IS INTRISIC SPIN REALLY A QUANTUM MECHANICAL CONCEPT?

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Abstract

The prevalent view today is that electron spin, for example, must be considered to be a quantum concept without detailed classical analogy. The author simply did not know if this proposition was true or false, and, subsequently, embarked upon a program (irregardless of whether the spin is quantized or not) to determine if the concept of 'intrinsic spin' (i.e. spin which is independent of a coordinate system) could be derived from ideas not considered Quantum Mechanical in nature. The author intuitively felt that the greatest chance for success lay in a geometrical approach, and, as such, modifications to the classic equations of Gauss and Weingarten in differential geometry were made in the form of postulates. The most important postulate proposed assumes an Asymmetric Coefficient of the Second Fundamental Form. Surprisingly, this postulate seems to transform a dull and undistinguished geometry into one that appears to very roughly emulate some of the properties of the physical universe, including 'intrinsic spin'.

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1 Introduction

The usual equations describing a hypersurface in differential geometry are the following classic equations of Gauss (1) and Weingarten (2):

$\mathbf{X}_{/i/k} = \Gamma_i{}^r{}_k \mathbf{X}_{/r} + A_{ik}\mathbf{N}$ .... (1)

$\mathbf{N}_{/j} = -A^r{}_j\ \mathbf{X}_{/r}$ .... (2)

with

$\mathbf{X}_{/i} \cdot \mathbf{X}_{/j} = g_{ij}$

$\mathbf{X}_{/i} \cdot \mathbf{N} = 0$

$\mathbf{N} \cdot \mathbf{N} = 1$

where

$\mathbf{X} = \mathbf{X}(x^i)$ = a position vector to a point on the hypersurface with the coordinates x^i. In this case of the four dimensional hypersurface, all indices (whether Latin or Greek) will run from 1 to 4.

$\mathbf{X}_{/r}$ = a tangent vector to the hypersurface at x^i.

N = the normal vector to the hypersurface at x^i.

$\Gamma_i{}^r{}_k$ = symmetric coefficient of connection.

A_{ik} = symmetric coefficient of the **Second Fundamental Form.**

g_{ij} = symmetric coefficient of the **First Fundamental Form** or, more commonly, the **space-time metric.**

$\mathbf{X}_{/i} = \partial \mathbf{X}/\partial x^i$ and likewise for $\mathbf{N}_{/j}$.

The above equations result in the usual development of Riemannian geometry and of General Relativity.

2 Postulatory Basis of a New Geometry

The following assumptions will be necessary in order to achieve the author's goal of deriving the concept of 'intrinsic spin' from geometrical considerations.

Postulate I:

The program which will be followed for the remainder of this paper will be to postulate an alternate set of equations as follows:

$\mathbf{X}_{/i/k} = \Gamma_i{}^r{}_k \mathbf{X}_{/r} + A_{ik}\mathbf{N}$ .... (3)

$\mathbf{N}_{/k} = -B^r{}_k\ \mathbf{X}_{/r} + d_k\mathbf{N}$ .... (4)

$\mathbf{X}_{/i} \cdot \mathbf{X}_{/j} = g_{ij}$ .... (5)

$\mathbf{X}_{/i} \cdot \mathbf{N} = \alpha_i$ .... (6)

$N \cdot N = \gamma^2$ .... (7)

where α_i and γ are **very small.** N must "point outward" in the direction of one or a combination of dimensions of an exterior imbedding space.

In general, for small α_i and small γ, B_{ik} is assumed to be **asymmetric** and d_l is no longer considered zero, but both are restricted, for the sake of mathematical continuity with equations (1) and (2), by requiring that in the limit of large γ:

$$\lim_{\substack{\alpha_i \to 0 \\ \gamma \to 1}} B_{ik}(\alpha_i, \gamma) = A_{ik}$$

and

$$\lim_{\substack{\alpha_i \to 0 \\ \gamma \to 1}} d_k(\alpha_i, \gamma) = 0$$

The ultimate goal is to find first order **linear** differential equations for α_i and γ, somewhat similar in form to the Gauss and Weingarten equations. We begin by taking the partial derivative of equation (6).

$$\alpha_{i/k} = X_{/i/k} \cdot N + X_{/i} \cdot N_{/k}$$

$$\alpha_{i/k} = \Gamma_i{}^r{}_k X_{/r} \cdot N + A_{ik} N \cdot N - B^r{}_k X_{/i} \cdot X_{/r} + d_k X_{/i} \cdot N$$

$$\alpha_{i/k} = \Gamma_i{}^r{}_k \alpha_r + A_{ik} \gamma^2 - B_{ik} + d_k \alpha_i \quad \text{.............................} \quad (8)$$

The partial derivative of equation (7) is next.

$$(\gamma^2)_{/k} = (N \cdot N)_{/k} = 2 \gamma \gamma_{/k} = 2 N \cdot N_{/k}$$

$$\gamma \gamma_{/k} = - B^r{}_k N \cdot X_{/r} + d_k N \cdot N$$

$$\gamma \gamma_{/k} = - B^r{}_k \alpha_r + d_k \gamma^2 \quad \text{....................................} \quad (9)$$

Equations (8) and (9) are derived without making any presumptions regarding the form of B_{ik} or d_l.

<u>Postulate II:</u>

The assumption is made that we can represent

$$B_{ik}(\alpha_i, \gamma) = -D_{ik}(\alpha_i) + T_{ik}(\gamma) \quad \text{...............................} \quad (10)$$

where D_{ik} and T_{ik} are both **asymmetric.** Keeping in mind the following limiting condition:

$$B_{ik}(0,1) = -D_{ik}(0) + T_{ik}(1) = A_{ik} \quad \text{..........................} \quad (11)$$

Upon substituting equation (10) into equations (8) and (9), we have:

$$\alpha_{i/k} = \Gamma_i{}^r{}_k \alpha_r + d_k \alpha_i + D_{ik}(\alpha_i) + A_{ik} \gamma^2 - T_{ik}(\gamma) \quad \text{...............} \quad (12)$$

$$\gamma\gamma_{/k} = D^r{}_k(\alpha_i)\alpha_r + d_k\gamma^2 - T^r{}_k(\gamma)\alpha_r \quad (13)$$

Postulate III:

