The pseudo-tensor gives positive, mistaken value for gravitational energy

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Abstract
The well known calculation of the total mass-energy for the gravitation field of a liquid sphere plus the matter of this sphere, which uses the pseudo-tensor of gravitational energy and momentum, is mistaken. The correct calculation shows that the pseudo-tensor provides a positive contribution to the total mass-energy. So, the pseudo-tensor is pointless

Keywords: gravitational field, conservation law, curvilinear coordinates

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1. Mass-energy of the matter of an object

As is known, mass-energy of a body M equals an integral of the volume density ρ over the volume with regard to the metric coefficients if curvilinear coordinates x^1, x^2, x^3 are in use:

$$M = \int \rho dV = \int \rho\sqrt{g_{11}g_{22}g_{33}}dx^1dx^2dx^3 \,. \tag{1.1}$$

Here g_{ik} is the metric tensor of the coordinate system, $\sqrt{g_{11}g_{22}g_{33}}dx^1dx^2dx^3 = dV$ is the infinitesimal physical valume. Or $\rho\sqrt{g_{11}g_{22}g_{33}}$ is a tensor (scalar) density of the weight +1, and $dx^1dx^2dx^3$ is a tensor (scalar) density of the weight -1.

Equation (1.1) can be obtained also in a space-time with coordinates x^0, x^1, x^2, x^3 and a metric tensor $g_{\alpha\beta}$. On this way, the mass dM is the physical time-coordinate[1] of the 4-momentum dP_α

$$dM = dP_0 / \sqrt{g_{00}} \,, \tag{1.2}$$

the volume density ρ is the coordinate of the energy-momentum tensor $\rho = T_0^0$, and the covariant coordinate of the 4-momentum dP_0 is expressed through the covariant coordinate of the 3-volume dV_0

$$dV_0 = \sqrt{-g}dx^1dx^2dx^3, \quad dP_0 = T_0^0 dV_0 = \rho\sqrt{-g}dx^1dx^2dx^3 \,. \tag{1.3}$$

In a static case, one may integrate the physical coordinates of infinitesimal 4-momentum (1.2). This gives formula (1.1)

$$M = \int \frac{dP_0}{\sqrt{g_{00}}} = \int \frac{\rho dV_0}{\sqrt{g_{00}}} = \int \frac{\rho\sqrt{-g}dx^1dx^2dx^3}{\sqrt{g_{00}}} = \int \rho\sqrt{-g_{11}g_{22}g_{33}}dx^1dx^2dx^3 \tag{1.4}$$

[1] We prefer vectors, covectors, *etc*. have coordinates, not components, as well as points have coordinates, not components: coordinates are numbers. Coordinates of a geometric object are not its components because a component is one of several parts of which sth is made, e.g., the components of a machine [Longman Advanced American Dictionary]. A geometric object is not made of numbers.

Here $\rho\sqrt{-g} = T_0^0\sqrt{-g}$ may be considered as the coordinate of a tensor density of weight +1, and $dx^1dx^2dx^3$ may be considered as the time-coordinate of a tensor density of weight -1. The physical volume dV is expressed through the covariant coordinate of the 3-volume dV_0

$$dV = dV_0 / \sqrt{g_{00}} = \sqrt{-g_{11}g_{22}g_{33}}dx^1dx^2dx^3. \qquad (1.5)$$

Equation (1.1) looks like

$$M = \int \rho\sqrt{-g_{rr}}r^2 4\pi\, dr \qquad (1.6)$$

if a spherical coordinates with the metric

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2,\ \ \sqrt{-g} = \sqrt{-g_{tt}g_{rr}}r^2\sin\theta. \qquad (1.7)$$

are in use (see the end of § 100 in [1])

2. The mass of a sphere of perfect fluid and the gravitational energy-momentum pseudo-tensor

Equation (1.6) is applied for a calculation of the mass M of the liquid sphere *matter* by the use of the Schwarzschild's interior solution. This solution is depended on two parameters, R and r_1, where $0 \le r \le r_1 < R$ [2 § 96]:

$$ds^2 = \left(\frac{3}{2}\sqrt{1-\frac{r_1^2}{R^2}} - \frac{1}{2}\sqrt{1-\frac{r^2}{R^2}}\right)^2 dt^2 - \frac{1}{1-\frac{r^2}{R^2}}dr^2 - r^2d\Theta^2 - r^2\sin^2\Theta\, d\varphi^2, \qquad (2.1)$$

Here R is the radius of curvature of the space, which is determined by the constant density of the liquid $T_t^t = \rho = 3/(8\pi R^2)$, and r_1 is the coordinate of the boundary of the sphere, where we make the interior solution (2.1) agree with the exterior solution,

$$ds^2 = \left(1-\frac{2m}{r}\right)dt^2 - \frac{1}{1-\frac{2m}{r}}dr^2 - r^2d\Theta^2 - r^2\sin^2\Theta\, d\varphi^2, \qquad (2.2)$$

which is dependent of one paremeter $m = r_g/2$: One can see $m = r_1^3/2R^2$.

