terminology

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [37, 38, 39, 40], and Salama et al. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. Smarandache introduced the neutrosophic components *T, I, F* — which represent the membership, indeterminacy and non-membership values respectively, which are included into the nonstandard unit interval.

2.1 Example 2.1 [37, 39]

Let us consider a neutrosophic set, a collection of possible locations (positions) of particle *x* and let A and B two neutrosophic sets.

One can say, by language abuse, that any particle *x* neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between $^{-}0$ and 1^{+}.

For example: *x* (0.5, 0.2, 0.3) belongs to A (which means a probability of 50% that the particle *x* is in A, a probability of 30% that *x* is not in A, and the rest is undecidable); or *y* (0, 0, 1) belongs to A (which normally means *y* is not for sure in A); or *z* (0, 1, 0) belongs to A (which means one does know absolutely nothing about *z* affiliation with A).

More general, *x*((0.2-0.3), (0.4—0.45) $\cup$ [0.50-0.51,{0.2,0.24,0.28}) belongs to the set, which means: with a probability in between 20-30%, the particle *x* is in a position of A (one cannot find an exact approximation because of various sources used); with a probability of 20% or 24% or 28%, *x* is not in A; the indeterminacy related to the appurtenance of *x* to A is in between 40-45% or between 50-51% (limits included).

The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and, in this case, *n-sup* = 30% + 51% + 28% > 100.

Definition 2.1 [14, 15, 21]

A neutrosophic crisp set (NCS for short) $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ which are subsets on X, and every crisp set in X is obviously a NCS having the form $\langle A_1, A_2, A_3 \rangle$.

Definition 2.2 [21]

The object having the form $A = \langle A_1, A_2, A_3 \rangle$ is called

1) *Neutrosophic Crisp Set with Type I* if it satisfies $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$ (NCS-Type I for short).
2) *Neutrosophic Crisp Set with Type II* if it satisfies $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$ and $A_1 \cup A_2 \cup A_3 = X$ (NCS-Type II for short).
3) *Neutrosophic Crisp Set with Type III* if it satisfies $A_1 \cap A_2 \cap A_3 = \phi$ and $A_1 \cup A_2 \cup A_3 = X$ (NCS-Type III for short).

Definition 2.3

1. *Neutrosophic Set* [7]: Let X be a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$, where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-membership (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where

$$0^{-} \leq \mu_A(x), \sigma_A(x), \nu_A(x) \leq 1^{+}$$

and

$$0^{-} \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^{+}.$$

2. *Neutrosophic Intuitionistic Set of Type 1* [8]: Let X be a non-empty fixed set. A neutrosophic intuitionistic set of type 1 (NIS1 for short) set A is an object having the form $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$, where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-membership (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where

$$0^{-} \leq \mu_A(x), \sigma_A(x), \nu_A(x) \leq 1^{+}$$

and the functions satisfy the condition

$$\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \leq 0.5$$

and

$$0^{-} \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^{+}.$$

3. *Neutrosophic Intuitionistic Set of Type 2* [41]: Let X be a non-empty fixed set. A neutrosophic intuitionistic set of type 2 A (NIS2 for short) is an object having the form $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of membership function (namely $\mu_A(x)$), the degree of

indeterminacy (namely $\sigma_A(x)$), and the degree of non-membership (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where

$$0.5 \le \mu_A(x), \sigma_A(x), \nu_A(x)$$

and the functions satisfy the condition

$$\mu_A(x) \wedge \sigma_A(x) \le 0.5,\ \mu_A(x) \wedge \nu_A(x) \le 0.5,\ \sigma_A(x) \wedge \nu_A(x) \le 0.5,$$

and

$$^-0 \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 2^+.$$

A neutrosophic crisp with three types the object $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ which are subsets on X, and every crisp set in X is obviously a NCS having the form $\langle A_1, A_2, A_3 \rangle$. Every neutrosophic set $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ on X is obviously a NS having the form $\langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$.

Salama et al in [14, 15, 21] constructed the tools for developed neutrosophic crisp set and introduced the NCS ϕ_N, X_N in X.

Remark 2.1

The neutrosophic intuitionistic set is a neutrosophic set, but the neutrosophic set is not a neutrosophic intuitionistic set in general. Neutrosophic crisp sets with three types are neutrosophic crisp set.

