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Restricted Interval Valued Neutrosophic Sets and Restricted Interval Valued Neutrosophic Topological Spaces

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Abstract: In this paper we introduce the concept of restricted interval valued neutrosophic sets (RIVNS in short). Some basic operations and properties of RIVNS are discussed. The concept of restricted interval valued neutrosophic topology is also introduced together with restricted interval valued neutrosophic finer and restricted interval valued neutrosophic coarser topology. We also define restricted interval valued neutrosophic interior and closer of a restricted interval valued neutrosophic set. Some theorems and examples are cites. Restricted interval valued neutrosophic subspace topology is also studied.

Keywords: Neutrosophic Set, Interval Valued Nuetrosophic Set, Restricted Interval Valued Neutrosophic Set, Restricted Interval Valued Neutrosophic Topological Space.

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1 Introduction

In 1999, Molodtsov [10] introduced the concept of soft set theory which is completely new approach for modeling uncertainty. In this paper [10] Molodtsov established the fundamental results of this new theory and successfully applied the soft set theory into several directions. Maji et al. [8] defined and studied several basic notions of soft set theory in 2003. Pie and Miao [14], Aktas and Cagman [1] and Ali et al. [2] improved the work of Maji et al. [9]. The intuitionistic fuzzy set is introduced by Atanaasov [4] as a generalization of fuzzy set [19] where he added degree of non-membership with degree of membership. Neutrosophic set introduced by F. Smarandache in 1995 [16]. Smarandache [17] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistant data. Maji [9] combined neutrosophic set and soft set and established some operations on these sets. Wang et al. [18] introduced interval neutrosophic sets. Deli [7] introduced the concept of interval-valued neutrosophic soft sets.

In this paper we introduce the concept of restricted interval valued neutrosophic sets (RIVNS in short). Some basic operations and properties of RIVNS are discussed. The concept of restricted interval valued neutrosophic topology is also introduced together with restricted interval valued neutrosophic finer and restricted interval valued neutrosophic coarser topology. We also define restricted interval valued neutrosophic interior and closer of a restricted interval valued neutrosophic set. Some theorems and examples are cited. Restricted interval valued neutrosophic subspace topology is also studied. We establish some properties of restricted interval valued neutrosophic soft topological space with supporting proofs and examples.

2 Preliminaries

Definition 2.1[17] A neutrosophic set A on the universe of discourse U is defined as

$A=\left\{\left\langle x,\mu_A(x),\gamma_A(x),\delta_A(x)\right\rangle : x\in U\right\}$, where

$\mu_A,\gamma_A,\delta_A : U \to]^-0,1^+[$ are functions such that the condition:

$\forall x \in U$, ${}^{-}0 \le \mu_A(x) + \gamma_A(x) + \delta_A(x) \le 3^{+}$ is satisfied.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0,1^{+}[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^{-}0,1^{+}[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0,1]$.

Definition 2.2 [6] An interval valued neutrosophicset A on the universe of discourse U is defined as $A = \{\langle x, \mu_A(x), \gamma_A(x), \delta_A(x)\rangle : x \in U\}$, where $\mu_A, \gamma_A, \delta_A : U \to Int]^{-}0,1^{+}[$ are functions such that the condition: $\forall x \in U$, ${}^{-}0 \le sup\mu_A(x) + sup\gamma_A(x) + sup\delta_A(x) \le 3^{+}$ is satisfied.

In real life applications it is difficult to use interval valued neutrosophic set with interval-value from real standard or non-standard subset of $Int(]^{-}0,1^{+}[)$. Hence we consider the interval-valued neutrosophic set which takes the interval-value from the subset of $Int([0,1])$ (where $Int([0,1])$ denotes the set of all closed sub intervals of $[0,1]$).

Definition 2.3 [15] Let *X* be a non-empty fixed set. A generalized neutrosophic set (*GNS* in short) *A* is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ Where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set *A*where the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \wedge \gamma_A(x) \le 0.5$.

We call this generalizedneutrosophic set**[15]** as **restricted neutrosophic set**.