Formula (1.6) with the use of the interior solution gives the mass of the fluid [3]:

$$M = \int_0^{r_1} T_t^t\sqrt{-g_{rr}}r^2dr4\pi = \int_0^{r_1}\frac{3}{2R}\frac{r^2dr}{\sqrt{R^2-r^2}} = \frac{3R}{4}(\arcsin\xi - \xi\sqrt{1-\xi^2}),\quad \xi = \frac{r_1}{R}. \qquad (2.3)$$

Restricting to two terms of the expantion in terms of ξ, formula (2.3) gives

$$M = m(1+\frac{3}{10}\frac{r_1^2}{R^2}+\ldots) = m + \frac{3m^2}{5r_1}+\ldots \qquad (2.4)$$

So, $M > m$. This excess of the matter mass M over the Schwarzschild parameter m was named (positive) gravitational mass defect [1 § 100]. The point is the parameter m is the *total* mass, i.e. mass of matter *and* of its gravitational field, and this total mass m does not change when gravitational contracting, according to the Birkhoff"s theorem. However the matter mass M increases when gravitational contracting, and, in the same time, the gravitational field is strengthened. Therefore we have to ascribe a *negative* mass to gravitational field in order to satisfy the conservation law of the total mass-energy.

To take into account this negative gravitational mass-energy, physicists use the gravitational energy-momentum pseudo-tensor t_β^α. So, the total mass-energy J equals integral (1.1) of the sum $T_0^0 + t_0^0 = \rho + t_0^0$:

$$J = \int (T_0^0 + t_0^0)dV = \int (T_0^0 + t_0^0)\sqrt{g_{11}g_{22}g_{33}}dx^1dx^2dx^3 = M + \int t_0^0\sqrt{g_{11}g_{22}g_{33}}dx^1dx^2dx^3. \qquad (2.5)$$

This total mass-energy must equal m for our liquid sphere. I.e. it must be $J = m$. Therefore coordinate t_0^0 of the pseudo-tensor must be negative.

3. The pseudo-tensor is in use

In reality, the contrary is the case. The standard expression for t_β^α [2 (87.12)] has positive coordinate $.t_0^0 > 0$ for the gravitational field of our liquid sphere. The standard integral of the sum $T_0^0 + t_0^0$, which equals m and which is given e.g. in [2 (91.1), (92.1)]

$$J_0 = \int (T_0^0 + t_0^0)\sqrt{g}\, dx^1 dx^2 dx^3 = m, \tag{3.1}$$

is incorrect [3,4]. Quantity J_0 is not the total mass J (2.5). Formula (3.1) implies an ariphmetic addition of covariant time-coordinates of the infinitesimal total 4-momentums

$$dJ_0 = (T_0^0 + t_0^0)\sqrt{g_{00}}\, dV = (T_0^0 + t_0^0)\sqrt{g}\, dx^1 dx^2 dx^3, \tag{3.2}$$

which belong to different spatial points where the coordinate vectorial bases can be not parallel. Therefore, **there is no basis** to which the the integral coordinates J_α could belong. Such an integral do not form a geometric quantity (covector).

The total mass J given by the integral (2.5) of the sum $T_0^0 + t_0^0$ is considerable more, than m, and this fact discredits the pseudo-tensor t_β^α. Coordinate t_0^0 is positive. It is proved by changing the integrand in (3.1) [5, 2 (92.4),(97.3)]:

$$J_0 = \int (T_0^0 + t_0^0)\sqrt{g}\, dx^1 dx^2 dx^3 = \int (T_0^0 + 3p)\sqrt{g}\, dx^1 dx^2 dx^3 . \tag{3.3}$$

Here p is the isophropic presure in the liquid. This substitution is true when we are interested in quiescent states of temporary or permanent equilibrium. This means that t_0^0 is a substantially positive quantity, and t_0^0-contribution to the total mass of the system "matter + its gravitational field" is also a substantially positive quantity, though this contribution must be negative. And the cherished quantity m belongs not to the total mass J (2.5), but to a senseless quantity J_0 ($J_0 < J$). It is because the integrand in (3.1) contains an extra term $\sqrt{g_{00}} < 1$ in compare with the integrand in (2.5).