The assumption is made that for small α_i that

$$D^r{}_k(\alpha_i) = \Sigma^{rj}{}_k\,\alpha_j \quad (14)$$

It then follows that in the equation $B_{ik}(0,1) = -D_{ik}(0) + T_{ik}(1)$, that $D_{ik}(0) = 0$ thus causing $T_{ik}(1) = A_{ik}$. Thus, we now have

$$\alpha_{i/k} = \Gamma_i{}^r{}_k\,\alpha_r + \Sigma_i{}^r{}_k\,\alpha_r + d_k\,\alpha_i + A_{ik}\gamma^2 - T_{ik}(\gamma) \quad (15)$$

$$\gamma\gamma_{/k} = \Sigma^{rj}{}_k\,\alpha_r\,\alpha_j + d_k\gamma^2 - T^r{}_k(\gamma)\alpha_r \quad (16)$$

From equation (5)

$\mathbf{X}_{/i}\cdot\mathbf{X}_{/j} = g_{ij}$, we can relate the coefficient of connection to the Christoffel symbol. Upon taking the partial derivative of both sides, we have

$$\mathbf{X}_{/i/k}\cdot\mathbf{X}_{/j} + \mathbf{X}_{/i}\cdot\mathbf{X}_{/j/k} = g_{ij/k}$$

$$\Gamma_i{}^r{}_k\mathbf{X}_{/r}\cdot\mathbf{X}_{/j} + A_{ik}\mathbf{N}\cdot\mathbf{X}_{/j} + \Gamma_j{}^r{}_k\mathbf{X}_{/i}\cdot\mathbf{X}_{/r} + A_{jk}\mathbf{X}_{/i}\cdot\mathbf{N} = g_{ij/k}$$

$$\Gamma_i{}^r{}_k\,g_{rj} + A_{ik}\,\alpha_j + \Gamma_j{}^r{}_k\,g_{ir} + A_{jk}\,\alpha_i = g_{ij/k}$$

$$g_{ij/k} = (\Gamma_i{}^r{}_k + A_{ik}\,\alpha^r)\,g_{rj} + (\Gamma_j{}^r{}_k + A_{jk}\,\alpha^r)g_{ir}$$

$g_{ij/k} = \{{}_i{}^r{}_k\}\,g_{rj} + \{{}_j{}^r{}_k\}\,g_{ir}$, where $\{{}_i{}^r{}_k\}$ is the Christoffel symbol, which means that the covariant derivative of the metric tensor is zero or $g_{ij//k} = 0$, where // means the covariant derivative. Hence, the Christoffel symbol is related to the coefficient of connection as follows.

$$\{{}_i{}^r{}_k\} = \Gamma_i{}^r{}_k + A_{ik}\,\alpha^r \quad (17)$$

Restating equations (15) and (16) in light of (17), we now have:

$$\alpha_{i/k} = \{{}_i{}^r{}_k\}\,\alpha_r + \Sigma_i{}^r{}_k\,\alpha_r + d_k\,\alpha_i - A_{ik}\,\alpha^r\,\alpha_r + A_{ik}\gamma^2 - T_{ik}(\gamma) \quad (18)$$

$$\gamma\gamma_{/k} = \Sigma^{rj}{}_k\,\alpha_r\,\alpha_j + d_k\gamma^2 - T^r{}_k(\gamma)\alpha_r \quad (19)$$

Postulate IV:

We have to make some kind of assumption about the form of $T_{ik}(\gamma)$, remembering that $T_{ik}(1) = A_{ik}$. To this end, a simple form taken will be simply a truncated power series in γ sufficient to satisfy imposed conditions.

$$T_{ik}(\gamma) = -\gamma L_{ik} + \gamma^2(A_{ik} + L_{ik}) \quad (20)$$

which satisfies $T_{ik}(1) = A_{ik}$. Upon rearranging, we have:

$$\gamma^2 A_{ik} - T_{ik}(\gamma) = \gamma(1-\gamma)L_{ik} \quad (21)$$

Since $T_{ik}(\gamma)$ is asymmetric and A_{ik} is symmetric, we are allowed to decompose the asymmetric tensor into its symmetric and antisymmetric parts, by concluding that L_{ik} is antisymmetric. Upon substitution from (20) or (21) into (18) and (19), we have:

$$\alpha_{i/k} = \{{}_i{}^r{}_k\}\,\alpha_r + \Sigma_i{}^r{}_k\,\alpha_r + \gamma\,L_{ik} + d_k\,\alpha_i - A_{ik}\,\alpha^r\alpha_r - \gamma^2 L_{ik} \;\ldots\ldots\; (22)$$

$$\gamma\gamma_{/k} = \Sigma^{rj}{}_k\,\alpha_r\,\alpha_j + d_k\,\gamma^2 + \gamma\,L^r{}_k\,\alpha_r - \gamma^2 A^r{}_k\,\alpha_r - \gamma^2 L^r{}_k\,\alpha_r \;\ldots\ldots\; (23)$$

Postulate V:

In general, $d_k = d_k(\alpha_i, \gamma)$, however, we will assume that an expansion similar to that in Postulate III is the simplest, i.e. linear in α_i. We would be faced with terms like $d_k\,\alpha_i$ and $d_k\,\gamma^2$ and since we want a linear equation, we can drop these as well as all other non-linear terms such as $\alpha_r\alpha_j$, $\alpha_r\,\alpha^r$, γ^2, $\gamma\alpha_r$, etc. We are then left with:

$$\alpha_{i/k} = \Gamma_i{}^r{}_k\,\alpha_r + \Sigma_i{}^r{}_k\,\alpha_r + \gamma\,L_{ik} \;\ldots\ldots\; (24)$$

$$\gamma_{/k} = L^r{}_k\,\alpha_r \;\ldots\ldots\; (25)$$

To avoid a non-linear conflict in (24), the α_r in $\Gamma_i{}^r{}_k$ in (17) will be assumed from this point to be an average over the region of interest. This will maintain a flexibility and convenience of switching back and forth between using $\Gamma_i{}^r{}_k$ or $\{{}_i{}^r{}_k\}$ depending upon the problem. Previously, all non-linearities were dropped, but the dependence of equation (17) upon α_r is unavoidable. We seem to be on the border between the linearity sought after in (24) and the non-linearity imposed on (24) by the dependence of $\Gamma_i{}^r{}_k$ upon α_r.