Definition 2.4 [15] Let *A* and *B* be two *RNS*s on *X* defined by $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$. Then union, intersection, subset and complement may be defined as

(i) The union of *A* and *B* is denoted by $A \cup B$ and is defined as

$$A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$$

or

$$A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}.$$

(ii) The intersection of *A* and *B* is denoted by $A \cap B$ and is defined as

$$A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$$

or

$$A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}.$$

or

$$A \cap B = \{\langle x, \mu_A(x).\mu_B(x), \sigma_A(x).\sigma_B(x), \gamma_A(x).\gamma_B(x)\rangle : x \in X\}$$

(iii) *A* is called subset of *B*, denoted by $A \subseteq B$ if and only if

$\mu_A(x) \le \mu_B(x)$, $\sigma_A(x) \ge \sigma_B(x)$, $\gamma_A(x) \ge \gamma_B(x)$

or

$\mu_A(x) \le \mu_B(x)$, $\sigma_A(x) \le \sigma_B(x)$,
$\gamma_A(x) \ge \gamma_B(x)$.

(iv) The complement of *A* is denoted by A^c and is defined as

$$A = \{\langle x, \gamma_A(x), 1-\sigma_A(x), \mu_A(x)\rangle : x \in X\}$$

or

$$A = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$$

or

$$A = \{\langle x, 1-\mu_A(x), \sigma_A(x), 1-\gamma_A(x)\rangle : x \in X\}$$

Definition 2.5: [15] A restricted neutrosophic topology (*RN*-topology in short) on a non empty set *X* is a family of restricted neutrosophic subsets in *X* satisfying the following axioms

(i) $0_N, 1_N \in \tau$

(ii) $\bigcup_i G_i \in \tau, \forall \{G_i : i \in J\} \subseteq \tau$

(iii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.

The pair (X, τ) is called restricted neutrosophic topological space (*RN*-topological space in short). The members of τ are called restricted neutrosophic open sets. A *RNS F* is closed if and only if F^c is *RN* open set.

3 Restricted Interval Valued Neutrosophic Set

In this section we introduce the concept of restricted interval valued neutrosophic set along with some examples, operators and results.

Definition 3.1 Let *X* be a non empty set. A restricted interval valued neutrosophic set (*RIVNS* in short) *A* is an object having the form $A = \{\langle x, \mu_A(x), \gamma_A(x), \delta_A(x)\rangle : x \in X\}$, where $\mu_A(x), \gamma_A(x), \delta_A(x) : X \to Int\,]^-0, 1^+[$ are functions such that the condition: $\forall x \in X$, $sup\mu_A(x) \wedge sup\gamma_A(x) \wedge sup\delta_A(x) \le 0.5$ is satisfied.

Here $\mu_A(x), \gamma_A(x)$ and $\delta_A(x)$ represent truth-membership interval, indeterminacy-membership interval and falsity- membership interval respectively of the element $x \in X$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A, \delta_A\rangle$ for the *RIVNS* $A = \{\langle x, \mu_A(x), \gamma_A(x), \delta_A(x)\rangle : x \in X\}$.

Example 3.2 Let $X = \{x_1, x_2, x_3\}$, then the *RIVNS* $A = \{\langle x, \mu_A(x), \gamma_A(x), \delta_A(x)\rangle : x \in X\}$ can be represent by the following table

X	$\mu_A(x)$	$\gamma_A(x)$	$\delta_A(x)$	$sup\mu_A(x) \wedge sup\gamma_A(x) \wedge sup\delta_A(x)$
x_1	$[.2,.3]$	$[0,.1]$	$[.4,.5]$	.1
x_2	$[.3,.5]$	$[.1,.4]$	$[.5,.6]$	.4
x_3	$[.4,.7]$	$[.2,.4]$	$[.6,.8]$	.4

The *RIVNS*s $\tilde{0}$ and $\tilde{1}$ are defined as $\tilde{0} = \{\langle x, [0,0], [1,1], [1,1]\rangle : x \in X\}$ and $\tilde{1} = \{\langle x, [1,1], [0,0], [0,0]\rangle : x \in X\}$.

Definition 3.3 Let $J_1 = [inf\, J_1, sup\, J_1]$ and $J_2 = [inf\, J_2, sup\, J_2]$ be two intervals then

(i) $J_1 \le J_2$ iff $inf\, J_1 \le inf\, J_2$ and $sup\, J_1 \le sup\, J_2$.

(ii) $J_1 \vee J_2 = [max(inf\, J_1, inf\, J_2), max(sup\, J_1, sup\, J_2)]$.

(iii) $J_1 \wedge J_2 = [min(inf\, J_1, inf\, J_2), min(sup\, J_1, sup\, J_2)]$.

