Kinematic Solutions to the Twin Paradox in Special Relativity

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This paper deals with the twin paradox within special relativity. The paper reveals the cause of paradoxical time dilation for an inertial stay-at-home twin, occurring, as believed by a non-inertial travelling twin, throughout his motion except for a short-time turn, though by the return of the travelling twin it is the stay-at-home twin who has aged more. This cause is the unconditional approach to the individual observer's inherent state of rest. Certain kinematic solutions to the paradox are given without resorting to a non-inertial reference frame. The existence of such solutions is shown both in special relativity and in Lorentz ether theory.

Introduction

The source of the twin paradox is an assertion that when there is relative motion of two twins – a stay-at-home twin and a travelling twin – they experience reciprocal time dilation (slowing of time relative to each other). The time dilation for the stay-at-home twin, occurring as seen by the travelling twin throughout his journey except for a short-time turn, seems paradoxical, as by the time of return of the traveler the stay-at-home twin has aged more.

As one of the solutions to the twin paradox, an argument is often put forward that the use of inertial reference frames cannot be extended to a twin who if only for a brief period becomes non-inertial. This argument is not actually a solution but rather recognition that it is impossible to solve the paradox applying methods of special relativity. Another solution proceeds from an assumption that time flow rates are different at the points of a traveler-related reference frame, non-inertial during the turn. However, if resorting to such a reference frame is necessary, it only confirms the fact that the solution to the paradox requires going beyond special relativity. With all that, certain kinematic solutions to the twin paradox do exist. The present paper offers two of such solutions. One of them will be dealt with within special relativity, whereas the other will be discussed going back to Lorentz ether theory. In so doing, we are going to demonstrate

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that the ether-related solution is not only free from incongruity but is particularly simple and illustrative.

1. The Ambiguity of the Notion of Relative Motion of Two Point Objects

Before proceeding to the solution to the twin paradox in special relativity, we will focus on the notion of relative motion of two point objects, which can also be represented by observers moving, as is often said, relative to each other. It is the looseness of the notion of relative motion of two point objects that results in quite a number of logical inconsistencies in special relativity.

Imagine two observers, observer *A* who is at rest at point *a* of a certain inertial reference frame, and observer *B* located at some moment at point *b* of this reference frame at a distance l_0 from observer *A*. Observer *B*, for a short time finding himself at a given moment at point *b* (flying through it), has velocity *v* directed perpendicular to segment *ab*, as shown in the figure below.