If the non-linear term $A_{ik}\,\alpha^r\,\alpha_r$ is dropped, then there is no practical difference between $\Gamma_i{}^r{}_k$ and $\{{}_i{}^r{}_k\}$. Equation (17) should **now** be read as $\{{}_i{}^r{}_k\} = \Gamma_i{}^r{}_k + A_{ik}\,\bar{\alpha}^r$. However, there is some loss in generality by using this definition, as $\bar{\alpha}^r$ becomes a sort of free vector, which transforms as a vector only under linear transformations.

Under these conditions, Equations (24) and (25) then form a new set of coupled, first order, linear partial differential equations describing this new geometry.

3 Integrability Conditions

It is now necessary to investigate an equation of the form $c^i \alpha_i + b\,\gamma = 0$ which is important in evaluating the integrability conditions.

$c^i \mathbf{X}_{/i} \cdot \mathbf{N} + b\,(\mathbf{N} \cdot \mathbf{N})^{1/2} = 0$

$b^2 \mathbf{N} \cdot \mathbf{N} = c^i c^j (\mathbf{X}_{/i} \cdot \mathbf{N})(\mathbf{X}_{/j} \cdot \mathbf{N}) = c^i c^j \mathbf{N} \cdot (\mathbf{X}_{/i} \mathbf{X}_{/j}) \cdot \mathbf{N}$

$\mathbf{N} \cdot (b^2 \gamma^{-2}\ \mathbf{N}\,\mathbf{N}) \cdot \mathbf{N} = \mathbf{N} \cdot (c^i c^j\ \mathbf{X}_{/i} \mathbf{X}_{/j}) \cdot \mathbf{N}$

$\mathbf{N} \cdot (c^i c^j\ \mathbf{X}_{/i} \mathbf{X}_{/j} - b^2 \gamma^{-2}\ \mathbf{N}\,\mathbf{N}) \cdot \mathbf{N} = \mathbf{N} \cdot \mathbf{S} \cdot \mathbf{N}$

If the tensor **S** is carefully chosen such that $\mathbf{S} = d^i \mathbf{X}_{/i} \mathbf{N} + f^i \mathbf{N} \mathbf{X}_{/i}$, then we can define another tensor **T** satisfying

$\mathbf{N} \cdot (c^i c^j\ \mathbf{X}_{/i} \mathbf{X}_{/j} - b^2 \gamma^{-2}\ \mathbf{N}\,\mathbf{N} - \mathbf{S}) \cdot \mathbf{N} = 0$ such that $\mathbf{T} = 0$.

$\mathbf{T} = c^i c^j\ \mathbf{X}_{/i} \mathbf{X}_{/j} - b^2 \gamma^{-2}\ \mathbf{N}\,\mathbf{N} - d^i \mathbf{X}_{/i} \mathbf{N} - f^i \mathbf{N} \mathbf{X}_{/i} = 0.$ This is a dyad in a five space which has the bases $\mathbf{X}_{/i} \mathbf{X}_{/j}$, $\mathbf{N}\,\mathbf{N}$, $\mathbf{X}_{/i} \mathbf{N}$ and $\mathbf{N} \mathbf{X}_{/i}$.

Such a tensor has $5^2 = 25$ components

$\mathbf{X}_{/i} \mathbf{X}_{/j}$	=	16 components
$\mathbf{X}_{/i} \mathbf{N}$	=	4 components
$\mathbf{N} \mathbf{X}_{/i}$	=	4 components
$\mathbf{N}\,\mathbf{N}$	=	1 component
		25 components

Since $\mathbf{T} = 0$, the coefficients of the basis are zero. Hence, $b=0$, $c^i = 0$, $d^i = 0$ and $f^i = 0$.

We now give the results of evaluating the **integrability conditions**:

$\alpha_{i/k/j} - \alpha_{i/j/k} = 0$ and

$\gamma_{/k/j} - \gamma_{/j/k} = 0.$

(A) <u>The integrability condition on γ</u>

$\gamma_{/k/j} - \gamma_{/j/k} = 0$ yields:

<u>coefficient of γ</u> results in

$L^r{}_k L_{rj} = L^r{}_j L_{rk}$ .... (26)

<u>coefficient of α_r</u> results in

$L^i{}_{k/j} - L^i{}_{j/k} + L^r{}_k \Gamma_r{}^i{}_j - L^r{}_j \Gamma_r{}^i{}_k + L^r{}_k \Sigma_r{}^i{}_j - L^r{}_j \Sigma_r{}^i{}_k = 0$ .... (27)

or if we locally assume $\Gamma_r{}^i{}_k \approx \left\{{}_r{}^i{}_k\right\}$ then we can use the usual covariant derivative from this point through equation (35).

$$L_{ik//j} - L_{ij//k} - \Sigma_{rik} L^r{}_j + \Sigma_{rij} L^r{}_k = 0 \quad (28)$$

(B) The integrability condition on α_i

$\alpha_{i/k/j} - \alpha_{i/j/k} = 0$ yields:

coefficient of γ results in

$$L_{ik//j} - L_{ij//k} + \Sigma_{irk} L^r{}_j - \Sigma_{irj} L^r{}_k = 0 \quad (29)$$

coefficient of α_r results in

$$-R^b{}_{ikj} = L_{ik} L^b{}_j - L_{ij} L^b{}_k + \Delta^b{}_{ikj} \quad (30)$$

where

$$R^b{}_{ikj} = \text{Riemann curvature tensor} = \left\{{}_i{}^b{}_k\right\}_{/j} - \left\{{}_i{}^b{}_j\right\}_{/k} + \left\{{}_i{}^r{}_k\right\}\left\{{}_r{}^b{}_j\right\} - \left\{{}_i{}^r{}_j\right\}\left\{{}_r{}^b{}_k\right\} \quad (31)$$

$$\Delta^b{}_{ikj} = \Sigma_i{}^b{}_{k//j} - \Sigma_i{}^b{}_{j//k} + \Sigma_i{}^r{}_k \Sigma_r{}^b{}_j - \Sigma_i{}^r{}_j \Sigma_r{}^b{}_k \quad (32)$$

Equations (28) and (29) are compatible if $\Sigma_{rik} = -\Sigma_{irk}$

4 Ricci Tensor Development

The Ricci tensor is $G^{ij} = R^{ij} - \frac{1}{2} g^{ij} R$ or $G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R$ where $R_{ij} = R^k{}_{ikj}$ and $R = R^i{}_i$; also $\Delta_{ij} = \Delta^k{}_{ikj}$ and $\Delta = \Delta^i{}_i$.

