A Comment on the Dimensionless Gravitational Coupling Constant

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Abstract

In this document we suggest that the definition of the Gravitational Coupling Constant, be expanded to include its significance as a universal scaling factor.

1. INTRODUCTION

The Gravitational Coupling Constant has been theoretically discussed in a number of physics papers, such as Silk (1977), Rozental (1980), Burrows and Ostriker (2013) and more recently by Haug (2016). [1 2 3 4]

The gravitational coupling constant (α_G) is typically defined in terms of the gravitational attraction between a pair of electrons.[5]

It is defined by the expression,

$$(\alpha_G) = \frac{G\, m_e^2}{\hbar\, c} = \frac{m_e^2}{m_P^2} \approx 1.7518\text{e-}45 \qquad (1)$$

where:

- G is the gravitational constant
- m_e is the electron rest mass
- c is the speed of light in a vacuum
- $\hbar$ ("h-bar") is the reduced Planck constant
- m_P is the Planck mass

2. The Gravitational Coupling Constant as a Scaling Ratio

We would like to point out in this paper, that the gravitational coupling constant (α_G), appears to be more significant than just a ratio between the electron rest mass and the Planck mass.

We suggest that the gravitational coupling constant is a scaling ratio that can be used to compare similar unit values from different scaling systems (Planck scale, the atomic scale, and the cosmological scale).

In the next few examples we will show that the gravitational coupling constant can be used to compare values of length, area and force.

(The Appendix contains a list and description of the symbols used in this document, along with recommended values used to evaluate all expressions.)

Length

Looking at lengths associated with the electron, we show that the reduced Compton wavelength of the electron ($r_{c,e}$), and the Schwarzschild radius of the electron (r_s) are related by the Gravitational coupling constant (α_G):

$$\alpha_G = \frac{r_s}{2r_{c,e}} = 1.7518e\text{-}45 \qquad (2)$$

where,

α_G = dimensionless Gravitational Coupling Constant
$r_{c,e}$ = reduced Compton wavelength of electron
r_s = Schwarzschild radius of electron

In expression (2) we show that the Schwarzschild radius of the electron represents a measurement of length related to gravity at a cosmological scale. The reduced Compton wavelength represents a length at the atomic scale and α_G represents the ratio between these two values.

Area

In a recent paper by Adler[6], it is shown that values at a cosmological scale are related to the Planck scale by a new physical constant called the Gravitational fine structure constant.

We suggest that the addition of a new physical constant is not required, since the ratios described in the Adlers' paper, can be transformed into ratios of area which describe the Gravitational coupling constant (α_G).

$$\alpha_G = \frac{L_p^2}{r_{c,e}^2} = \frac{4}{3}\frac{L_p^2 L_H^2}{r_{c,e}^2 R_d^2} = \frac{4 L_p^2 L_H^2 \Lambda}{9 r_{c,e}^2} = 1.7518\text{e-}45 \qquad (3)$$

where,

α_G = dimensionless Gravitational Coupling Constant
L_p = Planck length
$r_{c,e}$ = reduced Compton wavelength of electron
L_H = Hubble length
R_d = de Sitter radius
Λ = cosmological constant

In (3) we show relationships of area between the Planck scale, the atomic scale and the cosmological scale all within the same expression.

Force

In the next example we give an expression equating gravitational force with electrostatic force:

$$\frac{\alpha \cdot G \cdot m_e^2}{r^2} = \frac{\alpha_G \cdot e^2}{4\pi\varepsilon_o r^2} \qquad (4)$$

where, α = fine structure constant
G = Gravitational constant
m_e = rest mass of electron
α_G = Gravitational coupling constant
e = elementary charge of electron
r = distance between particles
ε_o= permittivity of free space

Solving for the gravitational coupling constant (α_G) we get

$$\alpha_G = \frac{\alpha \cdot G \cdot m_e^2 \cdot 4\pi\varepsilon_o}{e^2} = 1.7518e\text{-}45 \qquad (5)$$

Expression (5) shows that the ratio of the gravitational force and the electrostatic force is the Gravitational coupling constant (α_G). This implies that the gravitational force is a force representing the cosmological scale while the electrostatic force is a force representing the atomic scale.

