Number pi , formulas , curves , surfaces

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abstract

In this note we give formulas , curves and surfaces related with the constant pi.

$\pi = 3.1415926535\ ....$

keywords:number pi , formulas , curves, surfaces.

1. Introduction: formulas

Recordamos dos resultados clásicos :

Teorema 1. Sea $m \in \mathbb{N} = \{1, 2, 3, ...\}$, $x, y \in \mathbb{R}$ tales que :

$$x^2 + y^2 < 1 \,,\ \operatorname{Im}((x + i\,y)^m) = 1 + \operatorname{Re}((x + i\,y)^m) \tag{1}$$

se tiene :

$$\pi = 4 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \operatorname{Im}((x + i\,y)^{m\,n}) \tag{2}$$

$$\pi = 4 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \sum_{k=1}^{[(m\,n+1)/2]} (-1)^{k-1} \binom{m\,n}{2\,k-1} x^{m\,n-2\,k+1}\, y^{2\,k-1} \tag{3}$$

Teorema 2. Sea $m \in \mathbb{N} = \{1, 2, 3, ...\}$, $x, y \in \mathbb{R}$, $z > 0$, tales que :

$$x^2 + y^2 < z^2 \,,\ \operatorname{Im}((x + i\,y)^m) = z^m + \operatorname{Re}((x + i\,y)^m) \tag{4}$$

se tiene :

$$\pi = 4 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\,z^{m\,n}} \operatorname{Im}((x + i\,y)^{m\,n}) \tag{5}$$

$$\pi = 4 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\,z^{m\,n}} \sum_{k=1}^{[(m\,n+1)/2]} (-1)^{k-1} \binom{m\,n}{2\,k-1} x^{m\,n-2\,k+1}\, y^{2\,k-1} \tag{6}$$

una clásica fórmula es :

$$\pi = 4 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\,2^n} \operatorname{Im}((-1 + i)^n) \tag{7}$$

Sea $m \in \mathbb{N} = \{1, 2, 3, ...\}$, C_m , S_m las regiones definidas por :

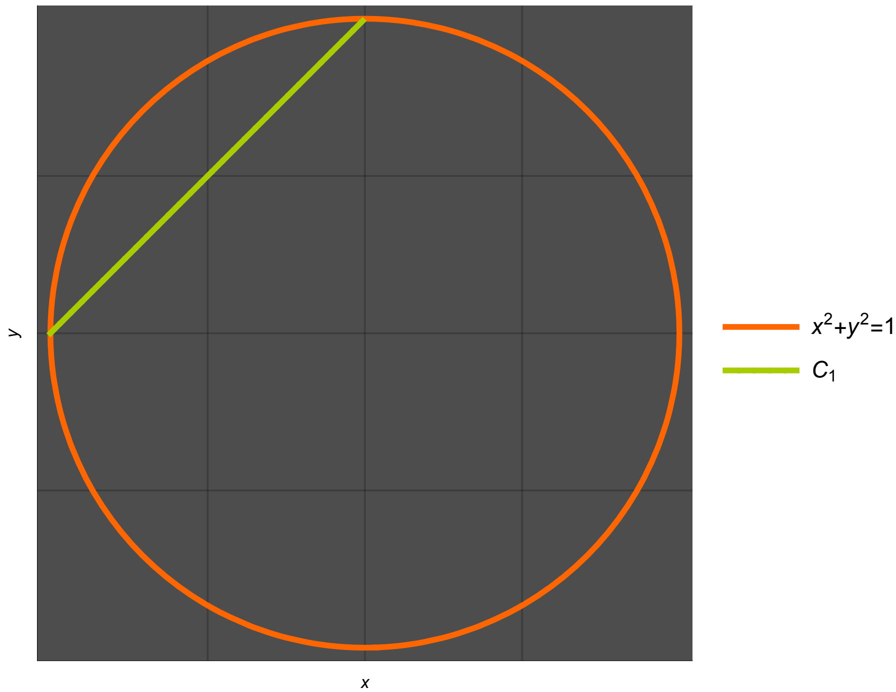
$$C_m = \left\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1 \ :\ \operatorname{Im}((x + i\,y)^m) = 1 + \operatorname{Re}((x + i\,y)^m)\right\} \tag{8}$$

$$S_m = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 < z^2 \ : z > 0 : \operatorname{Im}((x + i\,y)^m) = z^m + \operatorname{Re}((x + i\,y)^m)\right\} \tag{9}$$

