The Dual Nature of Bekenstein-Hawking Entropy

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If the Bekenstein-Hawking entropy of a black hole is expressed as Bekenstein's area quantum 8π multiplied by a power of 2, then exponents of 2 that are multiples of precisely 50 are found for particular black holes of well-defined mass, and for the Hubble sphere. Motivated by T-duality, the analysis is extended to particles. Exponents calculated for the particles of the Standard Model lie in symmetric arrangement about the number 125, which is the exponent found for the lightest tensor mesons. Exponents that are multiples or simple binary (improper) fractions of 25 are favoured.

On the basis of the 10D/4D correspondence [1], an alternative equation has been proposed for the Bekenstein-Hawking entropy $\mathrm{S_{BH}} = \mathrm{A}/4$ of a black hole [2]:[1]

$$\mathrm{S_{BH}} = 8\pi.\,2^{\mathrm{N}} \tag{1}$$

where

$$2^{\mathrm{N}} = \frac{1}{2}\mathrm{m_{BH}}^2 \tag{2}$$

The quantity 8π in (1) is equal to the area quantum proposed by Bekenstein [3].

N in (1) has been calculated from (2) [2] for (i) the Tolman-Oppenheimer-Volkoff (TOV) limit on the mass of a neutron star that is able to resist collapse to a black hole, as originally calculated at ~0.7 $\mathrm{M_\Theta}$ for non-interacting neutrons [4], (ii) the supermassive black holes (SMBH) within the Milky Way and M31 galaxies and (iii) the Hubble sphere.

For a TOV limit of 0.70±0.05 $\mathrm{M_\Theta}$, N equals 250.18±0.20.

For a Milky Way SMBH mass of 4.1×10^6 $\mathrm{M_\Theta}$ [5], N equals 295.1. For an M31 SMBH mass of 1.4×10^8 $\mathrm{M_\Theta}$ [6], N equals 305.3. Uncertainties in SMBH mass of ~10% give rise to an uncertainty in N of ~0.3. The Milky Way and M31 galaxies form a binary system. In the Planck Model they constitute a 'galactic doublet' [7]. The galactic doublet is represented in mass by the geometric mean of the two SMBH masses, 2.4×10^7 $\mathrm{M_\Theta}$, for which N equals 300.2.

For the Hubble sphere, N is calculated from the equation $2^{\mathrm{N}} = \Omega_\mathrm{m}/8\mathrm{H_0}^2$ [2], where Ω_m is the matter density parameter and H_0 is the Hubble constant. The cosmological parameters $\Omega_\mathrm{m} = 0.308\pm0.012$ and $H_0 = 67.8\pm0.9$ km/s/Mpc of the Planck Collaboration (2015) [8] imply that N equals 400.09±0.10 [2].

All the above values of N are multiples of 5 or 50.

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[1] We will be working in natural units: c=G=ħ=1.

Motivated by T-duality, we conjecture that the entropy S_{SP} of a particle of sub-Planckian mass m_{SP} is given by

$$S_{SP} = 8\pi.2^{N} \tag{3}$$

where

$$2^{N} = \frac{1}{2} m_{SP}{}^{-2} \tag{4}$$

Values of N calculated for specific particle mass scales are presented in Figure 1. Each value is a simple binary (improper) fraction of 25, e.g. 137.5 = 11/2 × 25.