4. Conclusion

The standard pseudo-tensor of gravitational energy and momentum ascribes a *positive* value of the gravitational energy for an *isolated system* in general relativity. So, this pseudo-tensor is a mistake.

References

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2. R. C. Tolman, *Relativity, Thermodynamics and Cosmology* (Oxford, Clarendon 1969).
3. R. I. Khrapko, The Truth about the Energy-Momentum Tensor and Pseudotensor *Gravitation and Cosmology*, **20**, 4 (2014). http://khrapkori.wmsite.ru/ftpgetfile.php?id=132&module=files
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5. R. C. Tolman, *Phys. Rev.* **35**, 875 (1930).

History of the submissions

This paper http://khrapkori.wmsite.ru/ftpgetfile.php?id=149&module=files is ignored by AJP, GRG, C&QG, NJP

AJP

We have reviewed your submission (#28707) and determined that it is not appropriate for publication in the **American Journal of Physics**. Please refer to the "Information for Contributors"
Sincerely David Jackson

GRG

In this paper (GERG-D-16-00195) the author makes another attempt to deal with the gravitational pseudo-tensor for energy-momentum. As with previous attempts by the author, the paper is based on confusion and errors of understanding of general relativity, and of the substantial literature on the topic. This paper should be rejected.
Roy Maartens, Editor-in-Chief **General Relativity and Gravitation**

C&QG (IOP)

To be publishable in this journal, articles must be of high quality and scientific interest, and be recognised as an important contribution to the literature. Your paper (CQG-102765) has been assessed and has been found not to meet these criteria.
Jennifer Sanders – Editor, **Classical and Quantum Gravity**.

My appeal to C&QG, which was ignored:
The paper proves the popular concept of the pseudo-tensor by Einstein, Eddington, Tolman and others is false. This is of high scientific interest, and must be recognised as an important contribution to the literature.

NJP (IOP)

To be publishable in this journal, articles must be of high quality and scientific interest, and be recognised as an important contribution to the literature. Your paper (NJP-105153) has been assessed and has been found not to meet these criteria".
Editor-in-Chief Barry C Sanders, **New Journal of Physics.**

My appeal to NJP:
The paper proves the popular concept of the pseudo-tensor by Einstein, Eddington, Tolman and others is false. This is of high scientific interest, and must be recognised as an important contribution to the literature.

A message from NJP:
We are unable to consider an appeal for this article. As we explained previously**,** New Journal of Physics does not reconsider articles which were previously rejected from a specialised, fellow **IOP** Publishing journal (in this case Classical and Quantum Gravity).
Ben Sheard – Editor

My letter to IOP, which was ignored:
The IOP infallibility dogma is "It is company policy that once an article has been rejected from one Institute of Physics journal, we cannot consider it for another". Thus, IOP is interested in its own infallibility rather than in scientific results.

PRD

This manuscript (DS11970) is closely related to your previously rejected manuscripts DAJ1097 and DJ11389, and shares their faults. Furthermore, the arguments and conclusions that you present here have already been published in your Ref. 3, which appeared in the journal Gravitation and Cosmology. In view of the above, we will not consider this manuscript.

Erick J. Weinberg Editor **Physical Review D"**

DAJ1097: "Goodbye, the Pseudotensor!" http://viXra.org/abs/1501.0173

DJ11389 "The energy-momentum pseudo-tensor of the gravitational field is a mistake" http://khrapkori.wmsite.ru/ftpgetfile.php?id=114&module=files

Ref. 3: "The Truth about the Energy-Momentum Tensor and the Pseudotensor" *Gravitation and Cosmology*, 20, 4 (2014), p. 264 http://khrapkori.wmsite.ru/ftpgetfile.php?id=132&module=files

My appeal to PRD:

The point is you rejected the paper because of two causes:

1) This manuscript is closely related to my previously rejected manuscripts, and shares their faults

2) The arguments and conclusions that are presented here have already been published in G&C.

But, sorry, if the paper is mistaken, you must not refer to its another publication in order to reject the paper.