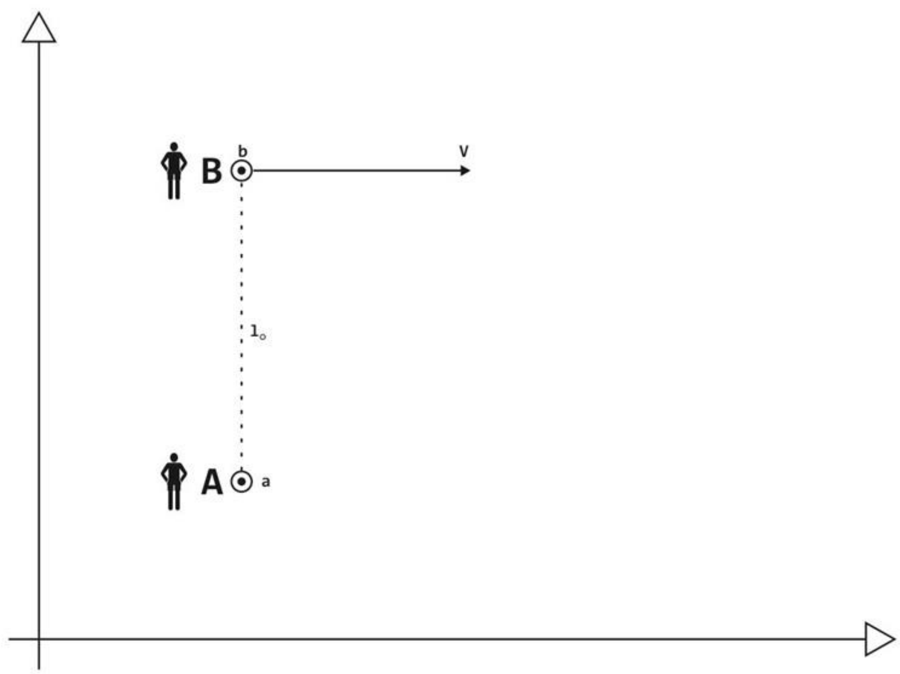

Fig. 1. Observer *A* is at rest at point *a* of an inertial reference frame. The velocity of observer *B* located at point *b* is equal to *v*. The distance *ab* between the observers is equal to l_0.

Let each of the observers have an observation tube and a source of equidirectional monochromatic green colored radiation, and each observer register a change of the wavelength of another observer's emitter.

At first glance, each of the observers, owing to the transverse Doppler effect, having directed the tube along the line segment *ab*, will register reddening of radiation from another observer's emitter. After all, it seems apparent that if observer *B* is moving relative to observer *A*, then observer *A* is moving relative to observer *B*. In fact, the answer is not that simple. The above statement of the problem does not maintain that the observers are moving "relative to each other". It is only the speed *v* of observer *B*, which is spoken about. With this problem formulation, the observers may, for example, stand on a rotating disk – observer *A* in the centre of the disk at point *a*, and observer *B* on its edge. In so doing the non-inertial observer *B*, circling in this reference frame at velocity *v*, has this velocity *v* at any point on the circumference, among them at point *b*, in which he finds himself at some point in time, as shown in Fig. 2.

Regardless of observer *B*'s motion, we cannot say he is moving relative to observer *A*, or that observers *A* and *B* are moving relative to each other, since the distance between the observers remains unchanged. In general, it makes no sense talking about relative motion of any points on a rotating disk because such "relative motion" has more in common with relative rest.

A. Observer *B* is moving in a circle, however, not relative to observer *A*, but in the inertial system *K* (or relative to it), wherein observer *A* is at rest. In this case, it does not matter whether the observers find themselves on a rotating disk or observer *B* is moving in a circle around its center in any other way.

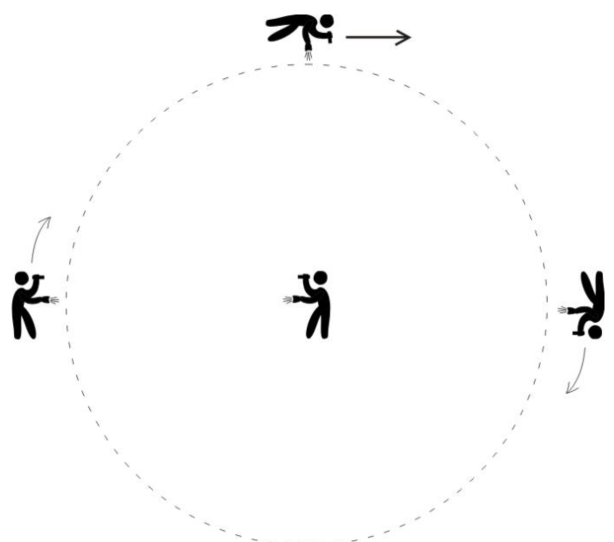

Fig. 2. Observer *B* is moving in a circle at velocity *v* around observer *A*, which is at rest in the inertial reference frame *K*.

2. The Observer's Circular Motion

If observer *A* located in the centre of a circle at point *a* of the reference frame *K* tilts the tube along the line segment *ab* to point *b* through which at some moment observer *B* has flown, he will see the source of a "warmer" yellow color, for example, at a certain velocity *v* of observer *B*. This wavelength shift is equivalent to slowing down of the clock of the moving observer *B*.

Moving along a circular path crosswise relative to the flux of monochromatic emission, observer *B* will perceive the color of the source in the centre of the circle as blue and not yellow. Due to aberration, a flow of light falls on the observer not at a right angle relative to its direction of motion, but at an angle θ, acute to this direction. This results in a shift of the emission spectral line to shorter wavelengths.

