Divide Beal's Conjecture into Several Parts Gradually to Prove the Beal's Conjecture

Zhang Tianshu

Zhanjiang city, Guangdong province, China
Email: chinazhangtianshu@126.com
china.zhangtianshu@tom.com

Introduction: The Beal's Conjecture was discovered by Andrew Beal in 1993. Later the conjecture was announced in December 1997 issue of the Notices of the American Mathematical Society. Yet it is still both unproved and un-negated a conjecture hitherto.

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Abstract

In this article, we first classify A, B and C according to their respective odevity, and thereby get rid of two kinds from $A^X+B^Y=C^Z$. Then, affirmed $A^X+B^Y=C^Z$ in which case A, B and C have at least a common prime factor by several concrete equalities. After that, proved $A^X+B^Y\neq C^Z$ in which case A, B and C have not any common prime factor by mathematical induction with the aid of the symmetric relations of positive odd numbers concerned after divide the inequality in four. Finally, reached a conclusion that the Beal's conjecture holds water via the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements.

Keywords: Beal's conjecture; indefinite equation; inequality; odevity; mathematical induction; symmetry.

The Proof

The Beal's Conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.

We consider the limits of values of aforesaid A, B, C, X, Y and Z as given requirements for hinder concerned indefinite equations and inequalities.

First we classify A, B and C according to their respective odevity, and thereby remove following two kinds from $A^X+B^Y=C^Z$.

1. If A, B and C all are positive odd numbers, then A^X+B^Y is an even number, yet C^Z is an odd number, so there is only $A^X+B^Y\neq C^Z$ due to an odd number ≠ an even number.

2. If any two of A, B and C are positive even numbers, yet another is a positive odd number, then when A^X+B^Y is an even number, C^Z is an odd number; yet when A^X+B^Y is an odd number, C^Z is an even number, so there is only $A^X+B^Y\neq C^Z$ due to an odd number ≠ an even number.

Thus, we merely continue to have following two kinds of $A^X+B^Y=C^Z$ under the given requirements.

1. A, B and C all are positive even numbers.

2. A, B and C are two positive odd numbers and a positive even number.

For indefinite equation $A^X+B^Y=C^Z$ which satisfies aforementioned either set of qualifications, in fact, it has many sets of the solution with A, B and C which are positive integers. Let us instance two concrete equalities as follows respectively to explain this proposition.

When A, B and C all are positive even numbers, if let A=B=C=2 and X=Y ≥3, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $2^X+2^X=2^{X+1}$. Obviously indefinite equation $A^X+B^Y=C^Z$ at the here has a set of the solution with A, B and C which are positive integers 2, 2 and 2, and that A, B and C have common prime factor 2.

In addition, if let A=B=162, C=54, X=Y=3 and Z=4, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $162^3+162^3=54^4$. So indefinite equation $A^X+B^Y=C^Z$ at the here has a set of the solution with A, B and C which are positive integers 162, 162 and 54, and that A, B and C have common prime factors 2 and 3.

When A, B and C are two positive odd numbers and a positive even number, if let A=C=3, B=6, X=Y=3 and Z=5, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $3^3+6^3=3^5$. So indefinite equation $A^X+B^Y=C^Z$ at the here has a set of the solution with A, B and C which are positive integers 3, 6 and 3, and that A, B and C have common prime factor 3.

In addition, if let A=B=7, C=98, X=6, Y=7 and Z=3, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $7^6+7^7=98^3$. So indefinite equation $A^X+B^Y=C^Z$ at the here has a set of the solution with A, B and C which are positive integers 7, 7 and 98, and that A, B and C have common prime factor 7.

Therefore, indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either set of qualifications is able to hold water, but A, B and C must have at least a common prime factor.

By now, if we can prove that there is only $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor, then the conjecture is tenable definitely.

Since A, B and C have common prime factor 2 when A, B and C all are positive even numbers, so these circumstances that A, B and C have not a common prime factor can only occur in which case A, B and C are two positive odd numbers and a positive even number.

If A, B and C have not a common prime factor, then any two of them have not a common prime factor either, because in case any two have a common prime factor, namely A^X+B^Y, C^Z-A^X or C^Z-B^Y has a common prime factor, yet remaining one has not the common prime factor, then it will directly lead up to $A^X+B^Y\neq C^Z$, $C^Z-A^X\neq B^Y$ or $C^Z-B^Y\neq A^X$ according to the unique factorization theorem of natural number.

Unquestionably, let following two inequalities add together to replace $A^X+B^Y\neq C^Z$ under the given requirements plus the set of qualifications that A, B and C are two positive odd numbers and a positive even number without a common prime factor, this is possible affirmatively.

1. $A^X+B^Y\neq 2^ZG^Z$ under the given requirements plus the set of qualifications that A and B are two positive odd numbers, G is a positive integer, and that A, B and G have not a common prime factor.

2. $A^X+2^YD^Y\neq C^Z$ under the given requirements plus the set of qualifications that A and C are two positive odd numbers, D is a positive integer, and that A, C and D have not a common prime factor.

For aforesaid $A^X+B^Y\neq 2^ZG^Z$, when G=1, it is exactly $A^X+B^Y\neq 2^Z$. When G>1: if G is an odd number, then the inequality changes not, namely it is still $A^X+B^Y\neq 2^ZG^Z$; if G is an even number, then the inequality is expressed by $A^X+B^Y\neq 2^W$ or $A^X+B^Y\neq 2^WH^Z$, where H is an odd number ≥3, and W is an integer > Z.