3 The Probability of Neutrosophic Crisp Sets

If an experiment produces indeterminacy, that is called a neutrosophic experiment. Collecting all results, including the indeterminacy, we get the neutrosophic sample space (or the neutrosophic probability space) of the experiment. The neutrosophic power set of the neutrosophic sample space is formed by all different collections (that may or may not include the indeterminacy) of possible results. These collections are called neutrosophic events.

In classical experimental, the probability is

$$\left(\frac{\text{number of times event A occurs}}{\text{total number of trials}} \right).$$

Similarly, Smarandache in [16, 17, 18] introduced the Neutrosophic Experimental Probability, which is:

$$\left(\frac{\text{number of times event A occurs}}{\text{total number of trials}}, \frac{\text{number of times indeterminacy occurs}}{\text{total number of trials}}, \frac{\text{number of times event A does not occur}}{\text{total number of trials}}\right)$$

Probability of NCS is a generalization of the classical probability in which the chance that an event $A=\langle A_1, A_2, A_3\rangle$ to occur is:

$$P(A_1)\ true, P(A_2)\ \text{indeterminate}, P(A_3)\ false,$$

on a sample space X, or $NP(A)=\langle P(A_1), P(A_2), P(A_3)\rangle$.

A subspace of the universal set, endowed with a neutrosophic probability defined for each of its subsets, forms a probability neutrosophic crisp space.

Definition 3.1

Let X be a non- empty set and A be any type of neutrosophic crisp set on a space X, then the neutrosophic probability is a mapping $NP: X \to [0,1]^3$, $NP(A)=\langle P(A_1), P(A_2), P(A_3)\rangle$, that is the probability of a neutrosophic crisp set that has the property that —

$$NP(A)=\begin{cases}(p_1, p_2, p_3) & \text{where } p_{1,2,3}\in[0,1]\\ 0 & \text{if } p_1, p_2, p_3 < o\end{cases}.$$

Remark 3.1

1. In case if $A=\langle A_1, A_2, A_3\rangle$ is NCS, then

$$^-0 \le P(A_1)+P(A_2)+P(A_3) \le 3^+.$$

2. In case if $A=\langle A_1, A_2, A_3\rangle$ is NCS-Type I, then $0 \le P(A_1)+P(A_2)+P(A_3) \le 2$.

3. The Probability of NCS-Type II is a neutrosophic crisp set where

$$^-0 \le P(A_1)+P(A_2)+P(A_3) \le 2^+.$$

4. The Probability of NCS-Type III is a neutrosophic crisp set where

$$^-0 \le P(A_1)+P(A_2)+P(A_3) \le 3^+.$$

Probability Axioms of NCS Axioms

1. The Probability of neutrosophic crisp set and NCS-Type III A on X $NP(A)=\langle P(A_1), P(A_2), P(A_3)\rangle$ where $P(A_1)\ge 0, P(A_2)\ge 0, P(A_3)\ge 0$ or

$$NP(A)=\begin{cases}(p_1, p_2, p_3) & \text{where } p_{1,2,3}\in[0,1]\\ 0 & \text{if } p_1, p_2, p_3 < o\end{cases}.$$

2. The probability of neutrosophic crisp set and NCS-Type IIIs A on X $NP(A)=\langle P(A_1), P(A_2), P(A_3)\rangle$ where $^-0 \le p(A_1)+p(A_2)+p(A_3) \le 3^+$.

3. Bounding the probability of neutrosophic crisp set and NCS-Type III
$NP(A) = \langle P(A_1), P(A_2), P(A_3) \rangle$ where $1 \geq P(A_1) \geq 0, P(A_2) \geq 0, P(A_3) \geq 0$.

4. Addition law for any two neutrosophic crisp sets or NCS-Type III

$$NP(A \cup B) =< (P(A_1) + P(B_1) - P(A_1 \cap B_1),$$
$$(P(A_2) + P(B_2) - P(A_2 \cap B_2), \ (P(A_3) + P(B_3) - P(A_3 \cap B_3) >$$

if

$A \cap B = \phi_N$, then $NP(A \cap B) = NP(\phi_N)$.

$$NP(A \cup B) =< NP(A_1) + NP(B_1) - NP(\phi_{N_1}), NP(A_2) + NP(B_2) - NP(\phi_{N_2}),$$
$$NP(A_3) + NP(B_3) - NP(\phi_{N_3}).$$