In order to see an observer in the centre, an observer on the circle has to look at an angle, acute to the direction of his motion (or align the tube at this angle). Observer *B* can find the acute angle experimentally. In the reference frame *K*, the tube tilt angle θ to the direction of motion of observer *B* is equal to arccos(*v/c*).

Formally, this fact can be expressed using the known ratio $f = \dfrac{f_0\sqrt{1-v^2/c^2}}{1-(v/c)\cos\theta}$, linking frequency *f* of the received light signal and natural frequency f_0 of the emitted signal at an angle θ between the tube tilt angle and the velocity vector. Since the tube tilt angle θ to the direction of motion of observer B is equal to arccos(v/c), then after substituting the angle θ with arccos(*v/c*) the mentioned ratio becomes

$$f = f_0\Big/\sqrt{1-v^2/c^2}\ . \qquad (1)$$

The frequency of the source emission is proportional to the clock rate, and the natural frequency of the green sources is the same for both observers. Hence, if at the initial moment of time the readings τ of observer *B*'s clock and the readings *t* of observer *A*'s clock were zero, then by analogy with formula (1) we can write

$$t = \tau\Big/\sqrt{1-v^2/c^2} \qquad (2)$$

If observers *A* and *B* had clocks in their hands, then observer *A* would find a slower rate of observer *B*'s clock and observer *B* would find a faster rate of observer *A*'s clock. To explain this fact without going beyond kinematics, observer *B* should acknowledge the fact of his motion relative to a reference frame conventionally at rest, i.e. the fact that at any time he is himself moving in the reference frame *K* and observer *A* is at rest in it. He will then state, "I am moving in the reference frame *K*; hence, my time and my clock rate are slowing down. Moving crosswise to a beam of light, I have to consider light aberration and tilt my tube to see the light. Because of the slowness of my clock and of the rate of my time flow, I fix the seemingly accelerated motion of observer *A'*s clock, which is actually not present in the reference frame *K*, since observer *A* in this reference frame is at rest".

Observer *B* on the circle for obvious reasons cannot regard himself at rest in an inertial frame and cannot observe the effects of the slowness of the clock and yellowing of observer *A*'s source. He likewise has no right to explain the blue color of the source in terms of the longitudinal component of Doppler effect caused by the central observer's counter motion. It is he who is moving, crosswise to the beam, and not the central observer. He can account for the blue color by the fact that moving crosswise to the green beam, he experiences time dilation and, being in delayed time conditions, perceives green light as blue. This can also be explained by the following circumstance.

If a great number of observers with green light sources are moving in a circle, and one of them has emitted a light pulse, which, having passed the center of the circle, hits the diametrically opposite point on the circle, where at this time another observer is moving, the latter will perceive the received pulse as green. This will take place despite the fact that each of the observers, having exchanged the pulse, is moving in the reference frame *K* with velocity *v*, in opposing motion. In other words, the value often called "relative velocity" of observers, according to the rule of velocity addition is equal to $2v/(1+v^2/c^2)$. The received pulse will evidently be green, given the fact that the pulse sent to a diametrically opposite point passes through the center and acquires the yellow color for the central observer. {On the other hand, the frequency of the yellow pulse emitted from the center of the circle, which has reached the circling observer, increases, according to the latter, $1/\sqrt{1-(v/c)^2}$ times, as it follows from formula (1), and the pulse becomes green.

If the observers, having exchanged the light pulse, acknowledge the fact of their motion, then they will attribute the invariance of the pulse colour to their identical time dilation due to the same speed in the reference frame *K*. The pulse from the green source of one observer, which has turned yellow in the reference frame *K* due to time dilation, is perceived by another observer as green again due to his time dilation.

3. The Rectilinear Motion of the Observer

Now consider a case when both observers are inertial.