3. Discussion

Equating all of the expressions previously discussed, we give an expression that contains most of the major physical constants.

$$\alpha_G = \frac{r_s}{2r_{c,e}} = \frac{L_p^2}{r_{c,e}^2} = \frac{4}{3}\frac{L_p^2 L_H^2}{r_{c,e}^2 R_d^2} = \frac{4}{9}\frac{L_p^2 L_H^2 \Lambda}{r_{c,e}^2} = \frac{\alpha G m_e^2 4\pi\varepsilon_o}{e^2} = \frac{G m_e^2}{h c} = 1.7518e\text{-}45 \qquad (6)$$

Finally, we can take expression (6) and take it to unity which is shown in expression (7).

$$\frac{R_d^2 \Lambda}{3} = \frac{4L_H^2 (Gh\Lambda)}{9L_p^2 2\pi c^3} = \frac{\alpha G 4\pi\varepsilon_o m_e^2 r_{c,e}^2}{e^2 \cdot L_p^2} = \frac{Gh}{2\pi L_p^2 c^3} = 2\frac{\alpha G m_e^2 4\pi\varepsilon_o r_{c,e}}{r_s e^2} = \varepsilon_o \mu_o c^2 = 1 \qquad (7)$$

In this form the expression gives additional meaning.

For example, taking a portion of the expression and comparing the numerator with the denominator (8), we show that the gravitational energy between two electrons at half the Schwarzschild radius is equal in magnitude to the electrostatic energy between two electrons at the reduced Compton wavelength of an electron

$$2\frac{\alpha G m_e^2}{r_s} = \frac{e^2}{4\pi\varepsilon_o r_{c,e}} \qquad (8)$$

What has been demonstrated in expression (6) and (7) can be referred to as a **'principle of opposites' [9].** The product or quotient of two opposite values representing two different scales and of the same unit is a constant.

4) Summary

In this paper we show that the Gravitational coupling constant (α_G) can be perceived as a scaling constant that equates values of similar unit found within the Planck scale, the atomic scale and the cosmological scale.

This scaling relationship can be shown to exist between all major physical constants.

$$\alpha_G = \frac{r_s}{2r_{c,e}} = \frac{L_p^2}{r_{c,e}^2} = \frac{4}{3}\frac{L_p^2 L_H^2}{r_{c,e}^2 R_d^2} = \frac{4 L_p^2 L_H^2 \Lambda}{9 r_{c,e}^2} = \frac{\alpha G m_e^2 4\pi\varepsilon_o}{e^2} = \frac{G m_e^2}{h c} = 1.7518e\text{-}45$$

Appendix

We include the values used to evaluate expression (6) and (7) to at least eight decimal places. Many of these values are comparable to the 2010 CODATA [7] values.

We also include a specific value for G.

These values also reflect a Hubble constant (Ho) of 73.2 kilometers per second per megaparsec [8].

$G = 6.673308846076e\text{-}11\ N{\cdot}m^2/kg^2$ (Gravitational constant)
$H_o = 4.215851602024e{+}17\ s$ (Hubble constant)
$L_H = 1.263880514334e{+}26\ m$ (Hubble length)
$R_d = 1.459403510348e{+}26\ m$ (de Sitter radius)
$\Lambda = 1.408544206420e\text{-}52\ m^{-2}$ (cosmological constant)
$L_p = 1.616134940431e\text{-}35\ m$ (Planck length)
$r_{c,e} = 3.861592679954e\text{-}13\ m$ (reduced Compton wavelength of electron)
$r_s = 1.35e\text{-}57\ m$ (Schwarzschild radius of electron)
$c = 2.99792458e8\ m/s$ (speed of light in a vacuum)
$\varepsilon_o = 8.854187817620e\text{-}12\ C^2/(N{\cdot}m^2)$ (permittivity of free space)
$m_e = 9.109382910054e\text{-}31\ kg$ (rest mass of electron)
$e = 1.602176565e\text{-}19\ C$ (elementary charge)
$h = 6.626069574902e\text{-}34\ J \cdot s$ (Planck constant)
$\alpha = 7.297352570124e\text{-}3$ (fine structure constant)
$\alpha_G = 1.751549092643e\text{-}45$ (Gravitational coupling constant)

References

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