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# Neutrosophic Goal Programming applied to Bank: Three Investment Problem

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**Abstract:** This paper represents a new multi-objective Neutrosophic goal programming and Lexicographic goal programming to solve a multi-objective linear programming problem. Here we describe some basic properties of Neutrosophic sets. We have considered a multi-objective Bank Three Investment model to get optimal solution for different weights.

**Keywords-** Neutrosophic goal programming, Lexicographic goal programming, Bank Three model.

## 1. Introduction

The concept of fuzzy sets was introduced by Zadeh in 1965 [1]. Since the fuzzy sets and fuzzy logic have been applied in many real applications to handle uncertainty. The traditional fuzzy sets uses one real value $\mu_A(x) \in [0, 1]$ to represents the truth membership function of fuzzy set A defined on universe X. Sometimes $\mu_A(x)$ itself is uncertain and hard to be defined by a crisp value. So the concept of interval valued fuzzy sets was proposed [2] to capture the uncertainty of truth membership. In some applications we should consider not only the truth membership supported by the evident but also the falsity membership against by the evident. That is beyond the scope of fuzzy sets and interval valued fuzzy sets. In 1986, Atanassov **[3]**, **[5]** devolved the idea of intuitionistic fuzzy set A characterized by the membership degree $\mu_A(x) \in [0, 1]$ as well as non-membership degree $\nu_A(x) \in [0, 1]$ with some restriction $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Therefore certain amount of indeterminacy 1 - $(\mu_A(x) + \nu_A(x))$ remains by default. However one may also consider the possibility $\mu_A(x) + \nu_A(x) > 1$, so that inconsistent beliefs are also allowed. In neutrosophic sets indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are independent. Neutrosophic set (NS) was introduced by Smarandache in 1995 **[4]** which is actually generalization of different types of FSs and IFSs. In 1978 a paper Fuzzy linear programming with several objective functions has been published by H.J Zimmermann **[11]**. In 2007 B.Jana and T.K.Roy **[9]** has studied multi-objective intuitionistic fuzzy linear programming problem and its application in Transportation model. In 1961 goal programming was introduced by Charnes and Cooper **[13]**, Aenaida and kwak **[14]** applied goal programming to find a solution for multi-objective transportation problem. Recently the authors used the fuzzy goal programming approach to solve multi-objective transportation problem **[15]**. Other authors used fuzzy goal programming technique to solve different types of multi-objective linear programming problems **[16, 17, 18, 19]**. Recently a paper named "neutrosophic goal programming" has published by Mohamed Abdel-Baset, Ibrahim M. Hezam and Florentin Smarandache in the journal Neutrosophic sets and sysatems **[20]**. The motivation of the present study is to give computational algorithm for solving multi-objective linear goal programming problem and Lexicographic goal programming problem by single valued neutrosophic optimization approach. We also aim to study the impact of truth membership, indeterminacy membership and falsity membership functions in such optimization process.

## 2. Some preliminaries

### 2.1 Definition -1 (Fuzzy set) [1]

Let X be a fixed set. A fuzzy set A of X is an object having the form $\tilde{A}$ = {(x,$\mu_A$ (x)), x ∈ X} where the function $\mu_A(x)$ : X → [0, 1] define the truth membership of the element x ∈ X to the set A.

### 2.2 Definition-2 (Intuitionistic fuzzy set) [3]

Let a set X be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}^i$ in X is an object of the form $\tilde{A}^i = \{< X, \mu_A(x), \nu_A(x) > / x \in X\}$ where $\mu_A(x)$ : X→ [0, 1] and $\nu_A(x)$ : X→ [0, 1] define the Truth-membership and Falsity-membership respectively, for every element of x∈ X , $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ .

### 2.3 Definition-3 (Neutrosophic set) [4]

Let X be a space of points (objects) and $x \in X$. A neutrosophic set $\tilde{A}^n$ in X is defined by a Truth-membership function$\mu_A(x)$, an indeterminacy-membership function $\sigma_A(x)$ and a falsity-membership function $\nu_A(x)$ and having the form $\tilde{A}^n = \{< X, \mu_A(x),\ \sigma_A(x),\ \nu_A(x) > / x \in X\}$.

