Can GR Lack "Dark Energy" or Abide a "Big Bang"?

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Abstract

In 1922 Alexandre Friedmann obtained, in the context of an unusual spherically-symmetric metric form, formal solutions of the Einstein equation for dust of uniform energy density which as well apply within spherically-symmetric dynamic dust balls of uniform energy density. The resulting Friedmann equation for the dynamical behavior of these ostensibly general-relativistic dust-ball solutions exclusively reflects, however, completely non-relativistic Newtonian gravitational dynamics, with no trace at all of the purely relativistic phenomenon of gravitational time dilation, notwithstanding that gravitational time dilation inescapably accompanies gravitation's presence in GR. That paradox wasn't noticed by Friedmann, nor has it since been consciously addressed. As a consequence, accepted dust-ball behavior is Newtonian gravitational in every respect, notably including compulsory deceleration of dust-ball expansion, as well as compulsory assumption by every expanding dust ball of a singular, zero-radius "Big Bang" configuration at a finite earlier time—despite both behaviors being incompatible with the implications of gravitational time dilation. The source of these inconsistencies is the GR-incompatible nature of Friedmann's unusual metric form, which extinguishes relativistic gravitational and speed time dilation by implicitly utilizing the GR-inaccessible set of clock readings of an infinite number of different observers. However in 1939 Oppenheimer and Snyder carried out a tour-de-force analytic space-time transformation of a Friedmann GR-unphysical dust-ball solution which satisfies a particular initial condition to fully GR-physical "standard" metric form. That Oppenheimer-Snyder transformation was recently extended to arbitrary dust-ball initial conditions, yielding the equation of motion in fully GR-physical "standard" coordinates of any dust ball's radius. This non-Newtonian GR-physical dust-ball radius equation of motion fully conforms to the implications of gravitational time dilation: it in no way forbids acceleration of dust-ball expansion, but it prevents, at any finite "standard" time whatsoever, any dust ball's radius from being smaller than or equal to its Schwarzschild radius-value. Full GR conformity thus needs no "dark energy", but can't support a "Big Bang".

When Einstein was developing his relativistic generalization of Newton's gravity, he as well was reflecting on cosmology. To accommodate the seemingly natural static cosmology of the day, Einstein included in his Theory of General Relativity a nonzero cosmological constant, whose presence isn't necessary to fulfill any theoretical-physics requirement of gravitation, relativity or Einstein's crucial principle of equivalence [1]. When Edwin Hubble published evidence that the universe is apparently expanding rather than static, Einstein abandoned his nonzero cosmological constant with alacrity, characterizing its initial inclusion in his General Relativity theory as his greatest mistake.

The 1998 publication by A. G. Riess, et al. [2] and S. Perlmutter, et al. [3] of evidence that the expansion of the universe is apparently accelerating—instead of decelerating in conformity with bedrock-fundamental Newtonian gravitational principle—triggered the precipitous reinsertion into GR theory of a nonzero cosmological constant, this time ad hoc fitted to produce the appropriate value of an ether-reminiscent "expansive pressure" everywhere in space-time, which is dubbed "dark energy" [4].

The cosmological "dark energy" enigma was in 1998 deposited atop another cosmological riddle of much longer standing, that of "dark matter". It had long been observed that the orbital velocities of stars within galaxies require those galaxies to have vastly more mass than can be accounted for by their observable luminous matter. The seemingly plausible suggestion that the greatly preponderant non-luminous mass might be altogether prosaic stuff such as tenuous widespread atomic hydrogen was, however, "vetoed" by the thermodynamic "primordial nucleosynthesis" abundance ratios which ostensibly stem from the "Big Bang fireball" [5]. An upshot is the idea that the preponderant dark mass must be stable non-baryonic matter. In that case this preponderant stable non-baryonic dark matter is truly extraordinarily exotic, since it has so far successfully defied empirical detection and classification despite its vastly greater total mass in galaxies than that of those galaxies' baryonic matter!