Definition 3.4 Let *A* and *B* be two *RIVNS*s on *X* defined by

$A = \{\langle x, \mu_A(x), \gamma_A(x), \delta_A(x)\rangle : x \in X\}$ and

$B=\{\langle x,\mu_B(x),\gamma_B(x),\delta_B(x)\rangle : x\in X\}$. Then we can define union, intersection, subset and complement in several ways.

(i) The *RIVN*union of *A* and *B* is denoted by $A\cup B$ and is defined as

$$A\cup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\gamma_A(x)\wedge \gamma_B(x),\delta_A(x)\wedge\delta_B(x)\rangle : x\in X\}$$

or

$$A\cup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\gamma_A(x)\vee \gamma_B(x),\delta_A(x)\wedge\delta_B(x)\rangle : x\in X\}$$

We take first definition throughout the paper.

(ii) The*RIVN*intersection of *A* and *B* is denoted by $A\cap B$ and is defined as

$$A\cap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\gamma_A(x)\vee \gamma_B(x),\delta_A(x)\vee\delta_B(x)\rangle : x\in X\}$$

or

$$A\cap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\gamma_A(x)\wedge \gamma_B(x),\delta_A(x)\vee\delta_B(x)\rangle : x\in X\}$$

We take first definition throughout the paper.

(iii) *A* is called *RIVN* subset of *B*, denoted by $A\subseteq B$ if and only if

$\mu_A(x)\le\mu_B(x)$, $\gamma_A(x)\ge\gamma_B(x)$, $\delta_A(x)\ge\delta_B(x)$

or

$\mu_A(x)\le\mu_B(x)$, $\gamma_A(x)\le\gamma_B(x)$, $\delta_A(x)\ge\delta_B(x)$.

We take first definition throughout the paper.

(iv) The *RIVN* complement of *A* is denoted by A^c and is defined as

$$A=\{\langle x,\delta_A(x),[1-sup\gamma_A(x),1-inf\,\gamma_A(x)],\mu_A(x)\rangle : x\in X\}$$

or

$$A=\{\langle x,\delta_A(x),\gamma_A(x),\mu_A(x)\rangle : x\in X\}$$

We take first definition throughout the paper.

Definition 3.5 Let $\{A_i : i\in J\}$ be an arbitrary family of *RIVNS*s in *X*, then $\bigcup A_i$ and $\bigcap A_i$ can be respectively defined as

$$\bigcup A_i=\{\langle x,\vee_{i\in J}\mu_{A_i}(x),\wedge_{i\in J}\gamma_{A_i}(x),\wedge_{i\in J}\delta_{A_i}(x)\rangle : x\in X\}$$

or

$$\bigcup A_i=\{\langle x,\vee_{i\in J}\mu_{A_i}(x),\vee_{i\in J}\gamma_{A_i}(x),\wedge_{i\in J}\delta_{A_i}(x)\rangle : x\in X\}$$

$$\bigcap A_i=\{\langle x,\wedge_{i\in J}\mu_{A_i}(x),\vee_{i\in J}\gamma_{A_i}(x),\vee_{i\in J}\delta_{A_i}(x)\rangle : x\in X\}$$

or

$$\bigcap A_i=\{\langle x,\wedge_{i\in J}\mu_{A_i}(x),\wedge_{i\in J}\gamma_{A_i}(x),\vee_{i\in J}\delta_{A_i}(x)\rangle : x\in X\}$$

.

Theorem 3.6 Let*A*, *B* and *C*be three *RIVNS*s then

(1) $A\cup A=A$
(2) $A\cap A=A$
(3) $A\cup B=B\cup A$
(4) $A\cap B=B\cap A$
(5) $(A\cup B)^c=A^c\cap B^c$
(6) $(A\cap B)^c=A^c\cup B^c$
(7) $(A\cup B)\cup C=A\cup(B\cup C)$
(8) $(A\cap B)\cap C=A\cap(B\cap C)$
(9) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$
(10) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Proof: Let $A=\langle x,[a_1,a_2],[a_3,a_4],[a_5,a_6]\rangle$, $B=\langle x,[b_1,b_2],[b_3,b_4],[b_5,b_6]\rangle$ and $C=\langle x,[c_1,c_2],[c_3,c_4],[c_5,c_6]\rangle$

(1) - **(4)**Straight forward.

