The Holochronous Universe: time to shed some light on the Dark Sector?

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Abstract. We describe a novel interpretation of the time dimension in General Relativity: the Holochronous Principle, and show that the application of this principle in standard cosmological situations is able fully to account for the effects currently attributed to Dark Matter in observational phenomena such as galactic rotation curves and gravitational lensing. We re-evaluate the role of the Friedman equations in defining a time varying spacetime metric, and in their place postulate a model that is based on the 'shrinkage' of baryons in a gravitational field to account for the dynamical behaviour of the cosmic scale factor. We show that integrating the Holochronous Principle into this model gives rise to a solution that takes the form of a resonant universe, in which the resultant damped oscillations can account for the observed accelerating expansion rate of the universe, to a greater level of precision than the standard ΛCDM model. The Holochronous model obviates the need for Dark Energy in the form of a cosmological constant, Λ, and also resolves other issues associated with the ΛCDM model, including the $\Omega = 1$ flatness problem.

1. Introduction

The so-called ΛCDM 'Concordance Model' of cosmology is currently our best attempt to describe the origin, evolution, and dynamics of the universe. (See [1] for an overview). These is considerable prima facie evidence to support the main constituents of this model, namely 'Dark Energy' in the form of of a cosmological constant(Λ), and cold non-baryonic 'Dark Matter'. However, the model is not without its problems. Indeed, the continuing inability of the scientific community to identify the origins of Dark Energy and Dark Matter constitutes arguably the two biggest unresolved questions in physics today.

In this section we briefly review the evidence for a cosmological constant and for Dark Matter, and outline some of the main issues inherent in the ΛCDM model, including the apparent cosmological coincidences that arise from the model's parameters. The next section discusses the concept of time within General Relativity (GR) and introduces the concept of a Holochronous universe. In Section 3 we apply the Holochronous principle to large scale cosmological phenomena, and in particular, galaxy evolution and galactic rotation curves. Section 4 evaluates how the Holochronous principle acts at the the sub-atomic scale, and shows how this can account for the observed dynamics of the universe. Finally we examine some of the wider implications of the Holochronous Universe, including predictions that are relevant to Black Holes, the Newtonian gravitational constant, and Earth satellite orbits.

1.1. Dark Matter

The history of Dark Matter dates back to 1933, when its existence was inferred by Zwicky from the dynamics of galaxy clusters (see [2] for example). Since then, the evidence for the pervasive presence of dark matter has become overwhelming and includes galactic rotation curves, the structure of galaxy groups and clusters, large-scale cosmic flows, and gravitational lensing. Possible candidates for Dark Matter include WIMPs, axions, MACHOs, and gravitinos. However, in spite of the attempts of numerous research projects to detect these hypothetical particles, to date none have proved successful, and the origins of Dark Matter remain as elusive as ever.

1.2. The Accelerating Universe

The expansion history of the universe can be explored by measuring the relationship between luminosity distance and redshift for a light source with a known intrinsic magnitude. Just such an ideal 'standard candle' has been identified in the form of Type Ia supernovae (SNe Ia). Several studies have now been undertaken by various groups, including the Supernova Cosmology Project [3] and the High-Z Supernova Search Team [4], to measure the distances of a relatively large sample of supernovae at redshifts extending up to $z > 2$. In 1998 these research teams independently identified an apparent acceleration in the cosmic expansion rate, commencing at an epoch corresponding to a redshift of $z \simeq 0.5$. The generally favoured candidate for the origin of this acceleration is a cosmological constant, corresponding to the Λ term in the Einstein Field Equations of General Relativity. In the context of the ΛCDM model, calculations indicate that the best fit with the observed SNe Ia results is obtained with $\Omega_\Lambda \simeq 0.7$ and $\Omega_m \simeq 0.3$, where Ω_Λ, Ω_m are the respective contributions of the cosmological constant and all forms of matter to the critical energy density Ω.

The most obvious explanation for a cosmological constant is thought to be vacuum energy arising from quantum loop corrections at the Planck scale, which is of the order of 10^{19} GeV, where gravity should unify with the other fundamental forces. However, the value of Λ required to account for the observed cosmological acceleration is a factor of 10^{120} smaller than this, a not insignificant discrepancy in need of an explanation!

1.3. Supporting Evidence

In addition to the evidence for Dark Matter and Dark Energy described above, other sources would appear to corroborate the existence of these two phenomena. Arguably the most significant of these are the measurements of the Cosmic Microwave Background (CMB) by the successive COBE, WMAP and Planck satellite missions. Results from analysis of the most recent Planck dataset [5] give a value for $\Omega_m = 0.308$, with a spatial curvature of zero, implying that $\Omega_{tot} = 1$ and hence that $\Omega_\Lambda = 0.69$. These are in close agreement with the values obtained from the SNe Ia measurements, hence justifying the description of the current ΛCDM cosmology as the 'Concordance Model'.

1.4. Fine Tuning Problems

The Concordance Model gives estimates for the constituent components of the critical energy density of $\Omega_{bm} = 0.049$, $\Omega_{dm} = 0.268$, and $\Omega_\Lambda = 0.683$, where Ω_{bm}, Ω_{dm} are respectively the proportions due to baryonic matter and dark matter, and

$$\Omega_{tot} = \Omega_{bm} + \Omega_{dm} + \Omega_\Lambda = 1$$

$$\Omega_M = \Omega_{bm} + \Omega_{dm} \equiv \left(\frac{8\pi G}{3H_0^2}\right)\rho_0$$

$$\Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}$$

The first fine tuning problem is known as the flatness problem: why is the present day density of the universe ρ_0 so close to the critical value required for $\Omega = 1$? Cosmic inflation has been postulated as a solution to this problem [6], but even this mechanism fails adequately to explain how it acts on the three *individual* components that constitute Ω so as to make $\Omega = 1$.

The second fine tuning problem is less well documented [7], but is equally intriguing: why is the age of the universe $\approx 1/H_0$, where H_0 is the present day value of the Hubble parameter? To restate the issue more succinctly, why are the observed proportions of Ω_{bm}, Ω_{dm} and Ω_Λ *precisely* those that give rise to a linearly expanding universe, similar to the so-called empty Milne universe in which the age does indeed $= 1/H_0$? The chances of this arising by coincidence must be extremely small, which suggests the existence of some currently unexplained underlying mechanism. We will refer to this as the 'linearity problem' throughout the rest of this article.

2. The Problem of Time

2.1. Time in General Relativity

The prevailing Standard Model of cosmology is the Friedman-Lemâitre-Robertson-Walker (FLRW) model. This provides the basis for the Hot Big Bang ΛCDM model that has been used to explain many of the observable features in the universe. The main components of the FLRW model are the Robertson Walker metric

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \tag{1}$$

and the Einstein field equation of GR

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{2}$$

Combing these equations for the 00 and 11 components of equation(2) gives the standard Friedman equations that describe the dynamics of the universe

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho_{tot} \tag{3}$$

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi G p \tag{4}$$

These dynamical equations imply that the universe is not static: it must either be expanding or contracting. The absence of a static solution prompted Einstein to introduce the cosmological constant, Λ, into his gravitational field equations. This so-called 'Dark Energy' has the property of negative pressure, which causes an acceleration in the growth of the scale factor.

However, the field equation of General Relativity (2) does not explicitly incorporate time. The 2nd order Ricci tensor $R_{\mu\nu}$ and its derivatives employ 4 covariant indices in the context of GR, but there is no implication or requirement that any of these should necessarily relate to time. Time, and the concept of a dynamical universe, only comes about because of the imposition of a time dependent scale factor in the FLRW metric.

