New spherical static solution in Gravity field

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ABSTRACT

In the general relativity theory, we discover new solution in gravity field by Einstein's gravity field equation with cosmological constant term. We treats curvature tensor in new solution.

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1.Introduction

We solve new solution in gravity field by gravity field equation with cosmological constant term.

Gravity field equation with cosmological constant term is in vacuum

$$R_{\mu\upsilon} = \Lambda g_{\mu\upsilon} \tag{1}$$

The spherical coordinate is

$$d\tau^2 = W(r,t)dt^2 - \frac{1}{c^2}[U(t,r)dr^2 + V(t,r)\{d\theta^2 + \sin^2\theta d\phi^2\}] \tag{2}$$

In this time, Einstein's gravity equation is

$$R_{tt} = \frac{1}{2}\frac{\ddot{U}U - \dot{U}^2}{U^2} + \frac{1}{2}\frac{-W''U + W'U'}{U^2} + \frac{1}{4}\frac{W'^2}{UW} + \frac{\dot{U}^2}{4U^2} - \frac{\dot{U}\dot{W}}{4UW} - \frac{1}{4}\frac{U'W'}{U^2}$$
$$+ \frac{\ddot{V}V - \dot{V}^2}{V^2} + \frac{1}{2}\frac{\dot{V}^2}{V^2} - \frac{\dot{W}\dot{V}}{2WV} - \frac{W'V'}{2UV} = -\Lambda W \tag{3}$$

$$R_{rr} = \frac{1}{2}\frac{W''W - W'^2}{W^2} - \frac{\ddot{U}W - \dot{U}\dot{W}}{2W^2} + \frac{W'^2}{4W^2} + \frac{\dot{U}^2}{4UW} - \frac{\dot{U}\dot{W}}{4W^2} - \frac{U'W'}{4WU}$$
$$+ \frac{V''V - V'^2}{V^2} + \frac{1}{2}\frac{V'^2}{V^2} - \frac{U'V'}{2UV} - \frac{\dot{U}\dot{V}}{2WV} = \Lambda U \tag{4}$$

$$R_{\theta\theta} = \frac{1}{2}\frac{-\ddot{V}W + \dot{V}\dot{W}}{W^2} - \frac{\dot{W}\dot{V}}{4W^2} + \frac{W'V'}{4UW} + \frac{V''U - V'U'}{2U^2} - \frac{\dot{U}\dot{V}}{4UW} + \frac{U'V'}{4U^2} - 1$$
$$= \Lambda V \tag{5}$$

$$R_{\phi\phi} = \sin^2\theta R_{\theta\theta} \tag{6}$$

$$R_{tr} = \frac{\dot{V}'}{V} - \frac{\dot{V}V'}{2V^2} - \frac{\dot{U}V'}{2UV} - \frac{W'\dot{V}}{2WV} = 0 \tag{7}$$

In this time, $A' = \frac{\partial A}{\partial r}, \dot{A} = \frac{1}{c}\frac{\partial A}{\partial t}$ (8)

2. New spherical static solution in Gravity field

We think

$$W(r,t) = g(r),\ U(t,r) = 1,\ V(t,r) = h(r) \tag{9}$$

In vacuum, Eq(7) is

$$R_{tr} = \frac{\dot{V}'}{V} - \frac{\dot{V}V'}{2V^2} - \frac{\dot{U}V'}{2UV} - \frac{W'\dot{V}}{2WV} = 0 \tag{10}$$

In vacuum, Eq(3)-(5) is

$$R_{tt} = \frac{1}{2}\frac{-W''}{U} + \frac{1}{4}\frac{W'^2}{W} - \frac{W'V'}{2UV} = -\Lambda W$$

$$R_{tt} = -\frac{g''}{2} + \frac{g'^2}{4g} - \frac{g'h'}{2h} = -\Lambda g \tag{11}$$

$$R_{rr} = \frac{1}{2}\frac{W''}{W} - \frac{W'^2}{4W^2} + \frac{V''}{V} - \frac{1}{2}\frac{V'^2}{V^2} = \Lambda U$$

$$R_{rr} = \frac{1}{2}\frac{g''}{g} - \frac{1}{4}\frac{g'^2}{g^2} + \frac{h''}{h} - \frac{1}{2}\frac{h'^2}{h^2} = \Lambda \tag{12}$$

$$R_{\theta\theta} = \frac{W'V'}{4UW} + \frac{V''}{2U} - 1 = \Lambda V$$

$$R_{\theta\theta} = \frac{g'h'}{4g} + \frac{h''}{2} - 1 = \Lambda h \tag{13}$$

Therefore, Eq(11)-Eq(13) is

$$-\frac{g''}{2g} + \frac{g'^2}{4g^2} - \frac{g'h'}{2hg} = -\Lambda \tag{14}$$

$$\frac{g''}{2g} - \frac{g'^2}{4g^2} + \frac{h''}{h} - \frac{1}{2}\frac{h'^2}{h^2} = \Lambda \tag{15}$$

$$\frac{g'h'}{4g} + \frac{h''}{2} - 1 = \Lambda h \tag{16}$$

In Eq(16), if h is constant, the equation (14)-(16) solved.

$$h = -\frac{1}{\Lambda} \tag{17}$$

Hence, Eq(14)-(15) is

$$\frac{g''}{2g} - \frac{g'^2}{4g^2} = \Lambda \tag{18}$$

The solution of Eq(18) is

$$g = \exp(2\sqrt{\Lambda}r)$$

Therefore, new solution is in vacuum in gravity field

$$ds^2 = -c^2 d\tau^2 = -c^2 \exp(2\sqrt{\Lambda}r)dt^2 + dr^2 - \frac{1}{\Lambda}(d\theta^2 + \sin^2\theta d\phi^2) \tag{19}$$

We treat curvature tensor of new solution.

$$g_{tt} = -\exp(2\sqrt{\Lambda}r) \ , \ g_{rr} = 1, \quad g_{\theta\theta} = -\frac{1}{\Lambda}$$

$$\Gamma^r{}_{tt} = \frac{1}{2}g^{rr}\left(-\frac{\partial g_{tt}}{\partial r}\right) = \sqrt{\Lambda}\exp(2\sqrt{\Lambda}r)$$

$$R^r{}_{ttr} = \frac{\partial \Gamma^r{}_{tt}}{\partial r} = 2\Lambda \exp(2\sqrt{\Lambda}r) \ , \ R_{rttr} = g_{rr}R^r{}_{ttr} = 2\Lambda \exp(2\sqrt{\Lambda}r) \tag{20}$$

3. Conclusion

Therefore, new spherical solution in gravity field is

$$d\tau^2 = \exp(2\sqrt{\Lambda}r)dt^2 - \frac{1}{c^2}[dr^2 - \frac{1}{\Lambda}(d\theta^2 + \sin^2\theta d\phi^2)] \tag{21}$$

According to the variable r , the observer's light speed is over light velocity C in vaccum.

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