Since our main purpose is to construct the tools for developing probability of neutrosophic crisp sets, we must introduce the following —

1. Probability of neutrosophic crisp empty set with three types ($NP(\phi_N)$ for short) may be defined as four types:

Type 1: $NP(\phi_N) = \langle P(\phi), P(\phi), P(X) \rangle =< 0,0,1 >$;

Type 2: $NP(\phi_N) = \langle P(\phi), P(X), P(X) \rangle =< 0,1,1 >$;

Type 3: $NP(\phi_N) = \langle P(\phi), P(\phi), P(\phi) \rangle =< 0,0,0 >$;

Type 4: $NP(\phi_N) = \langle P(\phi), P(X), P(\phi) \rangle =< 0,1,0 >$.

2. Probability of neutrosophic crisp universal and NCS-Type III universal sets ($NP(X_N)$ for short) may be defined as four types —

Type 1: $NP(X_N) = \langle P(X), P(\phi), P(\phi) \rangle =< 1,0,0 >$;

Type 2: $NP(X_N) = \langle P(X), P(X), P(\phi) \rangle =< 1,1,0 >$;

Type 3: $NP(X_N) = \langle P(X), P(X), P(X) \rangle =< 1,1,1 >$;

Type 4: $NP(X_N) = \langle P(X), P(\phi), P(X) \rangle =< 1,0,1 >$.

Remark 3.2

$NP(X_N) = 1_N$, $NP(\phi_N) = O_N$, where $1_N, O_N$ are in Definition 2.1 [6], or equals any type for 1_N.

Definition 3.2 (Monotonicity)

Let X be a non-empty set, and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$ with

$$NP(A) = \langle P(A_1), P(A_2), P(A_3) \rangle, NP(B) = \langle P(B_1), P(B_2), P(B_3) \rangle,$$

then we may consider two possible definitions for subsets ($A \subseteq B$) —

Type1:

$$NP(A) \leq NP(B) \Leftrightarrow P(A_1) \leq P(B_1), P(A_2) \leq P(B_2) \text{ and } \mathrm{P}(A_3) \geq P(B_3),$$

or Type2:

$$NP(A) \leq NP(B) \Leftrightarrow P(A_1) \leq P(B_1), P(A_2) \geq P(B_2) \text{ and } \mathrm{P}(A_3) \geq P(B_3).$$

Definition 3.3

Let X be a non-empty set, and NCSs A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$ be NCSs.

Then —

1. $NP(A \cap B)$ may be defined two types as —

Type1:

$$NP(A \cap B) = \langle P(A_1 \cap B_1), P(A_2 \cap B_2), P(A_3 \cup B_3) \rangle \text{, or}$$

Type2:

$$NP(A \cap B) = \langle P(A_1 \cap B_1), P(A_2 \cup B_2), P(A_3 \cup B_3) \rangle.$$

2. $NP(A \cup B)$ may be defined two types as:

Type1:

$$NP(A \cup B) = \langle P(A_1 \cup B_1), P(A_2 \cap B_2), P(A_3 \cap B_3) \rangle,$$

or Type 2:

$$NP(A \cup B) = \langle P(A_1 \cup B_1), P(A_2 \cup B_2), P(A_3 \cap B_3) \rangle.$$

3. $NP(A^c)$ may be defined by three types:

Type1:

$$NP(A^c) = \left\langle P(A_1{}^c), P(A_2^c), P(A_3^c) \right\rangle = < (1 - A_1), (1 - A_2), (1 - A_3) >$$

or Type2:

$$NP(A^c) = \left\langle P(A_3), P(A_2^c), P(A_1) \right\rangle$$

or Type3:

$$NP(A^c) = \left\langle P(A_3), P(A_2), P(A_1) \right\rangle.$$

Proposition 3.1

Let A and B in the form $A=\langle A_1,A_2,A_3\rangle$, $B=\langle B_1,B_2,B_3\rangle$ be NCSs on a non-empty set X.