Curved space can be represented as a 3-sphere $\mathcal{S}^3$ embedded in four-dimensional Euclidean space $\mathcal{E}^4$, such that

$$R^2 = x^2 + y^2 + z^2 + w^2$$

and the line element is given by

$$d\sigma^2 = |(dx, dy, dz, dw)|^2 = R^2[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)]$$

where R is the radius of the 3-sphere in spherical space.

Transforming to elliptical space, where $r = \sin\chi$ gives

$$d\sigma^2 = R^2 \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi) \right) \tag{5}$$

This describes a homogeneous, isotropic 3D space of curvature $1/R^2$, where $k = -1, 0$, or $+1$. Note that at this point in the derivation of the FLRW metric, there is no implication or requirement that the curvature R should be a function of time. The crucial step taken by Friedmann, Lemâitre, Robertson and Walker to go from (5) to (1) is to assert that R *should* be a function of time, and introduce the concept of a time varying scale factor defined as

$$a(t) \equiv \frac{R(t)}{R_0}$$

The Friedman equations (3) and (4) follow directly from this assertion. The main objection to this construction is that it imbues the FLRW metric with a spurious time dependency without proposing any physical mechanism or property of spacetime that might give rise to such a time dependent scale factor.

2.2. Holochronous Time

Having suggested in the preceding section that there is no inherent requirement for the metric of GR to incorporate time, we must consider how this fourth dimension should in fact be interpreted. We conjecture that this dimension is best visualised as being an ordering dimension, similar in principle to the concept of a configuration superspace in [8]. This superspace can be thought of as containing multiple foliations of 3-spaces, each representing successive configurations of matter and/or quantum fields. The key feature of this configuration superspace, in contrast to the spacetime of GR, is that it contains the cumulative histories of the metric. To illustrate this concept using a visual metaphor, consider the bending of spacetime due to a matter field. With the notion of a time dimension in standard GR spacetime, this curvature will evolve over time as the configuration of the matter field changes, and all traces of its past configuration will be erased (ignoring for now the concept of gravitational waves). This is illustrated by the successive snapshots in Figure 1.

Contrast this picture with the one illustrated in Figure 2, where the time dimension is replaced by the ordering dimension in configuration superspace, w. Now we see that the curvature of the metric induced by the evolving matter field leaves a permanent imprint on the 3-space slice. The cumulative effect is that of an aggregaation of successive spacetime distortions, *all* of which will contribute to the gravitational mass associated with the matter field that originally gave rise to this deformation.

If we now project the 4D configuration space onto conventional 3D space, we can see that, in the case of a matter field that is shrinking in size, the historic effects of the field on the fabric of space extend well beyond the present day confines of the field, as illustrated in Figure 3. We introduce here the term Holochronous (from the Greek

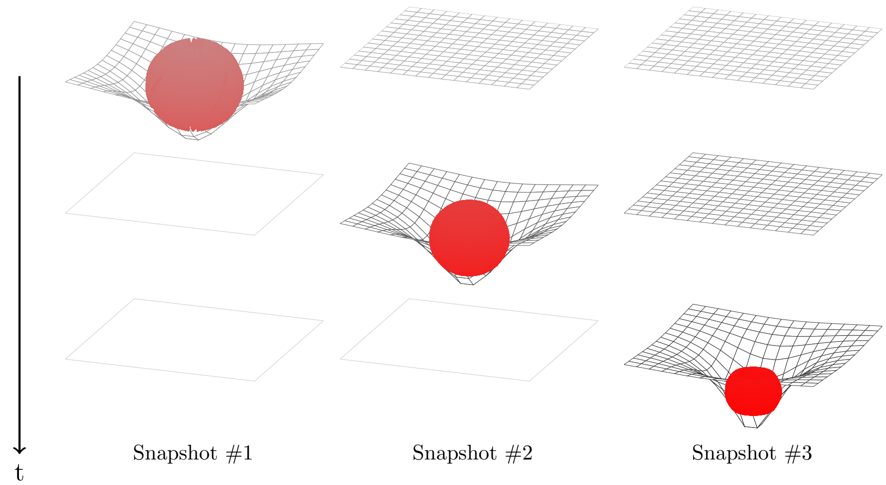

Figure 1. Standard Spacetime

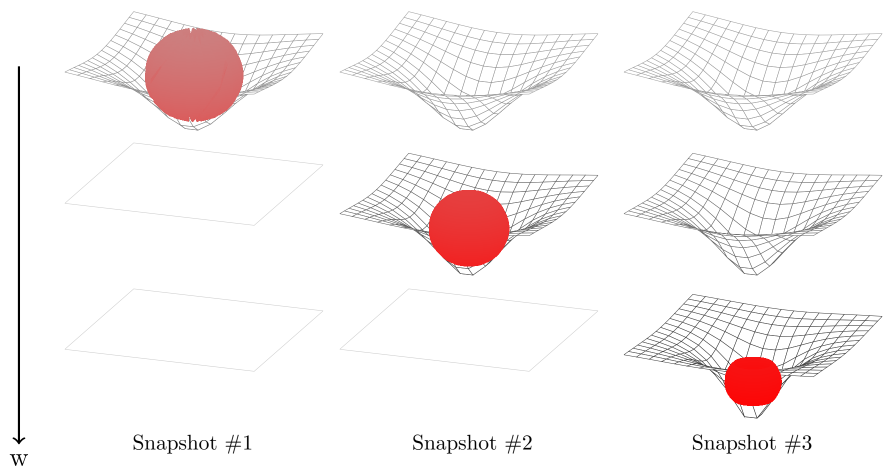

Figure 2. Holochronous Superspace

holo, meaning entire or whole, and *chronos*, meaning time) to describe this principle, whereby the 4th dimension represents the whole history of a system as opposed to a single instant on a continuous arrow of time.

The implication of this reinterpretation of spacetime is that the gravitational potential ϕ arising from a body of matter (or indeed energy) is dependent on the sum of the matter density ρ over each time slice w

$$\phi(\mathbf{x}, w) \propto \sum_{w=1}^{n} \rho(\mathbf{x}) \tag{6}$$

There is no reason why the Holochronous Principle should not apply irrespective of the scale of the matter field or fields involved. In Section 3 we examine the application of this principle on the largest scales: the evolution of galaxies. In Section 4 we apply it to the smallest scales: the behaviour of sub-atomic particles.

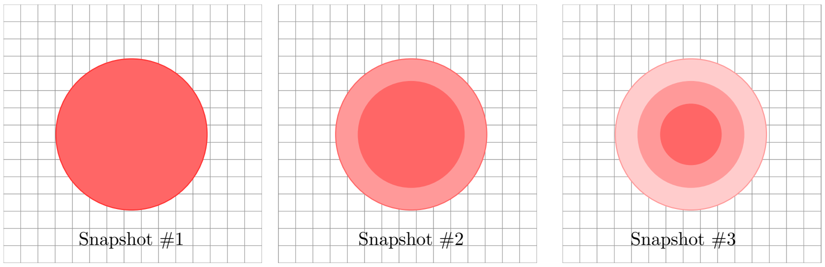

Figure 3. Superspace Projection onto 3D Space

3. Galactic Rotation Curves

3.1. Analysis

As mentioned in Section 1.1, the effects of Dark Matter manifest themselves in virtually all large scale observable phenomena in the universe. In this section we will concentrate on an analysis of galactic rotation curves, and how these could be accounted for by application of the Holochronous principle described in Section 2.2. There are two reasons for selecting this phenomenon rather than, for example, galaxy lensing. Firstly, the physics of galaxy formation is relatively straightforward to model, at least in a simplistic qualitative way, and secondly, there is an abundance of readily available rotation curve data that can be used to validate any model.