(5) $A\cup B=\langle x,[max(a_1,b_1),max(a_2,b_2)], [min(a_3,b_3),min(a_4,b_4)], [min(a_5,b_5),min(a_6,b_6)]\}$

$$(A\cup B)^c=\langle x,[min(a_5,b_5),min(a_6,b_6)], [1-min(a_4,b_4),1-min(a_3,b_3)], [max(a_1,b_1),max(a_2,b_2)]\rangle$$

Now

$$A^c = \langle x,[a_5,a_6],[1-a_4,1-a_3],[a_1,a_2]\rangle$$
$$B^c = \langle x,[b_5,b_6],[1-b_4,1-b_3],[b_1,b_2]\rangle$$
$$A^c \cap B^c = \langle x,[min(a_5,b_5),min(a_6,b_6)], [max(1-a_4,1-b_4),max(1-a_3,1-b_3)], [max(a_1,b_1),max(a_2,b_2)]\rangle$$
$$= \langle x,[min(a_5,b_5),min(a_6,b_6)], [1-min(a_4,b_4),1-min(a_3,b_3)], [max(a_1,b_1),max(a_2,b_2)]\rangle$$

(6) Same as **(5).**

(7) $A\cup B = \langle x,[max(a_1,b_1),max(a_2,b_2)], [min(a_3,b_3),min(a_4,b_4)], [min(a_5,b_5),min(a_6,b_6)]\rangle$

$$(A\cup B)\cup C = \langle x,[max(max(a_1,b_1),c_1), max(max(a_2,b_2),c_2)],[min(min(a_3,b_3),c_3), min(min(a_4,b_4),c_4)],[min(min(a_5,b_5),c_5), min(min(a_6,b_6),c_6)]\rangle$$
$$= \langle x,[max(a_1,b_1,c_1),max(a_2,b_2,c_2)], [min(a_3,b_3,c_3),min(a_4,b_4,c_4)], [min(a_5,b_5,c_5),min(a_6,b_6,c_6)]\rangle$$
$$B\cup C = \langle x,[max(b_1,c_1),max(b_2,c_2)], [min(b_3,c_3),min(b_4,c_4)], [min(b_5,c_5),min(b_6,c_6)]\rangle$$

$$A\cup(B\cup C) = \langle x,[max(a_1,max(b_1,c_1)), max(a_2,max(b_2,c_2))],[min(a_3,min(b_3,c_3)), min(a_4,min(b_4,c_4))],[min(a_5,min(b_5,c_5)), min(a_6,min(b_6,c_6))]\rangle$$
$$= \langle x,[max(a_1,b_1,c_1),max(a_2,b_2,c_2)], [min(a_3,b_3,c_3),min(a_4,b_4,c_4)], [min(a_5,b_5,c_5),min(a_6,b_6,c_6)]\rangle$$

(8) Same as **(7)**.

(9) $B\cap C = \langle x,[min(b_1,c_1),min(b_2,c_2)], [max(b_3,c_3),max(b_4,c_4)], [max(b_5,c_5),max(b_6,c_6)]\rangle$

$$A\cup(B\cap C) = \langle x,[max(a_1,min(b_1,c_1)), max(a_2,min(b_2,c_2))],[min(a_3,max(b_3,c_3)), min(a_4,max(b_4,c_4))],[min(a_5,max(b_5,c_5)), min(a_6,max(b_6,c_6))]\rangle$$
$$A\cup B = \langle x,[max(a_1,b_1),max(a_2,b_2)], [min(a_3,b_3),min(a_4,b_4)], [min(a_5,b_5),min(a_6,b_6)]\rangle$$

$$A\cup C = \langle x,[max(a_1,c_1),max(a_2,c_2)], [min(a_3,b_3),min(a_4,c_4)], [min(a_5,c_5),min(a_6,c_6)]\rangle$$
$$(A\cup B)\cap(A\cup C) = \langle x,[min(max(a_1,b_1),max(a_1,c_1)), min(max(a_2,b_2),max(a_2,c_2))], [max(min(a_3,b_3),min(a_3,c_3)), max(min(a_4,b_4),min(a_4,c_4))], [max(min(a_5,b_5),min(a_5,c_5)), max(min(a_6,b_6),min(a_6,c_6))]\rangle$$