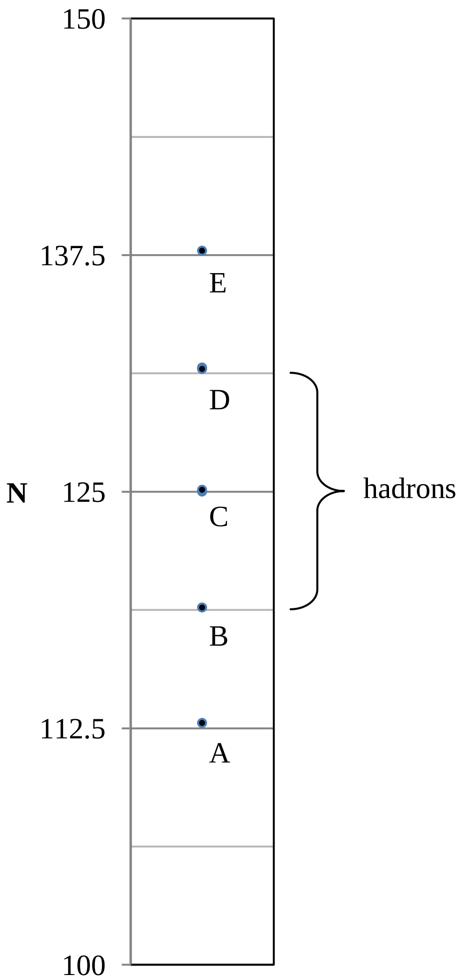

Figure 1: N-values of particles of the Standard Model.

A Z boson

B $\Upsilon(11020)$, the most massive known hadron

C $f_2(1270)$ and $a_2(1320)$, the lightest tensor mesons

D π^0 and $\pi^{\pm}$, the least massive hadrons

E The electron and the up quark both participate in symmetric partnerships about 16 MeV, the scale marking the centre of a symmetric arrangement of coincident mass levels within the Planck Model. [9, 10].

The arrangement of Figure 1 encompasses the particles of the Standard Model from the electron to the Z boson and is centred on N=125, the value found for the lightest tensor mesons. The factor 2^{125} recalls the primary length scale of the Planck Model, the Bohr radius, which precisely equals $(\pi/2)^{125} l_{\mathrm{Planck}}$ [11]. In the Planck Model, the Bohr radius is the characteristic length scale of the vacuum [12]. Do the ground state tensor mesons for which N=125 act as the lightest Kaluza-Klein gravitons? After all, the Planck Model offers a unified description of nature. The more massive[2] unflavoured and strange spin-2 resonances occupy the levels and sub-levels of the Planck sequences, as do all particles [13, 14], and constitute a tower of sorts. In Figure 2, the resonances are shown on the levels and sublevels of Sequences 2 and 3, which descend from Planck scale with common ratio 2/π and 1/e respectively. The mass levels may derive from the geometry of spacetime [15].

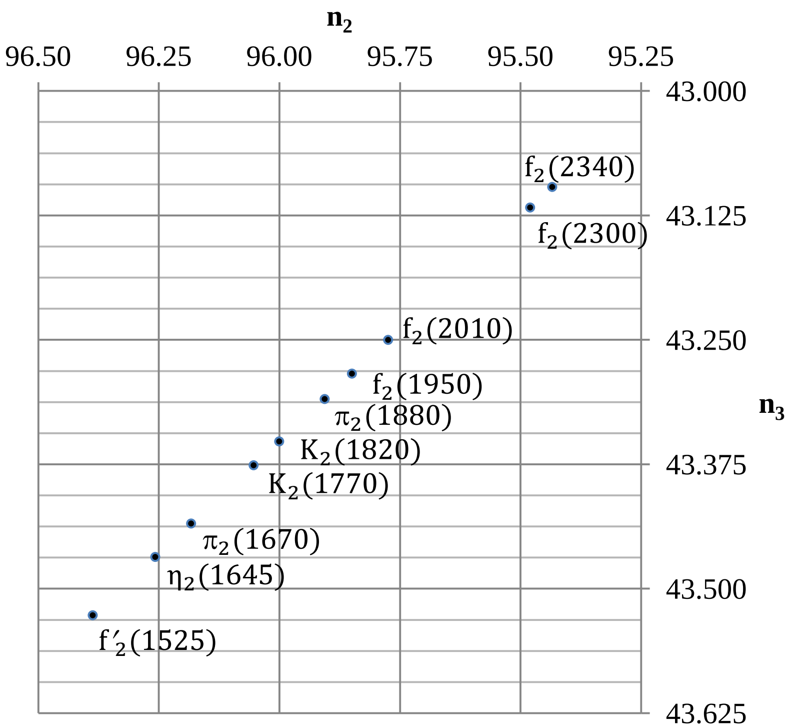

Figure 2: A tower of light unflavoured and strange spin-2 resonances on the mass levels and sublevels of Sequences 2 and 3 of the Planck Model. The dots mark the central values of Particle Data Group evaluation [16].

