Is the Schwarzschild Radius Truly a Radius?

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Abstract

This paper questions the assumption that the Schwarzschild radius actually represents a radius. It has recently been shown by Haug [1] that the Schwarzschild radius for any object can simply be written as $N2l_p$, where N is the number of Planck masses into which we can "hypothetically" pack an object of interest and l_p is the well known Planck length. The Schwarzschild radius seems to represent the length of the number of Planck mass objects we can "hypothetically" pack a planet or star into and then we can place them in perfect alignment along a single strand (of single particles) in a straight line.

Key words: Schwarzschild radius, Planck mass, Planck particle, Black hole, Sphere packing.

1 Introduction

The Schwarzschild radius is normally given by the following formula

$$r_s = \frac{2GM}{c^2} \tag{1}$$

Haug [1, 2, 3] has suggested that Newton's gravitational constant can be written as:

$$G_p = \frac{l_p^2 c^3}{\hbar} \tag{2}$$

where $\hbar$ is the reduced Planck's constant, c is the well tested round-trip speed of light, and l_p is the Planck length. We could call this Planck's form of the gravitational constant. This way of writing Newton's gravitational constant does not change the value of the constant. If one knows the Planck length, then the gravitational constant is known, or alternatively and more practically one can calibrate the Planck length based on empirical measurements of the gravitational constant. There is still considerable uncertainty in the exact measurement of the gravitational constant. Experimentally, substantial progress has been made in recent years based on various methods. See, for example, [4, 5, 6, 7, 8].

Based on this, the Planck mass can be written as

$$m_p = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{l_p}\frac{1}{c} \tag{3}$$

and, as shown by [1], the Schwarzschild radius can now be written as

$$\begin{aligned} r_s &= \frac{2GM}{c^2} \\ r_s &= \frac{2\frac{l_p^2 c^3}{\hbar} N \frac{\hbar}{l_p}\frac{1}{c}}{c^2} \\ r_s &= N2l_p \end{aligned} \tag{4}$$

Just to show that the formula works, let's look at the Schwarzschild radius of the Sun. The solar mass is $M_s \approx 1.988 \times 10^{30}$ kg. The Sun's mass in terms of the number of Planck masses must therefore be $\frac{1.988\times10^{30}}{2.17651\times10^{-8}} \approx 9.134 \times 10^{37}$. This means the Schwarzschild radius of the Sun is

$$r_s = N2l_p = 2 \times 9.134 \times 10^{37} \times 1.616199 \times 10^{-35} = 2952.47 \text{ meter}$$

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This is the well known Schwarzschild radius of the Sun, which one would also obtain from equation 1. Still, the equation 4 gives us additional insight into what the Schwarzschild radius potentially represents. First, we have hypothetically packed the mass of the Sun into a series of Planck masses. Each Planck mass has a reduced Compton wavelength of l_p, and we will consider this as the hypothetical radius of a hypothetical Planck mass "object". Later we will discuss slightly different alternatives for what we can call a Planck particle. We will at least initially assume this Planck mass object (particle) has a mass equal to the Planck mass and a diameter of $2l_p$. So we can pack the entire mass of the Sun into 9.134×10^{37} Planck mass objects. Next we lay these Planck masses out next to each other in perfect alignment along a single straight line. Then surprisingly we get a length exactly equal to the Schwarzschild radius. In other words, we have not packed the Planck mass particles into a formation that turned them into a larger sphere, but rather, we have aligned them in a single strand along a straight line. In this scenario, the Schwarzschild radius is no longer related to the radius of a spherical object.

We could however taken the 9.134×10^{37} Planck masses that give a mass equal to the Sun and sphere-packed them as densely as possible. In 1831, Gauss [9] proved that the most densely one could pack spheres amongst all possible lattice packings was given by

$$\frac{\pi}{3\sqrt{2}} \approx 0.74048 \tag{5}$$

In 1611, Johannes Kepler suggested that this was the maximum possible density for both regular and irregular arrangements; this is known as the Kepler conjecture. The Kepler conjecture was supposedly finally proven in 2014 by Hale [10]. Based on this form of sphere packing, the radius of the 9.134×10^{37} Planck spheres gives a Sun with radius (see the Appendix for detailed derivation of this result)

$$R = 2l_p \sqrt[3]{\frac{N}{\pi}} \sqrt[6]{18} = 1.616199 \times 10^{-35} \sqrt[3]{\frac{9.134 \times 10^{37}}{\pi}} \sqrt[6]{18} \approx 8.04533 \times 10^{-23} \text{ meter} \tag{6}$$

This is much smaller than the Schwarzschild radius of the Sun, which is about 2952.5 meters. The fact that this radius is much smaller than that does not prove that the Schwarzschild radius is not a radius. The main point here is that if we align all of the Planck masses (that we hypothetically can pack a mass into) perfectly into a single straight line, then this will correspond to the Schwarzschild radius exactly. At the same time, it is interesting to note that if we sphere-pack the Planck masses, then we will get a radius that is only 2.73×10^{-26} of the Schwarzschild radius.