And if the paper is true, then a publication of its arguments and conclusions in G&C (Ref. 3) must not prevent its publication in PRD because the publication in PRD will help Editors and readers of PRD to recognize that the paper is of high scientific interest, and is an important contribution to the literature. Now readers of *Gravitation and Cosmology* know that Einstein, Tolman, etc. were mistaken. Furthermore, the readers know "**The poverty of the PRD Editorial Board"** http://khrapkori.wmsite.ru/ftpgetfile.php?id=134&module=files . On the contrary, you, Professor Neil Cornish, and readers of PRD are deceived. The deception is simple: The mass M of liquid of the sphere is greater than the Schwarzschild parameter m, $M > m$. But the sum equals m:

$$M + \int t_0^0 dV = m\text{, though } t_0^0 > 0\,!$$

The answer from PRD:

Dear Dr. Khrapko, You appear to not dispute my assertion that the arguments and conclusions that are presented in the manuscript submitted to PRD have already been published in G&C. The Editorial Guidelines state that The Physical Review publish new results. Thus, prior publication of the same results generally will preclude consideration of a later paper. Because of this, we do not understand the grounds on which you are appealing. Are you claiming

1) that DS11970 contains significant new and correct results that were not in the article in G&C or

2) that even though the results in DS11970 have already appeared in G&C, they should also be published in PRD, despite the policy quoted above.

Erick J. Weinberg

My elucidation for PRD:

Dear Erick J. Weinberg, Now, when reading DS11970, you appear to not dispute that DS11970 proves a mistake in the classical calculation of the total energy, which uses the gravitational pseudo-tensor, and so discredits the Einstein's pseudo-tensor.

But you insist that the proving of this mistake presented in the prior publications, DAJ1097, G&C, DJ11389, is false.

And Professor Neil Cornish, when reading G&C, insists that integrals of the pseudo tensor yield invariant veritable results for isolated systems.

It means that the proof presented in DS11970 is a new result, and that DS11970 contains a significant new and correct proof, which was not in the G&C article. And this result is very useful for you, Professor Neil Cornish, and for PRD-readers because of a widespread delusion that the positive pseudo-tensor provides the negative gravitational binding energy. So, it seems worthwhile to publish the result.

Editorial Appeal DS11970

The pseudo-tensor gives positive, mistaken value for gravitational energy, by Radi I. Khrapko

In the following I will show that there are serious shortcomings in the manuscript which will render the manuscript not suitable for publication in PRD or any other journal.

(i) First, there do exist infinitely many pseudotensors for the gravitional field and not only “The pseudo-tensor ...” as given in the title of the manuscript (cf. parameter γ below). The one in the manuscript is just the Einsteinian (E) one. Only in the Conclusion section the specification “standard” is used.

(ii) Second, it is well known that for an *isolated system* the E-pseudotensor itself gives positive gravitational field energy. In the Newtonian limit (just for simplicity), with velocity v, mass density ϱ, Newtonian potential φ, and specific internal energy Π, one gets, also following from Eq. (3.3) of the manuscript,

$$E_{\mathrm{E}} = \int d^3x \left[\varrho \left(\frac{v^2}{2} + \Pi + \varphi \right) + \frac{1}{8\pi G} (\nabla\varphi)^2 \right],$$

(in the problem 1 of paragraph 106 in Ref. [1], german ed. 1976, the action of the manifest positive field-energy part can be found).
On the contrary, the Landau-Lifshitz (LL) pseudotensor itself gives negative gravitational field energy, again in the Newtonian limit, cf. the textbook by Misner/Thorne/Wheeler, p. 470,

$$E_{\mathrm{LL}} = \int d^3x \left[\varrho \left(\frac{v^2}{2} + \Pi - 3\varphi \right) - \frac{7}{8\pi G} (\nabla\varphi)^2 \right].$$

Obviously,

$$E_{\mathrm{E}} = E_{\mathrm{LL}} = \int d^3x \left[\varrho \left(\frac{v^2}{2} + \Pi \right) - \frac{1}{8\pi G} (\nabla\varphi)^2 \right],$$

which shows the negative gravitational field energy in the both approaches, E and LL; cf. footnote in the problem 1 mentioned above. The pseudotensors are not unique and their gravitational field energy content is not unique either, even ranging negative, zero, and positive. The very reason for that is the physical equivalence of the gravitational field energy densities

$$w_\gamma = \frac{\gamma}{2}\varrho\varphi - \frac{1-\gamma}{8\pi G}(\nabla\varphi)^2,$$

with arbitrary constant γ ($\gamma = 2$ for E, $\gamma = -6$ for LL). Physical statements can only be those where γ does not show up. One always has to keep in mind that for *isolated systems* pseudotensors do contain only part of the gravitational field energy, the other part enters via the energy-momentum tensor of the system (exception is $\gamma = 0$, at least on the Newtonian level). The concern of the author about the positivity in question is 'pointless'.