Let observer *C′* move tangentially to the circle through point *b* at a constant speed *v*. At first, the inertial observers *A* and *C′* approach each other, then, as observer C′ appears at point *b*, for a short time find themselves at a distance l_0, which is minimal from each other throughout the whole motion period, whereupon they begin to move away from each other.

When observer *C′* arrives at point *b*, observers *A* and *C′* do not come closer and do not move away from each other, and the distance *l* between observers *A* and *C′* for a short period remains unchanged. The short-term invariance of the distance between observers *A* and *C′* indicates a transient state of relative rest of these observers. Yet the motion at the moment of their "relative rest" does take place, but it is not the motion of the observers relative to each other, but the motion (or rest) of the observers in different reference frames.

As distinct from observer *B*, observer *C′* is capable to view both the slowing down and the acceleration of the central observer's clock rate, as well as to see the central observer's source in different colors, specifically, in yellow and in blue. The color of the viewed source depends on the direction of the observer's gaze (or on the direction of the tube).

An ability of observer *C′* to see different colors of the source belonging to observer *A*, in physical terms is explained by the fact that, unlike observer *B*, observer *C′*

accepts emission from the central source not only while stationed on the circle, but also outside it. Observer *A* is also capable of recording different colors of light coming from observer *C′*, which is also due to his ability to receive light emitted by observer *C′*s source outside the circle.

The question arises: how are observers *A* and *C′* supposed to carry out their observations, if they can see virtually anything?

First, they must choose a reference frame. If they are not interested in the interrelationship of the results of physical observations, then each of them can choose their own system. This is exactly what observers in special relativity do. None of Einsteinian observers acknowledges the fact of their motion relative to other observers. However, if observers *A* and *C′* wish to obtain interrelated results of their observations, they must choose one, and only one, inertial frame, and all further actions to be carried out given this choice. For example, they can choose the reference frame *K* wherein observer *A* is at rest and observer *C′* is in motion.

If observer *C′* as well as observer *B* who is moving in a circle, would assume that he, finding himself at point *b*, is moving in the reference frame *K* at velocity *v* crosswise to the beam, and would vector the tube in the same direction in which observer *B* stationed on the circle is looking, then, as is the case with the latter, he will see a blue source in the hands of observer *A*.

Assume now that observer *C′* is flying at velocity *v* parallel to the *X*-axis of the reference frame *K* at some distance from it and monitoring the flow of time of the system *K* on the clocks placed along the *X*-axis.

If observer *C′* will look in one direction at the clock in the stationary reference frame *K* he is flying by, then, consecutively fixing the readings of a great number of clocks passing the point on which his eyes are focused, he will fix an accelerated flow of time *t* in the reference frame relative to which he is moving. The rate of the flow of time will not depend on the direction of his gaze, although the color of the dials depends on it. With the initial readings of the clocks of the reference system *K* and of observer C′ equal to zero, the time acceleration is expressed as a ratio $t = \tau' \big/ \sqrt{1-(v/c)^2}$, where τ′ is the time shown by the flying observer C′'s own clock. Such accelerated time flow in the reference frame *K* is not consistent with the behavior of each individual clock or each individual emitter in the reference frame *K*, if the observer C′, believing that he is at rest and the reference frame *K* is moving relative to him, will tilt the tube perpendicular to its motion. Given this orientation of the tube, while tracking the behavior of the source emitting, for example, the green light, he will detect a shift of the wavelength of the emitter according to the transverse Doppler effect to the long-wave region and turning of the green beam to a yellow one. The shift of the wavelength will correspond to the slowness of the rate of each individual clock flying past observer *C′*. This inconsistency in the rate of an individual clock and of a great number of clocks is no less paradoxical than the twin paradox. How does time flow in the reference frame *K*, if, according to successive readings of a great number of clocks which observer C′ is flying by in the reference frame *K*, the time in the reference frame *K* passes faster, whereas according to the time of each of the clocks flying past observer *C′*, the time in the system *K* passes slower?