Without doubt, $A^X+B^Y\neq 2^W$ can represent $A^X+B^Y\neq 2^Z$, and $A^X+B^Y\neq 2^WH^Z$ can represent $A^X+B^Y\neq 2^ZG^Z$, where H is an odd numbers ≥3, and W is an integer ≥3. So $A^X+B^Y\neq 2^ZG^Z$ is expressed by two inequalities as follows.

(1) $A^X+B^Y\neq 2^W$, where A and B are positive odd numbers without a common prime factor, and that X, Y and W are integers ≥3.

(2) $A^X+B^Y\neq 2^WH^Z$, where A, B and H are positive odd numbers without a common prime factor, H ≥3, and that X, Y, Z and W are integers ≥3.

Again go back to aforementioned $A^X+2^YD^Y\neq C^Z$ to say, when D=1, it is exactly $A^X+2^Y\neq C^Z$. When D>1: if D is an odd number, then the inequality changes not, namely it is still $A^X+2^YD^Y\neq C^Z$; if D is an even number, then the inequality is expressed by $A^X+2^W\neq C^Z$ or $A^X+2^WR^Y\neq C^Z$, where R is an odd number ≥3, and W is an integer >Y.

Without doubt, $A^X+2^W \neq C^Z$ can represent $A^X+2^Y \neq C^Z$, and $A^X+2^W R^Y \neq C^Z$ can represent $A^X+2^Y D^Y \neq C^Z$, where R is an odd number ≥3, and W is an integer ≥3. So $A^X+2^Y D^Y \neq C^Z$ is expressed by two inequalities as follows.

(3) $A^X+2^W \neq C^Z$, where A and C are positive odd numbers without a common prime factor, and that X, W and Z are integers ≥3.

(4) $A^X+2^W R^Y \neq C^Z$, where A, R and C are positive odd numbers without a common prime factor, R ≥3, and that X, Y, Z and W are integers ≥3.

We regard the limits of values of A, B, C, H, R, X, Y, Z and W in above-listed four inequalities plus their co-prime relation in each of inequalities as known requirements thereinafter.

Thus it can be seen, the proof of $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor is changed to prove the existence of above-listed four inequalities under the known requirements. Such being the case, we shall first prove $A^X+B^Y \neq 2^W$. For this purpose, we should beforehand expound some circumstances and terminologies relating to the proof.

First let us divide all positive odd numbers into two kinds, i.e. Φ and Ω. Namely the form of Φ is 1+4n, and the form of Ω is 3+4n, where n ≥ 0. Odd numbers of Φ and Ω respectively arrange as the follows orderly.

Φ: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61…1+4n …

Ω: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63…3+4n …

Besides, we likewise use sign Φ to denote one of kind Φ, and use sign Ω to denote one of kind Ω in the sequence of non-concrete odd numbers and on the formulations concerning the symmetry of odd numbers.

After that, let us list from small to large positive odd numbers plus $2^{W-1}H^{Z}$ among them below, where H is an odd number ≥1, and W, Z ≥ 3. Also label the belongingness of each of odd numbers alongside itself.

1^{W}∈Φ, 3∈Ω; (2^{2}), 5∈Φ, 7∈Ω, (2^{3}), 9∈Φ, 11∈Ω, 13∈Φ, 15∈Ω, (2^{4}), 17∈Φ, 19∈Ω, 21∈Φ, 23∈Ω, 25∈Φ, 3^{3}∈Ω, 29∈Φ, 31∈Ω, (2^{5}), 33∈Φ, 35∈Ω, 37∈Φ, 39∈Ω, 41∈Φ, 43∈Ω, 45∈Φ, 47∈Ω, 49∈Φ, 51∈Ω, 53∈Φ, 55∈Ω, 57∈Φ, 59∈Ω, 61∈Φ, 63∈Ω, (2^{6}), 65∈Φ, 67∈Ω, 69∈Φ, 71∈Ω, 73∈Φ, 75∈Ω, 77∈Φ, 79∈Ω, 3^{4}∈Φ, 83∈Ω, 85∈Φ, 87∈Ω, 89∈Φ, 91∈Ω, 93∈Φ, 95∈Ω, 97∈Φ, 99∈Ω, 101∈Φ, 103∈Ω, 105∈Φ, 107∈Ω, $(2^{2}\times3^{3})$, 109∈Φ, 111∈Ω, 113∈Φ, 115∈Ω, 117∈Φ, 119∈Ω, 121∈Φ, 123∈Ω, 5^{3}∈Φ, 127∈Ω, (2^{7}), 129∈Φ, 131∈Ω, 133∈Φ, 135∈Ω, 137∈Φ, 139∈Ω, 141∈Φ, 143∈Ω, 145∈Φ, 147∈Ω, 149∈Φ, 151∈Ω, 153∈Φ, 155∈Ω, 157∈Φ, 159∈Ω, 161∈Φ, 163∈Ω, 165∈Φ, 167∈Ω, 169∈Φ, 171∈Ω, 173∈Φ, 175∈Ω, 177∈Φ, 179∈Ω, 181∈Φ, 183∈Ω, 185∈Φ, 187∈Ω, 189∈Φ, 191∈Ω, 193∈Φ, 195∈Ω, 197∈Φ, 199∈Ω, 201∈Φ, 203∈Ω, 205∈Φ, 207∈Ω, 209∈Φ, 211∈Ω, 213∈Φ, 215∈Ω, $(2^{3}\times3^{3})$, 217∈Φ, 219∈Ω, 221∈Φ, 223∈Ω, 225∈Φ, 227∈Ω, 229∈Φ, 231∈Ω, 233∈Φ, 235∈Ω, 237∈Φ, 239∈Ω, 241∈Φ, 3^{5}∈Ω, 245∈Φ, 247∈Ω, 249∈Φ, 251∈Ω, 253∈Φ, 255∈Ω, (2^{8}), 257∈Φ, 259∈Ω, 261∈Φ, 263∈Ω, 265∈Φ, 267∈Ω, 269∈Φ, 271∈Ω …

