Title   The Rydberg Constant and Rydberg Series

How a detailed look at the Rydberg Series can lead to a new model for the hydrogen atom

Author Norman Graves

Abstract

Consideration of the Rydberg Series leads to a model for the hydrogen atom in which the variable of quantisation is the Lorentz factor, Gamma, and not angular momentum as postulated by Niels Bohr and integral to current theories. This is consistent with an atom in which the electron is seen as an objectively real particle having deterministic position and velocity. The electron is seen to orbit at a constant radius irrespective of the energy state of the atom and so there is no change in potential energy between energy states. All of the energy changes are therefore kinetic in nature. The model also provides a simple physical explanation of the hitherto mysterious Fine Structure Constant.

Johannes Rydberg (1854 - 1919) developed a formula to describe the emission and absorption spectra of the hydrogen atom which was first published in 1888. The Rydberg series derives directly from the Rydberg formula and is a useful practical tool for calculating the emission or absorption energy between any two energy states and can be used to generate all of the other series: Lyman, Balmer, Paschen, Brackett, Pfund, Humphreys etc.

Rydberg's formula is the more general case of a formula developed by Balmer to describe the emission spectra of hydrogen. Where Balmer only dealt with one set of spectral lines, Rydberg dealt with all the emission lines of the hydrogen atom.

Balmer's chose to express his formula in terms of the wavelength of the emitted light, whereas Rydberg found it more convenient to deal with the reciprocal of the wavelength or wavenumber (that is the number of wavelengths that fit into unit length).and developed the formula:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$
Equation 1

R_H is known as the Rydberg constant. Rydberg originally used a value based on Balmer's formula, which was derived empirically, but Niels Bohr was able to calculate a value analytically for this constant based on his model for the hydrogen atom.

$$R_\infty = \frac{mq^4}{8 h^3 c\varepsilon_o^2}$$
Equation 2

Bohr had assumed that the electron orbiting the hydrogen atom was much more massive than the proton around which it orbits. In such a case the electron orbits around the centre of gravity of the proton. In fact the electron orbits around the combined centre of gravity of the proton and the electron, which is somewhat closer to the electron than the centre of gravity of the proton alone. In 1917 Arnold Somerfield provided a correction called the *reduced mass*:

$$R_H = R_\infty \left(\frac{m_p}{m_p + m_e} \right)$$
Equation 3

Where m_p is the rest mass of the proton and m_e is the rest mass of the electron and m_e/m_p=1/1847.

The use of wavenumber in Rydberg's formula provides very little insight into what is actually going on and so it is useful to express the formula in terms of frequency or energy multiplying both sides by either by *c* for frequency or by *2πℏc* for energy. Expressing the Rydberg formula in terms of energy is particularly useful:

$$\Delta E = 2\pi\,\hbar c\, R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Equation 4

If we ignore the slight error due to the effect of the reduced mass then:

$$\Delta E = 2\pi\,\hbar c\, \frac{m\,q^4}{8\,h^3\,c\varepsilon_0^2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Equation 5

Recognizing that *K*, the Coulomb constant or electrostatic force constant, is given by:

$$K = \frac{1}{4\pi\varepsilon_0}$$

Equation 6

That

$$h = 2\pi\,\hbar$$

Equation 7

And that Alpha, the fine structure constant is given by:

$$\alpha = \frac{K q^2}{\hbar c}$$

Equation 8

Equation 5 can be simplified to:

$$\Delta e = \frac{1}{2}\,m\,c^2\,\alpha^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Equation 9

This in itself is interesting because ½mc² is the kinetic energy that an electron would have if it were travelling at the speed of light and cα is the Bohr velocity, the velocity at which the electron supposedly travels in the Bohr model.

By setting $n_2 = \infty$ and allowing n_1 to take on successive integer values we obtain the difference between the energy level in any particular energy state and the maximum possible energy that the atom can contain. This forms a series of values called the Rydberg series.

$$\Delta E_n = \frac{1}{2} m c^2 \frac{\alpha^2}{n^2}$$

Equation 10

The Rydberg series is useful because it allows us to calculate the energy, frequency or wavelength associated with any energy transition within the atom simply by taking the difference between two terms in the Rydberg Series as appropriate.

The values of the Rydberg Series represent the *energy potential* [1] of the hydrogen atom in any particular energy state. By setting n =1 in the Rydberg series we obtain the maximum possible energy that the hydrogen atom can absorb or emit, thus the maximum energy potential of the atom is:

$$E = \frac{1}{2} m c^2 \alpha^2$$

Equation 11

All of this is consistent with the Bohr model, however the Bohr model is fraught with difficulties which are also present in other later models. Most notable in the Bohr model is the *quantum leap*; the need for the electron to move instantly from one place to another without occupying anywhere in between. Other, later models seek to avoid this by asserting that the state of the electron is inherently uncertain, until it is subject to some sort of observing process, at which time its *wave function*(whatever that is) collapses (however that works) to reveal either its position or velocity. Of course the collapsing wave function is merely a euphemism for the quantum leap and is equally unrealizable and unrealistic.