Although the exotic non-baryonic view of galactic "dark matter" is very widely accepted, it nonetheless can hardly be categorized as a view whose plausibility is self-evident. Thus it clearly is only prudent to review the degree to which the ostensible existence of the "Big Bang", which "vetoes" galaxies' dark matter being baryonic, is actually airtight vis-à-vis the fundamental principles of GR. A time-reversed expanding dust-ball model of the universe of course contracts to become progressively denser, thus progressively increasing its associated gravitational time dilation, which must eventually slow its rate of contraction. Indeed, as the

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contracting dust ball's radius approaches its Schwarzschild radius-value, its associated gravitational time dilation approaches infinity. Oppenheimer and Snyder showed that in "standard" coordinates such a dust ball's radius therefore never in any finite time contracts to its Schwarzschild radius-value [6]. It is only in the GR-unphysical non-relativistic Newtonian gravitational limit, wherein gravitational time dilation simply doesn't exist and the dust ball's Schwarzschild radius-value is zero, that the time-reversed expanding dust ball contracts within a finite time to zero radius, which is the time-reversed version of a "Big Bang". In other words, the gravitational time-dilation aspect of GR stymies the culmination of a time-reversed "Big Bang", a concept which is gravitationally consistent only in the non-relativistic Newtonian gravitational limit. Since the achievement of a "Big Bang" is inherently frustrated by the gravitational time dilation which is inseparable from GR, the prevailing idea that a galaxy's dark matter must be non-baryonic is shorn of its underlying "Big Bang" basis by a fundamental property of GR.

Just as the "Big Bang" concept is in fact rooted in non-relativistic Newtonian gravitation, and its GR-physical realization is actually stymied by gravitational time dilation, so the concept that a dust ball's expansion invariably decelerates is as well rooted in non-relativistic Newtonian gravitation, and its GR-physical realization is likewise overruled by gravitational time dilation whenever the dust ball's radius isn't adequately larger than its Schwarzschild radius-value: in such cases the diminishing of the dominant gravitational time dilation effect with the dust ball's expansion causes that expansion to accelerate rather than decelerate. In addition, recent work which extends the Oppenheimer-Snyder description in "standard" coordinates of dust balls that have certain restricted initial conditions to arbitrary dust-ball initial conditions [7] has brought to light an infinite range of dust-ball initial conditions for which the dust balls in question undergo acceleration rather than deceleration of their expansion regardless of how large their radii are. Therefore the ad hoc fitted nonzero "dark energy" cosmological constant fulfills no irreplaceable role whatsoever in actual GR gravitational physics.

The foregoing two paragraphs point out that both the "Big Bang" dust-ball scenario and the scenario of compulsory deceleration of dust-ball expansion are in fact non-relativistic Newtonian gravitational physics scenarios which as such don't and indeed can't take account of the gravitational time dilation that is inescapable in general-relativistic gravitational physics. The historical-mathematical reason that the full general-relativistic physics of dust balls, which by its intrinsic nature always includes the relativistic phenomenon of gravitational time dilation, has since the early 1920's in fact been counterproductively confounded with non-relativistic Newtonian dust-ball gravitational physics, which by its non-relativistic nature is of course wholly incapable of accommodating any such relativistic time dilation effect, comprises a subtle and fascinating cautionary scientific story.

In 1922 the mathematician and engineer Alexandre Friedmann realized that he surprisingly could analytically formally solve the Einstein equation for dynamic spherically-symmetric dust of uniform energy density, provided that he did so by using a very unusual spherically-symmetric metric form, namely [8],

$$ds^2 = c^2 dt^2 - U(r,t)dr^2 - V(r,t)[(d\theta)^2 + (\sin\theta d\phi)^2]. \tag{1}$$

This metric form, which fixes the value of g_{00} to be unity, at first sight appears somewhat analogous to the spherically-symmetric "standard" metric form, which fixes the values of $g_{\theta\theta}$ and $g_{\phi\phi}$ to be r^2 and $r^2(\sin\theta)^2$ respectively. But after careful study it becomes apparent that Friedmann's fixing of g_{00} to unity can only be arranged by accessing the clock readings of an infinite number of different observers [9]. However the need to do that to determine a metric and coordinate system isn't compatible with Einstein's observer-to-coordinate-system paradigm, nor is such access even physically possible. Therefore Friedmann's $g_{00} = 1$ so-called "comoving" metric forms, whose spherically-symmetric version is written down in Eq. (1) above, are in fact all GR-unphysical, and consequently incubate GR-unphysical solutions of their associated GR-unphysical Einstein equations.