By this token, the permutation of positive odd numbers from small to large has infinitely many cycles of Φ plus Ω, to wit Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; ⋯

Let us regard each of $2^{W-1}H^{Z}$ as a symmetric center of positive odd numbers concerned, then positive odd numbers on the left side of the symmetric center and positive odd numbers near the symmetric center on the right side of the symmetric center are one-to-one bilateral symmetries, whether they are at the number axis or in the sequence of natural numbers.

For example, if regard 2^{W-1} as a symmetric center, then $2^{W-1}-1\in\Omega$ and $2^{W-1}+1\in\Phi$, $2^{W-1}-3\in\Phi$ and $2^{W-1}+3\in\Omega$, $2^{W-1}-5\in\Omega$ and $2^{W-1}+5\in\Phi$ etc are bilateral symmetry respectively.

Such symmetric relations of positive odd numbers indicate that for any of $2^{W-1}H^{Z}$ as a symmetric center, it can only symmetrize one of kind Φ and one of kind Ω, yet can not symmetrize two of either kind.

After regard any of $2^{W-1}H^{Z}$ as a symmetric center, from the symmetric center start out, there are both finitely many cycles of Ω plus Φ leftwards until Ω=3 with Φ=1, and infinitely many cycles of Φ plus Ω rightwards.

According to the symmetric relations of positive odd numbers, two distances from a symmetric center to bilateral symmetric Φ and Ω on two sides of the symmetric center are two equilong segments at the number axis or two identical odd differences in the sequence of natural numbers.

Thus, the sum of two bilateral symmetric odd numbers Φ and Ω is equal to the double of the even number which the symmetric center expresses.

Yet over the left, a sum of two non-symmetric odd numbers is affirmatively unequal to the double of the even number which the symmetric center expresses. In other words, after regard any of $2^{W-1}H^{Z}$ as a symmetric center, not just each other's symmetric odd numbers can only be Φ and Ω, but also $2^{W}H^{Z}$ as the sum of two odd numbers can only be got from the addition of bilateral symmetric two odd numbers.

Attention please, the conclusion is a significant evidence for latter proofs.

Next, we list orderly many kinds of odd numbers which have a common odd number as the base number, and that label the belongingness of each of them alongside itself.

$1^{1}\in\Phi$, $3^{1}\in\Omega$, $5^{1}\in\Phi$, $7^{1}\in\Omega$, $9^{1}\in\Phi$, $11^{1}\in\Omega$,

$1^{2}\in\Phi$, $3^{2}\in\Phi$, $5^{2}\in\Phi$, $7^{2}\in\Phi$, $9^{2}\in\Phi$, $11^{2}\in\Phi$,

$1^{3}\in\Phi$, $3^{3}\in\Omega$, $5^{3}\in\Phi$, $7^{3}\in\Omega$, $9^{3}\in\Phi$, $11^{3}\in\Omega$,

$1^{4}\in\Phi$, $3^{4}\in\Phi$, $5^{4}\in\Phi$, $7^{4}\in\Phi$, $9^{4}\in\Phi$, $11^{4}\in\Phi$,

$1^{5}\in\Phi$, $3^{5}\in\Omega$, $5^{5}\in\Phi$, $7^{5}\in\Omega$, $9^{5}\in\Phi$, $11^{5}\in\Omega$,

… … … … … …

$13^{1}\in\Phi$, $15^{1}\in\Omega$, $17^{1}\in\Phi$, $19^{1}\in\Omega$, $21^{1}\in\Phi$, $23^{1}\in\Omega$…

$13^{2}\in\Phi$, $15^{2}\in\Phi$, $17^{2}\in\Phi$, $19^{2}\in\Phi$, $21^{2}\in\Phi$, $23^{2}\in\Phi$ …

$13^{3}\in\Phi$, $15^{3}\in\Omega$, $17^{3}\in\Phi$, $19^{3}\in\Omega$, $21^{3}\in\Phi$, $23^{3}\in\Omega$ …

$13^{4}\in\Phi$, $15^{4}\in\Phi$, $17^{4}\in\Phi$, $19^{4}\in\Phi$, $21^{4}\in\Phi$, $23^{4}\in\Phi$ …

$13^{5}\in\Phi$, $15^{5}\in\Omega$, $17^{5}\in\Phi$, $19^{5}\in\Omega$, $21^{5}\in\Phi$, $23^{5}\in\Omega$ …

… … … … … …

From above-listed many kinds of odd numbers, we are not difficult to see that odd numbers whereby each of Φ as a base number belong still within Φ, and odd numbers which every even power of Ω forms belong within Φ; yet odd numbers which every odd power of Ω forms belong to Ω, i.e. $\Phi^X \in \Phi$, $\Omega^{2n} \in \Phi$ and $\Omega^{2n-1} \in \Omega$, where $X \geq 1$ and $n \geq 1$.