Now let us consider a_1,b_1 and c_1, six cases may arise as

$a_1 \ge b_1 \ge c_1$, for this

$$max(a_1,min(b_1,c_1)) = min(max(a_1,b_1),max(a_1,c_1)) = a_1$$

$a_1 \ge c_1 \ge b_1$, for this

$$max(a_1,min(b_1,c_1)) = min(max(a_1,b_1),max(a_1,c_1)) = a_1$$

$b_1 \geq a_1 \geq c_1$, for this

$$max\left(a_1, min\left(b_1, c_1\right)\right) = min\left(max\left(a_1, b_1\right), max\left(a_1, c_1\right)\right) = a_1$$

$b_1 \geq c_1 \geq a_1$, for this

$$max\left(a_1, min\left(b_1, c_1\right)\right) = min\left(max\left(a_1, b_1\right), max\left(a_1, c_1\right)\right) = c_1$$

$c_1 \geq a_1 \geq b_1$, for this

$$max\left(a_1, min\left(b_1, c_1\right)\right) = min\left(max\left(a_1, b_1\right), max\left(a_1, c_1\right)\right) = a_1$$

$c_1 \geq b_1 \geq a_1$, for this

$$max\left(a_1, min\left(b_1, c_1\right)\right) = min\left(max\left(a_1, b_1\right), max\left(a_1, c_1\right)\right) = b_1.$$

Similarly it can be shown that other results are true for a_2, b_2, c_2 ; a_3, b_3, c_3 ; a_4, b_4, c_4 ; a_5, b_5, c_5 ; a_6, b_6, c_6 . Hence

$$A \cup \left(B \cap C\right) = \left(A \cup B\right) \cap \left(A \cup C\right) .$$

(10) Same as **(9)**.

4. Restricted Interval Valued neutrosophic Topological Spaces

In this section we give the definition of restricted interval valued Neutrosophic topological spaces with some examples and results.

Definition 4.1A restricted interval valued neutrosophic topology (*RIVN*-topology in short) on a non empty set *X* is a family of restricted interval valued neutrosophic subsets in *X* satisfying the following axioms

(iv) $\tilde{0}, \tilde{1} \in \tau$

(v) $\bigcup_i G_i \in \tau, \forall \left\{G_i : i \in J\right\} \subseteq \tau$

(vi) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.

The pair $\left(X, \tau\right)$ is called restricted interval valued neutrosophic topological space (*RIVN*-topological space in short). The members of τ are called restricted interval valued neutrosophic open sets. A *RIVNSF* is closed if and only if F^c is *RIVN* open set.

Example 4.2 Let*X* be a non-empty set. Let us consider the following *RIVNS*s

$G_1 = \left\{\left\langle x, [.5,.8], [.2,.3], [.2,.5] \right\rangle : x \in X \right\}$,

$G_2 = \left\{\left\langle x, [.6,.7], [.5,.6], [.3,.4] \right\rangle : x \in X \right\}$,

$G_3 = G_1 \cup G_2 = \left\{\left\langle x, [.6,.8], [.2,.3], [.2,.4] \right\rangle : x \in X \right\}$,

$G_4 = G_1 \cap G_2 = \left\{\left\langle x, [.5,.7], [.5,.6], [.3,.5] \right\rangle : x \in X \right\}$.

The family $\tau_1 = \left\{\tilde{0}, \tilde{1}, G_1, G_2, G_3, G_4\right\}$ is a *RIVN*-topology in *X* and $\left(X, \tau_1\right)$ is called a *RIVN*-topological space. But $\tau_2 = \left\{\tilde{0}, \tilde{1}, G_1, G_2\right\}$ is not a *RIVN*-topology as $G_1 \cup G_2 = G_3 \notin \tau_2$.

Definition 4.3 The two *RIVN* subsets $\tilde{0}, \tilde{1}$ constitute a *RIVN*-topology on *X*, called indiscrete *RIVN*-topology. The family of all *RIVN* subsets of *X* constitutes a *RIVN*-topology on*X*, such topology is called discrete *RIVN*-topology.

Theorem 4.4 Let $\left\{\tau_j : j \in J\right\}$ be a collection of *RIVN*-topologies on*X*. Then their intersection $\bigcap_{j \in J} \tau_j$ is also a*RIVS*-topology on *X*.

Proof: (i) Since $\tilde{0}, \tilde{1} \in \tau_j$ for each $j \in J$. Hence $\tilde{0}, \tilde{1} \in \bigcap_{j \in J} \tau_j$.