Upon evaluating G_{ij} we obtain :

$$G_{ij} = -\left(\mathbf{L_{ib} L^b{}_j + \tfrac{1}{4} g_{ij} L_{rb} L^{rb}}\right) - \tfrac{1}{4} g_{ij} L_{rb} L^{rb} - (\Delta_{ij} - \tfrac{1}{2} g_{ij} \Delta) .$$

The bold portion resembles the electromagnetic energy-momentum tensor since L^{rb} is antisymmetric. The quantity $L_{rb} L^{rb}$ represents the Lagrangian of this supposed 'electromagnetic' field. Equation (26) shows that $L_{ib} L^b{}_k$ is symmetric in the indices i and k.

Let $h^{ij} = \Delta^{ij} - \frac{1}{2} g^{ij} \Delta$ which we can break down further into the following:

$h^{ij} = Z^{ij} - \frac{1}{2} g^{ij} Z + (V^{rij})_{//r}$

where $Z^{ij} = \Sigma^{ir}{}_b \Sigma_r{}^{bj} - \Sigma^{irj} \Sigma_r{}^b{}_b$

$Z = Z^i{}_i$ and

$V^{rij} = \Sigma^{rij} + g^{jr} \Sigma^{ib}{}_b - g^{ij} \Sigma^{rb}{}_b = -V^{irj}$

$G^{ij} = -\left(\mathbf{L^i{}_b L^{bj} + \tfrac{1}{4} g^{ij} L_{rb} L^{rb}}\right) - \tfrac{1}{4} g^{ij} L_{rb} L^{rb} - h^{ij}$

$G^{ij} = -\left(\mathbf{L^i{}_b L^{bj} + \tfrac{1}{4} g^{ij} L_{rb} L^{rb}}\right) - \tfrac{1}{4} g^{ij} L_{rb} L^{rb} - (Z^{ij} - \tfrac{1}{2} g^{ij} Z) - (V^{rij})_{//r}$

$G^{ij}{}_{//i} = 0 = [(-g)^{\frac{1}{2}} (G^{ij} + t^{ij})]_{/i} = 0$ where t^{ij} is the pseudo-tensor of the gravitational field.

5 Angular Momentum Development

The **total** angular momentum would be proportional to M^{ik} which is:

$M^{ik} = \int \{(G^{jk} + t^{jk})\, x^i - (G^{ji} + t^{ji})\, x^k\} (-g)^{½} dS_j$

$M^{ik} = \int \{(G^{4k} + t^{4k})\, x^i - (G^{4i} + t^{4i})\, x^k\} (-g)^{½} dx^3$

$dS_4 = dx^3$ when integrating over a hypersurface in which the time coordinate is constant.

$G^{ik} = -(L^{i}{}_{b} L^{bk} + ¼\, g^{ik} L_{ab} L^{ab}) - ¼\, g^{ik} L_{ab} L^{ab} - (Z^{ik} - ½\, g^{ik} Z) - (V^{rik})_{//r}$

$G^{ik} = \mathcal{G}^{ik} - (V^{rik})_{//r}$

$M^{ik} = \int \{(\mathcal{G}^{4k} + t^{4k} - (V^{r4k})_{//r})\, x^i - (\mathcal{G}^{4i} + t^{4i} - (V^{r4i})_{//r})\, x^k\} (-g)^{½} dx^3$

$M^{ik} = \int \{(\mathcal{G}^{4k} + t^{4k})\, x^i - (\mathcal{G}^{4i} + t^{4i})\, x^k\} (-g)^{½} dx^3 + \int \{(V^{r4i})_{//r}\, x^k - (V^{r4k})_{//r}\, x^i\}(-g)^{½} dx^3$

$(V^{rik})_{//r} = (-g)^{-½} [(-g)^{½} V^{rik}]_{/r} + \{{}_{a}{}^{k}{}_{r}\} V^{ria}$ .... (33)

Looking only at the second integral of M^{ik}:

$\int \{(V^{r4i})_{//r}\, x^k - (V^{r4k})_{//r}\, x^i\}(-g)^{½} dx^3$ and making the substitution from (33) while noting that there will be substitutions like

$x^i [(-g)^{½} V^{r4k}]_{/r} = [x^i (-g)^{½} V^{r4k}]_{/r} - (-g)^{½} V^{r4k} \delta^i_{\ r}$. We also note that there will be integrals of the type:

$\int [x^i (-g)^{½} V^{r4k}]_{/r}\, dx^3$ which can be considered as zero because the fields vanish on 2 dimensional surfaces very far away as a result of the divergence theorem.

$\int \{(V^{r4i})_{//r}\, x^k - (V^{r4k})_{//r}\, x^i\}(-g)^{½} dx^3 = \int [\{{}_{a}{}^{i}{}_{r}\}\, x^k - \{{}_{a}{}^{k}{}_{r}\}\, x^i] V^{r4a} (-g)^{½} dx^3$
$+ \int [V^{i4k} - V^{k4i}](-g)^{½} dx^3$

The **total** angular momentum is then proportional to:

$M^{ik} = \int \{(\mathcal{G}^{4k} + t^{4k})\, x^i - (\mathcal{G}^{4i} + t^{4i})\, x^k\} (-g)^{½} dx^3 +$
$+ \int [\{{}_{a}{}^{i}{}_{r}\}\, x^k - \{{}_{a}{}^{k}{}_{r}\}\, x^i] V^{r4a} (-g)^{½} dx^3$
$+ \int [V^{i4k} - V^{k4i}](-g)^{½} dx^3$

$M^{ik} = N^{ik} + C^{ik} + S^{ik}$ = proportional to the **total** angular momentum.

$N^{ik} = \int \{(\mathcal{G}^{4k} + t^{4k})\, x^i - (\mathcal{G}^{4i} + t^{4i})\, x^k\} (-g)^{½} dx^3$ = proportional to the **orbital angular momentum.**

$$C^{ik} = \int [\{{}^{\ i}_{a\,r}\}\, x^k - \{{}^{\ k}_{a\,r}\}\, x^i] V^{r4a} (-g)^{1/2}\, dx^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (34)$$

and is a **correction term** to the **orbital angular momentum** due to the non-flatness of space.

$$S^{ik} = \int [V^{i4k} - V^{k4i}] (-g)^{1/2}\, dx^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (35)$$

and is proportional to **intrinsic spin momentum,** which is independent of any coordinate system.