Also two adjacent odd numbers which have an identical exponent or a common odd number as the base number except for 1 are an even number apart, and that such even numbers are getting greater and greater along with their exponents or base numbers are getting greater and greater.

Altogether, positive odd numbers of odd exponents plus even exponents are all positive odd numbers of Φ plus Ω. Yet positive odd numbers whose exponents≥ 3 are merely a part of all positive odd numbers, and that this part is dispersed among all positive odd numbers. Therefore, without exception the part positive odd numbers with other positive odd numbers are possessed of the symmetric relations whereby each of $2^{W-1}H^Z$ as a symmetric center likewise.

For any positive odd number, it can be expressed either as O^V, where $V \geq 3$ or V=1 or 2, where V expresses the greatest common divisor of exponents of distinct prime divisors of the positive odd number, and O is another positive odd number. For O^V with V=1or 2, we may write it down $O^{1\sim2}$.

By now, we just begin to prove aforementioned four inequalities by mathematical induction with the aid of the symmetric relations of positive odd numbers thereinafter, one by one.

Firstly, Let us regard 2^{W-1} as a symmetric center of positive odd numbers concerned to prove $A^X+B^Y\neq 2^W$ under the known requirements by mathematical induction.

(1) When W-1=2, 3, 4, 5 and 6, bilateral symmetric odd numbers on two sides of symmetric center 2^{W-1} are listed as follows successively.

1^6, 3, (2^2), 5, 7, (2^3), 9, 11, 13, 15, (2^4), 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127

By this token, there are not two O^V with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^{W-1}, where W-1=2, 3, 4, 5 and 6. So there are $A^X+B^Y\neq 2^3$, $A^X+B^Y\neq 2^4$, $A^X+B^Y\neq 2^5$, $A^X+B^Y\neq 2^6$ and $A^X+B^Y\neq 2^7$ under the known requirements according to the conclusion about the double of the even number as a symmetric center.

(2) Suppose that when W-1=K with $K\geq 6$, there are not two O^V with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^K. Namely suppose $A^X+B^Y\neq 2^{K+1}$ under the known requirements.

(3) Prove that when W-1=K+1, there are not two O^V with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^{K+1}. Namely prove $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

Proof *We known that odd numbers whereby 2^{W-1} including 2^K plus 2^{K+1} as a symmetric center are possess of the bilateral symmetric relations.

From this, let us list the form of permutation of positive odd numbers whereby 2^{K+1} including 2^K as a symmetric center as follows.

1^{K+1}, 3, 5, 7, …**Φ, Ω, Φ, Ω,** (2^K),**Φ, Ω, Φ, Ω,** …**Φ, Ω, Φ, Ω,** …**Φ, Ω, Φ, Ω,** (2^{K+1}),**Φ, Ω, Φ, Ω,**…**Φ, Ω, Φ, Ω,** …**Φ, Ω, Φ, Ω, Φ, Ω, Φ, Ω,**…**Φ, Ω, Φ, Ω.**

Since every pair of bilateral symmetric odd numbers for symmetric center 2^{W-1} belongs to one of kind Φ and one of kind Ω, thus two differences from 2^{W-1} to bilateral symmetric Φ and Ω are an identical odd number.

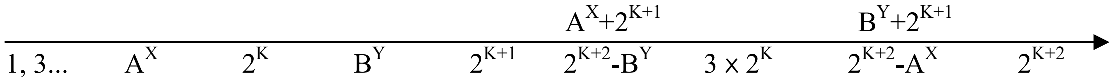
In reality, positive odd numbers whereby 2^K as a symmetric center are exactly positive odd numbers on the left side of symmetric center 2^{K+1}. Thus, for positive odd numbers whereby 2^{K+1} as a symmetric center, their a half retains still original places after move symmetric center to 2^{K+1} from 2^K, and that the half lies on the left side of 2^{K+1}. While another half is formed from 2^{K+1} plus each of positive odd numbers whereby 2^K as a symmetric center, and that the half lies on the right side of 2^{K+1}.

Suppose that A^X and B^Y are a pair of bilateral symmetric positive odd numbers for symmetric center 2^K, then there is $A^X+B^Y=2^{K+1}$ according to the conclusion about the double of the even number as a symmetric center.

Since there are not two O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^K according to second step of the mathematical induction, thus tentatively let A^X as one O^V

with $V \geq 3$, and let B^Y as one $O^{1\sim 2}$, i.e. let $X \geq 3$ and $Y = 1$ or 2.

By now, let B^Y plus 2^{K+1} to make B^Y+2^{K+1}. Please, see also a simple illustration at the number axis as follows.



