Proof Bill hypothesis - a consequence of the properties of invariant identity of a certain type

(Elementary Aspect).

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Annotation. A variant of the solution with the help of Bill hypothesis direct evidence "Great" Fermat's theorem elementary methods rows. New are "invariant identity" (keyword) and obtained by us in the text, the identity of the work, which allowed directly to solve the FLT, and several others.
Also proposed a new formulation of the theory (clause 2.1.4.), the evidence for n = 1,2,3, .. n> 2 and x, y, z> 2.

§1

The proof of FLT

1.1. We obtain the following identity:

$$\boldsymbol{m} = \frac{(x_1 + x_2)^2 + (x_1 + x_3)^2 + \cdots + (x_1 + x_{m+2})^2 + (x_2 + x_3)^2 + \cdots}{x_1^2 + x_2^2 + \cdots + x_{m+2}^2}$$

$$\frac{\ldots + (x_2 + x_{m+2})^2 + \cdots + (x_{m+1} + x_{m+2})^2 - (x_1 + x_2 + \cdots + x_{m+2})^2}{x_1^2 + x_2^2 + \cdots + x_{m+2}^2}$$

Here,
$0 \le m < \infty$- are arbitrary positive integers, including zero;
x_i-are arbitrary elements of arbitrary numerical systems, including zero;
$1 \le i \le m+2$ - are indexes. The value of each "m" is not dependent on the set values of the elements included in the invariant identity.

1.2. Fermat's Last Theorem - "The equation " $a^n + b^n = c^n$ "has no solutions when a, b, c, and n are all positive integers and n is greater than 2."

1.2.1. The proof for n = 1 and, for example, m=1.

$$m = 1 \equiv \frac{(x_1 + x_2)^1 + (x_1 + x_3)^1 + (x_2 + x_3)^1 - (x_1 + x_2 + x_3)^1}{x_1^1 + x_2^1 + x_3^1} =$$

$$= \frac{2(x_1^1 + x_2^1 + x_3^1) - (x_1^1 + x_2^1 + x_3^1)}{x_1^1 + x_2^1 + x_3^1} = 1 - \frac{A_1 = 0}{x_1^1 + x_2^1 + x_3^1}$$

$A_1 = 0$ – is a necessary condition.

1.2.1.1. Let $x_1 = a^1, x_2 = b^1, a^1 + b^1 = z$ – is a positive integer for arbitrary natural «a» and «b». But $a^1 + b^1 = c^1 = z$,then $c = z^{\frac{1}{1}}$– a positive integer - is sufficient condition.

1.2.2. The proof for n = 2 and m=1.

$$m = 1 \equiv \frac{(x_1 + x_2)^2 + (x_1 + x_3)^2 + (x_2 + x_3)^2 - (x_1 + x_2 + x_3)^2}{x_1^2 + x_2^2 + x_3^2} =$$
$$= \frac{2(x_1^2 + x_2^2 + x_3^2) - (x_1^2 + x_2^2 + x_3^2)}{x_1^2 + x_2^2 + x_3^2} = 1 - \frac{A_2 = 0}{x_1^2 + x_2^2 + x_3^2}$$

and $A_2 = 0$ - necessary condition.

1.2.2.1. Let $x_1 = a^2, x_2 = b^2, a^2 + b^2 = z$ – a positive integer when «a» and «b» arbitrary natural numbers. And $A_2 = 0$. But if $a^2 + b^2 = c^2$, c = $p^2 + q^2$ - is natural when $a = p^2 - q^2$ and $b = 2\,p\,q$ (p and q – arbitrary coprime positive integers). Therefore, $c^2 = z^2$ and $c = z^{\frac{2}{2}}$ – will be natural is sufficient condition, because, as you know, these expressions give all solutions of $a^2 + b^2 = c^2$ in c2.oprime natural numbers.

1.2.2.2. Suppose that $a_1^2 + b_1^2 = z_1$ for all other relatively prime positive integers that can be the solutions of the equation in positive integers $a_1^2 + b_1^2 = c_1^2$. Then, $c_1^4 = z_1^2$ and $c_1 = z_1^{\frac{1}{2}}$ cannot be a natural number - the sufficiency of the condition is not satisfied. Thus, for n = 2 when $A_2 = 0$ and $z_1^{\frac{1}{2}}$- solutions in natural numbers there. This suggests the need to consider for n ≥ 1, both conditions: $A_i = 0$, or $A_j \neq 0$ - necessary, $z_n^{\frac{1}{n}}$ - sufficient.