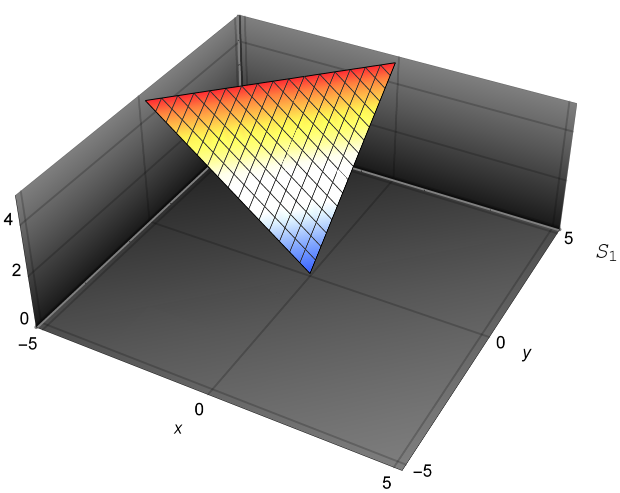
en esta nota mostramos la representación gráfica de las regiones : $C_m, S_m,\ m = 1, 2, 3, 4.$

2. Curve C_1 , Surface S_1

$$C_1 = \{(x,\ y) \in \mathbb{R}^2 :\ x^2 + y^2 < 1 : y = 1 + x\} \tag{10}$$

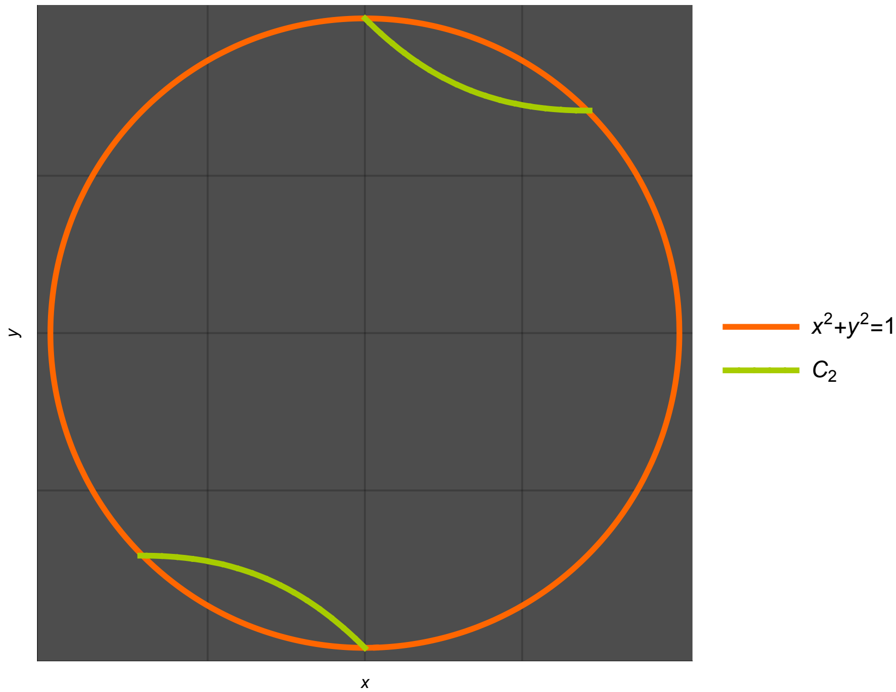


$$S_1 = \{(x,\ y,\ z) \in \mathbb{R}^3 :\ x^2 + y^2 < z^2 : z > 0 :\ y = z + x\} \tag{11}$$