Since there is only $A^X+B^Y \neq 2^{K+1}$ under the known requirements according to second step of the mathematical induction, therefore there is inevitably $A^X+B^Y=2^{K+1}$ under the known requirements except for Y, and Y=1 or 2. And that it has $B^Y+2^{K+1}=A^X+2B^Y=2^{K+2}-A^X$ further. Evidently A^X and $2^{K+2}-A^X$ are a pair of bilateral symmetric odd numbers for symmetric center 2^{K+1} due to $A^X+(2^{K+2}-A^X)=2^{K+2}$ according to the conclusion about the double of the even number as a symmetric center. So A^X and A^X+2B^Y are bilateral symmetric odd numbers for symmetric center 2^{K+1}, and that it has $A^X+(2^{K+2}-A^X)=A^X+(A^X+2B^Y)=2^{K+2}$ under the known requirements except for Y, and Y=1 or 2. Of course, A^X and A^X+2B^Y in this case are a pair of bilateral symmetric Φ and Ω for symmetric center 2^{K+1} still.

But then, there is only $A^X+B^Y \neq 2^{K+1}$ under the known requirements, thus it has $A^X+ [A^X+2B^Y] =2[A^X+B^Y] \neq 2^{K+2}$ in that case.

In any case, A^X+2B^Y is a positive odd number, so let $A^X+2B^Y=D^E$, where E expresses the greatest common divisor of exponents of distinct prime divisors of the positive odd number, and D is another positive odd number, then we get $A^X+ [A^X+2B^Y] =A^X+D^E \neq 2^{K+2}$ under the known requirements.

That is to say, no matter what positive integer which E equals and no matter what positive odd number which D equals from $A^X+2B^Y=D^E$, there is $A^X+[A^X+2B^Y]=A^X+D^E\neq 2^{K+2}$ under the known requirements invariably. Namely A^X and D^E under the known requirements are not two bilateral symmetric odd numbers for symmetric center 2^{K+1}.

Whereas A^X and A^X+2B^Y, i.e. A^X and D^E under the known requirements except for Y and Y=1 or 2 are indeed a pair of bilateral symmetric odd numbers for symmetric center 2^{K+1} due to $A^X+[A^X+2B^Y]=A^X+D^E=2^{K+2}$ according to the conclusion about the double of the even number as a symmetric center. Such being the case, provided slightly change the evaluation of any letter of A^X+2B^Y, then it at once is not original that A^X+2B^Y under the known requirements except for Y and Y=1 or 2. Naturally, now it lies not on the place of the symmetry of A^X either. Namely A^X and A^X+2B^Y under the known requirements are not bilateral symmetric odd numbers for symmetric center 2^{K+1} because the value of Y has changed, i.e. from Y=1 or 2 to Y $\geq$3. Thus there is $A^X+ [A^X+2B^Y]= A^X+D^E\neq 2^{K+2}$ under the known requirements according to the conclusion about the double of the even number as a symmetric center.

Besides, A^X is supposed as one O^V with V≥3 on the left side of symmetric center 2^{K+1}. Also there is $A^X+B^Y=2^{K+1}$ under the known requirements except for Y and Y=1 or 2, thereby it has $A^X+2B^Y=2^{K+1}+B^Y$. Thus it can be seen, A^X+2B^Y i.e. D^E lies on the right side of symmetric center 2^{K+1}.

For inequality $A^X+D^E\neq 2^{K+2}$ under the known requirements, let us substitute

D by B, since B and D can express any identical positive odd number, and substitute Y for E where E ≥ 3, since $Y \geq 3$.

Consequently, we obtain $A^X+B^Y \neq 2^{K+2}$ under the known requirements.

In this proof, if B^Y is one O^V with $V \geq 3$, then A^X is surely one $O^{1\sim 2}$, yet a conclusion concluded on such a premise is really one and the same with $A^X+B^Y \neq 2^{K+2}$ under the known requirements.

If A^X and B^Y are bilateral symmetric two $O^{1\sim 2}$ for symmetric center 2^K, then whether A^X and A^X+2B^Y, or B^Y and B^Y+2A^X, they are still a pair of bilateral symmetric odd numbers for symmetric center 2^{K+1}. But, no matter what positive odd number which A^X+2B^Y or B^Y+2A^X equal, it can not turn the pair of bilateral symmetric odd numbers into two O^V with $V \geq 3$, because A^X or B^Y in the pair is not one O^V with $V \geq 3$ originally.

To sum up, we have proven that when W-1=K+1 with K ≥ 6, there is only $A^X+B^Y \neq 2^{K+2}$ under the known requirements. In other words, there are not two O^V with $V \geq 3$ on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^{K+1}.

Apply the preceding way of doing, we can continue to prove that when W-1=K+2, K+3…up to every integer ≥ 3, there are merely $A^X+B^Y \neq 2^{K+3}$, $A^X+B^Y \neq 2^{K+4}$ … up to $A^X+B^Y \neq 2^W$ under the known requirements.

Secondly, Let us successively prove $A^X+B^Y \neq 2^W H^Z$ under the known requirements by mathematical induction, and point out H ≥ 3 emphatically.

(1) When H=1, $2^{W-1}H^Z$ i.e. 2^{W-1}, we have proven $A^X+B^Y \neq 2^W$ under the known requirements in the preceding section. Namely there are not two O^V with $V \geq 3$ on two places of every pair of bilateral symmetric odd numbers for symmetry center 2^{W-1}.

(2) Suppose that when H=J and J is an odd number ≥1, $2^{W-1}H^Z$ i.e. $2^{W-1}J^Z$, there is $A^X+B^Y \neq 2^W J^Z$ under the known requirements. Namely suppose that there are not two O^V with $V \geq 3$ on two places of every pair of bilateral symmetric odd numbers for symmetry center $2^{W-1}J^Z$.

