Is Electric Field Vector Field or Covector Field?

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There are different fields in electrodynamics: vector fields and covector fields.

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As is well known, "the force $\mathbf{F}$ experienced by a small test charge is directly proportional to the charge e of the test body

$$\mathbf{F} = e\mathbf{E}, \quad F^i = eE^i \tag{1}$$

where $\mathbf{E}$ is called the electric field strength ($i = x, y, z$, or $i = r, \theta, \varphi$, or $i = 1,2,3, \ldots$). It is characteristic of the Maxwell theory that a reality independent of the existence of the probe charge is immediately ascribed to this vector field $\mathbf{E}$, and that it forms the actual object of the investigation" [1, p. 59].

In the same time [1, p. 60], "for every electrostatic field the electric field strength must be expressible in the form"

$$\mathbf{E} = -\mathrm{grad}\,\phi, \quad E_i = -\partial_i \phi \text{ ''}. \tag{2}$$

However, a (contravariant) vector and a gradient, which is a covariant vector (covector), are different "actual objects". A vector is represented as a "stick" or as an arrow (e.g., "$\rightarrow$"). The basis representation of a covector gradient is a system of the equipotential surfaces. The magnitude of a vector is proportional to the length of the stick. The arrow demonstrates the inner orientation of the stick. Some sense of the covector direction might be indicated by a wavy line with an arrow at the end (Fig. 1). The magnitude of a covector is proportional to the density of the sheets [2]

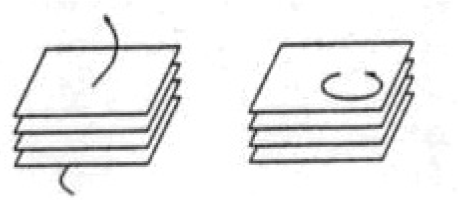

Figure 1. Covectors with outer and inner orientations respectively. (It is from [2]).

Misner et al. [3] give the same representation of a vector and of a covector (Fig. 2).

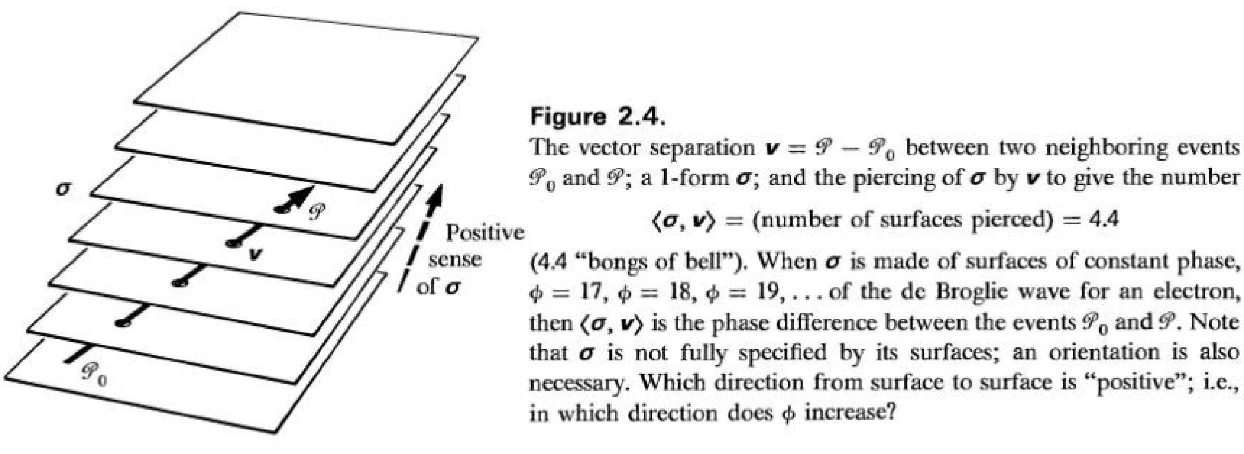

Figure 2. A vector $\mathbf{V}$ and a covector σ. (It is from [3]).

Schouten, when depicting electromagnetic fields, represents a covector by a biplane element [4,5]

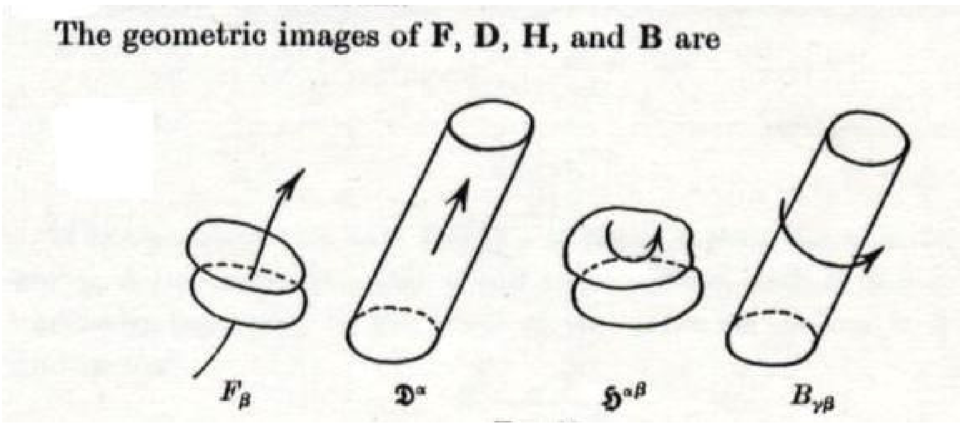

Figure. **3.** Covector $\mathbf{E} = \mathbf{F}$ is represented by two parallel plane elements equipped with an outer orientation. Vector density $\mathbf{D}$ is represented by a cylinder with an inner orientation. Bivector density **H** is represented by two parallel plane elements equipped with an inner orientation. Covariant bivector $\mathbf{B}$ is represented by a cylinder with an outer orientation.

The same depicting of a covector by a biplane element is used in [6].

So, we must recognize that there are two different electric fields in the electrodynamics: vector field E^i and covector field E_i. Given one of the fields, we can obtain other field by the use of the metric tensor g_{ik} of the coordinate system. The metric tensor establishes a one-to-one correspondence between co- and contravariant vectors:

$$E_i = g_{ik}E^k, \quad E^i = g^{ik}E_k \tag{3}$$

The metric tensor determins the mutual orthotogonality of the stick and the sheets (Fig. 4).

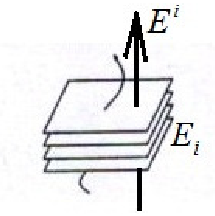

Figure 4. The vector E^i and covector E_i, which are connected by a metric tensor. Inner orientation of the vector is agreed with outer orientation of the covector

References

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Электрическое поле является векторным полем или ковекторным?

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В электродинамике присутствуют различные типы полей, в частности, векторные и ковекторные

Ключевые слова: электромагнетизм, градиент, геометрические объекты
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Addition

Nobody knows and nobody wants to know the difference between vector and covector.
This paper was rejected without reviewing by **AJP, TPT, EJP, УФН:**

We have reviewed your submission, our manuscript #28607, and determined that it is not appropriate for publication in the American Journal of Physics. Please refer to the "Information for Contributors" and the "Statement of Editorial Policy" at the AJP home page. Sincerely, David Jackson

I do not see that this material is useful enough to the practicing introductory physics teacher to warrant sending it out for review, Gary White Editor the Physics Teacher

The Editor-in-Chief of EJP recommended that, based on your past **misconduct**, you should not be allowed to submit any work to EJP. Dr Ben Sheard – Editor.

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