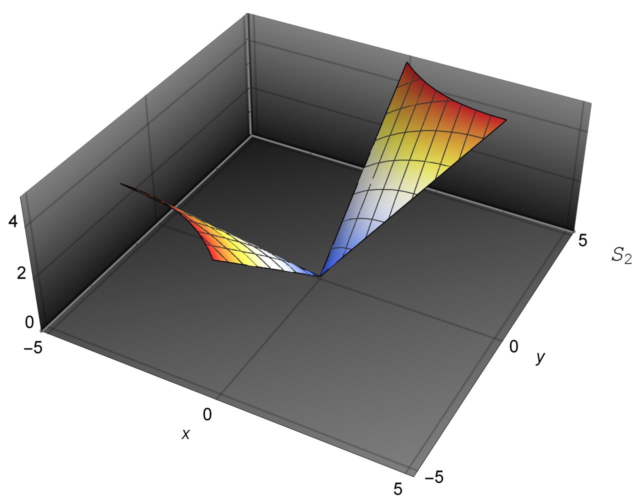


3. Curve C_2 , Surface S_2

$$C_2 = \{(x,\ y) \in \mathbb{R}^2 :\ x^2 + y^2 < 1 : 2\,x\,y = 1 + x^2 - y^2\} \tag{12}$$

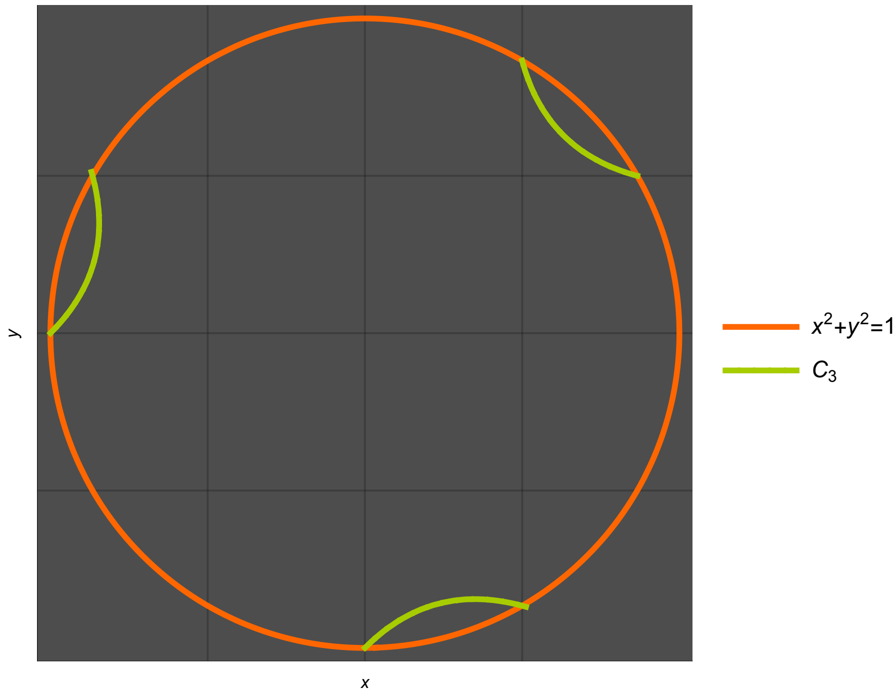


$$S_2 = \{(x,\ y,\ z) \in \mathbb{R}^3 :\ x^2 + y^2 < z^2 : z > 0 :\ 2\,x\,y = z^2 + x^2 - y^2\} \tag{13}$$

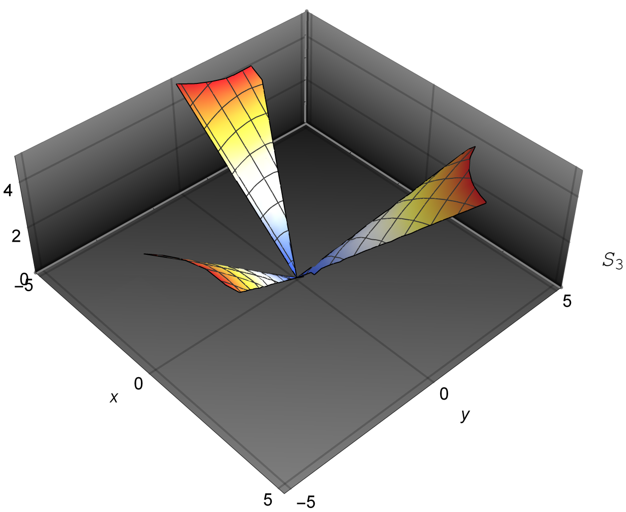


4. Curve C_3 , Surface S_3

$$C_3 = \{(x,\ y) \in \mathbb{R}^2 :\ x^2 + y^2 < 1 : 3\,x^2\,y - y^3 = 1 + x^3 - 3\,x\,y^2\} \tag{14}$$