(3) Prove that when H=K with K=J+2, $2^{W-1}H^Z$ i.e. $2^{W-1}K^Z$, there is $A^X+B^Y \neq 2^W K^Z$ under the known requirements too. Namely prove that there are not two O^V with $V \geq 3$ on two places of every pair of bilateral symmetric odd numbers for symmetry center $2^{W-1}K^Z$.

Proof * We known that after regard $2^{W-1}H^Z$ as a symmetric center, the sum of every pair of bilateral symmetric odd numbers is equal to $2^W H^Z$, yet a sum of two odd numbers of no symmetry is unequal to $2^W H^Z$ affirmatively.

In addition, there are not two O^V with $V \geq 3$ on two places of every pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}J^Z$. Namely there is $A^X+B^Y \neq 2^W J^Z$ under the known requirements according to second step of the mathematical induction.

Such being the case, let us suppose that A^X and B^Y are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}J^Z$, also tentatively let Y ≥3, and let X=1 or 2, then there is surely $A^X+B^Y=2^W J^Z$.

On the other, after regard $2^{W-1}K^Z$ as a symmetric center, B^Y and $2^W K^Z-B^Y$

are a pair of bilateral symmetric odd numbers due to $B^Y+ (2^WK^Z-B^Y)=2^WK^Z$ according to the conclusion about the double of the even number as a symmetric center.

By now let A^X plus $2^W(K^Z-J^Z)$ to make $A^X+2^W(K^Z-J^Z)$, also $A^X+2^W(K^Z-J^Z)= A^X+2^WK^Z-2^WJ^Z=2^WK^Z-(2^WJ^Z-A^X)=2^WK^Z-B^Y$ under the known requirements except for X, and X=1 or 2, also due to $A^X+B^Y=2^WJ^Z$ in this case.

Now that there is $A^X+2^W(K^Z-J^Z)=2^WK^Z-B^Y$ under the known requirements except for X, and X=1 or 2; in addition B^Y and $2^WK^Z-B^Y$ are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$, then B^Y and $A^X+2^W(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$. Thus we get $B^Y+[A^X+2^W(K^Z-J^Z)]=2^WK^Z$ under the known requirements except for X, and X=1 or 2.

Of course, B^Y and $A^X+2^W (K^Z-J^Z)$ in this case are still a pair of bilateral symmetric Φ and Ω for symmetric center $2^{W-1}K^Z$.

From $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^W(K^Z-J^Z)$ and beforehand supposed $A^X+B^Y\neq 2^WJ^Z$ under the known requirements, we get $B^Y+[A^X+2^W(K^Z-J^Z)]= [A^X+B^Y]+2^WK^Z-2^WJ^Z \neq 2^WK^Z$ under the known requirements.

Thus it can be seen, B^Y and $A^X+2^W (K^Z-J^Z)$ under the known requirements are not two bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$ due to $B^Y+[A^X+2^W (K^Z-J^Z)] \neq 2^WK^Z$.

It is obvious that $A^X+2^W (K^Z-J^Z)$ in the aforesaid two cases expresses two disparate odd numbers, due to $X \geq 3$ in one, and X=1 or 2 in another.

From $A^X+2^W(K^Z-J^Z)=2^WK^Z-(2^WJ^Z-A^X)$ and $2^WJ^Z-A^X\neq B^Y$ under the known requirements, we get $A^X+2^W(K^Z-J^Z)\neq 2^WK^Z-B^Y$.

In any case, $A^X+2^W(K^Z-J^Z)$ is a positive odd number, thus let $A^X+2^W(K^Z-J^Z)=F^V$, where V expresses the greatest common divisor of exponents of distinct prime divisors of the positive odd number, and F is another positive odd number, so there is $F^V\neq 2^WK^Z-B^Y$ due to $A^X+2^W(K^Z-J^Z)\neq 2^WK^Z-B^Y$ under the known requirements. Namely there is $B^Y+F^V\neq 2^WK^Z$ under the known requirements.

Since B^Y and $A^X+2^W(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$ due to $B^Y+[A^X+2^W(K^Z-J^Z)]=2^WK^Z$ under the known requirements except for X and X=1 or 2, according to the conclusion reached previously. Such being the case, provided slightly change the evaluation of any letter of $A^X+2^W(K^Z-J^Z)$, then it at once is not original that $A^X+2^W(K^Z-J^Z)$ under the known requirements except for X and X=1 or 2. Naturally, now it lies not on the place of the symmetry of B^Y either. Namely B^Y and $A^X+2^W(K^Z-J^Z)$ under the known requirements are not two bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$ because the value of X has changed, i.e. from X=1 or 2 to $X\geq 3$.

Thereby there is $B^Y+[A^X+2^W(K^Z-J^Z)]\neq 2^WK^Z$ under the known requirements according to the conclusion about the double of the even number as a symmetric center. Namely there is $B^Y+F^V\neq 2^WK^Z$ under the known requirements due to $A^X+2^W(K^Z-J^Z)=F^V$.

For inequality $B^Y+F^V\neq 2^WK^Z$, let us substitute F by A, since A and F can express any identical positive odd number, and substitute X for V where $V \geq 3$, since $X \geq 3$.

Consequently, we obtain $A^X+B^Y\neq 2^WK^Z$ under the known requirements.