Then —

$$NP(A)^c + NP(A) =< (1,1,1> \text{ or } NP(X_N) = 1_N \text{, or = any type of} 1_N .$$
$$NP(A-B) =< (P(A_1) - P(A_1 \cap B_1)), (P(A_2) - P(A_2 \cap B_2)),$$
$$(P(A_3) - P(A_3 \cap B_3) >$$
$$NP(A/B) =< \frac{NP(A_1)}{NP(A_1 \cap B_1)}, \frac{NP(A_2)}{NP(A_2 \cap B_2)}, \frac{NP(A_3)}{NP(A_3 \cap B_3)} >.$$

Proposition 3.1

Let A and B in the form $A=\langle A_1,A_2,A_3\rangle$, $B=\langle B_1,B_2,B_3\rangle$ are NCSs on a non-empty set X and p , p_N are NCSs.
Then

$$NP(p) = \left\langle \frac{1}{n(X)}, \frac{1}{n(X)}, \frac{1}{n(X)} \right\rangle;$$
$$NP(p_N) = \left\langle 0, \frac{1}{n(X)}, 1 - \frac{1}{n(X)} \right\rangle.$$

Example 3.1

1. Let $X = \{a,b,c,d\}$ and A , B are two neutrosophic crisp events on X defined by $A = \langle \{a\}, \{b,c\}, \{c,d\} \rangle$, $B = \langle \{a,b\}, \{a,c\}, \{c\} \rangle$, $p = \langle \{a\}, \{c\}, \{d\} \rangle$ then see that $NP(A) = \langle 0.25, 0.5, 0.5 \rangle$, $NP(B) = \langle 0.5, 0.5, 0.25 \rangle$, $NP(p) = \langle 0.25, 0.25, 0.25 \rangle$, one can compute all probabilities from definitions.

2. If $A = \langle \{\phi\}, \{b,c\}, \{\phi\} \rangle$ and $B = \langle \{\phi\}, \{d\}, \{\phi\} \rangle$ are neutrosophic crisp sets on X. Then —

$$A \cap B = \langle \{\phi\}, \{\phi\}, \{\phi\} \rangle \text{ and } NP(A \cap B) = \langle 0,0,0 \rangle = 0_N ,$$

$$A \cap B = \langle \{\phi\}, \{b,c,d\}, \{\phi\} \rangle \text{ and } NP(A \cap B) = \langle 0, 0.75, 0 \rangle \neq 0_N .$$

Example 3.2

Let $X = \{a,b,c,d,e,f\}$,

$A = \langle \{a,b,c,d\}, \{e\}, \{f\} \rangle$, $D = \langle \{a,b\}, \{e,c\}, \{f,d\} \rangle$ be a NCS-Type 2,

$B = \langle \{a,b,c\},\{d\},\{e\} \rangle$ be a NCT-Type I but not NCS-Type II, III,

$C = \langle \{a,b\},\{c,d\},\{e,f,a\} \rangle$ be a NCS-Type III, but not NCS-Type I, II,

$E = \langle \{a,b,c,d,e\},\{c,d\},\{e,f,a\} \rangle$,

$F = \langle \{a,b,c,d,e\},\phi,\{e,f,a,d,c,b\} \rangle$.

We can compute the probabilities for NCSs by the following:

$$NP(A) = \left\langle \frac{4}{6},\frac{1}{6},\frac{1}{6} \right\rangle,$$

$$NP(D) = \left\langle \frac{2}{6},\frac{2}{6},\frac{2}{6} \right\rangle,$$

$$NP(B) = \left\langle \frac{3}{6},\frac{1}{6},\frac{1}{6} \right\rangle,$$

$$NP(C) = \left\langle \frac{2}{6},\frac{2}{6},\frac{3}{6} \right\rangle,$$

$$NP(E) = \left\langle \frac{4}{6},\frac{2}{6},\frac{3}{6} \right\rangle,$$

$$NP(F) = \left\langle \frac{5}{6},0,\frac{6}{6} \right\rangle.$$

Remark 3.3

The probabilities of a neutrosophic crisp set are neutrosophic sets.

Example 3.3

Let $X = \{a,b,c,d\}$, $A = \langle \{a,b\},\{c\},\{d\} \rangle$, $B = \langle \{a\},\{c\},\{d,b\} \rangle$ are NCS-Type I on X and $U_1 = \langle \{a,b\},\{c,d\},\{a,d\} \rangle$, $U_2 = \langle \{a,b,c\},\{c\},\{d\} \rangle$ are NCS-Type III on X; then we can find the following operations —

1. *Union, intersection, complement, difference and its probabilities.*

a) Type1: $A \cap B = \langle \{a\},\{c\},\{d,b\} \rangle$, $NP(A \cap B) = \langle 0.25, 0.25, 0.5\} \rangle$ and
Type 2,3: $A \cap B = \langle \{a\},\{c\},\{d,b\} \rangle$, $NP(A \cap B) = \langle 0.25, 0.25, 0.5\} \rangle$.