There exist many references in published literature to the issue of anomalous galactic rotation curves, and the implications for the existence of Dark Matter. (See [9] for example, for a pedagogical overview). The issue can be summarised reasonably succinctly: the observed velocities of stars orbiting in galaxies, instead of falling off in proportion to $1/\sqrt{R}$ (where R is the radial distance of the star from the galactic centre), as might be expected with a conventional Keplerian model in Newtonian gravity, appear to remain virtually constant extending out to distances several times greater than the radius of the galaxy's luminous core. This phenomenon appears to be ubiquitous to all observable galaxies. Although various theories have been put forward to explain these anomalous rotation curves, including MOND [10] and other modified gravity theories such as [11], the currently favoured explanation is that of non-baryonic Dark Matter. Not only does solution provide a good fit with observation, but it is also consistent with the behaviour of other large scale cosmological phenomena, such as galaxy clustering.

We will now look in more detail at the application of the Holochronous principle to the process of galaxy formation in order to evaluate how this might impact on galactic rotation curves. Consider a spherical hydrogen cloud of initial radius R_0, and initial density ρ_0 (see Figure 4). Over time this cloud will progressively collapse under its own gravitational attraction, with an increasing density given by

$$\rho(t) = \rho_0 \left(\frac{R_0}{r(t)} \right)^3 \qquad (7)$$

Recalling from equation (6) that in Holochronous superspace, the gravitational

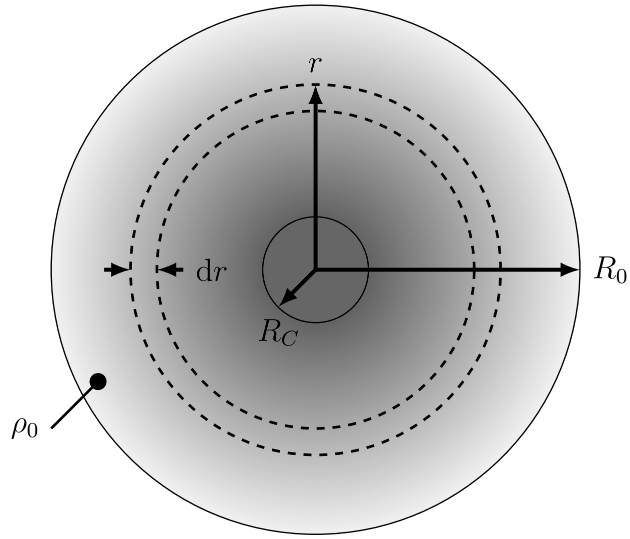

Figure 4. Galaxy collapse

effects of a body of mass have to be summed over all time slices, then for a spherical shell of gas at a radius r from the centre of the proto-galaxy we need to calculate the density integral over time.

$$\rho_{shell}(r) = \int_{t_1}^{t_2} \rho(t,r)\mathrm{d}t \tag{8}$$

For simplicity of analysis, and in order to make the model independent of any specific timescale, we will assume here that the rate of galaxy contraction under gravity is linear, such that $\mathrm{d}r/\mathrm{d}t = k$, where k is a constant velocity. Then, using (7), equation (8) becomes

$$\rho_{shell}(r) = \frac{\rho_0 R_0^3}{k} \int_{R}^{R_0} \frac{1}{r^3}\mathrm{d}r \tag{9}$$

where R_0 is the initial radius of the proto-galaxy gas cloud. Evaluating (9) gives an expression for the shell density-time integral as a function of the radial distance R from the galactic centre

$$\rho(R) = \frac{\rho_0 R_0^3}{2k} \left(\frac{1}{R^2} - \frac{1}{R_0^2} \right) \tag{10}$$

This density as a function of radial distance from the centre of the galaxy is plotted in Figure 5.

Next we integrate again to give the total mass of the galaxy halo that lies within a sphere of radius R, but beyond the radius of the visible core R_C

$$M_{halo} = \int_{R_C}^{R} 4\pi r^2 \rho(r)\mathrm{d}r$$

Using (10), this gives

$$M_{halo} = \frac{2\pi\rho_0 R_0^3}{k} \int_{R_C}^{R} r^2 \left(\frac{1}{r^2} - \frac{1}{R_0^2} \right) \mathrm{d}r$$

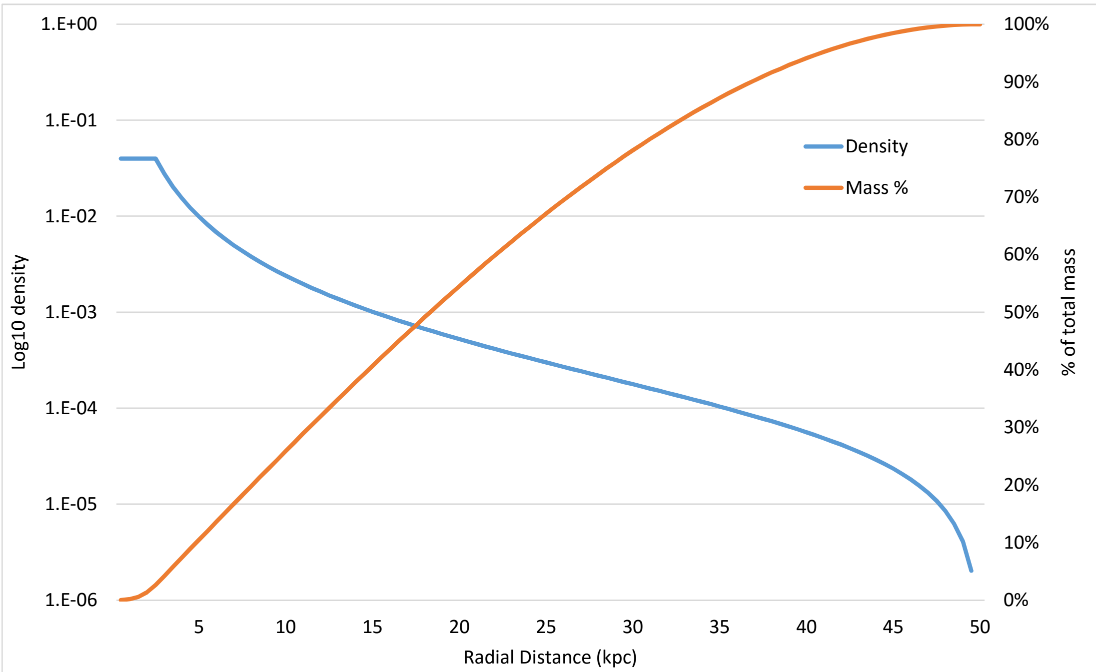

Figure 5. Mass Fraction and Density v. Radial Distance

which evaluates to

$$M_{halo} = \frac{2\pi\rho_0 R_0^3}{k}\left(R\left(1-\frac{R^2}{3R_0^2}\right) - R_C\left(1+\frac{R_C^2}{3R_0^2}\right)\right) \tag{11}$$

We need to add in the core mass to (11) in order to calculate the total mass within a radial distance R

$$M_R = M_{halo} + M_{core}$$

In this simple model, we assume that the density ρ_C within the core radius R_C is constant, so that the mass within the core, using (10) is given by

$$M_{core} = \int_0^{R_C} 4\pi r^2 \rho_C \mathrm{d}r$$

$$M_{core} = \frac{2\pi\rho_0 R_0 R_C}{3k}\left(R_0^2 - R_C^2\right) \tag{12}$$

Adding the core mass from (12) to the halo mass (11) gives us the total mass M_R within a sphere of radius R. Figure 5 plots this as a percentage of the total galactic mass M_{R_0}, as a function of radial distance.