In this proof, if A^X is one O^V with $V\geq 3$, then B^Y is surely one $O^{1\sim 2}$, yet a conclusion concluded on such a premise is really one and the same with $A^X+B^Y\neq 2^WK^Z$ under the known requirements.

If A^X and B^Y are bilateral symmetric two $O^{1\sim 2}$ for symmetric center $2^{W-1}J^Z$, then whether B^Y and $A^X+2^W(K^Z-J^Z)$, or A^X and $B^Y+2^W(K^Z-J^Z)$, they are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$ too. But, no matter what positive odd number which $A^X+2^W(K^Z-J^Z)$ or $B^Y+2^W(K^Z-J^Z)$ equal, it can not turn the pair of bilateral symmetric odd numbers into two O^V with $V\geq 3$, since B^Y or A^X in the pair is not one O^V with $V\geq 3$ originally.

To sum up, we have proven $A^X+B^Y\neq 2^WK^Z$ with $K=J+2$ under the known requirements. Namely when $H=J+2$, there are not two O^V with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}(J+2)^Z$.

Apply the above-mentioned way of doing, we can continue to prove that when H=J+4, J+6… up to every positive odd number, there are merely $A^X+B^Y\neq 2^W(J+4)^Z$, $A^X+B^Y\neq 2^W(J+6)^Z$… up to $A^X+B^Y\neq 2^WH^Z$ under the known requirements, and point out $H \geq 3$ emphatically.

Thirdly, On the basis of the anterior conclusion got, we continue to prove $A^X+2^W\neq C^Z$ under the known requirements by mathematical induction.

(1) When W=3, 4, 5, 6 and 7, bilateral symmetric odd numbers on two sides of symmetric center 2^3, 2^4, 2^5, 2^6 or 2^7 are listed successively below.

1^7, 3, 5, 7, (2^3), 9, 11, 13, 15, (2^4), 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127, (2^7), 129, 131, 133, 135, 137, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 159, 161, 163, 165, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225,227, 229, 231, 233, 235, 237, 239, 241, 3^5, 245, 247, 249, 251, 253, 255.

By this token, there is 1^7 on the left side of 2^3;

There is 1^7 on the left side of 2^4;

There are 1^7 and 3^3 on the left side of 2^5;

There are $1^7$and 3^3 on the left side of 2^6;

There are 1^7, 3^3, 3^4 and 5^3 on the left side of 2^7.

It is observed that $1^7+2^3\neq C^Z$; $1^7+2^4\neq C^Z$; $1^7+2^5\neq C^Z$, $3^3+2^5\neq C^Z$; $1^7+2^6\neq C^Z$, $3^3+2^6\neq C^Z$; $1^7+2^7\neq C^Z$, $3^3+2^7\neq C^Z$, $3^4+2^7\neq C^Z$ and $5^3+2^7\neq C^Z$.

Thus there are $A^X+2^3\neq C^Z$, $A^X+2^4\neq C^Z$, $A^X+2^5\neq C^Z$, $A^X+2^6\neq C^Z$ and $A^X+2^7\neq C^Z$ under the known requirements.

(2) Suppose that when W=N with N≥7, there is $A^X+2^N\neq C^Z$ under the known requirements, where $A^X<2^N$ and $C^Z>2^N$.

(3) Prove that when W=N+1, there is $A^X+2^{N+1}\neq C^Z$ under the known requirements too, where $A^X<2^{N+1}$ and $C^Z>2^{N+1}$.

Proof* Since $(2^{N+1}+A^X)+(2^{N+1}-A^X)=2^{N+2}$, so $2^{N+1}+A^X$ and $2^{N+1}-A^X$ are a pair of bilateral symmetric odd numbers for symmetric center 2^{N+1} according to the conclusion about the double of the even number as a symmetric center.

Also there is $2^{N+1}-A^X\neq O^V$ i.e. $A^X+O^V\neq 2^{N+1}$ where V≥3 according to proven $A^X+B^Y\neq 2^W$ under the known requirements, so $2^{N+1}-A^X$ can only be one $O^{1\sim2}$.

Now that $2^{N+1}-A^X$ is one $O^{1\sim2}$, then $2^{N+1}-A^{1\sim2}$ contain both some $O^{1\sim2}$ and all O^V with V≥3 under even number 2^{N+1}.

In addition, $2^{N+1}+A^{1\sim2}$ and $2^{N+1}-A^{1\sim2}$ are bilateral symmetric odd numbers for symmetric center 2^{N+1} due to $(2^{N+1}+A^{1\sim2})+(2^{N+1}-A^{1\sim2})=2^{K+2}$ according to the conclusion about the double of the even number as a symmetric center.

Therefore $2^{N+1}+A^{1\sim2}$ contain both some $O^{1\sim2}$ and all O^V with V≥3 under even number 2^{N+2}.

Since $2^{N+1}-A^X$ in $(2^{N+1}+A^X) + (2^{N+1}-A^X)=2^{N+2}$ is one $O^{1\sim2}$, then $2^{N+1}+A^X$ is either one $O^{1\sim2}$ or one O^V with V≥3 under even number 2^{N+2} according to the conclusion got that there are not two O^V with V≥3 on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^{W-1}.

But $2^{N+1}+A^{1\sim2}$ contained all O^V with V≥3 under even number 2^{N+2}, therefore $2^{N+1}+A^X$, i.e. A^X+2^{N+1} can only be one $O^{1\sim2}$.