In the left-right symmetric Planck Model [17, 18], the gravitino $\tilde{G}$ and the gluino $\tilde{g}$ are of mass 35 TeV and 1.5 TeV, respectively, after Ellis, Kane and Zheng [19], while the right-handed Higgs VEV υ_R is of value ~7.7 TeV. Values of N calculated for the three scales are shown in Figure 3, where they lie symmetrically about N=100. Once more, a value of entropy is revealed that is equal to 8π multiplied by 2^{25} raised to an integer power. Such powers recall how $(\pi/2)^{25}$ raised to integer

[2] i.e. other than the ground state tensor mesons f_2, a_2 and K_2*

powers features prominently in the Planck Model, where the powers relate mass scales on both sub-Planckian mass scales [11] and super-Planckian mass scales [20].

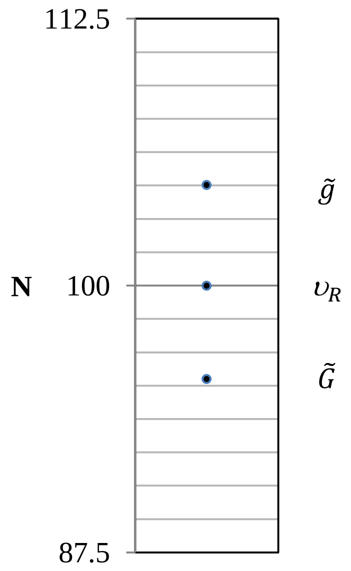

Figure 3: N-values for the right-handed VEV υ_R (7.7 TeV) of the left-right symmetric Planck Model, the conjectured gravitino $\tilde{G}$ (35 TeV) and gluino $\tilde{g}$ (1.5 TeV).

N-values for the left-handed VEV (246 GeV) and right-handed VEV (7.7 TeV) of the left-right symmetric Planck Model are shown in Figure 4.

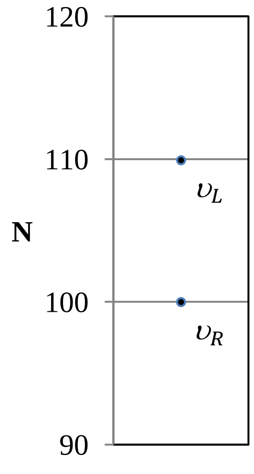

Figure 4: N-values for the left-handed VEV υ_L (246 GeV) and right-handed VEV υ_R (7.7 TeV) of the left-right symmetric Planck Model.

The entropic arrangement of Figure 1, which is centred on N=125, is similar to an arrangement of principal scales on the mass/energy (or length scale) levels of Planck Sequence 2, which descends (or ascends) from Planck scale with common ratio $2/\pi$ (or $\pi/2$). The arrangement of scales shown in Figure 5 is centred on Level 125, which is equal to the Bohr radius a_0 [11], the characteristic length scale of the vacuum [12]. Included in Figure 5 are the pion charge radius r_π, the Rydberg constant R_∞, the scale m_{e-u} associated with (equal to the geometric mean of the masses of) the electron and the up quark, and a neutrino mass scale m_ν of 0.05 eV. Level 150 (of the neutrino mass scale?) is of value 0.047 eV.