(ii) Let $\left\{G_k : k \in K\right\}$ be an arbitrary family *RIVNS*s where $G_k \in \bigcap_{j \in J} \tau_j$ for each $k \in K$. Then for each $j \in J$, $G_k \in \tau_j$ for $k \in K$ and since for each $j \in J$, τ_j ia a *RIVN*-topology, therefore $\bigcup_{k \in K} G_k \in \tau_j$ for each $j \in J$. Hence $\bigcup_{k \in K} G_k \in \bigcap_{j \in J} \tau_j$.

(iii) Let $G_1, G_2 \in \bigcap_{j \in J} \tau_j$, then $G_1, G_2 \in \tau_j$ for each $j \in J$. Since for each $j \in J$, τ_j is an *RIVN*-topology, therefore $G_1, G_2 \in \tau_j$ for each $j \in J$. Hence $G_1 \cap G_2 \in \bigcap_{j \in J} \tau_j$.

Thus $\bigcap_{j\in J}\tau_j$ forms a *RIVN*-topology as it satisfies all the axioms of RIVN-topology. But union of R*IVN*-topologies need not be a R*IVN*-topology.

Let us show this with the following example.

Example 4.5 In example 4.2, let us consider two *RIVN*- topologies τ_3 and τ_4 on *X* as $\tau_3=\{\tilde{0},\tilde{1},G_1\}$ and $\tau_4=\{\tilde{0},\tilde{1},G_2\}$. Here their union $\tau_3\cup\tau_4=\{\tilde{0},\tilde{1},G_1,G_2\}=\tau_2$ is not a *RIVN*-topology on*X*.

Definition 4.6 Let (X,τ) be an R*IVN*-topological space over X . A R*IVN* subset G of X is called restricted intervalvalued neutrosophicclosed set (in short R*IVN*-closed set) if its complement G^c is a member of τ .

Definition 4.7 Let (X,τ_1) and (X,τ_2) be two R*IVN*-topological spaces over X . If each $G\in\tau_2$ implies $G\in\tau_1$, then τ_1 is called restrictedinterval valued neutrosophic finer topology than τ_2 and τ_2 is called restricted interval valued neutrosophic coarser topology than τ_1 .

Example 4.8 In example 4.2 and 4.5, τ_1 is restricted interval valued neutrosophic finer topology than τ_3 and τ_3 is called restricted interval valued neutrosophic coarser topology than τ_1 .

Definition 4.9 Let τ be a *RIVN*-topological space on X and ß be a subfamily of τ . If every element of τ can be express as the arbitrary restrictedinterval valued neutrosophic union of some elements of ß, then ß is called restricted interval valued neutrosophic basis for the R*IVN*-topology τ .

5 Some Properties of Restricted Interval Valued Neutrosophic Soft Topological Spaces

In this section some properties of *RIVN*-topological spaces are introduced. Some results on *RIVNInt* and *RIVNCl* are also introduced.Restricted interval valued neutrosophic subspace topology is also studied.

Definition 5.1 Let (X,τ) be a *RIVN*-topological space and *A* be a *RIVNS* in *X*. The restrictedinterval valued neutrosophic interior and restrictedinterval valued neutrosophiccloser of*A* is denoted by *RIVNInt(A)* and *RIVNCl(A)* are defined as

$$RIVNInt(A)=\bigcup\{G:G\text{ is an }RIVN\text{ open set and }G\subseteq A\}$$

and

$$RIVNCl(A)=\bigcap\{F:F\text{ is an }RIVN\text{ closed set and }F\supseteq A\}$$

respectively.

Theorem 5.2 Let (X,τ) be a *RIVN*-topological space and G and H be two *RIVNS*s then the following properties hold

(1) $RIVNInt(G)\subseteq G$

(2) $G\subseteq H\Rightarrow RIVNInt(G)\subseteq RIVNInt(H)$

(3) $RIVNInt(G)\in\tau$

(4) $G\in\tau\Leftrightarrow RIVNInt(G)=G$

(5) $RIVNInt(RIVNInt(G))=RIVNInt(G)$

(6) $RIVNInt(\tilde{0})=\tilde{0}$, $RIVNInt(\tilde{1})=\tilde{1}$

Proof:

(1) Straight forward.

