the general science journal

# The relativistic length contraction rebutted

**Radwan M. Kassir** ©2016

radwan.elkassir@dar.com


## Abstract

In this paper, it is established that Einstein's interpretation of the Lorentz transformation in predicting the length contraction is based on invalid assumption with respect to the reference frame in which the simultaneity of events needs to be considered. It is shown that the correct assumption would lead, within the special relativity framework, to contradictory length expansion through the same Lorentz transformation.


## Introduction

In special relativity, the length of a moving body is measured based on the assumption that for an observer in the "stationary" frame to measure the length of a body fixed in the "traveling" frame, along the motion direction, he or she must receive simultaneously two signals, each emitted from a different end of the body; the measured distance in the stationary frame between the two signal spatial coordinates would be equal to the body length. It will be shown, however, that such an assumption results in an erroneous length, as it is demonstrated that the actual requirement for measuring the length of the moving body would be to determine in the "stationary" frame the distance between two signals emitted simultaneously from the ends of the body in the traveling frame (i.e., its rest frame). The result would be a length expansion when the Lorentz transformation equation is applied accordingly.

## Misinterpretation of length contraction in Special Relativity

In his 1907 paper[1] on special relativity, Einstein derived the Lorentz transformation equations, and carried out some interpretations deducing the length contraction and time dilation predictions. Considering two inertial frames in relative motion, he concluded his length contraction prediction as per the following excerpt from the above referenced paper:—

> Consider a body at rest relative to $S'$. Let $x_1', y_1', z_1'$ and $x_2', y_2', z_2'$ be the coordinates of two of its material points referred to $S'$. Between the coordinates $x_1, y_1, z_1$ and $x_2, y_2, z_2$ of these points relative to $S$, there obtain at each time $t$ of $S$, according to the above-derived transformation equations, the relations

$$x_2 - x_1 = [1-(v/c)^2]^{1/2}(x_2' - x_1'), \tag{1}$$

$$y_2 - y_1 = y_2' - y_1', \quad z_2 - z_1 = z_2' - z_1'.$$

Where, $S$ and $S'$ represent the stationary and moving frame, respectively, $v$ the relative speed, and $c$ the speed of light in empty space.

There's trickery in the above Einstein's deduction. The assertion that the above $x$-$x'$ coordinates relation is obtained "at each time *t* of $S$" is misleading. Indeed, care should be taken that the Lorentz transformation gives the relation between events space and time coordinates measured from the perspective of each of the two reference frames. To the events coordinates $x_1', y_1', z_1', t_1'$ and $x_2', y_2', z_2', t_2'$, connected to the above body material points, referred to $S'$, there corresponds their coordinates $x_1, y_1, z_{1,} t_1$ and $x_2, y_2, z_2, t_2$ with respect to $S$, in such a way that the following relations hold according to the Lorentz transformation:–

With reference to the $S$ coordinates of the vents, their $S'$ coordinates are given by

$$x_2' - x_1' = [1-(v/c)^2]^{-1/2}[(x_2 - x_1) - v(t_2 - t_1)], \tag{2}$$

$$y_2' - y_1' = y_2 - y_1, \quad z_2' - z_1' = z_2 - z_1; \tag{3}$$

$$t_2' - t_1' = [1-(v/c)^2]^{-1/2}[(t_2 - t_1) - v(x_2 - x_1)/c^2], \tag{4}$$

and, with reference to their $S'$ coordinates, the events $S$ coordinates are given by

$$x_2 - x_1 = [1-(v/c)^2]^{-1/2}[(x_2' - x_1') + v(t_2' - t_1')], \tag{5}$$

$$y_2 - y_1 = y_2' - y_1', \quad z_2 - z_1 = z_2' - z_1'. \tag{6}$$

$$t_2 - t_1 = [1-(v/c)^2]^{-1/2}[(t_2' - t_1') + v(x_2' - x_1')/c^2]. \tag{7}$$

Evidently, as illustrated in Figs. 1 and 2, the distance $\Delta x' = x_2' - x_1'$ (wagon length in $S'$) is independent of the time $t'$ in the body [wagon] rest frame $S'$. i.e., when two different light signals are emitted each from one of the two considered body's material points, the distance (spatial coordinates difference) between the two signals emission points will be the same in $S'$ regardless of at what times $t_1'$ and $t_2'$ the two signals are emitted. However, when the distance between the signals emission points is measured from the stationary frame $S$, it would depend on the time interval $t_2' - t_1'$, which must be equal to zero (simultaneous events in $S'$) in order for the measured distance $\Delta x = x_2 - x_1$ to represent the length of the wagon (Figs. 1c and 2c). Therefore, Lorentz transformation Eq. (5) must be used for the case under consideration, yielding the following length expansion equation:

$$x_2 - x_1 = [1-(v/c)^2]^{-1/2}(x_2' - x_1'). \tag{8}$$

To obtain Einstein's length contraction Eq. (1) above, we must set $t_2 = t_1$ in Eq. (2), which means that the above time coordinates $t'_1$ and $t'_2$ of the above events in $S'$ must be so carefully chosen that their corresponding time coordinates $t_1$ and $t_2$ in S must be equal; i.e. the above events must be perceived simultaneously in $S$. For this to be logically feasible from the perspective of $S'$, i.e., for the two signals to arrive simultaneously to an observer in $S$, the signal $\sigma'_2$ must be emitted, relative to $S'$, at the time instant when the first signal $\sigma'_1$ emitted from the far end of the wagon reaches its other end (Figs. 1b and 2b). This is equivalent to imposing the condition that $t'_2 = t'_1 + L'/c$, or $\Delta t' = L'/c$, and $\Delta x' = -L'$.