3.2. Results

From the relationship between mass M and radial distance r, we can calculate the gravitational potential Φ and hence the rotational velocity v as a function of r, using

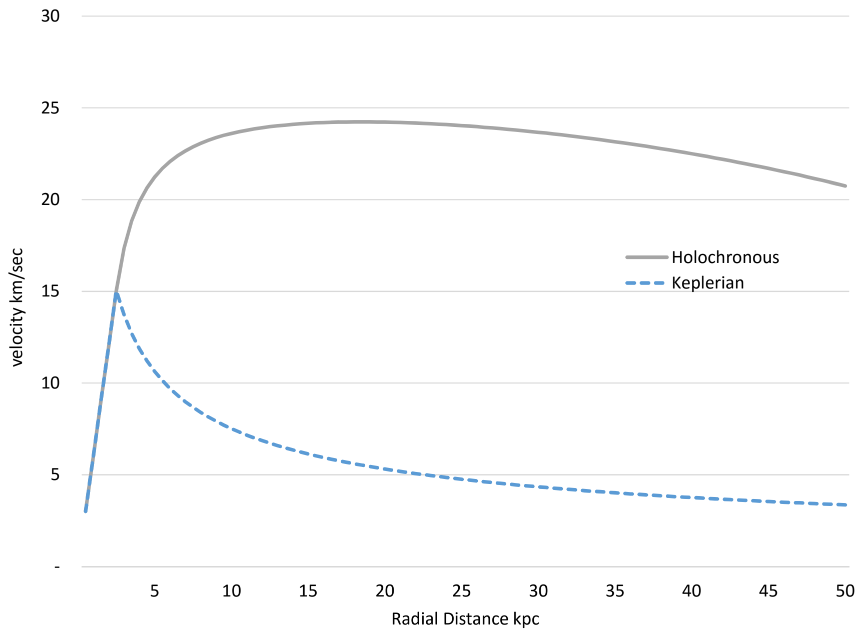

Figure 6. Holochronous Rotation Curve

the standard formula for a Keplerian potential

$$v = \left(\frac{GM}{r}\right)^{1/2}$$

where M here represents the total gravitational mass of the galaxy obtained by the addition of the core mass (12) to the halo mass (11). The resulting rotation curve is plotted in Figure 6, together with the equivalent curve for a galaxy whose gravitational mass is contained entirely within the visible region.

The overall shape of this curve is dependent only on the density-time function. The scale of the curve, in terms of the radial extent and the range of orbital velocities, is determined solely by three parameters: initial radius of the gas cloud (R_0), initial 4-density (ρ_0), and the radius of the luminous galaxy core (R_C). Using these three parameters, we can now fit the Holochronous model rotation curve to the observed galactic rotation curves for a sample of galaxies.

For this analysis we have chosen to use the sample of galaxies in the Ursa Major cluster provided in [12]. Within this sample, we have selected galaxies that meet the criteria of having a minimum of 15 data points spread evenly over the radial distance range, and which also have at least 3 data points within the galactic core region. Figure 7 shows the results of fitting the Holochronous rotation curve model to 6 of these galaxies.

3.3. Discussion

From this curve fitting exercise it is evident that, in spite of the simplifying assumptions made in respect of galaxy contraction rates and core density, the Holochronous model is

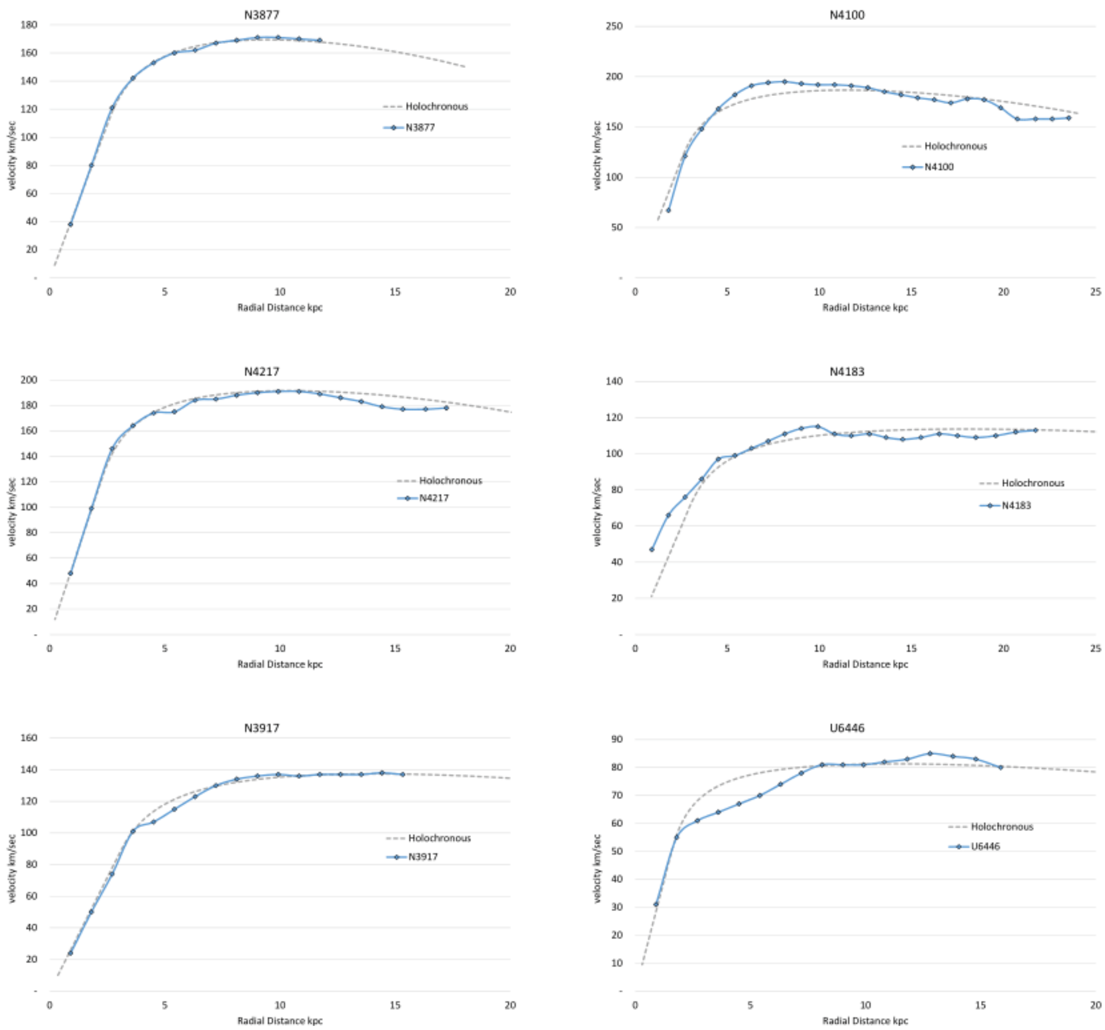

Figure 7. Sample of Galaxy Rotation Curves in Ursa Major Cluster

able to account very precisely and predictably for the observed flattened rotation curves. Essentially what we are seeing is the effects of a galactic halo comprised of what might be more accurately termed 'Ghost Matter' - the legacy of layers of gravitationally warped spacetime laid down over time by the primordial galactic gas cloud as it condenses and contracts.