Friedmann's $g_{00} = 1$ "comoving" metric constraint in fact inherently suppresses relativistic gravitational and speed time dilation in its associated Einstein-equation solutions. In the particular case of a dust ball of uniform energy density, the upshot of that suppression is GR-unphysical non-relativistic Newtonian gravitational physics which, inter alia, is compatible with a "Big Bang", and as well dictates the compulsory deceleration of such a dust ball's expansion.

From the standpoint of the seeker of analytic solutions of the Einstein equation, the GR-unphysical nature of Friedmann's $g_{00} = 1$ "comoving" metric forms is a wickedly deceptive double-edged sword. On one hand the absence of relativistic time dilation from those solutions may well simplify them sufficiently that they can be analytically worked out, as is indeed the case for spherically-symmetric dynamic dust balls of uniform energy density. On the other hand any nontrivial $g_{00} = 1$ analytic solution of the Einstein equation is with certainty GR-unphysical. Unfortunately that last fact wasn't explicitly pointed out either by Friedmann in the 1920's or by his successors up to the present time.

However, notwithstanding the GR-unphysical nature of any nontrivial Einstein-equation analytic solutions associated with Friedmann's $g_{00} = 1$ "comoving" metric forms, those analytic solutions can nevertheless serve as key formal mathematical building blocks of the associated GR-legitimate physics—insofar as it is possible to transform those solutions from a GR-unphysical Friedmann $g_{00} = 1$ "comoving metric" to a GR-physical metric. Precisely that "transformation salvation" of a GR-unphysical $g_{00} = 1$ "comoving" spherically-symmetric dust-ball Einstein-equation solution with a particular initial condition was achieved in 1939 by Oppenheimer and Snyder [10] by tour-de-force analytic transformation to GR-physical spherically-symmetric "standard" coordinates $(\bar{r}, \bar{t})$ which are defined by the GR-physical metric form,

$$ds^2 = B(\bar{r}, \bar{t})c^2 d\bar{t}^2 - A(\bar{r}, \bar{t})d\bar{r}^2 - \bar{r}^2[(d\theta)^2 + (\sin\theta d\phi)^2]. \tag{2}$$

Recently this Oppenheimer-Snyder analytic transformation from GR-unphysical $g_{00} = 1$ "comoving" spherically-symmetric coordinates to GR-physical "standard" spherically-symmetric coordinates has been extended from Oppenheimer and Snyder's particular dust-ball initial condition to any arbitrary dust-ball initial condition [7], at long last completely freeing dust-ball models from the imprisonment in GR-unphysical non-relativistic Newtonian gravitational physics in which they have heretofore languished ever since Alexandre Friedmann first brought them into the light of day in 1922.

We stated above that Friedmann's $g_{00} = 1$ "comoving" metric constraint inherently extinguishes relativistic gravitational and speed time dilation. That fact is readily verified in the static spherically-symmetric special case, wherein such a $g_{00} = 1$ metric has the form,

$$ds^2 = c^2 dt^2 - U(r)dr^2 - V(r)[(d\theta)^2 + (\sin\theta d\phi)^2]. \tag{3}$$

For any such a static spherically-symmetric metric it is well-known that the general-relativistic gravitational time dilation factor is given by $\left(1/\sqrt{g_{00}(r)}\right)$ [11]. For the above $g_{00} = 1$ Friedmann metric form that gravitational time dilation factor is of course equal to unity for every value of r. Therefore gravitational time dilation is indeed completely extinguished in this static spherically-symmetric special case of the $g_{00} = 1$ Friedmann metric form.

We also stated above that Friedmann's $g_{00} = 1$ "comoving" metric constraint imposes non-relativistic Newtonian gravitational physics on dynamic spherically-symmetric dust balls of uniform energy density $\rho(t)$. We will now write down the dust-ball solution of the Einstein equation for Friedmann's GR-unphysical $g_{00} = 1$ spherically-symmetric "comoving" metric form of Eq. (1), which is,

$$ds^2 = c^2 dt^2 - U(r, t)dr^2 - V(r, t)[(d\theta)^2 + (\sin\theta d\phi)^2],$$

in order to explicitly show that that is the case.