1.2.3. The prof for n = 3, m = 1.

$$m = 1 \neq \frac{(x_1 + x_2)^3 + (x_1 + x_3)^3 + (x_2 + x_3)^3 - (x_1 + x_2 + x_3)^3}{x_1^3 + x_2^3 + x_3^3} =$$
$$= \frac{2(x_1^3 + x_2^3 + x_3^3) - (x_1^3 + x_2^3 + x_3^3)}{x_1^3 + x_2^3 + x_3^3} = 1 - \frac{A_3 = 6\,x_1 x_2 x_3}{x_1^3 + x_2^3 + x_3^3}$$

$A_3 = 6\,x_1 x_2 x_3 \neq 0$- is necessary condition .

1.2.3.1. Let $x_1 = a^3, x_2 = b^3, a^3 + b^3 = z$ –is a positive integer for arbitrary natural a , b and $A_3 \neq 0$. Suppose that $a^3 + b^3 = c^3$. Then, $c^6 = z^2$, c = $z^{\frac{2}{6}} = z^{\frac{1}{3}}$ - It cannot be a natural number - a sufficient condition.

1.2.4. The proof for n > 2 and m = 1 .

$$m = 1 \neq \frac{(x_1+x_2)^n + (x_1+x_3)^n + (x_2+x_3)^n - (x_1+x_2+x_3)^n}{x_1^3+x_2^3+x_3^3} =$$

$$= \frac{2(x_1^n+x_2^n+x_3^n) - (x_1^n+x_2^n+x_3^n)}{x_1^n+x_2^n+x_3^n} = 1 - \frac{A_n \neq 0}{x_1^n+x_2^n+x_3^n}$$

If n $>$ 2 $A_n \neq 0$ - is a necessary condition

1.2.5.

$$m \neq \frac{\sum_{i=1,i<j}^{i=m+1,j=m+2}\left(x_i+x_j\right)^n - (x_1+x_2+\cdots+x_{m+2})^n}{x_1^n+x_2^n+\cdots+x_{m+2}^n} =$$

$$= \frac{(m+1)(x_1^n+x_2^n+\cdots+x_{m+2}^n) - (x_1^n+x_2^n+\cdots+x_{m+2}^n+A_n \neq 0)}{x_1^n+x_2^n+\cdots+x_{m+2}^n} =$$

$$= m - \frac{A_n \neq 0}{x_1^n+x_2^n+\cdots+x_{m+2}^n}$$

for $n > 2$ $A_n \neq 0$ – is a necessary condition.

1.2.5. 1.

Let $x_1 = a^n, x_2 = b^n, a^n + b^n = z$ - is a positive integer for arbitrary

natural «a» and «b». Suppose that n > 2 $a^n + b^n = c^n$. Then,

$c^{2n} = z^2$ and c = $z^{\frac{1}{n}}$, which is only possible for n = 1 and n = 2 (with considering 1.2.2.)- is a sufficient condition.

1.3. Thus, for n >2 $A_n \neq 0$ *and* c = $z^{\frac{1}{n}}$ are necessary and sufficient condition for insolvability of the equation $a^n + b^n = c^n$ in the natural number a,b,c .

1.4. From §1, in the end, it follows that for n> 2, $A_n \neq 0$ is a necessary and sufficient condition for unsolvability of equations

$a^n + b^n = c^n$ in the natural numbers a, b, c. The proof is complete.

1.5. Another variant of the proof of the FLT. example.(3) ,item..2.2.

§2

The proof of Beal's Conjecture

2.1. Beal conjecture : «If A^x + B^y =C^z,where A,B,C,x,y,z - are natural numbers with x,y,z>2 then A,B,C have a common prime factor » (Wikipedia. "Open mathematical problems," in particular, the open (unresolved) mathematical problems).

2.1.1. Let in addition to the .2.1. § 2 in the $A^x + B^y = C^z$ (A,B,C)=1- coprime (As will be shown in §3, addition significantly), $x_1 = A^x$, $x_2 = B^y$, $A^x + B^y = r_C$ a natural numbers for arbitrary natural A and B. Suppose that $A^x + B^y = C^z$ for x,y,z>2. Then, similar to the § 1 the above $C^{2z} = r_C^2$ and C $= r_C^{\frac{1}{z}}$- cannot be a natural number.

2.1.2. By analogy with 2.1.1. § 2 – operations with $C^z – B^y = A^x = r_A$ and $C^z – A^x = B^y = r_B$.

2.1.3. Thus, the equation $A^x + B^y = C^z$ for (A,B,C) = 1 and x,y,z >2 – natural insoluble in natural numbers, and therefore cannot have a common prime factor. The proof is complete.

