The number ϕ and the numbers π, ln2, G, $\zeta(k)$, k∈ℕ

Edgar Valdebenito

abstract

In this note we show series for the numbers : π , ln 2 , G , ζ(k) , k∈ℕ.

In this series appears the number ϕ :

$$\phi = \frac{1+\sqrt{5}}{2}$$

Keywords:series, numbers ϕ,π,G,ln 2,ζ(k).

1. Introducción

La sucesión de fibonacci es la $\{F_n\}$, $n \in \mathbb{N} = \{1, 2, 3, ...\}$, con :

$$F_{n+2} = F_{n+1} + F_n \ , \ F_1 = F_2 = 1 \tag{1}$$

el $n-$ ésimo termino esta dado por :

$$F_n = \frac{1}{\sqrt{5}}\left(\phi^n - \left(-\frac{1}{\phi}\right)^n\right) , \ n \in \mathbb{N} , \ \phi = \frac{1+\sqrt{5}}{2} \tag{2}$$

algunos valores de F_n son :

$$\{F_n\} = \{1, 1, 2, 3, 5, 8, ...\} \tag{3}$$

Las constantes π, $\ln 2$, G , se definen por :

$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} \tag{4}$$

$$\ln 2 = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n} \tag{5}$$

$$G = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2} \tag{6}$$

La función zeta de Riemann se define por :

$$\zeta(k) = \sum_{n=1}^{\infty} n^{-k} \ , \ k > 1 \tag{7}$$

En esta nota se muestran series para los números : π , $\ln 2$, G , $\zeta(k)$, $k \in \mathbb{N}$. las series involucran el número ϕ.

2. Series

2.1. Serie para π

$$\pi = \frac{8}{3} + \frac{8}{35}\,\phi^{-1} + 8\sum_{n=1}^{\infty}\frac{v_n\,\phi^{-n-1}}{(4\,n+1)\,(4\,n+3)\,(4\,n+5)\,(4\,n+7)} \tag{8}$$

$$v_n = \{169,\ 492,\ 1621,\ 4033,\ 9654,\ 21\,367,\ \ldots\} \tag{9}$$

$$v_n = \left(32\,n^2 + 64\,n + 38\right)F_n + (4\,n+1)\,(4\,n+3)\,F_{n+1} \tag{10}$$

$$v_{n+2} = \frac{A(n)\,v_n + B(n)\,v_{n+1}}{C(n)}\ ,\ \ n \in \mathbb{N} \tag{11}$$

$$A(n) = 49\,197 + 79\,200\,n + 47\,840\,n^2 + 12\,800\,n^3 + 1280\,n^4 \tag{12}$$

$$B(n) = 17\,517 + 37\,760\,n + 29\,920\,n^2 + 10\,240\,n^3 + 1280\,n^4 \tag{13}$$

$$C(n) = 6317 + 16\,800\,n + 17\,120\,n^2 + 7680\,n^3 + 1280\,n^4 \tag{14}$$

2.2. Serie para ln2

$$\ln 2 = \frac{1}{2} + \frac{\phi^{-1}}{12} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{v_n\,\phi^{-n-1}}{(n+1)\,(n+2)\,(2\,n+1)\,(2\,n+3)} \tag{15}$$

$$v_n = \{27,\ 73,\ 230,\ 558,\ 1313,\ 2871,\ 6069,\ \ldots\} \tag{16}$$

$$v_n = \left(4\,n^2 + 10\,n + 7\right)F_n + (n+1)\,(2\,n+1)\,F_{n+1} \tag{17}$$

$$v_{n+2} = \frac{A(n)\,v_n + B(n)\,v_{n+1}}{C(n)}\ ,\ \ n \in \mathbb{N} \tag{18}$$

$$A(n) = 1128 + 1650\,n + 905\,n^2 + 220\,n^3 + 20\,n^4 \tag{19}$$

$$B(n) = 453 + 855\,n + 595\,n^2 + 180\,n^3 + 20\,n^4 \tag{20}$$

$$C(n) = 183 + 420\,n + 365\,n^2 + 140\,n^3 + 20\,n^4 \tag{21}$$

2.3. Serie para G

$$G = 1 - \frac{\phi^{-1}}{3^2} + \sum_{n=1}^{\infty}\frac{(-1)^n\,v_n\,\phi^{-n-1}}{(2\,n+1)^2\,(2\,n+3)^2} \tag{22}$$

$$v_n = \{7,\ -26,\ -83,\ -285,\ -728,\ -1749,\ -3893,\ \ldots\} \tag{23}$$

$$v_n = 8\,(n+1)\,F_n - (2\,n+1)^2\,F_{n+1} \tag{24}$$

$$v_{n+2} = \frac{A(n)\,v_n + B(n)\,v_{n+1}}{C(n)}\ ,\ \ n \in \mathbb{N} \tag{25}$$

$$A(n) = 241 + 680\,n + 536\,n^2 + 160\,n^3 + 16\,n^4 \tag{26}$$

$$B(n) = 209 + 480\,n + 376\,n^2 + 128\,n^3 + 16\,n^4 \tag{27}$$

$$C(n) = -47 + 24\,n + 152\,n^2 + 96\,n^3 + 16\,n^4 \tag{28}$$

2.4. Serie para ζ(k)

$$\zeta(k) = 1 + \frac{\phi^{-1}}{2^k} + \sum_{n=1}^{\infty} \frac{v_n\,\phi^{-n-1}}{(n+1)^k\,(n+2)^k}\ ,\ k > 1 \tag{29}$$

$$v_n = \left((n+1)^k + (n+2)^k\right) F_n + (n+1)^k\,F_{n+1} \tag{30}$$

$$v_{n+2} = \frac{A(n)\,v_n + B(n)\,v_{n+1}}{C(n)}\ ,\ \ n \in \mathbb{N} \tag{31}$$

$$A(n) = (n+2)^k\,(n+3)^k + 2\,(n+3)^{2k} + (n+2)^k\,(n+4)^k + (n+3)^k\,(n+4)^k \tag{32}$$

$$B(n) = (n+1)^k\,(n+3)^k + 3\,(n+2)^k\,(n+3)^k + (n+2)^k\,(n+4)^k \tag{33}$$

$$C(n) = (n+1)^k\,(n+2)^k + 2\,(n+2)^{2k} + (n+1)^k\,(n+3)^k + (n+2)^k\,(n+3)^k \tag{34}$$

$$v_1 = 2^{k+1} + 3^k \tag{35}$$

$$v_2 = 2^{2k} + 3^{k+1} \tag{36}$$

Ejemplo $k = 3$:

$$\zeta(3) = 1 + \frac{\phi^{-1}}{2^3} + \sum_{n=1}^{\infty} \frac{v_n\,\phi^{-n-1}}{(n+1)^3\,(n+2)^3} \tag{37}$$

$$v_n = \{43,\ 145,\ 570,\ 1648,\ 4523,\ 11\,299,\ \ldots\} \tag{38}$$

Referencias

A. Abramowitz, M. and Stegun, I.A.: Handbook of Mathematical Functions.Nueva York:Dover,1965.

B. Gradshteyn, I.S. and Ryzhik, I.M.: Table of Integrals,Series and Products. 5th ed. , ed. Alan Jeffrey.Academic Press,1994.