$\mu_A(x), \sigma_A(x)$ *and* $\nu_A(x)$ are real standard or non-standard subsets of

] $0^-$, $1^+$ [. that is

$\mu_A(x)$ : X→ ] $0^-$, $1^+$ [

$\sigma_A(x)$ : X→ ] $0^-$, $1^+$ [

$\nu_A(x)$ : X→ ] $0^-$, $1^+$ [

There is no restriction on the sum of $\mu_A(x), \sigma_A(x)$ *and* $\nu_A(x)$, so

$0^- \leq \sup\mu_A(x) + \sup\sigma_A(x) + \sup\nu_A(x) \leq 3^+$

### 2.4 Definition-4 (Single valued Neutrosophic sets) [6]

Let X be a universe of discourse. A single valued neutrosophic set $\tilde{A}^n$ over X is an object having the form $\tilde{A}^n = \{< X, \mu_A(x), \sigma_A(x), \nu_A(x) > / x \in X\}$where$\mu_A(x)$ : X→ [0, 1],$\sigma_A(x)$ : X→[0, 1] and$\nu_A(x)$ : X→ [0, 1] with $0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3$ for all x ∈ X.

**Example 1** Assume that X = [$x_1$, $x_2$, $x_3$]. $X_1$ is capability, $x_2$ is trustworthiness and $x_3$ is price. The values of $x_1$, $x_2$and $x_3$ are in [0, 1]. They are obtained from the questionnaire of some domain experts, their option could be a degree of "good service", a degree of indeterminacy and a degree of "poor service". $\tilde{A}^n$ is a single valued neutrosophic set of X defined by

$\tilde{A}^n$ =⟨0.3,0.4,0.5⟩/$x_1$ + ⟨0.5,0.2,0.3⟩/$x_2$ + ⟨0.7,0.2,0.2⟩/$x_3$. $\tilde{B}^n$ is a single valued neutrosophic set of X defined by $\tilde{B}^n$=⟨0.6,0.1,0.2⟩/$x_1$ + ⟨0.3,0.2,0.6⟩/$x_2$ + ⟨0.4,0.1,0.5⟩/$x_3$

### 2.5 Definition 5(Complement): [6].

The complement of a single valued neutrosophic set $\tilde{A}^n$ is denoted by c($\tilde{A}^n$) and is defined by

$$\mu_{c(\tilde{A}^n)}(x) = \nu_{\tilde{A}^n}(x)$$

$$\sigma_{c(\tilde{A}^n)}(x) = 1 - \sigma_{\tilde{A}^n}(x)$$

$$\nu_{c(\tilde{A}^n)}(x) = \mu_{\tilde{A}^n}(x) \quad \text{for all x in X}$$

**Example 2** Let $\tilde{A}^n$ be a single valued neutrosophic set defined in example 1. Then

c($\tilde{A}^n$) = ⟨0.5,0.6,0.3⟩/$x_1$ + ⟨0.3,0.8,0.5⟩/$x_2$ + ⟨0.2,0.8,0.7⟩/$X_3$

**2.6 Definition 6 (Union): [6]** The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, Written as C = A ∪ B, whose truth-membership, indeterminacy-membership and falsity-membership functions are given by

$\mu_C(x)$ = max $\{\mu_A(x), \mu_B(x)\}$

$\sigma_C(x)$ = max $\{\sigma_A(x), \sigma_B(x)\}$

$\nu_C(x)$ = min $\{\nu_A(x), \nu_B(x)\}$ for all x in X

**Example 3** Let A and B be two single valued neutrosophic sets defined in example 1. Then A ∪ B = ⟨0.6,0.4,0.2⟩/$x_1$ + ⟨0.5,0.2,0.3⟩/$x_2$ + ⟨0.7,0.2,0.2⟩ /$X_3$

**2.7 Definition 7(Intersection): [7]** The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, Written as C = A ∩ B, whose truth-membership, indeterminacy-membership and falsity-membership functions are given by

$\mu_C(x)$ = min $\{\mu_A(x), \mu_B(x)\}$

$\sigma_C(x)$ = min $\{\sigma_A(x), \sigma_B(x)\}$

$\nu_C(x)$ = max $\{\nu_A(x), \nu_B(x)\}$ for all x in X

**Example 4** Let A and B be two single valued neutrosophic sets defined in example 1. Then A ∩ B = ⟨0.3,0.1,0.5⟩/$x_1$ + ⟨0.3,0.2,0.6⟩/$x_2$ + ⟨0.4,0.1,0.5⟩ /$X_3$

## 3. Multi-objective linear programming problem (MOLPP) [19]

A general multi-objective linear programming problem with n objectives, m constraints and q decision variables may be taken in the following form

Minimize $f_1(x) = c_1$ X

Minimize $f_2(x) = c_2$ X

……………………………….