2.1.4. Finally, taking into account §§1 and 2, “The equation $A^x + B^y = C^z$ at

x, y, z> 2 – natural numbers each, including x = y = z = n, has no solution in the coprime natural numbers (A, B, C) = 1”.

§3

3.1 If, in particular, A +B=C, (A, B, C) =1- is coprime, then the equation $A_1 + B_1 = C_1$ $((A_1, B_1, C_1) \neq 1$ – functions A, B, C) are infinite number of solutions in positive integers when, particularly, (x,y,z)=1- are arbitrary natural and have a common prime factor.

3.2.1. Let

$$A + B \equiv C,$$

where A, B - are arbitrary natural numbers, as

$$A^{\alpha x - pyz = 1} + B^{\beta y - qxz = 1} \equiv C^{\gamma z - mxy = 1} \quad [\mathbf{1}]$$

Multiplying $[\mathbf{1}]$ by

$$A^{pyz} B^{qxz} C^{mxy}$$

, we obtain

$$(A^{\alpha} B^{qz} C^{my})^x + (A^{pz} B^{\beta} C^{mx})^y \equiv$$

$$\equiv (A^{py} B^{qx} C^{\gamma})^z \quad [\mathbf{2}].$$

All values are indicators $[\mathbf{2}]$ we obtain from the equations

$$\alpha x - pyz = 1$$

$$\beta y - qxz = 1 \quad [\mathbf{3}]$$

$$\gamma z - mxy = 1$$

, where $x, y, z, \alpha, \beta, \gamma, p, q, m$ - corresponding solution $[\mathbf{3}]$ natural numbers.

3.2.2. If $\alpha_0, \beta_0, \gamma_0, p_0, q_0, m_0$ any (or minimal) solutions of equations positive integers for fixed values x, y, z
(G.Devenport,"THE HIGHER ARITHMETIC","Science",Fizmatgiz,Moscow, 1965,p.88-89,item5"),

then

$$\alpha = \alpha_0 + yzQ_1 \quad p = p_0 + xQ_1$$
$$\beta = \beta_0 + xzQ_2 \quad q = q_0 + yQ_2$$
$$\gamma = \gamma_0 + xyQ_3 \quad m = m_0 + zQ_3,$$

$Q_1, Q_2, Q_3 -$ are arbitrary natural (whole) numbers, or zero, and

$$(A^{\alpha_0+yzQ_1}B^{q_0 z+yzQ_2}C^{m_0 y+yzQ_3})^x +$$
$$+(A^{p_0 z+xzQ_1}B^{\beta_0+xzQ_2}C^{m_0 x+xzQ_3})^y = \qquad [4]$$
$$= (A^{p_0 y+xyQ_1}B^{q_0 x+xyQ_2}C^{\gamma_0+xyQ_3})^z.$$

3.3 Let $AP + BP \equiv CP$ $[\mathbf{5}]$ for arbitrary natural numbers A and B ,where P -is arbitrary prime number. Then, with respect to $[\mathbf{2}]$

$$(P^{\alpha+qz+my}A^{\alpha}B^{qz}C^{my})^x + (P^{pz+\beta+mx}A^{pz}B^{\beta}C^{mx})^y \equiv$$
$$\equiv (P^{py+qx+\gamma}A^{py}B^{qx}C^{\gamma})^z \ [\mathbf{6}]$$

3.3.1 $A = 2; B = 3; C = 5; P = 7$

$$x = 4; y = 5; z = 7$$

Since,

$$\alpha \times 4 - p \times 5 \times 7 = 1$$
$$\alpha = 9; p = 1$$
$$\beta \times 5 - q \times 4 \times 7 = 1$$
$$\beta = 17; q = 3$$
$$\gamma \times 7 - m \times 4 \times 5 = 1$$
$$\gamma = 3; m = 1.$$

Thus,

$$(7^{35} \times 2^9 \times 3^{21} \times 5^5)^4 + (7^{28} \times 2^7 \times 3^{17} \times 5^4)^5 =$$
$$= (7^{20} \times 2^5 \times 3^{12} \times 5^3)^7.$$

3.3.2. An identity: $[(2A^{xy+1})^y]^x + [(2A^{xy+1}))^x]]^y \equiv (2A^{xy})^{xy+1}$. Here, A, x, y - positive arbitrary integer numbers, including zero.