It should be noted that the foregoing analysis has only concentrated on a small sample of elliptic galaxies that have, presumably, followed the simple collapsing gas cloud model used here. However, it is reasonable to expect that the same concept of a galactic Ghost Matter halo will still be applicable to other more complex galactic evolution models such as, for example, galaxies formed from the merger of two smaller galaxies.

One final observation that may be significant. From the general shape of the Holochronous density profile (Figure 5), and empirically from analysis of the rotation curves in the sample used here, it would appear that the mass in the luminous core of the galaxy represents about 4% of the galaxy's total gravitational mass extending out to the initial radius R_0. The implication being that if this proportion were to be generally applicable to all galaxies, then the Holochronous principle would appear to account for 100% of the gravitational mass of the universe, without the requirement for any additional Dark Matter or Dark Energy to give us a closed universe with $\Omega_{tot} = 1$.

4. The Accelerating Universe

4.1. Baryonic scale evolution

In Section 2.2 it has been postulated that the metric of General Relativity should be time independent. However, there is an abundance of evidence indicating that the universe is expanding in the sense that the cosmic scale factor is increasing with time. The only conclusion that reconciles these two observations is that the other component of the scale factor is changing, i.e. baryonic matter on a sub-atomic scale is shrinking relative to the cosmological reference frame. Let us now take this concept at face value and see how it might work. We need to replace the cosmic dynamics derived from the Friedman equations (3, 4) with some alternative model that is based on the behaviour of sub-atomic particles in a gravitational field gradient. In such a model, the rate of change in the radius of, for example, a proton $\ddot{r_p}$, would be proportional to the difference in the gravitational potential arising from the proton's own mass, $\phi_p \propto m_p/r_p$, and the potential arising from all other matter in the universe, $\phi_U = \sum_0^{R_H} m_p/R$, where r_p is the proton radius, and R_H is the particle horizon. If we now consider a test particle orbiting in close proximity to the proton, the ratio of the gravitational force acting on the particle arising from the proton, to the gravitational force from the rest of the universe will (ignoring geometrical integration factors) be

$$\frac{NR_H^2}{r_p^2} \simeq 1 \tag{13}$$

where N is the baryon number of the universe within a sphere of radius R_H. Equilibrium will be achieved when this ratio is equal to unity. ‡ If we now replace our hypothetical test particle with a particle or field that is itself a constituent of the proton, a quark for example, we can extend the same equilibrium condition to apply to that particle also. In other words the dimensions of the proton will remain unchanged provided that the gravitation forces in (13) remain in equilibrium. (It is important to note that there is no suggestion here that gravitational forces play any part in the internal structure of a baryon, which in the standard model is determined by the forces between quarks and gluons with the domain of quantum chromodynamics).

As it stands, the equilibrium in (13) is inherently unstable. A small change in r_p will result in a runaway expansion or contraction in the cosmic scale factor, which is

‡ It is perhaps worth noting in passing that (13) is essentially a restatement of the Dirac Large Number Hypothesis (LNH) [13].

essentially the ratio R_H/r_p. What is needed in order to stabilise the system is some form of negative feedback, such that any change in r_p results in a counteracting change in ϕ_U. This would be difficult, if not impossible, to achieve with the standard concept of spacetime, since the proton mass would be effectively constant everywhere, and hence unable to make any time varying contribution to ϕ_U. At this point we need to revisit the Holochronous principle outlined in Section 2.2, and recall that the gravitational potential at any point in 3D space is proportional to the sum of the matter (or energy) density at that point over time. This was shown in Section 3 to work on the largest scales in the case of galactic rotation curves. If we apply the same principle to the atomic scale then we have a picture in which every sub-atomic particle in the universe is embedded in a superspace that retains its gravitational history. To complete the model, we need to take into account the fact that distortions in spacetime caused by mass/energy propagate, in the form of gravitational waves, at the speed of light. Thus the gravitational potential due to a particle at a distance R will be proportional to m_Δ/R, where m_Δ is the particle's effective gravitational mass at a time $\Delta = c/R$ in the past. We therefore end up with a situation in which the evolution of each individual proton is dependent on the time delayed effects of the gravitational potential arising from every other proton in the universe.

Essentially, we now have a model for the evolution of a particle matter field that incorporates a positive feedback component due to the particle's self-gravitation, and a negative feedback component due to the gravitational potential arising from the effects of all other matter in the universe. However, the feedback from matter in the rest of the universe is subject to a time delay that is proportional to distance (conformal time). The system described above can be considered to be analogous to an Infinite Impulse Response (IIR) digital filter, whereby the output at any time is determined by the system's output at previous times

$$y[n] = 1 + ay[n-1] - b\sum_{k=2}^{\infty} y[n-k] \tag{14}$$

where a, b are respectively positive and negative feedback coefficients, $y[n]$ is the current value of the output of the system, and $y[n-k]$ is the output of the system k sampling periods prior to the current period. Using this digital filter analogy, we can employ digital signal processing techniques to analyse the system's response in the time domain, and using the Z-transform, in the frequency domain (see Appendix A).

4.2. Simulation Solution

The filter response in equation (14) can be modelled by implementing the Direct Form I structure illustrated in Figure A1, using a simple numerical model such as a spreadsheet. In this model, the output value $y[n]$ represents the gravitational potential differential (13) that gives rise to changes in the particle's size, i.e. $\ddot{a}$. The results of this simulation are shown in Figure 8, in which the time scale and scale factor have been normalised such that $t_0 = 1$ represents the current time, and $a_0 = 1$ is the scale factor today.