Now, from the perspective of $S$, for $\Delta t = 0$, the Lorentz transformation Eq. (7) results in $\Delta t' = -v\Delta x'/c^2$, which leads to $L'/c = vL'/c^2$, or $v = c$, when the above simultaneity condition in $S$ from $S'$ perspective is applied. Therefore, for the considered events to be perceived simultaneously in $S$, the Special Relativity paradoxically requires that $v = c$.

Stating that the length contraction Eq. (1), for which $\Delta t = 0$, is valid at each time $t$ of $S$ is misleading, as it would rather be valid from the perspective of $S'$ under the above condition ($\Delta t' = L'/c$, and $\Delta x' = -L'$), for which the distance in $S$ between the two events under consideration would be shorter than the length of the wagon when it is receding (Fig. 1b), and longer when it is approaching the observer (Fig. 2b). Whereas, from the perspective of $S$, according to Lorentz transformation, the above simultaneity condition is equivalent to the contradicting result of $v = c$.

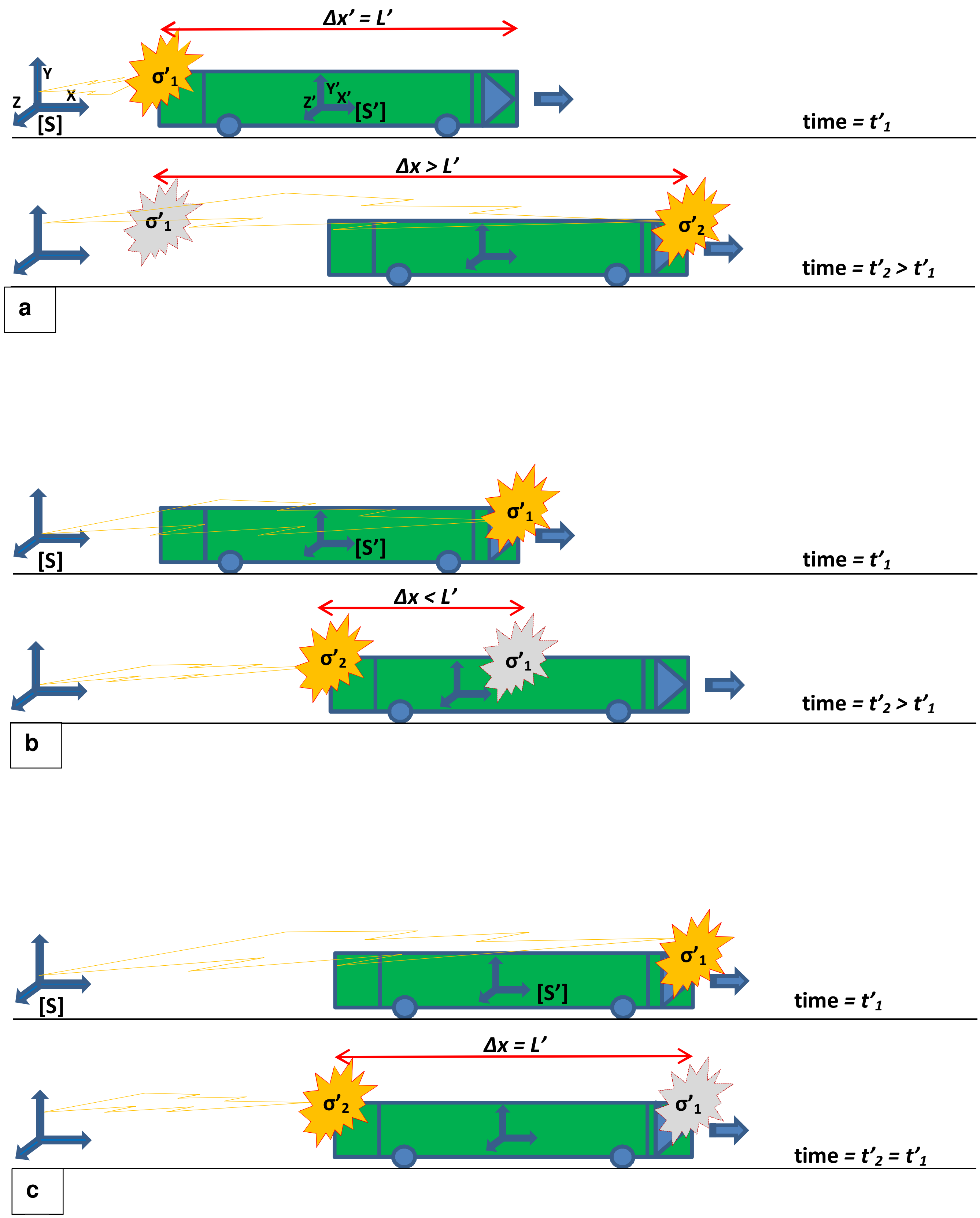


**Fig. 1 Light signals emitted from the ends of receding wagon at different time orders**

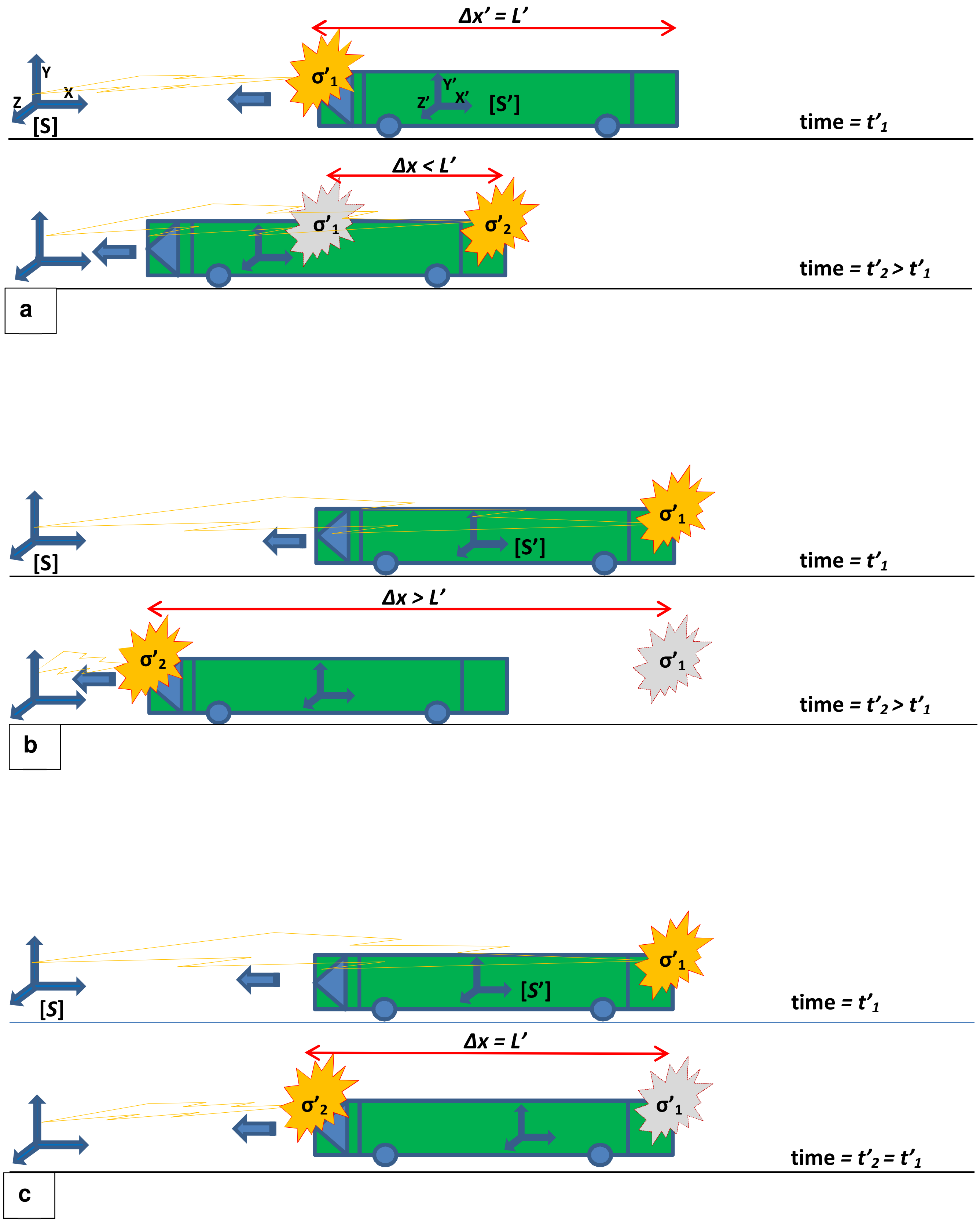


**Fig. 2 Light signals emitted from the ends of approaching wagon at different time orders**

## Conclusion

Einstein's interpretation of the Lorentz transformation to obtain the length contraction prediction is misleading, arriving at erroneous finding. Using the right criteria, within the special relativity framework, to measure the distance reflecting the moving object's length in the stationary frame, along with the appropriate Lorentz transformation equation, a contradictory finding of length expansion is established, thus refuting the tenability of the Lorentz transformation to confirm the special relativity's basic length contraction prediction.

1. Einstein A. Einstein's comprehensive 1907 essay on relativity, part I. *English translations in Am Jour Phys 45 (1977), Jahrbuch der Radioaktivitat und Elektronik* 1907; 4.