6 Development of a Force Equation

Let us derive what appears to be a force equation using equations (24) and (25).

$\alpha_{i/k} = \Gamma_i{}^r{}_k\, \alpha_r + \Sigma_i{}^r{}_k\, \alpha_r + \gamma\, L_{ik}$

$\gamma_{/k} = L^r{}_k\, \alpha_r$

Upon multiplying both of these equations by the four velocity u^k we obtain

$\alpha_{i/k}\, u^k = \Gamma_i{}^r{}_k\, u^k\, \alpha_r + \Sigma_i{}^r{}_k\, u^k\, \alpha_r + \gamma\, L_{ik}\, u^k$

$\gamma_{/k}\, u^k = L^r{}_k\, u^k\ \alpha_r$

$d\,\alpha_i /ds = \alpha_{i/k}\, u^k$

$d\,\gamma/ds = \gamma_{/k}\, u^k$

Noting that

$d(\alpha_i\, u^i) = u^i\, d\,\alpha_i /ds + \alpha_i\, d\,u^i/ds$ and

$d(\phi\,\gamma)/ds = \gamma\, d\phi/ds + \phi\, d\,\gamma/ds$ Where ϕ is some scalar function.

$u^i\, d\,\alpha_i /ds = d(\alpha_i\, u^i)/ds - \alpha_i\, du^i /ds$

$\phi\, d\,\gamma/ds = d(\phi\,\gamma)/ds - \gamma\, d\phi/ds$

$d(\alpha_i\, u^i)/ds - \alpha_i\, du^i /ds = u^i\, d\,\alpha_i /ds = \alpha_{i/k}\, u^i\, u^k = \Gamma_i{}^r{}_k\, u^k\, u^i\, \alpha_r + \Sigma_i{}^r{}_k\, u^k\, u^i\, \alpha_r + \gamma\, L_{ik}\, u^k\, u^i$

$L_{ik}\, u^k\, u^i = 0$ since L_{ik} is antisymmetric

$d(\phi\,\gamma)/ds - \gamma\, d\phi/ds = \phi\, d\,\gamma/ds = \phi\,\gamma_{/k}\, u^k = \phi\, L^r{}_k\, u^k\ \alpha_r$

$d(\alpha_i\, u^i)/ds = \alpha_r\, du^r /ds + \Gamma_i{}^r{}_k\, u^k\, u^i\, \alpha_r + \Sigma_i{}^r{}_k\, u^k\, u^i\, \alpha_r$

$d(\phi\,\gamma)/ds = \phi\, L^r{}_k\, u^k\ \alpha_r + \gamma\, d\phi/ds$

Upon adding, we obtain

$dF/ds = d[\alpha_i\, u^i + \phi\,\gamma]/ds = [\, du^r /ds + \Gamma_i{}^r{}_k\, u^k\, u^i + \Sigma_i{}^r{}_k\, u^k\, u^i + \phi\, L^r{}_k\, u^k\,]\alpha_r + \gamma\, d\phi/ds.$

If we assume as a conservation law that the scalar F is to be conserved across any arc length, then dF/ds = 0 implies that the coefficients of α_r and γ are zero. Hence we have the force equation

$du^r/ds + \Gamma_i{}^r{}_k u^k u^i + \Sigma_i{}^r{}_k u^k u^i + \phi L^r{}_k u^k = 0$ or

$du^r/ds + \Gamma_i{}^r{}_k u^k u^i + \Sigma_i{}^r{}_k u^k u^i = - \phi L^r{}_k u^k$ .... (36)

$d\phi/ds = 0$ or ϕ = constant.

From equation (36) we can see one feature which justifies our calling the term L_{ik} as indicative of a type of 'electromagnetic' field strength tensor since it enters the force equation contracted with the four velocity; i.e. $\phi L^r{}_k u^k$ represents a quantity similar to the Lorentz electromagnetic force, while ϕ resembles something like an electric charge, since it is constant over a differential displacement of the arc length.

From our previously modified equation (17) we have $\{_i{}^r{}_k\} = \Gamma_i{}^r{}_k + A_{ik}\,\bar{\alpha}^r$ and from equation (36)

$du^r/ds = - \Gamma_i{}^r{}_k u^k u^i - \Sigma_i{}^r{}_k u^k u^i - \phi L^r{}_k u^k$

$du^r/ds = - [\{_i{}^r{}_k\} - A_{ik}\bar{\alpha}^r\,] u^k u^i - \Sigma_i{}^r{}_k u^k u^i - \phi L^r{}_k u^k$

$du^r/ds = [A_{ik}\,\bar{\alpha}^r - \{_i{}^r{}_k\}] u^k u^i - \Sigma_i{}^r{}_k u^k u^i - \phi L^r{}_k u^k$ .... (37)

The term $- \{_i{}^r{}_k\} u^k u^i$ represents the usual gravitational force.

It is unclear what type of force the term $\Sigma_i{}^r{}_k u^k u^i$ represents, but it doesn't appear to be either gravitational or 'electromagnetic'. We can obtain some insight into this force if we look at either equation (28) or (29) and calculate the covariant divergence in the 'electromagnetic' field,

$\mathcal{S}_i = L_i{}^k{}_{//k} = \Sigma_r{}^k{}_k L^r{}_i - \Sigma_r{}^k{}_i L^r{}_k$ .... (38)

Thus, in this new geometry, this force may be involved in holding the 'electromagnetic' charge density together. Further, since equation (38) is the difference of two vectors, this may imply that $\mathcal{S}_i$ is not a point source.

The next force is $A_{ik}\,\bar{\alpha}^r\, u^i u^k$ which, by the postulated smallness of $\bar{\alpha}^r$, is considered to be very small, although this is a vector force. As we have seen from equation (20), this force is related to the 'electromagnetic' force, in that both A_{ik} and L_{ik} have the same origin when the asymmetric quantity $T_{ik}(\gamma) = - \gamma L_{ik} + \gamma^2 (A_{ik} + L_{ik})$ comes into being. In fact, it may be possible to assume that equations (1) and (2) represent a Reimannian geometry which is at a higher 'energy' level due to the symmetry of A_{ik} in equation (2); when this symmetry is broken, equation (4) comes into being causing 'electromagnetism' and the other forces to appear. Since there are more forces and seemingly more degrees of freedom or complexity available, a sort of geometric 'entropy' seems at work. Could the Big Bang have been caused by the changing of A_{ik} to B_{ik}? Could this be the geometric equivalent of 'symmetry breaking' in particle theory?