(iii) Third, the Eq. (2.5) in the manuscript is interpreted wrongly. It is not a proof that the energy from the Einsteinian pseudotensor must be negative. Thus, the standard pseudotensor is *not* a mistake.

(iv) Fourth, contrarily to the author's claim, the Eq. (3.1) is correct. J_0 correctly transforms as time component of a four-vector when asymptotic Lorentz transformations get applied. Calling J_0 a 'senseless quantity' is wrong. The statement $J_0 = m$ is correct, yet $J = m$, with J from Eq. (2.5), is incorrect. In Ref. [3] of the manuscript the author quotes the famous russian mathematician Ludwig Faddeev on the energy problem in Einstein's theory of gravity but he completely ignores the disproof therein of his claim.

(v) Fifth, the author ignores the usefulness of the pseudotensors, including the Einsteinian one, for the treatment of gravitational waves.

Differently posed in presentation but similarly based on the well known books by Tolman and Landau-Lifshiftz, the manuscript comes to the same erroneous conclusion as Ref. [3] of the manuscript, namely the Einsteinian pseudotensor being a mistake. The supposed additional support, as inferred from the Eq. (2.5), is empty.

Publication of the manuscript, I can not recommend.

Gerhard Schäfer
Member of the Editorial Board

Author's reply on Editorial Appeal DS11970

The fact,
"there do exist infinitely many pseudotensors for the gravitional field",
discredits the idea about the pseudotensor on its own account.

Gerhard Schafer's statement,
"it is well known that for an isolated system the E-pseudotensor itself gives positive gravitational field energy"
is simply wrong. In reality, all physicists are convinced that the Einstein-Tolman pseudo-tensor of gravitational energy provides a negative contribution for an isolated system. Please, read Landau & Lifshitz:

$$"m = \frac{4\pi}{c^2}\int_0^a T_0^0 r^2 dr\,. \qquad (100.23)$$

We call attention to the fact that the integration is taken with respect to $4\pi r^2 dr$, whereas an element of spatial volume for the metric (100.2) is $dV = 4\pi r^2 e^{\lambda/2} dr$, where, according to (100.20), $e^{\lambda/2} > 1$. This difference indicates the gravitational mass defect of the body".
Or please see Tolman:

$$"U = \int \rho_{00} dV_0 + \int \frac{1}{2}\rho\psi dV, \quad \psi = \frac{1}{2}(g_{44} - 1) < 0. \qquad (97.10)$$

We thus see, at the Newtonian level of approximation, that the relativistic formula for the total energy of a fluid sphere would reduce to the sum of the total proper energy and the usual Newtonian expression for potential gravitational energy".

Gerhard Schafer's statement,
"the Landau-Lifshitz (LL) pseudotensor itself gives negative gravitational field energy",
is simply wrong. Please, read Landau & Lifshitz:

$$"P^0 = \frac{1}{c}\int (T_0^0 - T_1^1 - T_2^2 - T_3^3)\sqrt{-g}\,dV, \quad -T_1^1 - T_2^2 - T_3^3 > 0\,. \qquad (105.23)$$

This formula expresses the total energy of matter and the constant gravitational field (R. Tolman, 1930). We recall that in the case of central symmetry we have another expressionfor this quantity – formula (100.23)".

The fact,
"The pseudotensors are not unique and their gravitational field energy content is not unique either, even ranging negative, zero, and positive",
discredits the idea about the pseudotensor, because, in reality, the gravitational field energy of an isolated system is negative.

The fundamental delusion of Gerhard Schafer and others is:
"the Eq. (3.1)

$$J_0 = \int dJ_0 = \int (T_0^0 + t_0^0)\sqrt{g}\,dx^1 dx^2 dx^3 = m\,, \qquad (3.1)$$

is correct. J_0 correctly transforms as time component of a four-vector when asymptotic Lorentz transformations get applied. Calling J_0 a 'senseless quantity' is wrong".
In reality, Eq. (3.1) implies an arithmetic addition of the infinitesimal time components dJ_0 belonging to different spatial points where the coordinate vectorial bases are not parallel, when using curvilinear coordinates. Therefore, **there is no basis** to which the integral component J_0 could belong. And at coordinate transformation $x^i = f(y^a)$ **there is no transformation law** for the integral components J_i to the components J_a because the coordinate transformation is different in different points of the integration domain. For these reasons, the quantity J_i is not a geometrical quantity, and J_0 is meaningless.

Publication of the manuscript is extremely usefull for Members of the PRD Editorial Board.

R. Khrapko, 07/10/2016