2. $NP(A - B)$ *may be equals.*

Type1: $NP(A - B) =< 0.25,0,0 >$, Type 2: $NP(A - B) =< 0.25,0,0 >$,
Type 3: $NP(A - B) =< 0.25,0,0 >$,
b) Type 2: $A \cup B = \langle \{a,b\},\{c\},\{d\} \rangle$, $NP(A \cup B) = \langle 0.5, 0.25, 0.25\} \rangle$ and
Type 2: $A \cup B = \langle \{a.b\},\{c\},\{d\} \rangle$ $NP(A \cup B) = \langle 0.5, 0.25, 0.25\} \rangle$.

c) Type1: $A^c = \langle \{c,d\} \ , \{a,b,d\}, \{a,b,c\} \rangle$ NCS-Type III set on X,

$NP(A^c) = \langle 0.5, 0.75, 0.75 \rangle$.

Type2: $A^c = \langle \{d\} \ , \{a,b,d\}, \{a,b\} \rangle$ NCS-Type III on X,

$NP(A^c) = \langle 0.25, 0.75, 0.5 \rangle$.

Type3: $A^c = \langle \{d\} \ , \{c\}, \{a,b\} \rangle$ NCS-Type III on X,

$NP(A^c) = \langle 0.75, 0.75, 0.5 \rangle$.

d) Type 1: $B^c = \langle \{b,c,d\}, \{a,b,d\}, \{a,c\} \rangle$ be NCS-Type III on X ,

$NP(B^c) = \langle 0.75, 0.75, 0.5 \rangle$

Type 2: $B^c = \langle \{b,d\}, \{c\}, \{a\} \rangle$ NCS-Type I on X, and $NP(B^c) =$
$\langle 0.5, 0.25, 0.25 \rangle$.

Type 3: $B^c = \langle \{b,d\}, \{a,b,d\}, \{a\} \rangle$ NCS-Type III on X and $NP(B^c) =$
$\langle 0.5, 0.75, 0.25 \rangle$.

e) Type 1: $U_1 \cup U_2 = \langle \{a,b,c\}, \{c,d\}, \{a,d\} \rangle$, NCS-Type III,

$NP(U_1 \cup U_2) = \langle \{0.75, 0.5, 0.5 \rangle$,

Type 2: $U_1 \cup U_2 = \langle \{a,b,c\}, \{c\}, \{a,d\} \rangle$, $NP(U_1 \cup U_2) = \langle \{0.75, 0.25, 0.5 \rangle$.

f) Type1: $U_1 \cap U_2 = \langle \{a,b\}, \{c,d\}, \{a,d\} \rangle$, NCS-Type III,

$NP(U_1 \cap U_2) = \langle 0.5, 0.5, 0.5 \rangle$,

Type2: $U_1 \cap U_2 = \langle \{a,b\}, \{c\}, \{a,d\} \rangle$, NCS-Type III, and

$NP(U_1 \cap U_2) = \langle 0.5, 0.25, 0.5 \rangle$,

g) Type 1: $U_1{}^c = \langle \{c,d\}, \{a,b\}, \{c,b\} \rangle$, NCS-Type III and

$NP(U_1{}^c) = \langle 0.5, 0.5, 0.5 \rangle$

Type 2: $U_1{}^c = \langle \{a,d\}, \{c,d\}, \{a,b\} \rangle$, NCS-Type III and

$NP(U_1{}^c) = \langle 0.5, 0.5, 0.5 \rangle$

Type3: $U_1{}^c = \langle \{a,d\}, \{a,b\}, \{a,b\} \rangle$, NCS-Type III and

$NP(U_1{}^c) = \langle 0.5, 0.5, 0.5 \rangle$.

h) Type1: $U_2{}^c = \langle \{d\}, \{a,b,d\}, \{a,b,c\} \rangle$ NCS-Type III and

$NP(U_2{}^c) = \langle 0.25, 0.75, 0.75 \rangle$, Type2: $U^c{}_2 = \langle \{d\}, \{c\}, \{a,b,c\} \rangle$
NCS-Type III and $NP(U_2{}^c) = \langle 0.25, 0.25, 0.75 \rangle$, Type3:
$U^c{}_2 = \langle \{d\}, \{a,b,d\}, \{a,b,c\} \rangle$ NCS-Type III. $NP(U_2{}^c) = \langle 0.25, 0.75, 0.75 \rangle$.