The solution to the paradox of inconsistent clocks is the right selection of the tilt angle of the tube moving in the system K by observer C'. The paradox occurs when observer C' forgets that he is himself moving, and referring the state of rest to himself, counts each clock of the system K as moving relative to him. If the moving observer C' remembers that it is not the clock moving past him, but he himself is moving past the clock, then he will tilt the tube at a needed angle and find that the light has become blue, and the emission frequency has exceeded that of the original green source $1\big/\sqrt{1-(v/c)^2}$ times. It means that in this position of the tube the pace of change of a great number of clock readings and the pace of a single clock prove consistent. It is just this position of the tube that corresponds to speed v of observer C', relative to the reference frame K. The tube tilt angle θ to the direction of motion of observer C′ is equal to arccos(v/c).

Due to consistency of the pace of movement of an individual clock and a great number of clocks of the stationary reference frame K, we can talk about acceleration of movement of the clock in the reference frame K as a whole, relative to the clock rate of observer C'. This acceleration also relates to the clock of observer A, which is an element of the reference frame K.

Thus, observer C′ may state, "I am moving in the stationary reference frame K, and because of my motion my time and the pace of my clock slow down in this system. Because of the slow pace of my clock and of the flow of my time I register a seemingly accelerated pace of all clocks in the reference system K, though actually in the stationary reference frame K no acceleration is present because my movement is in no way related to the movement of the clock in the reference frame K".

4. Regarding Relative Motion of Intrinsic Inertial Systems

Those used to absolute rest of intrinsic reference frames may assume that the recognition by a moving observer of the fact of his motion relative to a stationary reference frame is contrary to the principle of the equality of inertial systems. There is actually no contradiction here. Inertial reference frames are equal in that each of them can be at rest and in motion relative to other systems. To each independent inertial reference frame one can ascribe a state of proper rest and a state of proper motion relative to other systems.

Ascribing a state of rest to their proper reference frames is mathematically convenient because it ensures invariance of the mathematical notation of physical laws. Yet, ascribing a state of rest to all reference frames in motion relative to one another leads to inconsistency of physical quantities of the $A>B$ *type*, whereas $A<B$. Smoothing of such inconsistency with explanations "from the point of view of different observers" or "in different reference frames" is routine, though physically not always and not for everyone convincing.

This inconsistency may be eliminated, for example, by way of arbitrary selection of a cardinal stationary reference frame in relation to which all other systems acquire certain velocities. The assignment of the state of rest to a cardinal inertial reference frame can be done by way of the following definition: "The stationary inertial reference system is such a system in which for any pair of points belonging to this system the light propagation time from one point to another is equal to the light propagation time between these points in the opposite direction”.

Under this definition, the difference in velocities of light in opposite directions in the moving reference systems becomes dependent on the direction and on their speed relative to the cardinal reference system, while the average value of the speed of light in a closed path is maintained. The invariance of the mathematical notation of physical laws is broken, but the invariance and consistency of physical quantities emerge.

If observer C' who has acknowledged the fact of his motion in the above example will rigidly fix himself to the reference frame K'_{mov}, which is moving together with him in relation to the reference system K, then all the observers who have acknowledged their state of motion in the reference frame K'_{mov} will register an accelerated time flow in the stationary reference frame K. Moreover, if synchronization of clocks in a moving reference frame has been performed taking into account its own motion relative to the system K and compliance to a uniform simultaneity in the systems K and K'_{mov}, then the accelerated flow of time in the stationary system K can be detected not only by visual observations. This acceleration may as well be detected by way of comparison of the movement of each individual clock in the reference frame K with a pair of clocks in the system K'_{mov}. In turn, if each of the observers in the reference frame K, for example, observer A, looks in one direction at a clock of the reference system K'_{mov}, moving past him, then, consecutively fixing readings of a great number of clocks passing the point on which his eyes are focused, he will fix not the acceleration but the slowness of the flow of time in the reference frame K'_{mov}.