……………………………....

Minimize $f_n(x) = c_n$ X

Subject to A X ≤ $b$ and X ≥ 0

Where C= $(c_{i1}, c_{i2}, \dots\dots\dots, c_{iq})$ for i=1,2,……….n.

A= $[a_{ji}]_{m.n}$, X = $(x_1, x_2, \dots\dots\dots, x_q)^T$, b=$(b_1, b_2, \dots\dots\dots, b_m)^T$.

for j=1,2,………m; i=1,2,……….n.

## 4. A. Neutrosophic linear goal programming problem (NLGPP) [21]

Find X = $(x_1, x_2, \dots\dots\dots, x_q)^T$ (4.1)

So as to

Minimize $f_i(x)$

with target value $c_i$ , Truth tolerance $a_i$ , Falsity tolerance $t_i$, Indeterminacy tolerance $p_i$ i= 1, 2, ………..n.

subject to $g_j(x) \le b_j$ j= 1, 2, ………..m.

$x_k \ge 0$ k= 1,2, ………..q.

With Truth-membership, Falsity-membership, Indeterminacy-membership functions are

$$\mu_i(f_i(x)) = \begin{cases} 1 & if\ f_i(x) \le c_i \\ \frac{a_i+c_i-f_i(x)}{a_i} & if\ c_i \le f_i(x) \le a_i + c_i \\ 0 & if\ f_i(x) \ge a_i + c_i \end{cases} \quad (4.2)$$

$$\nu_i(f_i(x)) = \begin{cases} 0 & if\ f_i(x) \le c_i \\ \frac{f_i(x)-c_i}{t_i} & if\ c_i \le f_k(x) \le c_i + t_i \\ 1 & if\ f_i(x) \ge c_i + t_i \end{cases}$$

(4.3)

$$\sigma_i(f_i(x)) = \begin{cases} 1 & if\ f_i(x) \le c_i' \\ \frac{p_i+c_i'-f_i(x)}{p_i} & if\ c_i' \le f_i(x) \le p_i + c_i' \\ 0 & if\ f_i(x) \ge p_i + c_i' \end{cases} \quad (4.4)$$

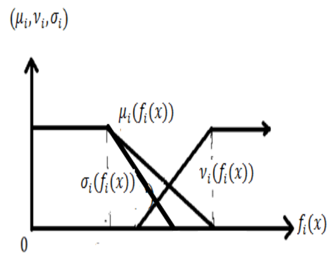


**Fig1. Truth membership, Falsity membership and Indeterminacy membership functions of a minimization-type objective function.**

Neutrosophic goal programming can be transformed into crisp linear programming problem using Truth-membership, Falsity-membership, Indeterminacy-membership functions as

Maximize $\sum_{i=1}^{n} w_i\, \mu_i(f_i(x))$

(4.5)

Minimize $\sum_{i=1}^{n} w_i\, \nu_i(f_i(x))$

Maximize$\sum_{i=1}^{n} w_i\, \sigma_i(f_i(x))$

i= 1, 2, ………..,n.

Subject to $g_j(x) \le b_j$

j= 1, 2,………..,m.

$x_k \ \ge 0$

k= 1,2, ………..q.

$\sum_{i=1}^{n} w_i \ = 1$

Which is equivalent to

Max $\sum_{i=1}^{n} w_i\, (\, \mu_i(f_i(x)) - \ \nu_i(f_i(x)) + \sigma_i\big(f_i(x)\big))$ (4.6)

Subject to $g_j(x) \le b_j$

j= 1, 2,………..,m.

$\sum_{i=1}^{n} w_i \ = 1$

**B. Neutrosophic Lexicographic goal programming**

The Lexicographic optimization takes objective in order: optimizing one, then a second subject to the first achieving its optimal value, and so on.