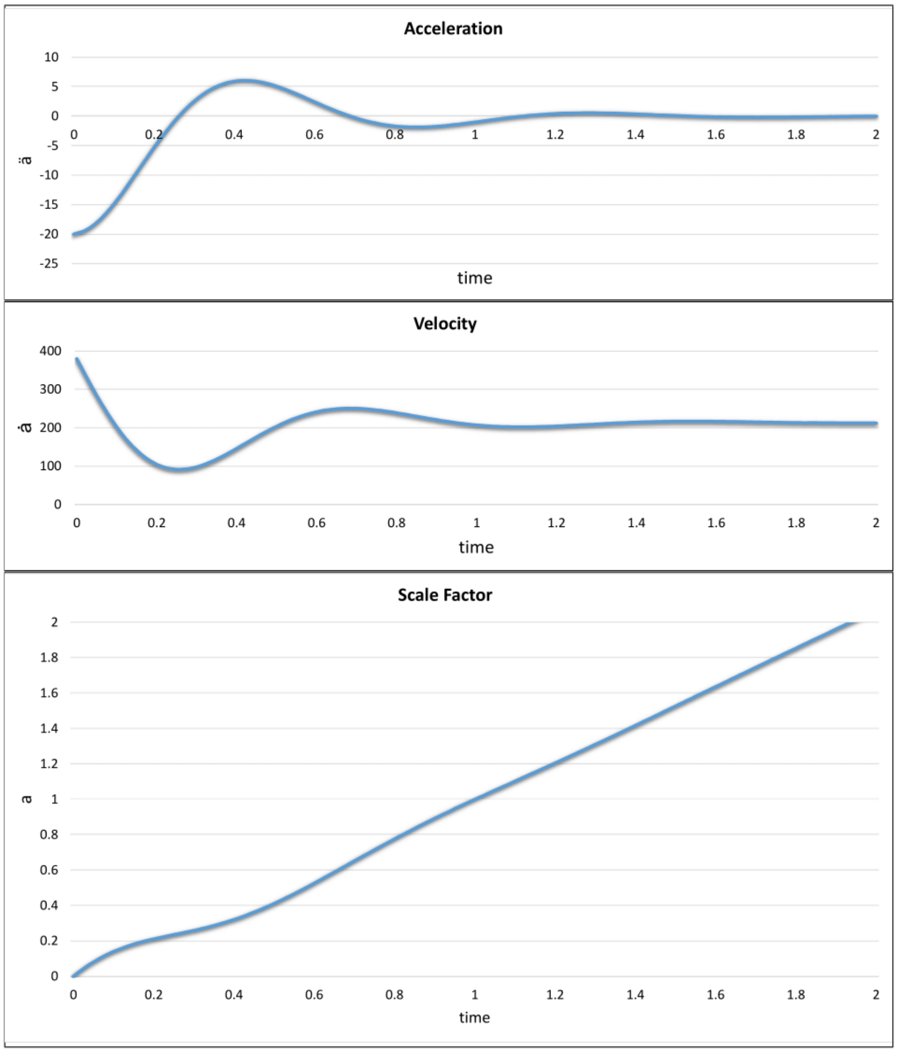

Figure 8. Cosmic Dynamics Simulation

From this output, it can be seen that a universe based on this Holochronous model behaves very much like a IIR filter, exhibiting a well defined resonant frequency with exponentially damped oscillations.

4.3. Analytic Solution

It is possible to derive an analytic solution to the filter equation in (14), at least for the simple case of a second order system, where the output is only dependent on terms up to $y[n-2]$, as in

$$y[n] = b_0 + a_1 y[n-1] - a_2 y[n-2] \tag{15}$$

It can be shown (Appendix A.3) that this corresponds to a system function of

$$H(z) = \frac{b_0}{z^2 + a_1 z + a_2}$$

which has an impulse response of

$$h[n] = 2\alpha r^n \cos(\theta n + \phi) u[n] \tag{16}$$

where ϕ is the initial phase angle and

$$r = \sqrt{-a_2}$$

$$\theta = \cos^{-1}\left(\frac{a_1}{2\sqrt{-a_2}}\right)$$

Figure 4.3 shows the output generated by (16), from which it is apparent that the general form of the impulse response derived by analysis is very similar to the acceleration calculated by numerical simulation, even though the analytic solution only includes second order terms, whereas the numerical solution is of nth order, where n is the number of timesteps used in the stimulation. This is not perhaps surprising when one considers the associative properties of digital filters, whereby one can approximate an nth order filter by chaining a sequence of $n/2$ 2nd order filters.

As with the simulated solution, we can integrate the impulse response (equivalent to $\ddot{a}$) to give the rate of expansion $\dot{a}$, and then integrate again to give the scale factor a, which is then normalised such that the present day value $a_0 = 1$.

In principle, it might be reasonable to expect that we should be able to derive the positive and negative feedback parameters in (15) from first principles, using the fundamental geometry of the Holochronous model, together with selected dimensionless numbers, of which the baryon number of the universe, N, is one of the most obvious candidates. However, such analysis is beyond the scope of this article.

A reasonable question to ask at this point might be: what gives rise to the initial impulse in this Holochronus universe? The short answer is: an initial, almost instantaneous, period of exponential scale factor growth, corresponding to a process of baryogenesis, which might be considered to be analogous to inflation in the standard model. This will leave the universe in a state that is far from equilibrium, with the ratio in (13) much less than unity. It is this imbalance in gravitational potentials that gives the resonant universe its initial 'kick' and sets it oscillating.

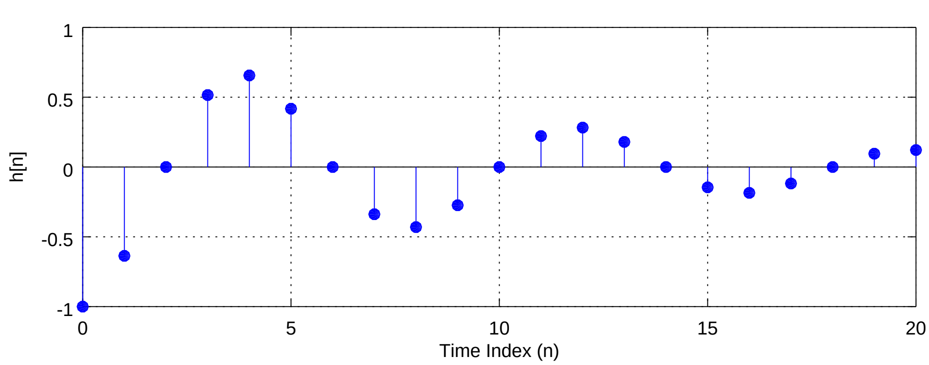

Figure 9. Cosmic Impulse Response

4.4. Comparison with observational results

In order to evaluate the results from Sections 4.2 and 4.3, we must convert the output, in the form of scale factor as a function of proper time, into an expression for luminosity distance as a function of conformal time. Luminosity distance as a function of redshift z is defined (see [14], for example, for derivation) as

$$d_L(z) = \frac{c\tau(1+z)}{H_0} \tag{17}$$

where τ is the conformal lookback time, and H_0 is the present day value of the Hubble parameter.

First we determine the redshift from the scale factor, using the definition of redshift

$$z + 1 = \frac{a_0}{a}$$

Next we must convert to conformal time, defined as

$$\tau \equiv \int_0^{t_0} \frac{\mathrm{d}t}{a(t)}$$

Finally, we need to convert the luminosity distance in (17) into bolometric magnitude in order to compare with observational measurements, using the definition for bolometric magnitude

$$m(z) = M + 5\log_{10} d_L(z) + 25$$

where M is the absolute magnitude.

The Hubble diagram is a plot of the distance modulus $\mu \equiv m - M$ as a function of redshift z. In order better to visualise the differences between various cosmological models, it is common practice to display the distance modulus residuals for each model as a delta against a base case. Figure 10 plots the Hubble diagram for the Holochronous model (simulation and analytic results), together with the outputs from various other models based on ΛCDM cosmology, as a delta against a base case of a flat, empty, ΛCDM universe with $\Omega_\Lambda = 0, \Omega_M = 0, \Omega_K = 1$.

The dataset used for this analysis comprises a total of 172 SNe Ia including:

- 146 Gold SNe Ia from [15] and [16]
- 23 new SNe Ia from Hubble Space Telescope (HST) measurements [17] plus 16 recalibrated SNe Ia from [15]
- two high-z SNe Ia from [18]
- one high-z SNe Ia from [19]

All the models have been optimised against this dataset, by varying Ω_Λ, Ω_M and H_0 in the case of the two ΛCDM based models, α, r, θ and H_0 in the case of the Holochronous analytic model, and b_0, a_1, a_2 and H_0 in the case of the Holochronous simulation model. Table 1 shows the χ^2 results from this exercise, from which it can be seen that both versions of the Holochronous model give appreciably better fits with the observational data than does the prevailing ΛCDM model.

Table 1. χ^2 comparison of data set to models

Model	χ^2 (172 SNe Ia)	H_0 ($\mathrm{kms^{-1}Mpc^{-1}}$)
$\Omega_M = 0.264, \Omega_\Lambda = 0.736$	213	64.1
$\Omega_M = 0, \Omega_\Lambda = 0, \Omega_k = 1$	231	62.2
Holochronous (analytic)	209	63.5
Holochronous (simulation)	208	64.4

Although the full dataset of 172 SNe Ia has been used for calculating the χ^2 fits, the results in Figure 10 are binned for display purposes, with a fixed bin size of $n\Delta z = 5$, where Δz is bin width in redshift and n is number of SNe in bin.