7 The Coefficient of Connection as an Ancillary Issue

From equation (37), we have

$du^r/ds = [A_{ik}\,\overline{\alpha}^r - \{{}_i{}^r{}_k\}]\,u^k u^i - \Sigma_i{}^r{}_k u^k u^i - \phi L^r{}_k u^k$

We have

$\Gamma_i{}^r{}_k = \{{}_i{}^r{}_k\} - A_{ik}\,\overline{\alpha}^r$ or

$-\Gamma_i{}^r{}_k = A_{ik}\,\overline{\alpha}^r - \{{}_i{}^r{}_k\}$

Even though $\overline{\alpha}^r$ is postulated to be minute, it may be possible to take advantage of the fact that $\overline{\alpha}^r$ is a vector and that vectors have the additive property. Therefore, over vast expanses of space, we might be able to observe the summative effects of this minute quantity. If we further consider the possibility of "umbilical", "navel" or "dimple" 'points' - possibly galaxies with their central 'black hole' causing tremendous deformations in space-time - then at such 'points'

$A_{ik} = \kappa g_{ik}$ (κ being a curvature constant and having dimensions of an inverse distance and with a value dependent upon the galaxy). Thus our equation becomes

$-\Gamma_i{}^j{}_k = \kappa g_{ik}\,\overline{\alpha}^j - \{{}_i{}^j{}_k\} = \kappa g_{ik}\,\overline{\alpha}^j - \tfrac{1}{2} g^{rj}(g_{ir/k} - g_{ik/r} + g_{rk/i})$.

Using a weak field and non-relativistic velocity approximation, we know that

$-\Gamma_o{}^j{}_o = \kappa g_{oo}\,\overline{\alpha}^j - \{{}_o{}^j{}_o\}$ where $g_{oo} = g_{oo\,(\mathrm{Lorentz})} + \epsilon\gamma_{oo}$

where ϵ = small constant. We assume that g_{ik} has a signature of (-1-1-1+1) in a Lorentzian space.

$\{{}_o{}^j{}_o\} \approx \tfrac{1}{2}\epsilon\gamma_{oo/j}$

$-\Gamma_o{}^j{}_o = \kappa(1 + \epsilon\gamma_{oo})\,\overline{\alpha}^j - \tfrac{1}{2}\epsilon\gamma_{oo/j}$ and looking only at $j = 1$ corresponding to the r coordinate in, e.g. a spherical coordinate system.

We know that in the limit of weak gravitational fields and small velocities

$\epsilon\gamma_{oo} = 2\phi/c^2$ where $\phi = -G_o \dfrac{M}{r}$

$$\epsilon\gamma_{oo/1} = \frac{2G_o}{c^2}\frac{M}{r^2}$$

$$-c^2\Gamma_o{}^1{}_o = c^2\kappa\overline{\alpha}^1 - 2\kappa\overline{\alpha}^1 G_o\frac{M}{r} - G_o\frac{M}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (39)$$

There are aspects of this equation which are not gravitational in origin, but which have to do with the **coefficient of connection** in this new geometry. **Equation (39) shows why it was essential not to lose the distinction between the coefficient of connection and the Christoffel symbol in this new geometry.**

The $\overline{\alpha}^r$ components are postulated to be very small, and necessarily of such a magnitude as to not be of any consequence (this is not the same as saying that this effect cannot be detected) on a planetary or solar system scale in order to be consistent with the experimental observation of the validity of Newtonian gravitation on such scales. However, as stated before, we may be able to see these effects on a galactic scale due to the additive property of vectors.

Theoretically, by solving equations (24) and (25), $\overline{\alpha}^r$ is determined from the distribution of gravity, L_{ik}, and $\Sigma_i{}^r{}_k$ and therefore must also be dependent upon the galactic distribution of matter and energy. When $\overline{\alpha}^r \neq 0$, we see a 'modification' to the Newtonian (r^{-2}) forces, as another hybrid (r^{-1}) acceleration (partly of gravitational origin and partly of non-gravitational origin) comes into play. As both of the (r^{-1}) and the (r^{-2}) forces die off with increased distance, there remains a residual acceleration of $c^2\,\kappa\,\overline{\alpha}^1$ at work.

If $\kappa\,\overline{\alpha}^1 > 0$, then this residual force is repulsive, and the (r^{-1}) force is attractive. This residual repulsive force may cause a volume of galaxies to repel each other, thus causing the volume to expand or inflate. The force $A_{ik}\,\overline{\alpha}^r$ which is generated by this new geometry is quite remarkeable. On the local scale level, (solar system and planetary) it shows up as an extremely weak force (geometrical equivalent of the weak force?), while on the postulated galactic scale, it breaks into two other forces, which may be the geometrical equivalents of 'dark matter' and 'dark energy'.

8 Conclusion

Although, this paper started out asking whether or not 'intrinsic spin' is unique to Quantum Mechanics per se, the conclusion has been reached that the concept of 'intrinsic spin' is not unique to Quantum Mechanics, as no concepts of Quantum Mechanics were introduced into the postulates of this paper. Thus, we can see that a change from a Symmetric Coefficient of the Second Fundamental Form to one of asymmetry seems to take us from a bland and sterile Reimannian geometry of possibly higher 'energy', from the point of view of physics, to a Reimannian geometry of a lower 'energy' which leads to or admits other geometrical structures which may, in a cursory fashion, be loosely identified with 'electromagnetic' structure, 'intrinsic spin' structure, as well as others. This theory, as it stands, is a theory of structure and not one of computation, as is Quantum Mechanics. In Quantum Mechanics, we know that the Coefficient of the First Fundamental Form - the space-time metric- can be represented by bilinear combinations of the 4 x 4 Dirac gamma matrices. A valid question to ask is what sort of representation would the coefficient of the Second Fundamental Form (symmetric or asymmetric) have in Quantum Mechanics? This answer may lie in electro-weak theory. We also know that the equation representing 'intrinsic spin', equation (35), can be represented by bilinear combinations of these same 4 x 4 gamma matrices, together with probability amplitudes. This seems to indicate that Quantum Mechanics is lurking on the periphery of this new geometry but not yet a part of it.