**Step-1.** Max $\mu_1$ - $\nu_1$ + $\sigma_1$

Subject to $g_j(x) \le b_j$ j= 1, 2,………..,m.

$x_k \ \ge 0$ k= 1,2, ………..q.

Solving we get optimal solution $f_1^* = F_1$

**Step-2** Max $\mu_2$ - $\nu_2$ + $\sigma_2$

Subject to $g_j(x) \le b_j$ j= 1, 2,………..,m.

$f_1 \le F_1$

$x_k \ \ge 0$ k= 1,2, ………..q.

Solving we get optimal solution $f_2^* = F_2$

**Step-3** Max $\mu_3$ - $\nu_3$ + $\sigma_3$

Subject to $g_j(x) \le b_j$ j= 1, 2,………..,m.

$f_1 \le F_1$

$f_2 \le F_2$

$x_k \ \ge 0$ k= 1,2, ………..q.

Solving we get optimal solution $f_3^* = F_3$

And so on.

Proceeding in this way finally we get optimal decision variables and all the optimal objective values.

## 5. Application of Neutrosophic goal programming to Bank Three Investment Problem

Every investor must trade off return versus risk in deciding how to allocate his or her available funds. The opportunities that promise the greatest profits are almost the ones that present the most serious risks.

Commercial banks must be especially careful in balancing return and risk because legal and ethical obligations demand that they avoid undue hazards, yet their goal as a business enterprise is to maximize profit. This dilemma leads naturally to multi-objective optimization of investment that includes both profit and risk criteria.

Our investment example **[12]** adopts this multi-objective approach to a fictitious Bank Three. Bank Three has a modest $ 20 million capital, with $ 150 million in demand deposits and $80 million in time deposits (savings accounts and certificates of deposit). Table 1 displays the categories among which the bank must divide its capital and deposited funds. Rates of return are also provided for each category together with other information related to risk.

**Table 1** Bank Three Investment Opportunities

| Investment Category, j | Return Rate (%) | Liquid Part (%) | Required Capital Asset (%) | Risk ? |
|---|---|---|---|---|
| 1: Cash | 0.0 | 100.0 | 0.0 | No |
| 2: Short term | 4.0 | 99.5 | 0.5 | No |
| 3: Government: 1 to 5 years | 4.5 | 96.0 | 4.0 | No |
| 4: Government: 5 to 10 years | 5.5 | 90.0 | 5.0 | No |
| 5: Government: over 10 years | 7.0 | 85.0 | 7.5 | No |
| 6: Installment loans | *10.5* | 0.0 | 10.0 | Yes |
| 7: Mortgage loans | 8.5 | 0.0 | 10.0 | Yes |
| 8: Commercial loans | 9.2 | 0.0 | 10.0 | Yes |

The first goal of any private business is to maximize profit. Using rates of return from table 1, this produces objective function

$$\text{Max} \quad 0.04x_2 + 0.045x_3 + 0.055x_4 + 0.070x_5 + 0.105x_6 + 0.085x_7 + 0.092x_8 \quad \text{(Profit)}$$

It is less clear how to quantify investment risk. We employ two common ratio measures.

One is the capital-adequacy ratio, expressed as the ratio of required capital for bank solvency to actual capital. A low value indicates minimum risk. The "required capital" rates of table 1 approximate U.S. government formulas used to compute this ratio, and Bank Three's present capital is $ 20 million. Thus we will express a second objective as

Min $\frac{1}{20}(0.005x_2 + 0.040x_3 + 0.050x_4 + 0.075x_5 + 0.100x_6 + 0.100x_7 + 0.100x_8)$

(Capital - adequacy)

Another measure of risk focuses on illiquid risk assets. A low risk asset/capital ratio indicates a financially secure institution. For our example, this third measure of success is expressed as

Min $\frac{1}{20}$ $(x_6 + x_7 + x_8)$ (Risk - asset)

To complete a model of Bank Three's investment plans, we must describe the relevant constraints.

1. Investments must sum to the available capital and deposit funds.
2. Cash reserves must be at least 14% of demand deposits plus 4 % of time Deposits.
3. The portion of investments considered liquid should be at least 47 % of Demand deposits plus 36 % of time deposits.
4. At least 5 % of funds should be invested in each of the eight categories.
5. At least 30 % of funds should be invested in commercial loans, to Maintain the bank's community status.