It turns out that the Friedmann $g_{00} = 1$ spherically-symmetric dynamical dust-ball Einstein-equation solution is for the most part described by the dimensionless function $R(t)$, which is defined to be the cube root of the reciprocal of the ratio of the uniform energy density $\rho(t)$ of the dynamical dust ball to its initial value $\rho(t_0)$,

$$R(t) \stackrel{\text{def}}{=} \left(\rho(t_0)/\rho(t)\right)^{\frac{1}{3}}. \tag{4a}$$

The Einstein equation solution gives the Friedmann metric functions $U(r, t)$ and $V(r, t)$ in terms of $R(t)$ and r as follows,

$$U(r, t) = (R(t))^2/[1 + \gamma(\omega r/c)^2] \text{ and } V(r, t) = (R(t))^2 r^2. \tag{4b}$$

The Einstein equation solution furthermore implies that $R(t)$ obeys the Friedmann equation,

$$(\dot{R}(t))^2 = \omega^2[(1/R(t)) + \gamma], \tag{4c}$$

where the initial condition for $R(t)$ is of course $R(t_0) = 1$ from its definition of Eq. (4a) in terms of $\rho(t)$ and $\rho(t_0)$. The constant ω^2 which appears in the Friedmann equation given by Eq. (4c) above and also in the $U(r, t)$ given by Eq. (4b) above is the convenient abbreviation,

$$\omega^2 \stackrel{\text{def}}{=} (8\pi/3)G\rho(t_0)/c^2, \tag{4d}$$

while the dimensionless constant γ that appears both in the Friedmann equation and in $U(r, t)$ is obtained by specializing the Friedmann equation to the initial time $t = t_0$,

$$\gamma = [\dot{R}(t_0)/\omega]^2 - 1 = [\dot{\rho}(t_0)/(3\omega\rho(t_0))]^2 - 1, \tag{4e}$$

where the second equality in Eq. (4e) follows from the relation of $R(t)$ to $\rho(t)$ and $\rho(t_0)$ that is given by Eq. (4a).

The Eq. (4c) Friedmann equation for $R(t)$ follows from the $g_{00} = 1$ "comoving" Einstein equation, which we know is GR-unphysical because fixing g_{00} to unity requires the set of clock readings of an infinite number of different observers [9], which of course isn't physically accessible, nor is the need for such a set to determine a metric and coordinate system compatible with Einstein's observer-to-coordinate-system paradigm. The Friedmann equation is consequently GR-unphysical; indeed it has a completely non-relativistic Newtonian gravitational nature that is made explicit upon defining the radial coordinate $r(t)$ of a Newtonian-analog "test mass" as,

$$r(t) \stackrel{\text{def}}{=} aR(t), \tag{5a}$$

where a is the radius of a dust ball of uniform energy density $\rho(t)$, and simultaneously defining the mass M of a Newtonian-analog "point mass" as the initial $t = t_0$ mass of that dust ball of radius a, namely,

$$M \stackrel{\text{def}}{=} (4\pi/3)\rho(t_0)a^3/c^2 = \omega^2 a^3/(2G). \tag{5b}$$

Noting that,

$$\omega^2 = 2GM/a^3, \quad R(t) = r(t)/a, \quad a = r(t_0), \text{ and } \gamma = [\dot{r}(t_0)/(a\omega)]^2 - 1, \tag{5c}$$

it is readily worked out that the Friedmann equation $(\dot{R}(t))^2 = \omega^2[(1/R(t)) + \gamma]$ becomes in terms of the "test mass" radial coordinate $r(t)$ and the "point mass" M,

$$\tfrac{1}{2}(\dot{r}(t))^2 - [GM/r(t)] = \tfrac{1}{2}(\dot{r}(t_0))^2 - [GM/r(t_0)], \tag{5d}$$

which when multiplied through by the arbitrary mass m of the "test mass" is clearly the first integral of the strictly Newtonian equation of the purely radial motion of the "test mass" in the gravitational field of the "point mass" M.

Thus we see that Friedmann's $g_{00} = 1$ GR-unphysical "comoving" solution of the Einstein equation for dust of uniform energy density is indeed strictly non-relativistic Newtonian gravitational in nature, without the slightest trace of the relativistic gravitational and speed time dilation which are inseparable from legitimate general-relativistic physics. We reiterate that the above-shown transparently GR-unphysical Newtonian gravitational nature of Friedmann's Einstein-equation solution is the consequence of Friedmann's imposition of the GR-unphysical $g_{00} = 1$ "comoving" condition on the metric, which can only be arranged by accessing the set of clock readings of an infinite number of different observers [9], a set that isn't physically accessible, nor is the need for such a set to determine Friedmann's metric and coordinate system compatible with Einstein's observer-to-coordinate-system paradigm.