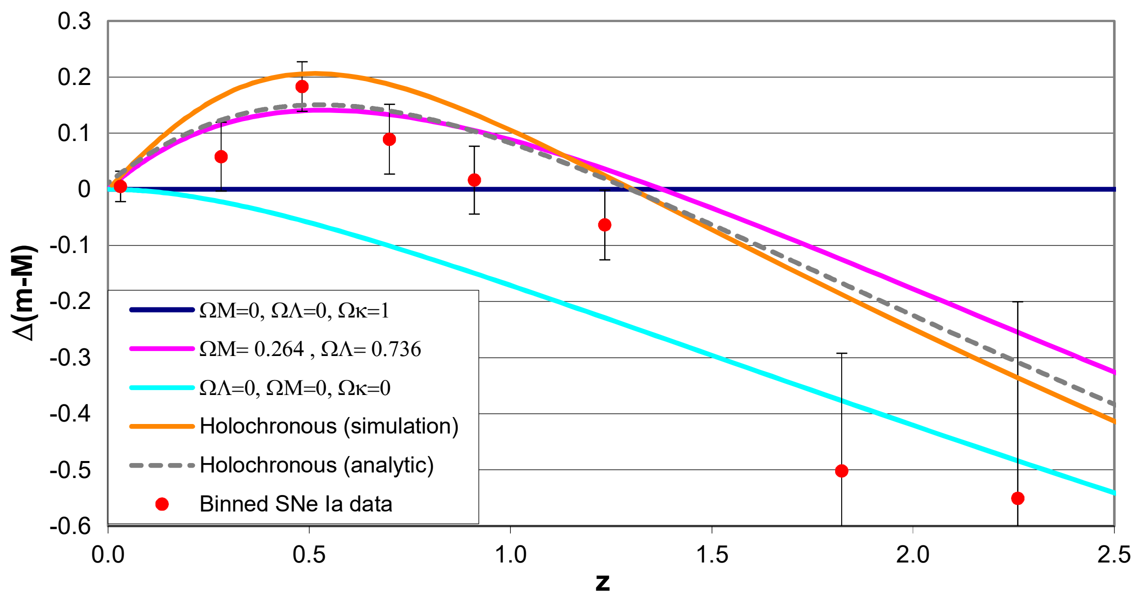

Figure 10. Hubble diagram showing SNe Ia results plotted against various cosmological models

5. Summary and Conclusions

We have introduced here the concept of the Holochronous Universe, in which the time dimension of GR is replaced by an ordering dimension in a form of configuration superspace, and shown that this leads to the concept of a static spacetime metric. This new paradigm has the ability to solve many of the outstanding problems associated with the current ΛCDM model of cosmology.

5.1. Summary

(i) The Holochronous model is able predictably to quantify the gravitational potential arising from the 'Ghost Matter' halo that is created by a galaxy condensing from a primordial gas cloud. The resultant galactic rotation curves provide a close fit with observational data, without the need to invoke Dark Matter. Perhaps significantly, if the ratio of gravitational mass due to luminous matter to that resulting from Ghost Matter (about 4%:96%) was applicable throughout the universe then this would account for the entire gravitational mass required for a flat $\Omega = 1$ universe.

(ii) Replacing cosmic dynamics based on the Friedman equations for a time varying metric, with a model that is based on the 'shrinkage' of matter on a sub-atomic scale within a gravitational field, and combining this with the Holochronous principle, whereby the universe retains a memory of its past gravitational states, results in a universe that exhibits the behaviour of a resonant system. The damped oscillatory response of this model to an initial impulse is able to achieve a better fit with SNe Ia data than the standard ΛCDM cosmology, in terms of its ability to account for the observed acceleration of the cosmic scale factor. The model obviates the need for Dark Energy in the form of a cosmological constant, Λ.

(iii) The Holochronous model is able to solve the cosmological flatness problem in three ways by:

- accounting for all the missing gravitating mass in the universe without the need for a cosmological constant or cold Dark Matter
- making the concept of a critical density, Ω, redundant by doing away with a time varying metric
- providing a mechanism for negative feedback such that the scale of gravitational forces between each matter particle and the rest of the universe is maintained in equilibrium

(iv) The negative feedback made possible by the Holochronous universe ensures that the cosmic scale factor will, over a long enough timescale, always evolve at a constant rate, thus solving the linearity problem associated with the ΛCDM universe. The model predicts that the next cosmic oscillatory cycle after the one that we are currently experiencing will be of a much smaller amplitude, with the amplitude of subsequent cycles being virtually negligible.

5.2. Speculative consequences

5.2.1. Einstein Field Equations We have seen that the Holochronous Universe obviates the need for a cosmological constant, making the Λ component of the Einstein field equation redundant.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} + \cancel{\Lambda g_{\mu\nu}} \tag{18}$$

However, it is perhaps worthwhile to examine the other elements of the left hand side of (18), to see if any other modifications would be merited by the adoption of Holochronous cosmology. In this context, it is interesting to note that the Newtonian gravitational constant, G, does not enter into any of the dynamical calculations used in Sections 3 or 4. The sole reason for its use within (18), apart from the need to convert the units of $T_{\mu\nu}$

from those of energy density to Gaussian curvature, is so that the equation will reproduce that of Newtonian gravity in the weak field limit. As such, G is a philosophically unsatisfactory concept, which arguably should not be present in an idealised theory of gravitation. (In contrast to the other main physical constants c and h, which merely serve to convert from one set of arbitrary units to another, and can be set equal to unity without affecting the underlying physics). Noting also that the methodology for calculating cumulative gravitation potentials used in Section 3 employs the concept of a 4-density, which appears very similar in principle to a Lagrangian density $\mathcal{L}(\mathbf{x})$. Integrating this density over 4 dimensions will in effect give us a scalar with the units of action

$$S = \int \mathrm{d}^4\mathbf{x}\mathcal{L}(\mathbf{x})$$

Combining these two observations suggest the conjecture that both G and $T_{\mu\nu}$ should be replaced in (18) by some construction based on the ratio of the action arising from a body of mass/energy $\mathcal{S}_M$, to the action resulting from all mass/energy within the universe $\mathcal{S}_U$, such that

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= \cancel{\frac{8\pi G}{c^4}T_{\mu\nu}} + \cancel{\Lambda g_{\mu\nu}} \\ \Rightarrow G_{\mu\nu} &= \frac{1}{R^2}\left(\frac{\mathcal{S}_M}{\mathcal{S}_U}\right)g_{\mu\nu} \end{aligned} \tag{19}$$

5.2.2. Newtonian Gravitational Constant Although the argument is made above that there is no place for the Newtonian gravitational constant G in a idealised formulation of GR, nevertheless the concept of such a 'constant' is perhaps useful in situations where Newtonian gravity is applicable. However, in the The Holochronous universe, G as measured by the usual experimental methods would not in fact be constant, either temporally or spatially. Specifically, $G(t)$ will be proportional to $\mathcal{S}_U^{-1}$ from equation (19), which in turn will vary over time, with damped sinusoidal oscillations around its long term mean value, determined by the dynamics of equation (16). Furthermore, $G(\mathbf{x})$ will tend towards zero when measured in proximity to massive bodies, where $\mathcal{S}_M/\mathcal{S}_U \to 1$.

5.2.3. Planetary Ghost Matter halos Sections 3 and 4 examined the implications of extended gravitational fields on the largest (galactic) and smallest (sub-atomic) scales respectively. The Holochronous principle should also be applicable to objects that lie between these two scales, which would include stars and planets. This raises the intriguing possibility that the Earth could posses a Ghost Matter halo, with potentially observable consequences that might be testable from an analysis of the behaviour of Low Earth Orbit (LEO) satellites. On the scale of the solar system, there might also be observable anomalous gravitational effects from a Ghost Matter halo created during the epoch of planetary formation.