Combining the 3 objective functions above with these 5 constraints completes a multi-objective linear programming model of Bank Three's investment problem:

Max $0.04x_2 + 0.045x_3 + 0.055x_4 + 0.070x_5 + 0.105x_6 + 0.085x_7 + 0.092x_8$ (Profit)

Min $\frac{1}{20}(0.005x_2 + 0.040x_3 + 0.050x_4 + 0.075x_5 + 0.100x_6 + 0.100x_7 + 0.100x_8)$

(Capital - adequacy)

Min $\frac{1}{20}$ $(x_6 + x_7 + x_8)$ (Risk - asset)

Such that $x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8= (20+150+80)$ (Invest all)

$x_1 \geq 0.14\ (150) + 0.04\ (80)$ (Cash reserve )

$1.00x_1+0.995x_2+0.960x_3+0.900x_4+0.850x_5 \geq 0.47(150) + 0.36(80)$

(Liquidity)

$x_j \geq 0.05\ (20+150+80)$ for all $j=1,\ldots,8$ (Diversification)

$x_8 \geq 0.30\ (20+150+80)$ (Commercial)

$x_1, x_2, \ldots, x_8 \geq 0$

## 6. Numerical Example

$c_1 = 12$, $a_1 = 6.67$, $t_1 = 3$, $c_1' = 13$, $p_1 = 5.67$

$c_2 = 0.58$, $a_2 = 0.22$, $t_2 = 0.20$, $c_2' = 0.60$, $p_2 = 0.20$

$c_3 = 5$, $a_3 = 1.5$, $t_3 = 1.0$, $c_3' = 5.5$, $p_3 = 1.0$

**Table 1. Goal Programming Solution of the Bank Three Problem**

| Weights | Optimal Primal Variables | Optimal Objectives |
|---|---|---|
| $W_1=0.8$, $w_2=0.1$, $w_3=0.1$ | $x_1=24.2$, $x_2=12.5$, $x_3=12.5$, $x_4=12.5$, $x_5=46.367665$, $x_6=54.4323$, $x_7=12.5$, $x_8=75$ | $f_1=18.67363$, $f_2=0.942915$, $f_3=7.096$ |
| $W_1=0.05$, $w_2=0.9$, $w_3=0.05$ | $x_1=100$, $x_2=12.5$, $x_3=12.5$, $x_4=12.5$, $x_5=12.5$, $x_6=12.5$, $x_7=12.5$, $x_8=75$ | $f_1=11.9$, $f_2=0.60625$, $f_3=5.00$ |
| $W_1=0.1$, $w_2=0.1$, $w_3=0.8$ | $x_1=24.2$, $x_2=88.30$, $x_3=12.5$, $x_4=12.5$, $x_5=12.5$, $x_6=12.5$, $x_7=12.5$, $x_8=75$ | $f_1=14.932$, $f_2=0.6252$, $f_3=5.00$ |
| $W_1=1/3$, $w_2=1/3$, $w_3=1/3$ | $x_1=24.2$, $x_2=88.30$, $x_3=12.5$, $x_4=12.5$, $x_5=12.5$, $x_6=12.5$, $x_7=12.5$, $x_8=75$ | $f_1=14.932$, $f_2=0.6252$, $f_3=5.00$ |

**Table 2. Lexicographic Goal Programming Solution of the Bank Three Problem**

| Optimal Primal variables | Optimal Objective functions |
|---|---|
| $x_1=24.2$, $x_2=22.51454$, $x_3=12.5$, $x_4=12.5$, $x_5=34.64474$, $x_6=56.14072$, $x_7=12.5$, $x_8=75$ | $f_1=18.43299$, $f_2=0.9100$, $f_3=7.182036$ |

## 7. Conclusion

In this paper, we presents simple Neutrosophic optimization approach to solve Multi-objective linear goal programming problem and Lexicographic goal programming problem. It can be considered as an extension of fuzzy and intuitionistic fuzzy optimization. This proposed method Neutrosophic Goal Programming can also be applied for multi-objective non-linear programming problem.

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