3. *Probabilities for events*.

$NP(A)=\langle 0.5,0.25,0.25\rangle$, $NP(B)=\langle 0.25,0.25,0.5\rangle$, $NP(U_1)=\langle 0.5,0.5,0.5\rangle$, $NP(U_2)=\langle 0.75,0.25,0.25\rangle$

$NP(U_1{}^c)=\langle 0.5,0.5,0.5\rangle$, $NP(U_2{}^c)=\langle 0.25,0.75,0.75\rangle$.

e) $(A\cap B)^c=\langle\{b,c,d\},\{a,b,d\},\{a,c\}\rangle$ be a NCS-Type III.

$NP(A\cap B)^c=\langle 0.75,0.75,0.25\rangle$ be a neutrosophic set.

f) $NP(A)^c\cap NP(B)^c=\langle 0.5,0.75,0.75\rangle$,

$NP(A)^c\cup NP(B)^c=\langle 0.75,0.75,0.5\rangle$

g) $NP(A\cup B)=NP(A)+NP(B)-NP(A\cap B)=\langle 0.5,0.25,0.25\}\rangle$

h) $NP(A)=\langle 0.5,0.25,0.25\rangle$, $NP(A)^c=\langle 0.5,0.75,0.75\rangle$,

$NP(B)=\langle 0.25,0.25,0.5\rangle$, $NP(B^c)=\langle 0.75,0.75,0.5\rangle$

4. *Probabilities for Products*. The product of two events given by —

$A\times B=\langle\{(a,a),(b,a)\},\{(c,c)\},\{(d,d),(d,b)\}\rangle$,

and $NP(A\times B)=\langle 2/16,1/16,2/16\rangle$

$B\times A=\langle\{(a,a),(a,b)\},\{(c,c)\},\{(d,d),(b,d)\}\rangle$

and $NP(B\times A)=\langle 2/16,1/16,2/16\rangle$

$A\times U_1=\langle\{(a,a),(b,a),(a,b),(b,b)\},\{(c,c),(c,d)\},\{(d,d),(d,a)\}\rangle$,

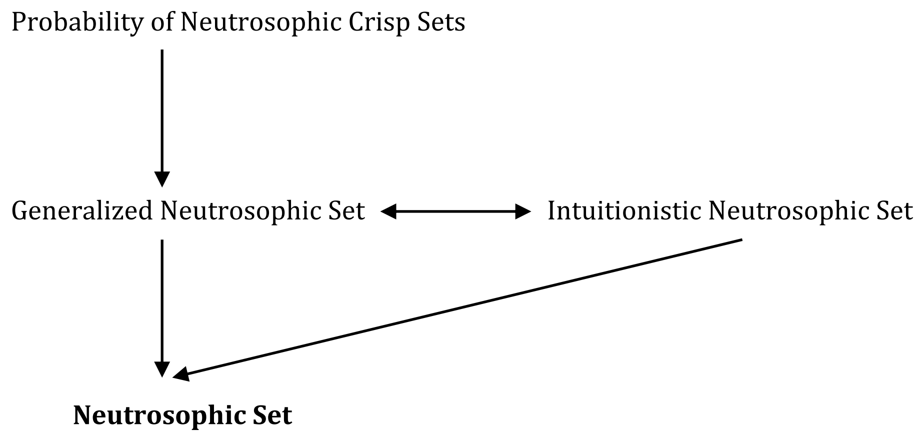
and $NP(A\times U_1)=\langle 4/16,2/16,2/16\rangle$

$U_1\times U_2=\langle\{(a,a),(b,a),(a,b),(b,b),(a,c),(b,c)\},\{(c,c),(d,c)\},\{(d,d),(a,d)\}\rangle$

and $NP(U_1\times U_2)=\langle 6/16,2/16,2/16\rangle$.

Remark 3.4

The following diagram represents the relation between neutrosophic crisp concepts and neutrosphic sets:



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