5.2.4. Black Holes Equation (19) has implications for massive cosmological objects such as Black Holes, where $\mathcal{S}_M/\mathcal{S}_U = 1$. Black Holes would still exist in the Holochronous Universe, but there would be no central singularity. All the mass and entropy associated with a Black Hole would be located at the event horizon, which would constitute an impenetrable barrier to in-falling matter and light, as opposed to being merely a boundary affecting outgoing photons in conventional GR.

5.2.5. Linkage to Baryogenesis In Section 4.3 mention was made of the possibility that the positive and negative feedback parameters associated with the resonant model of the Holochronous Universe might in principle be derived from basic cosmological quantities, such as baryon number density. Taking this conjecture one step further, it is possible to envisage that the scale of the initial impulse that sets the resonant universe into oscillation might also be directly derivable from the output of the period of exponential scale factor growth associated with the era of baryogenesis (and equivalent to the inflationary epoch in standard cosmology).

5.2.6. High Frequency Modes The long time sampling periods ($\simeq 10^7$ years) used in the simulations of cosmic dynamics in Sections 4.2 and 4.3 may well have suppressed higher frequency oscillatory modes that have periods shorter than this sampling period. A more detailed modelling and analysis exercise than the one carried out here could reveal the existence of such modes. These would manifest themselves as spatially periodic matter density fluctuations in the universe, which could in principle be observed from a Fourier analysis of cosmological matter distributions, obtained from detailed galaxy count surveys.

5.3. Conclusion

The Holochronous principle provides a unified model that would appear to account for the observed dynamical behaviour of the universe in an economical and elegant manner, without the need to invoke as yet unexplained phenomena such as Dark Energy and Dark Matter. As such, the Holochronous Universe model does perhaps offer us the opportunity to emerge from the cosmological 'Dark Ages' into a new age of enlightenment.

Appendix A. Digital Filter Analysis

Appendix A.1. Digital IIR filter

An Infinite Impulse Response (IIR) filter, with its output $y[n]$ at time period n dependent on its output at all preceding sampling periods, can be described with the general form

$$y[n] = b_0 x[n] + a_1 y[n-1] - a_k \sum_{k=2}^{\infty} y[n-k]$$

where a_1 and a_k are, respectively, positive and negative feedback coefficients.

This can be modelled directly using numerical simulation techniques that implement the Direct-Form I structure illustrated in Figure A1.

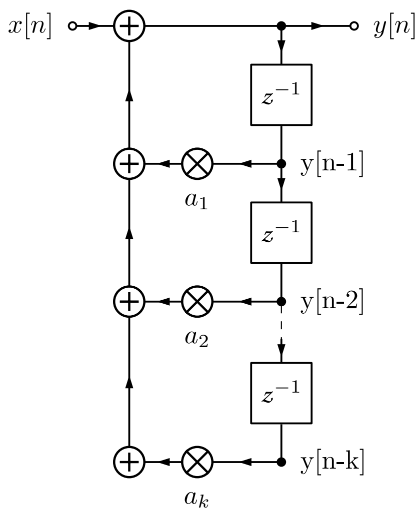

Figure A1. IIR Filter Direct-Form I structure

Appendix A.2. The z-Transform

Given a discrete-time signal x[n], we can determine its frequency response using the Discrete Fourier Transform (DFT)

$$X[k] \triangleq \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad k = 0, 1, 2, \ldots, N-1.$$

- The DFT can be interpreted as the sum of projections of x[n] onto a set of N sampled complex sinusoids or sinusoidal basis functions at (normalized) radian frequencies given by $\omega_k = 2\pi k/N$ with $k = 0, 1, 2, \ldots, N-1$.
- The frequency response of a digital filter can be found by taking the DFT of the filter impulse response.

- The unilateral z-Transform of a discrete-time signal x[n] is given by:

$$X(z) \triangleq \sum_{n=0}^{+\infty} x[n] z^{-n},$$

where z is a complex variable.
- The z-transform maps a discrete-time signal to a function of the complex variable z.
- A convenient property of the z-transform is given by the Shift Theorem,

$$x[n-\Delta] \leftrightarrow z^{-\Delta} X(z),$$

which says that a delay of Δ samples in the time domain corresponds to a multiplication by $z^{-\Delta}$ in the z domain.
- From the shift theorem, we can easily calculate the z-transform of a digital filter's difference equation. Given the following second-order difference equation, $y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] - a_1 y[n-1] - a_2 y[n-2]$,
- the z-transform can immediately be written (assuming the system is linear) $Y(z) = b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) - a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z)$.
- From this expression, we can determine the transfer function, H(z) = Y(z) / X(z), of the filter:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$H(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2}$$

- It is convenient to evaluate the z-transform of a system in the complex z-plane, as shown below:

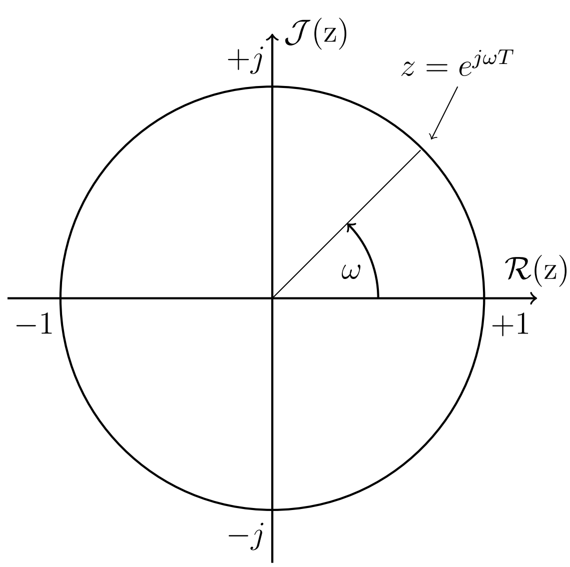

Figure A2. The complex z-plane

Appendix A.3. Impulse Response

Consider a system with a transfer function defined as

$$H(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2}$$

This has has a solution with poles at

$$p_{1,2} = \frac{a_1}{2} \pm j\frac{1}{2}\sqrt{-a_1^2 - 4a_2}$$

$$= re^{j\theta}$$

where

$$r = \sqrt{-a_2}$$

$$\theta = \cos^{-1}\left(\frac{a_1}{2\sqrt{-a_2}}\right)$$

The transfer function can therefore be rewritten in the form

$$H(z) = \frac{A_1}{1 - p_1 z^{-1}} + \frac{A_2}{1 - p_2 z^{-1}}$$

where

$$A_k = H(z)(1 - p_k z^{-1})\Big|_{z=p_k}$$

The impulse response then becomes

$$h[n] = A_1(p_1)^n u[n] + A_2(p_2)^n u[n] \tag{A.1}$$

For complex conjugate poles, A_k and p_k can more conveniently be expressed in polar coordinate form, with

$$A_k = \alpha e^{j\phi}$$

$$p_k = re^{j\theta}$$

giving

$$A_1(p_1)^n + A_2(p_2)^n = \alpha r^n e^{j(\theta n + \phi)} + \alpha r^n e^{-j(\theta n + \phi)}$$

$$= 2\alpha r^n \cos(\theta n + \phi)$$

The impulse response in (A.1) thus becomes

$$h[n] = 2\alpha r^n \cos(\theta n + \phi) u[n] \tag{A.2}$$

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