Since nothing can travel faster than light, the maximum kinetic energy that the orbiting electron in the hydrogen atom could ever possibly achieve is given by:

$$E = \frac{1}{2} m c^2$$

Equation 12

If the issue of the quantum leap is to be overcome, then the electron must orbit at the same orbital radius in all energy states, since anything else implies that the quantum leap exists. If the orbital radius remains constant for all energy states then there can be no change in potential energy between energy states and all of the change in energy must therefore be kinetic in nature. In which case the energy in the nth energy state must be given by

$$E = \frac{1}{2} m v_n^2$$

Equation 13

Where v_n is the orbital velocity in the nth energy state.

But the difference between these two values (Equation 12 and Equation 13) is the energy potential of the atom and so we can write:

$$\frac{1}{2} mc^2 - \frac{1}{2} m v_n^2 = \frac{1}{2} mc^2 \frac{\alpha^2}{n^2}$$

Equation 14

Simplifying and rearranging this gives:

$$\frac{1}{\sqrt{1 - \frac{v_n^2}{c^2}}} = \frac{n}{\alpha}$$

Equation 15

Readers will be familiar with the LHS of this equation as the Lorentz factor Gamma (γ) associated with the effects of special relativity. Hence

$$\gamma_n = \frac{n}{\alpha}$$

Equation 16

And from this we can calculate the value of v_1 as $0.999973371c$.

What this reveals is an atom in which the electron in the base energy state is orbiting at 99.9973371%c and in the maximum possible energy state is orbiting at velocity c. There is no substantial change in orbital radius across this dynamic range and hence no need to introduce the idea of the quantum leap or its latter day equivalent – collapsing wave functions. The angular momentum too remains substantially the same for all energy levels and if we equate that to Planck's constant we find that the orbital radius is

$$R = \frac{\hbar}{mc}$$

Equation 17

We stationary observers then see the orbital frequency as being substantially constant over the entire dynamic range:

$$\omega = \frac{c}{R} = \frac{mc^2}{\hbar}$$

Equation 18

However the orbiting electron is in an environment where time is slowed down due to the effects of relativity by a factor Gamma and therefore where frequency is multiplied by the same factor Gamma and hence, as far as the orbiting electron is concerned, its orbital frequency is

$$\omega_n = \frac{mc^3 n}{Kq^2}$$

Equation 19

Equation 16 shows us that the variable of quantisation within the atom is not angular momentum as postulated by Niels Bohr and incorporated into his and subsequent models, but instead is Gamma, the Lorentz factor. In the base energy state Gamma has a value of 137.03, in the second energy state this is 2*137.03 and so on. This gives a physical significance to α, the Fine Structure Constant. In the past the precise nature of what this constant represents has been missing. This has led to some wild speculation as to its true nature and significance. Here it is simply seen as the ratio of the orbital circumference as traversed at the relativistic speed of the orbiting electron, and so foreshortened, to that traversed at non relativistic speeds.

From Equation 19 it can be seen that the frequencies experienced by the orbiting electron form a harmonic series, starting with a base frequency experienced in the base energy state and rising in integer multiples with each succeeding energy state. We stationary observers, on the other hand, always see the orbital frequency as being substantially constant. This shows that at the heart of the discrete energy levels of the atom lies a harmonic series, much as de Broglie had suggested, only rather than appearing directly in our observing domain, it appears instead in the domain of the moving electron.

We also know that wherever we see a harmonic series in the frequency domain there must be a corresponding sampling process in the time domain. This comes about because a harmonic sequence in the Fourier space appears as a series of impulses or spikes, equally spaced along the frequency axis, forming what is known as a Dirac comb. The inverse Fourier transform of such a Dirac comb in the time domain is itself a Dirac comb formed of a series of impulses or spikes, only this

time in the time domain and spaced T seconds apart where T=1/F and where F is the fundamental frequency of the harmonic series. Such a Dirac comb in the time domain represents a sampling signal and so we can expect to find some sort of sampling process taking place in the time domain when we examine the electro dynamics of the atom.

The stability of such an atom depends on their being a force balance between the orbiting electron and the nuclear proton. It must be stable in each of the various energy states.

Such an analysis is beyond the scope of this paper, however it is dealt with in more detail elsewhere in https://quantum-reality.net/sampling-the-hydrogen-atom/ and also available at http://vixra.org/abs/1202.0072

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[1] Note that *energy potential* should not be confused with potential energy. Energy potential is the difference between the energy contained within the atom in a particular energy state and the maximum possible energy that the atom can possess, while potential energy is energy associated with the difference in orbital radius between a particular energy state and the base energy state in the presence of an electric field and is analogous to gravitational potential energy associated with the height of an object above some datum level.