The consistency of the speeds of time flow in different reference systems results from the unity of simultaneity. It is the relativity of simultaneity that leads to inconsistencies of the physical quantities in different reference frames. Note that we are talking about the unity, and not about the absoluteness of simultaneity. The choice of a cardinal stationary reference system is an arbitrary and conditional action, and when changing the cardinal system the nature of simultaneity changes because of relativity of the latter, the simultaneity becomes uniform again, though for other events. Only in case there existed an absolute reference frame or the stationary ether, with a cardinal reference frame rigidly fixed to it, we could talk about absolute simultaneity, and the physical quantities would acquire absolute character. Nevertheless, even in this case, the possibility of invariant recording of the laws of nature would remain, because, as you know [...], special relativity and Lorentz ether theory are mathematically and experimentally equivalent, differing only in philosophical content.

5. A Solution to the Twin Paradox in Special Relativity

Now we turn to the twin paradox.

Imagine twin B' escaping point g (with twin A remaining) to point h of the inertial system K, and who after a brief turn at point h has made a reverse journey and returned to twin A at point g. Assume now that twin B' is flying at velocity v, parallel to the X-axis of the reference frame K, at some distance from it and monitoring the flow of time of the system K on the clocks placed along the X-axis.

If twin B' acknowledges the fact of his motion at velocity v in the stationary reference frame K and directs his look or tilt the tube at an angle arccos(v/c), then he will fix the acceleration of time t in the reference frame relative to which he is moving. The acceleration is expressed as a ratio $\Delta t/\Delta\tau' = 1\big/\sqrt{1-(v/c)^2}$, where Δt = the time interval that has elapsed in the reference frame K during some time $\Delta\tau'$, which has elapsed for the flying twin according to his own clock. Twin B' will find the same acceleration, if he compares the pace of his clock with the flow of readings consecutively taken by him from the clocks past which he is flying, belonging to the reference frame K.

Thus, the travelling twin who has acknowledged the fact of his movement in the reference frame K, detects acceleration of the time flow of his inertial twin brother remaining at rest during the whole period of separation of the twins.

The ratio $\Delta t/\Delta\tau' = 1\big/\sqrt{1-(v/c)^2}$ can be viewed as a consequence of the inverse Lorentz transformation $t = (t' + vx'/c^2)\big/\sqrt{1-(v/c)^2}$. {Whatever point x' of the reference system K', which is in relative motion with the reference system K, would the twin B' mentally bind himself to, the time interval Δt at this point is equal to the quantity $\Delta t'\big/\sqrt{1-(v/c)^2}$.

6. A Solution to the Twin Paradox in the Ether Theory

There is actually no paradox related to the ether theory. In the ether theory, the paradox that has arisen in the depths of special relativity is resolved by means of elementary algebraic methods.

So what would happen if one of two twins who are at rest in the ether at one point, flies at speed v to a distant point and then after a while returns to twin A remaining at rest?

The answer is clear. If for the twin flying in the ether his “local time” characterizing the rate of physical processes in his body and the pace of the movement of his clock on both segments of his flight (there and back) slows down due to interaction with the ether, then the lapse of his “local time” will be $1\big/\sqrt{1-(v/c)^2}$ times less than for the twin at rest in the ether, and the “travelling” twin will get less “old”. The turn of the travelling twin, provided it is virtually instantaneous, has no practical effect on the ratio of times of both twins.

Moreover, what will happen if the two twins are flying side by side in the ether at speed v – with their “local time” passing slower – then one of them stops, staying at rest in the ether for some time, then catching up with the travelling twin? The twin who continued his flight in the ether with no information about the fact of his motion in the ether perceives this maneuver of his brother as a round trip to a distant point.

An obvious answer is that, since according to the ether theory after the twin’s stop in the ether his time will pass faster than the “local time” of his twin brother who continues his flight, and then when the twin stopping in the ether after some time catches up with the missing brother, he will age more than the latter. The “local” time of the twin catching up with his flying brother will actually flow slower than for the flying brother.

This is due to the faster speed of the twin catching up with his brother. As a result, the brother making a stop in the ether will age not more, but less than his twin brother who has not interrupted his flight.