It is worth saying a few words on Planck mass objects or Planck particles. It was likely Motz [11, 12, 13], based on the work of Planck [14], who first suggested that a particle with a mass equal to what today is known as the Planck mass, $m_p = \sqrt{\frac{\hbar c}{G}}$ might exist. Motz called this particle the Uniton and suggested that it could be a fundamental particle. Since then, modern physics has derived a Planck particle based on setting the Compton wavelength equal to the Schwarzschild radius and by extension, the Planck particle would have a mass of $\sqrt{\pi}$ times the Motz-Planck particle, that is

$$m = \sqrt{\frac{hc}{2G}} = \frac{h}{\sqrt{\pi}2l_p}\frac{1}{c} \tag{7}$$

where h is Planck's constant. Instead of using a Planck mass particle (the Motz particle), we could have worked with this modern Planck particle. This would mean the Sun's mass would be $\frac{9.134 \times 10^{37}}{\sqrt{\pi}}$ Planck particles. If we assume that the Planck particle has a diameter or length equal to its Compton wavelength of $\lambda = \frac{h}{mc} = \sqrt{\pi}2l_p$ and is perfectly aligned, then the Planck particles on a single strand would also add up to the Schwarzschild radius exactly. This gives a mass equal to the mass of the object of interest. In addition, this means that we are able to write the Schwarzschild "radius" as $r_s = N2l_p$. We also can write the Schwarzschild "radius" as

$$r_s = N_2\sqrt{\pi}2l_p \tag{8}$$

where N_2 is the number of modern Planck particles we can pack the object of interest (Sun or planet into). The Schwarzschild radius is considered the radius of a black hole and also a mini black hole. The mini black hole has an assumed mass equal to the Planck particle just mentioned. Still, if we pack the Sun into Planck particles, then there are just enough particles to make a straight line equal to the Schwarzschild radius.

One also has to question the idea that if the Schwarzschild radius represents the edge of a black hole, then it would necessarily be a linear function of the number of Planck particles one can hypothetically pack the mass into. The gravitational force follows the inverse square law, so why would the Schwarzschild radius be a linear function of the mass? We believe that our analysis of the Schwarzschild radius calls into question the nature and even the existence of black holes. At the very least, it makes the current black hole interpretation, which is highly dependent on the Schwarzschild radius being a true radius, subject to debate. This question has been posed by others as well, see for example [15, 17, 16]. Others [18, 19, 20] has also suggested that the Schwarzschild radius is not truly a radius, but their work is based on very different argumentation than what is represented in this paper. Nevertheless, the discussion over the Schwarzschild radius and even the existence of black holes is growing.

2 Conclusion

When compressing a massive object like the Sun into a series of Planck mass objects, and then aligning these objects perfectly one by one in a straight line, we get a length equal to the Schwarzschild radius. The Schwarzschild radius is widely viewed as representing the radius of a spherical object, but it also represents a length of a different variety.

Appendix: Radius Formula of Planck Mass Packed Sphere

Assume we first pack the entire mass of the Sun into some Planck mass objects with radius l_p and diameter $2l_p$. This means the volume of each Planck sphere is

$$V = \frac{4}{3}\pi l_p^3$$

When we pack the Planck spheres as densely as possible, then assuming that the Planck mass is a spherical shape, they will take up a volume of

$$V_t = \frac{\frac{4}{3}\pi l_p^3}{\frac{\pi}{3\sqrt{2}}} = l_p^3\sqrt{32}$$

The total volume is then NV_t. This means we need a larger sphere with radius

$$\begin{aligned} NV_t &= \frac{4}{3}\pi R^3 \\ R &= \sqrt[3]{\frac{\frac{3}{4}NV_t}{\pi}} \\ R &= \sqrt[3]{\frac{\frac{3}{4}Nl_p^3\sqrt{32}}{\pi}} \\ R &= l_p\sqrt[3]{\frac{N}{\pi}}\sqrt[6]{18} \end{aligned} \quad (9)$$

This is the formula to calculate the radius of a sphere consisting of maximum packed spheres with radius l_p.

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