3.3.2.1. This identity allows us to obtain the following equation:

$[(2A^{abxy+1})^{aby}]^x + [(2A^{abxy+1})^{abx}]^y = [(2A^{abxy})^c]^z$.

Here, (x, y, z) = 1- a, particularly arbitrary coprime integers,

cz = abxy + 1, a, b, c are found from the equation cz - abxy = 1 (example 3.2.2.).

For example: x = 5, y = 7, z = 11, 11c - 5.7.ab = 1 11.86 - 35.27 = 1, where,

a = 3, b = 9, c = 86 and

$$[(2A^{27.35+1})^{27.7}]^5 + [(2A^{27.35+1})^{27.5}]^7 = [(2A^{27.35})^{86}]^{11}.$$

Thus, you can get all the countless decisions that equation.

§ 4

4.1. One option of finding solutions in positive integers the equation

$A^4 + B^3 = C^2$ at (A, B, C) = 1, or A, B, C - of all even violating values

performance of the original equation degrees when cutting.

4.1.1. We have the identity: $[y(y^2 + 3)]^2 - (3y^2 + 1)^2 = (y^2 - 1)^3$.

Let $3y^2 + 1 = x^2$. Then, $x^2 - 3y^2 = 1$ and $[y(y^2 + 3)]^2 - x^4 = (y^2 - 1)^3$.

AccordingW.Sierpinski ("On reshengii equations in integers" Fizmatgiz, Moscow, 1961 str.29-30)o

$x_{k+1} = x_1 x_k + 3y_1 y_k \quad y_{k+1} = \boldsymbol{y_1 x_k + x_1 y_k}$ **for** $\mathbf{1 \leq \kappa < \infty}$**.** When $\boldsymbol{x_1}$**=2,** $\boldsymbol{y_1}$**=1**

$\mathbf{2^2 - 3 . 1 = 1}$, and tdi etc. recursively to infinity:

$7^4 + 15^3 = 76^2, \quad 26^4 + 224^3 = 3420^2 \quad , \quad 97^4 + 3135^3 = 175784^2$, etc.

4.2. Tam same (page 63) is a process for the preparation of similar solutions, such as:

$28^2 + 8^3 = 6^4$, $1176^2 + 49^3 = 35^4$and (method not specified) $27^2 + 18^3 = 9^4$,

$63^2 + 36^3 = 15^4$.

4.3. Another option to find solutions in natural numbers

equation $z^2 + x^3 = y^4$ (7).

4.3.1. From (7) $x^3 = y^4 - z^2$, $x = y^2 - z$, $x^2 = y^2 + z$, $z = y^2 - x$ (8) and

$x^2 + x - 2y^2 = 0$ (9).

4.3.2. According W.Sierpinski (item .4.1.1.) page 21-23, formula

$$x_{n+1} = 3x_n + 4y_n + 1 \qquad y_{n+1} = 2x_n + 3y_n + 1 \qquad n=1,2,3,\dots .$$

recurrently give all solutions in positive integers the equation (9).

4.3.3. Thus n=1 $28^2+8^3=6^4$ $7^2 . 2^4+4^3 . 2^3=3^4 . 2^4$,но $7^2+4^3 \neq 3^4$

4.3.4. From item 4.1.1. and 4.2.

$26^4 +224^3 = 3420^2$ $13^4 . 2^4 +7^3 . 2^{15} = 855^2 . 2^4$, but $13^4 +7^3 \neq 855^2$

$63^2 +36^3 =15^4$ $7^2 . 3^4+2^6 . 3^6 = 5^4 . 3^4$, but $7^2 +2^6 \neq 5^4$.

4.3.5. From item 4.3.1.,4.3.2. $[(2x_n + 3y_n)^2+ x_n+2y_n]^2 + (3x_n+4y_n + 1))^3= (2x_n + 3y_n + 1))^4$ -

- recurrence equation.

n =1,2 ,3,......, x_1=1 , y_1 =1 .

Thus n=3 $(41328)^2+ (288)^3 = (204)^4$, $287^2 . 2^8 .3^4 + 1^3 . 2^{15} . 3^6 = 17^4 . 2^8 . 3^4$,

but $287^2 + 1^3 \neq 17^4$.

References:

1. H.DABENPORT, "THE HIGHER ARITHMETIC", HARPER & SROTHERS, NEW YORK

2. W.Sierpinski, "On reshengii equations in integers" Fizmatgiz, Moscow,

1961 p.p. 29-30, 63, 21-23.

3. Reuven Tint (www.ferm-tint.blogspot.co.il) «Unique invariant identity and the ensuing unique consequences (elementary aspect)».