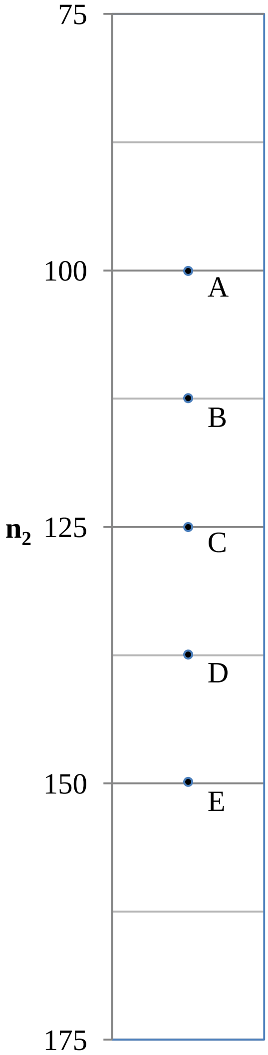

Figure 5: Principal scales on the levels of Sequence 2 of the Planck Model

A r_π, $0.672 \pm 0.008 \times 10^{-15}$ m [16]

B m_{e-u}, 1.08 MeV ($m_u = 2.3$ MeV [16])

C a_0, 0.529×10^{-10} m

D R_∞, 13.6 eV

E m_ν, 0.05 eV

Returning to the entropic sequence, we see (Figure 6) that N is of integer value for the proton and the deuteron, either side of N=125. Integer N-values are also found for atomic nuclei with A=4, 8, 16, 32, 64, 128 and 256.

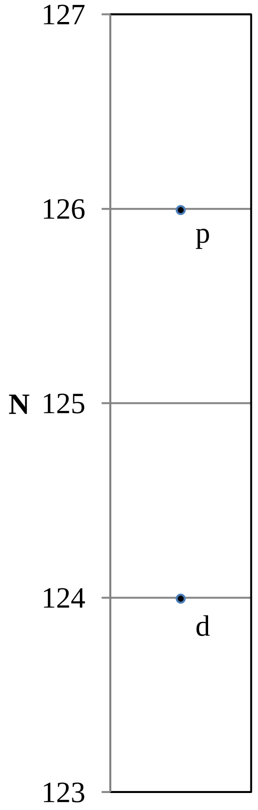

Figure 6: N-values of the proton and deuteron

Binding energies in general have been found to occupy the levels of the Planck sequences [21].Values of N, the entropic number, have now been calculated for the K-shell (1s) electron binding energies [22]. The entropic numbers corresponding to the K-shell binding energies of the lightest four elements lie in symmetric arrangement about N=175, another multiple of 25, as shown in Figure 7.

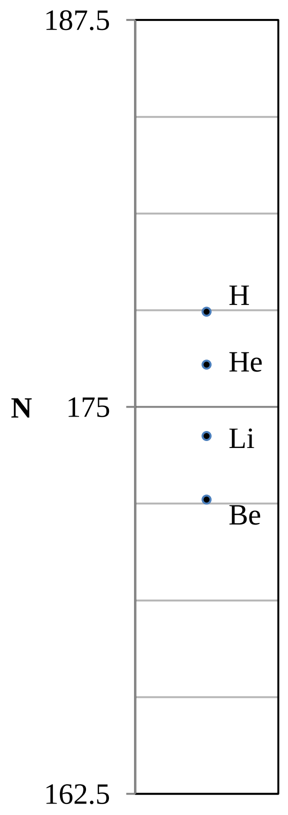

Figure 7: N-values for (i) the K-shell (1s) electron binding energies of the lightest four elements in their natural forms

Conclusions

If the Bekenstein-Hawking entropy (A/4) of a real black hole is written as 2^N times the area quantum 8π then N takes specific values. In the cases studied, N is a multiple of 5 or 50.

For particles, N again takes specific values. Multiples or simple binary (improper) fractions of 25 are favoured, although integers are also found.

Values of N calculated for several scales of particle physics take specific values, as above.

A connection has been found between the lightest tensor mesons and the Bohr radius, the characteristic length scale of the vacuum.

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