The new type of Reimannian geometry created by the postulates in this paper, may possibly be verified by astronomical observations. The coefficient of connection associated with this new geometry may present strange observational results, such as a repulsive acceleration that may cause a volume of galaxies to expand, in addition to an anomalous 'modification' of Newtonian mechanics from accelerations varying like r^{-2} to include accelerations varying like r^{-1}. A further prediction is that the additive properties of the geometrical equivalent of the 'weak' force may break down on a galactic scale into the geometrical equivalents of the 'dark energy' force and the 'dark matter' force.

The new Reimannian geometry of this paper, based solely on tensors, does not introduce spinors of any kind, yet an 'intrinsic spin' is derived! Equation (24) in this new geometry has a rough similitude with equation (2) of reference [7]:
$\gamma_{i/k} - \Gamma_{i\,k}^{\;\;\mu}\,\gamma_\mu - \Gamma_k\,\gamma_i + \gamma_i\,\Gamma_k = 0$ (see also equation (3.4) of reference [1])

. In this equation, γ_i is a generalized Dirac 4x4 matrix satisfying $\gamma_i \gamma_k + \gamma_k \gamma_i = 2g_{ik}\,1$, where g_{ik} is the metric tensor at that point and 1 is the unit matrix, $\Gamma_{i\ k}^{\ \mu}$ is the usual Christoffel symbol and the Γ_k are 4x4 matrices. The main thrust is to derive a generalization of the tensor covariant derivative to include spin. In the words of this paper, *"The $\Gamma_{i\ k}^{\ m}$ and Γ_k together permit one to define the covariant derivative of any object of which the transformation properties for general coordinate and similarity transformations are known."*
Equation (24) is

$$\alpha_{i/k} = \Gamma_{i\ k}^{\ r}\,\alpha_r + \Sigma_{i\ k}^{\ r}\,\alpha_r + \gamma\, L_{ik}$$

There is a rough correspondence as follows:

$$\Sigma_{i\ k}^{\ r}\,\alpha_r \sim \Gamma_k \gamma_i - \gamma_i \Gamma_k$$

$$\Gamma_k = g_{\mu\alpha}\,[b_v{}^{\beta}{}_{/k}\; a^{\alpha}{}_{\beta} - \Gamma_{v\ k}^{\ \alpha}]\, s^{\mu v} + a_k\, 1$$

where $s^{\mu v} = \frac{1}{2}(\gamma^\mu \gamma^v - \gamma^v \gamma^\mu)$ and $b_v{}^{\beta}$ and $a^{\alpha}{}_{\beta}$ are Vierbein components and a_k is arbitrary.

Previously, this new geometry has admitted equation (35)

$$S^{ik} = \int [V^{i4k} - V^{k4i}](-g)^{\frac{1}{2}}\, dx^3$$

where $V^{rij} = \Sigma^{rij} + g^{jr}\,\Sigma^{ib}{}_b - g^{ij}\,\Sigma^{rb}{}_b = -V^{irj}$
Comparing equations (3.44) and (3.45) of reference [1] the 'intrinsic spin' is

$$\int [U_M{}^{[\lambda 4]v} - U_M{}^{[v4]\lambda}](-g)^{\frac{1}{2}}\, dx^3 = \int \tfrac{1}{4}\, i\, c\, \hbar\, \Psi^\dagger \gamma^4 (\gamma^\mu \gamma^v - \gamma^v \gamma^\mu)\Psi (-g)^{\frac{1}{2}}\, dx^3$$

$U_M{}^{[\lambda\mu]v}$ is a matter super potential $= \frac{1}{2}\{Z^{\lambda[\mu v]} - Z^{v[\lambda\mu]} + Z^{\mu[v\lambda]}\}$

$$Z^{\lambda[\mu v]} = -\tfrac{1}{4}(\partial\mathfrak{M}/\partial\Psi_{/\lambda})(\gamma^\mu\gamma^v - \gamma^v\gamma^\mu)\Psi + \tfrac{1}{4}\Psi^\dagger(\gamma^\mu\gamma^v - \gamma^v\gamma^\mu)(\partial\mathfrak{M}/\partial\Psi^\dagger{}_{/\lambda})$$

where $\mathfrak{M}$ is the matter Lagrangian and is equal to a generalization of the Lagrangian for the Dirac electron.

The purpose of these two comparisons is to emphasize the similarity in $\Sigma_{i\ k}^{\ r}$, Γ_k, V^{rij}, $U_M{}^{[\lambda\mu]v}$, and $Z^{\lambda[\mu v]}$. Further, there is a striking similarity between the following three equations:
$V^{rij} = \Sigma^{rij} + g^{jr}\,\Sigma^{ib}{}_b - g^{ij}\,\Sigma^{rb}{}_b = -V^{irj}$ admitted in this new geometry.
$U_M{}^{[\lambda\mu]v} = \frac{1}{2}\{Z^{\lambda[\mu v]} - Z^{v[\lambda\mu]} + Z^{\mu[v\lambda]}\}$ equation (3.32) from Reference [1].
$\Gamma^i_{[jk]} = S^i_{jk} - \frac{1}{2}\delta_j^i S^l_{lk} - \frac{1}{2}\delta_k^i S^l_{jl}$ from page 474 of Reference [8] (here the authors are deriving spin by considering the spin flux of matter as related to an

antisymmetric connection and a Vierbein connection).

There is one major difference between the new Reimannian geometry presented in this paper and references [1], [7], and [8]. Section 1 of this paper presented the usual Gauss and Weingarten equations which lead to usual Reimannian geometry and the General Theory of Relativity. Reimannian geometry, as given by equations (1) and (2), has no innate facility for the introduction of spin. References [1], [7], and [8] (all three papers chosen because of the similarity with concepts introduced in this new geometry) introduce spin by introducing Vierbein/spinor transformations. The modified Gauss and Weingarten equations from Section 2 create a geometry in which spin is automatic and, theoretically, there is no need to introduce the Vierbein representation. As one can see, in order to introduce spin in the case of reference [1] and [8], there was a need to introduce the matter Lagrangian. The question is does this new Reimannian geometry introduce a type of structural matter Lagrangian in a hidden format?