Let us demonstrate that if the proper times of the motion there and back of the non-inertial twin relative to the inertial twin are equal, then for the non-inertial twin it will take $1\Big/\sqrt{1-(v/c)^2}$ times less time than for the moving inertial twin, and the non-inertial twin will age less.

Let at the time of stop of one of the twins in the ether the clocks of the parting twins show zeros. Suppose that after making a stop for some time, the twin who has lagged behind, at the moment t_1 of the ether time when his clock (because of the stop) was showing this time, left at speed *u*, such that $v<u<c$, following his brother flying away from him. The distance between the twins at the start of the twin who has left behind is equal to vt_1. Setting out, the twin left behind will catch up with the twin flying at a constant speed *v* at the point in time t_2, having spent the time equal to $vt_1/(u-v)$. During this period, by the clock of the twin following the flying away brother at speed *u*, there will be a lapse of proper time, which is $1\Big/\sqrt{1-(v/c)^2}$ times less than the ether time and equals $vt_1\sqrt{1-(u/c)^2}\Big/(u-v)$. Let us assume the velocity *u* such that the proper time t'_2-t'_1 of the catching up twin is numerically equal to the time t_1 of his stay at rest relative to the ether, i.e. t'_2-t'_1= t_1 or

$$t_1 = vt_1\sqrt{1-(u/c)^2}\Big/(u-v) \qquad (3)$$

This equality meets the condition under which the twin spends the same proper time on a trip to a distant point and back. By elementary transformations of the equality (3) we can obtain the value of velocity *u*, which is equal to $\dfrac{2v}{1+(v/c)^2}$. Substituting this value in the expression for the time $vt_1/(u-v)$ required for the return of the twin, and summing the time $vt_1/(u-v)$ and the time t_1, we obtain the ether time spent by the lagging behind twin on the stop and return to the flying twin. This time is equal to $2t_1\Big/(1-v^2/c^2)$. Since the clock of the inertial twin flying at a speed *v* go $1\Big/\sqrt{1-(v/c)^2}$ times slower than the clock at rest in the ether, the flying twin will determine the time spent by the lagging behind twin on the stop and return to the flying twin as a quantity meeting the equality:

$$t'_2 = 2t_1\Big/\sqrt{1-(v/c)^2}$$

Since the time elapsed for the non-inertial twin by the moment of his return is numerically equal to $2t_1$, and the time of the inertial twin is numerically equal to $2t_1\Big/\sqrt{1-(v/c)^2}$, then the lapse of time for the non-inertial twin is $1\Big/\sqrt{1-(v/c)^2}$ times shorter, and he has aged less than the inertial twin has.

Conclusion

One of the reasons for the paradoxical effects of special relativity is the unconditional approach to the state of proper rest by an observer moving relative to some reference frame. This also applies to the slowness of time flow for the inertial twin who has rapidly grown old. This slowness throughout the period of separation, except for the instantaneous turn, is observed by the twin who has made a round trip to a distant point. Declaring relativity of states of motion and rest, Einsteinian observers always assign the state of rest to themselves and to their reference frames and never do so for the state of motion. Assigning the state of rest to their own reference frames found in a state of mutual relative motion with other reference frames leads to inconsistency of physical quantities. This inconsistency is that each of two unequal quantities is simultaneously larger than the other one. The explanation of such an inconsistency with such comments as "from the point of view of different observers" or "in different reference frames" is routine, though physically not always and not for everyone convincing. The introduction to the relativistic theory of observers who recognize the state of proper motion relative to third party reference frames allows for the solution of the twin paradox to be confined to kinematics of their motion.

The ether theory does not need any tricks to account for the age difference between an inertial and a non-inertial twin who have met each other after parting. The fact that the inertial twin always turns out to have aged more than the non-inertial one at their meeting after parting, in the ether theory is an elementary consequence of the slowing down of the rate of processes in bodies moving in the ether, which can be shown by means of simple algebraic calculations.

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