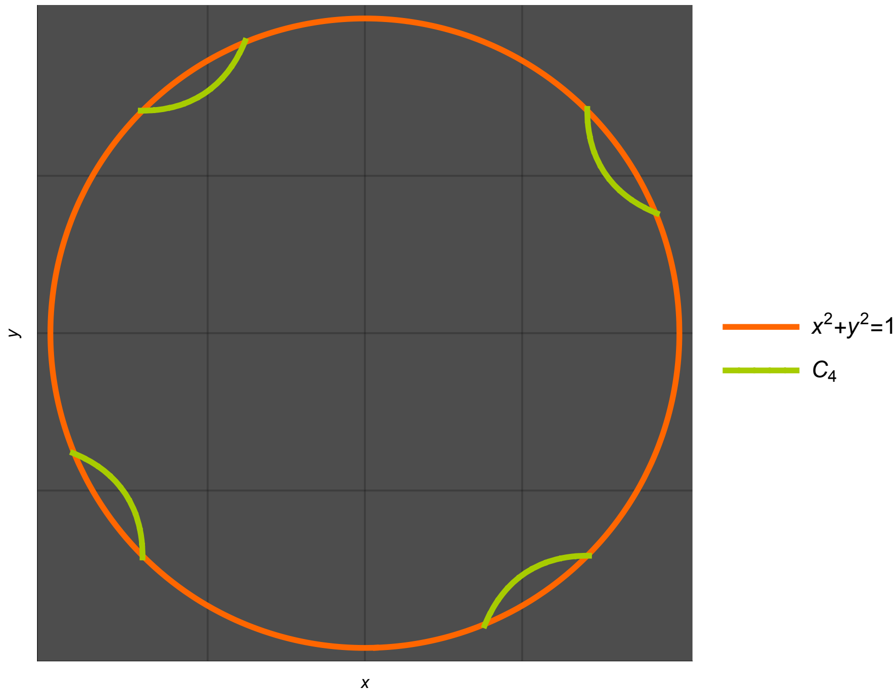


$$S_3 = \{(x,\ y,\ z) \in \mathbb{R}^3 :\ x^2 + y^2 < z^2 : z > 0 :\ 3\,x^2\,y - y^3 = z^3 + x^3 - 3\,x\,y^2\} \tag{15}$$

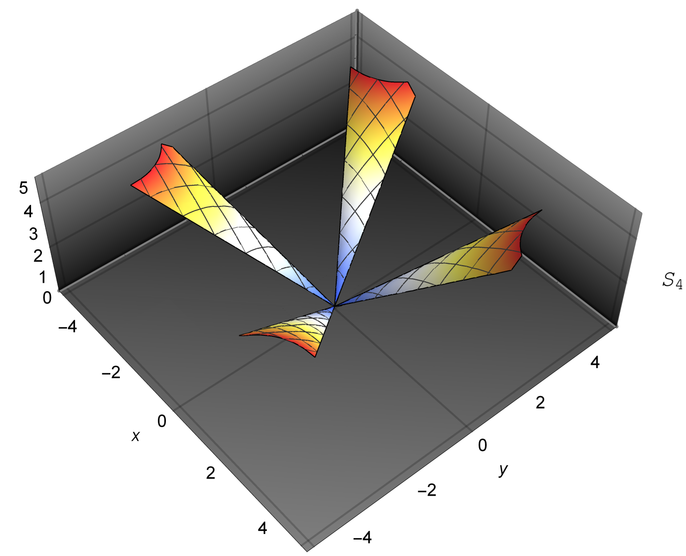


5. Curve C_4 , Surface S_4

$$C_4 = \{(x,\ y) \in \mathbb{R}^2 :\ x^2 + y^2 < 1 : 4\,x^3\,y - 4\,x\,y^3 = 1 + x^4 - 6\,x^2\,y^2 + y^4\} \tag{16}$$



$$S_4 = \{(x,\ y,\ z) \in \mathbb{R}^3 : \ x^2 + y^2 < z^2 : z > 0 : \ 4\,x^3\,y - 4\,x\,y^3 = z^4 + x^4 - 6\,x^2\,y^2 + y^4\} \tag{17}$$



References

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