It seems to be quite obvious from the previous discussion that $\Sigma_{i\ k}^{\ r}$, α_i, γ, $\mathbf{X}_{/r}$ and $\mathbf{N}$ are related in some esoteric manner to the 4x4 matrices γ^{μ} and the 4x1 matrices Ψ and $\Psi^{\dagger}$. The author is not claiming that this new geometry is the last word.....far from it! What the author is trying to show is that there does exist a spin based Reimannian geometry which naturally relates to ideas developed in the past which have introduced spin unnaturally (at least to this author) into a Reimannian geometry using Vierbein/spinor formalisms.

We started out questioning the uniqueness of 'intrinsic spin' as a Quantum Mechanical concept, but we seem to be opening the door to a whole range of other issues, including: (a) the origin of 'electromagnetic' structure itself, (b) whether the real physical universe is represented in whole or in part by a geometry with an Asymmetric Coefficient of the Second Fundamental Form, (c) whether or not the space-time manifold even has an external imbedding space due to the postulated smallness of $\mathbf{N} \cdot \mathbf{N} = \gamma^2$ (speculatively, if there is no imbedding space, then do the tangent and normal 'vectors' become matrices?), (d) whether or not there could be other effects from this new geometry, in particular the coefficient of connection, to be observed on a galactic scale, and (e) whether the six geometrical equivalent forces generated by this new geometry can be correlated with already existing forces in our real physical universe (see Diagram 1), (f) whether or not the Big Bang was caused by the change of symmetry of

$A_{ik} \rightarrow B_{ik}$, (g) whether the 'weak force' is actually three forces in one, and (h) can this new geometry be modified so as to include the complex Hilbert 'space' (in both its denumerable (separable) and nondenumerable (nonseparable) formats) as being representative of internal constraints/limits on degrees of freedom, thus allowing present day Quantum Mechanics to emerge in a natural way from geometric considerations?

D.R. Brill and J.A. Wheeler (see reference [7]) state in their conclusion

> *"...the mystery of why spinors occur in nature is left as pressing as ever. What is there about the description of geometry of space which is not already adequately covered by ordinary scalars, vectors, and tensors of standard tensor analysis? To this question the mathematics of spinor fields gives a well known answer: spinors allow one to describe rotations at one point in space completely independently of rotations at all other points in space - rotations that have nothing to do with the coordinate transformations that are treated in the usual tensor analysis. Fully to see at work this machinery of independent rotations at each point in space, we do best to consider the spinor field in a general curved space, as in this paper. But the deeper part of such rotations in the description of nature is still mysterious."*

This is a very powerful commentary! First of all, it expresses a generally held feeling by the physics community, not only then but now, of *'Abandon all hope ye who enter here (quantum mechancis) regarding the utility of tensors in regards to any hope of describing intrinsic spin!'* Brill and Wheeler are asking the question "Why does 'intrinsic spin' occur? " It is simply not possible to answer this question by introducing spinors or Vierbeins into the usual Reimannian geometry. These devices are phenomenological and, with all their success in describing, do not explain. This paper at least has made a full faith effort to answer the question of Brill and Wheeler, and any others, by offering the notion of the transition of the Coefficient of the Second Fundamental Form $A_{ik} \rightarrow B_{ik}$ by breaking its symmetry to achieve a state of asymmetry, thereby introducing 'intrinsic spin', in addition to a whole host of other geometric structures.

CHART OF THE SIX FORCES WITHIN THIS NEW GEOMETRY ALONG WITH THEIR POTENTIAL CORRESPONDENCE IN THE REAL PHYSICAL WORLD

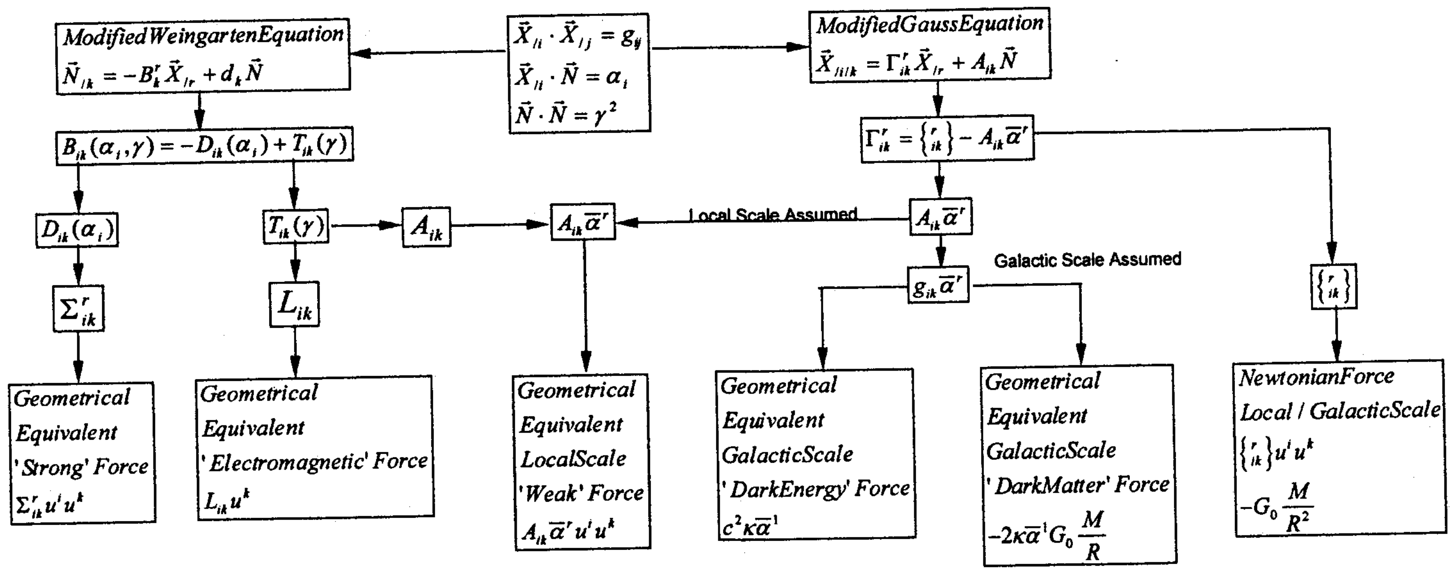

Diagram 1

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