However, in 1939 Oppenheimer and Snyder accomplished the tour-de-force analytic transformation of such a Friedmann $g_{00} = 1$ GR-unphysical "comoving" strictly non-relativistic Newtonian gravitational solution of the Einstein equation for a dust ball of radius a which has the particular initial condition $\dot{\rho}(t_0) = 0$ (i.e., $\gamma = -1$) to GR-physical spherically-symmetric "standard" coordinates $(\bar{r}, \bar{t})$ which are defined by the GR-physical metric form given by Eq. (2) [10]. Oppenheimer and Snyder's analytic transformation to GR-physical "standard" coordinates of Friedmann's $g_{00} = 1$ GR-unphysical "comoving" non-relativistic Newtonian gravitational dust-ball solution is a GR-physical solution of the Einstein equation in "standard" coordinates which definitely manifests general-relativistic gravitational time dilation. Indeed at any finite "standard" time that gravitational time dilation in fact prevents the "test mass" from getting as near to the "point mass" M as the Schwarzschild radius-value $2GM/c^2$ in "standard" coordinates of that "point mass" [6].

This analytic Oppenheimer-Snyder transformation to GR-physical "standard" coordinates of Friedmann's GR-unphysical "comoving" non-relativistic Newtonian gravitational dust-ball solution with the particular initial condition $\dot{\rho}(t_0) = 0$ (i.e., $\gamma = -1$) has recently been extended to any dust-ball initial condition whatsoever (i.e., to all $\gamma \geq -1$ and $\omega^2 > 0$), which extension enables the equation of motion in GR-physical "standard" coordinates of any dust ball's radius to be written down and studied [7]. That decidedly non-Newtonian equation of motion for any dust ball's radius in GR-physical "standard" coordinates confirms a fact concerning dust-ball acceleration versus deceleration of expansion which was stated in the fifth paragraph above, namely that general-relativistic gravitational time dilation ensures the existence of a range of dust-ball radius values that are all larger than $r_S = 2GM/c^2$, the Schwarzschild radius-value in "standard" coordinates which corresponds to the dust ball's mass M, for which the dust-ball radius undergoes acceleration instead of Newtonian deceleration of expansion. Furthermore, if the initial-condition parameter γ happens to be greater than or equal to half of the ratio of the the dust ball's initial radius value a to its Schwarzschild radius-value

r_S, then the dust ball's radius in fact undergoes acceleration instead of Newtonian deceleration of expansion regardless of how large that dust-ball radius is [7]. This last fact renders the ad hoc fitted nonzero "dark energy" cosmological constant completely unnecessary for accommodating the observed acceleration of the expansion of the universe.

As a result of its proper compliance with general-relativistic gravitational time dilation, the equation of motion in GR-physical "standard" coordinates of any dust ball's radius prevents that radius from, at any finite "standard" time whatsoever, being as small or smaller than the dust ball's Schwarzschild radius-value r_S [7]. Although putative occurrence of a zero-radius "Big Bang" dust ball configuration at any finite time is therefore the pure artifact of the completely GR-unphysical $g_{00} = 1$ Friedmann "comoving" non-relativistic Newtonian gravitational dust-ball solution which was discussed in detail above, the formation of stars nevertheless ought to increase in frequency with a time-reversed expanding dust-ball model universe's contraction, albeit only up to the point where gravitational time dilation overwhelms the density-related encouragement of star formation. Thus a star-formation-rate peak at some finite remote past time is a plausible supposition, and the effects of such a peak could conceivably have given rise to phenomena such as the observed cosmic microwave background. Stars are of course also thermodynamic-equilibrium generators of nucleosynthesis, and a plethora of extremely hot supergiants during peak star formation could conceivably have to a certain degree mimicked the production of supposedly "primordial" nuclear abundances.

Attempting in that spirit to properly sort out the myriad details of cosmology's incipient long-overdue reconciliation with general-relativistic gravitational time dilation will undoubtedly usher in a scientifically more productive period for cosmology than any previous chapter in its history.

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