(2) Let $G\subseteq H$, then all the R*IVN*-open sets Contained in G also contained in H .

i.e. $\{G^*\in\tau:G^*\subseteq G\}\subseteq\{H^*\in\tau:H^*\subseteq H\}$

i.e. $\bigcup\{G^*\in\tau:G^*\subseteq G\}\subseteq\bigcup\{H^*\in\tau:H^*\subseteq H\}$

i.e. $RIVNInt(G)\subseteq RIVNInt(H)$

(3) $RIVNInt(G)=\bigcup\{G^*\in\tau:G^*\subseteq G\}$

Now clearly $\bigcup\{G^*\in\tau:G^*\subseteq G\}\in\tau$

$\therefore RIVNInt(G)\in\tau$.

(4) Let $G\in\tau$, then by (1) $RIVNInt(G)\subseteq G$.

Now since $G\in\tau$ and $G\subseteq G$, therefore $G\subseteq\bigcup\{G^*\in\tau:G^*\subseteq G\}=RIVNInt(G)$

i.e, $G\subseteq RIVNInt(G)$

Thus $RIVNInt(G)=G$

Conversely, let $RIVNInt(G)=G$

Since by (3) $RIVNInt(G)\in\tau$

Therefore $G\in\tau$

(5) By (3) $RIVNInt(G)\in\tau$

$\therefore$ By (4)
$RIVNInt\left(RIVNInt\left(f_A, E\right)\right) = RIVNInt\left(f_A, E\right)$

(6) We know that $\tilde{0}, \tilde{1} \in \tau$
$\therefore$ By (4)
$RIVNInt\left(\tilde{0}\right) = \tilde{0},\ RIVNInt\left(\tilde{1}\right) = \tilde{1}$

Theorem 5.3 Let (X, τ) be a *RIVN*-topological space and G and H are two *RIVNS*s then the following properties hold

(1) $G \subseteq RIVNCl(G)$

(2) $G \subseteq H \Rightarrow RIVNCl(G) \subseteq RIVNCl(H)$

(3) $\left(RIVNCl(G)\right)^c \in \tau$

(4) $G^c \in \tau \Leftrightarrow RIVNCl(G) = G$

(5) $RIVNCl\left(RIVNCl(G)\right) = RIVNCl(G)$

(6) $RIVNCl\left(\tilde{0}\right) = \tilde{0},\ RIVNCl\left(\tilde{1}\right) = \tilde{1}$

Proof: straight forward.

Theorem 5.4 Let (X, τ) be an *RIVN*-topological space on X and A be a *RIVNS* of*X*and let $\tau_A = \{A \cap U : U \in \tau\}$. Then τ_A forms a *RIVN*-topology on A.

Proof:

(i) Clearly $\tilde{0} = A \cap \tilde{0} \in \tau_A$ and $\tilde{1} = A \cap \tilde{1} \in \tau_A$.

(ii) Let $G_j \in \tau_A, \forall j \in J$, then $G_j = A \cap U_j$ where $U_j \in \tau$ for each $j \in J$.

Now $\bigcup_{j \in J} G_j = \bigcup_{j \in J} \left(A \cap U_j\right) = A \cap \left(\bigcup_{j \in J} U_j\right) \in \tau_A$ (since $\bigcup_{j \in J} U_j \in \tau$ as each $U_j \in \tau$).

(iii) Let $G, H \in \tau_A$ then $G = A \cap U$ and $H = A \cap V$ where $U, V \in \tau$.
Now
$G \cap H = (A \cap U) \cap (A \cap V) = A \cap (U \cap V) \in \tau_A$
(since $U \cap V \in \tau$ as $U, V \in \tau$).

Definition 5.5 Let (X, τ) be an *RIVN*-topological space on X and A be a *RIVNS* of*X*. Then $\tau_A = \{A \cap U : U \in \tau\}$ is called restricted interval valued neutrosophic subspace topology and (A, τ_A) is called restricted interval valued neutrosophic subspace of *RIVN*-topological space (X, τ).

Conclusion: In this paper we introduce the concept of restricted interval valued neutrosophic set which is the generalization of restrictedneutrosophic set. We define some operators on *RIVNS*. We also introduce a topological structure based on this. *RIVN* interior and *RIVN* closer of a restricted interval valued neutrosophic set are also defined. Restricted interval valued neutrosophic subspace topology is also studied. In future combining the ideas presented in this paper with concept of soft set one can define a new concept named restricted interval valued Neutrosophic soft set and can define a topological structure too.

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