Also C^Z is one O^V with $V\geq3$ according to two of the known requirements. Consequently there is $A^X+2^{N+1}\neq C^Z$ under the known requirements.

Apply the preceding way of doing, we can continue to prove that when $W=N+2$, $N+3$…up to every integer≥3, there are merely $A^X+2^{N+2}\neq C^Z$, $A^X+2^{N+3}\neq C^Z$…up to $A^X+2^W\neq C^Z$ under the known requirements.

Fourthly, On the basis of the anterior conclusion got, we shall last prove $A^X+2^WR^Y\neq C^Z$ under the known requirements by mathematical induction.

(1) When $R=1$, 2^WR^Y to wit 2^W, we have proven $A^X+2^W\neq C^Z$ under the known requirements in the preceding section.

(2) Suppose that when $R=J$ where J is an odd number ≥1, 2^WR^Y i.e. 2^WJ^Y, there is $A^X+2^WJ^Y\neq C^Z$ under the known requirements, where $A^X<2^WJ^Y$ and $C^Z>2^WJ^Y$.

(3) Prove that when $R=K$ with $K=J+2$, 2^WR^Y i.e. 2^WK^Y, there is $A^X+2^WK^Y\neq C^Z$ under the known requirements, where $A^X<2^WK^Y$ and $C^Z>2^WK^Y$.

Proof* Since $(2^WK^Y+A^X)+(2^WK^Y-A^X)=2^{W+1}K^Y$, then $2^WK^Y+A^X$ and $2^WK^Y-A^X$ are a pair of bilateral symmetric odd numbers for symmetric center 2^WK^Y according to the conclusion about the double of the even number as a symmetric center.

Also there is $2^WK^Y-A^X\neq O^V$ i.e. $A^X+O^V\neq 2^WK^Y$ where $V\geq3$, according to proven $A^X+B^Y\neq 2^WH^Z$ under the known requirements, so $2^WK^Y-A^X$ can only be one $O^{1\sim2}$.

Now that $2^WK^Y-A^X$ is one $O^{1\sim2}$, then $2^WK^Y-A^{1\sim2}$ contain both some $O^{1\sim2}$ and all O^V with $V \geq 3$ under even number $2^W K^Y$.

In addition, $2^W K^Y + A^{1\sim2}$ and $2^W K^Y - A^{1\sim2}$ are bilateral symmetric odd numbers for symmetric center $2^W K^Y$ due to $(2^W K^Y + A^{1\sim2}) + (2^W K^Y - A^{1\sim2}) = 2^{W+1} K^Y$ according to the conclusion about the double of the even number as a symmetric center.

Therefore $2^W K^Y + A^{1\sim2}$ contain both some $O^{1\sim2}$ and all O^V with $V \geq 3$ under even number $2^{W+1} K^Y$.

Since $2^W K^Y - A^X$ in $(2^W K^Y + A^X) + (2^W K^Y - A^X) = 2^{W+1} K^Y$ is one $O^{1\sim2}$, then $2^W K^Y + A^X$ is either one $O^{1\sim2}$ or one O^V with $V \geq 3$ under even number $2^{W+1} K^Y$ according to the conclusion got that there are not two O^V with $V \geq 3$ on two places of every pair of bilateral symmetric odd numbers for symmetric center $2^W H^Z$.

But $2^W K^Y + A^{1\sim2}$ contained all O^V with $V \geq 3$ under even number $2^{W+1} K^Y$, therefore $2^W K^Y + A^X$, i.e. $A^X + 2^W K^Y$ can only be one $O^{1\sim2}$.

Also C^Z is one O^V with $V \geq 3$ according to two of the known requirements.

Consequently, there is $A^X + 2^W K^Y \neq C^Z$, i.e. $A^X + 2^W (J+2)^Y \neq C^Z$ under the known requirements.

Apply the preceding way of doing, we can continue to prove that when R=J+4, J+6…up to every positive odd number, there are $A^X + 2^W (J+4)^Y \neq C^Z$, $A^X + 2^W (J+6)^Y \neq C^Z$…up to $A^X + 2^W R^Y \neq C^Z$ under the known requirements.

To sun up, we have proven every kind of $A^X + B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor.

In addition to this, we have proven that $A^X+B^Y=C^Z$ under the given requirements plus the qualification that A, B and C have at least a common prime factor has many sets of the solution of positive integers, in the beginning of this article.

Last, let $A^X+B^Y=C^Z$ as compared $A^X+B^Y\neq C^Z$ under the given requirements, we reach inevitably such a conclusion that an indispensable prerequisite of the existence of $A^X+B^Y=C^Z$ under the given requirements is that A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, the Beal's conjecture holds water.

PS.1. If Beal's conjecture is proved to hold water, then let X=Y=Z, so indefinite equation $A^X+B^Y=C^Z$ is changed into $A^X+B^X=C^X$. In addition, divide three terms of $A^X+B^X=C^X$ by maximal common factor of the three terms, then get a set of the solution of positive integers without a common prime factor. Obviously this conclusion is in contradiction with proven Beal's conjecture, so we proved Fermat's last theorem as easy as the pie.

PS.2. Since $A^X+B^Y\neq C^Z$ under the given requirements plus a qualification that A, B and C haven't a common prime factor, so partial ABC conjecture is tenable in which case the greatest common divisor of exponents of distinct prime divisors of each of A, B and C is more than or equal to 3.

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