Mathematical Foundations of Unifying Gravitation and Electromagnetism

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Einstein showed that, the effect of gravitational field on a space-time is explained mathematically using Ricci tensor. Also, it is clear that the effect of electromagnetic filed on a space-time is explained with electromagnetic tensor which satisfies Maxwell's equations. In the real world of physics, both electromagnetic and gravitational fields exist in a space-time simultaneously. Therefore, the space-time should be considered simultaneously using two second rank tensors. In this paper, a new method is proposed for unifying gravitation with electromagnetism in a five-dimensional space-time. As a result, the Electrogravity theory is presented.

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1. Introduction

Since 1914, that Albert Einstein proposed a new and complete definition of gravitation, there have been several attempts to unify electromagnetism and gravitation, such as Kaluza theory with the fifth dimension and some other attempts in the area of Quantum mechanics, all of which were unsuccessful in unifying electromagnetism with gravitation[1]. For example, several years after the suggestion of Kaluza theory, in 1939 Einstein rejected his theory in a letter mentioning that [2]: "*As no arbitrary constants occur in the equations, the theory would lead to electromagnetic and gravitation fields of the same order of magnitude. Therefore one would be unable to explain the empirical fact that the electrostatic force between two particles is so much stronger than the gravitational force. This means that a consistent theory of matter could not be based on these equations*."

Einstein, himself, tried a lot to do this unification, as he commented [3]: "*It would be a great step forward to unify in a single picture the gravitational and electromagnetic fields, Then there would be a worthy completion of the epoch of theoretical physics …*".

Einstein was trying to unify these two fields in a theory based on fields and not particles, as it has been mentioned in his paper in 1935 : " *A complete field theory knows only fields and not the concepts of particles and motions*"[4]. Einstein wanted the fields to be absorbed in geometry and he wanted to formulate electromagnetism in geometry, same as gravitation. But electromagnetism has not been absorbed in geometry in any of the previous theories.

Einstein showed that, the effect of gravitational field on a space-time is explained with a symmetric rank 2 tensor, namely Ricci tensor. Also, it is known that, the effect of the electromagnetic filed on a space-time is explained with a rank 2 antisymmetric tensor, which satisfies Maxwell's equations.

In the real world of physics, both electromagnetic and gravitational fields exist in a space-time simultaneously. Therefore, the space-time should be considered, simultaneously by two second rank tensors, one of which is symmetric and the other antisymmetric. But what is the relationship between these two tensors and how can they be introduced in a single equation? In this paper, a new theory is proposed for unifying gravitation with electromagnetism in a five dimensional space-time, while considering the above mentioned tensors.

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In this theory, we need to modify the light speed constancy postulate. Let K be a system of reference in which a mass distant from other masses is moving with uniform motion on a straight line and K' be another system of reference which is moving relatively to K in accelerated translation. If a light ray propagates in a straight line with a constant velocity with respect to k, the path of light will be curvilinear with respect to K'. Here it is accepted the other principle of the relativity theory which is, the general laws of nature are to be expressed by equations which hold good for all systems of coordinates [5].

For better understanding, in the following at first we discuss on some necessary mathematical backgrounds.

2. Mathematical Background

The class of totally antisymmetric tensors is an important class of tensors of type (0, s). This class contains covariant tensors with antisymmetric property in every pair of their arguments, i.e.

$$T(X_1,...,X_\mu,...,X_\upsilon,...,X_s) = -T(X_1,...,X_\upsilon,...,X_\mu,...,X_s) \qquad (1)$$

for all pairs of indices μ and υ and for all X's. By applying the alternating operator A to a general tensor T of type (0,s), this kind of tensor can be formed. Applying operator A to T give the linear combination defined by

$$AT(X_1,...,X_s) = \frac{1}{s!}\sum_{\upsilon_1,...,\upsilon_s} \mathrm{sgn}(\upsilon_1,...,\upsilon_s)T(X_{\upsilon_1},...,X_{\upsilon_s}) \qquad (2)$$

where in this summation, $(\upsilon_1,...,\upsilon_s)$ are an even or an odd permutation of (1,…,s) integer numbers, and based on that, $\mathrm{sgn}(\upsilon_1,...,\upsilon_s) = \pm 1$, and equation (2) is to be valid for every $(X_1,...,X_s)$.

When T is totally antisymmetric, applying the A operator to it simply reproduces T. However, when $s \rangle n$ (the dimension of the vector space), applying the A operator to T reduces T to zero; simply put, there is no totally antisymmetric tensor of type (0, s) for $s \rangle n$.

Antisymmetric tensors of type (0,s) are called s-forms. They must vanish when any two of their arguments coincide. This space is denoted by $\Lambda^s T_p^*$.

By applying the A operator to the basis elements of the following tensor product, a basis for $\Lambda^s T_p^*$ can be obtained:

$$A(e^{\upsilon_1} \otimes ... \otimes e^{\upsilon_s}) \qquad (3)$$

The resulting basis elements can be written as the exterior or the wedge product of the e^{υ}'s as the following:

$$e^{\upsilon_1} \wedge e^{\upsilon_2} \wedge .... \wedge e^{\upsilon_s} \quad (\upsilon_1 \rangle \upsilon_2 \rangle .... \rangle \upsilon_s) \qquad (4)$$

By extending the summation only over strictly descending sequences, a general s-form can be written as:

$$\Omega = \Omega_{\upsilon_1...\upsilon_s} e^{\upsilon_1} \wedge e^{\upsilon_2} \wedge ... e^{\upsilon_s} \qquad (\upsilon_1 \rangle \upsilon_2 \rangle .... \rangle \upsilon_s) \qquad (5)$$

Considering that interchanging a pair of indices is equal to interchanging the corresponding elements in the wedge product, it can be deduced that interchanging the elements in a wedge product must be accompanied by a change of sign:

$$e^{\upsilon} \wedge e^{\tau} = -e^{\tau} \wedge e^{\upsilon} \qquad (6)$$

The expression for an s-form in a local coordinate basis is:

$$\Omega = \Omega_{\upsilon_1 \ldots \upsilon_s} dx^{\upsilon_1} \wedge \ldots \wedge dx^{\upsilon_s} \qquad (7)$$

To obtain a (p+q) form, the wedge product of any p-form Ω^1 and a q-form Ω^2 can be formed by the rule

$$\Omega^1 \wedge \Omega^2 = A(\Omega^1 \otimes \Omega^2) \qquad (8)$$

which must accordingly vanish identically if $(p+q)\rangle n$.

By definition,

$$\Omega^1 \wedge \Omega^2 = (\Omega^1_{\upsilon_1 \ldots \upsilon_p} e^{\upsilon_1} \wedge \ldots \wedge e^{\upsilon_p}) \wedge (\Omega^2_{\tau_1 \ldots \tau_q} e^{\tau_1} \wedge \ldots \wedge e^{\tau_q}) \qquad (9)$$

where $(\upsilon_1,\ldots,\upsilon_p)$ and $(\tau_1,\ldots,\tau_q)$ are strictly descending sequences. Since each of the q basis elements $e^{\tau_1},\ldots,e^{\tau_q}$ must go through p interchanges before $\Omega^1 \wedge \Omega^2$ can be brought to the form required of $\Omega^2 \wedge \Omega^1$, consequently:

$$\begin{aligned} \Omega^1 \wedge \Omega^2 &= (-1)^{pq} (\Omega^2_{\tau_1 \ldots \tau_q} e^{\tau_1} \wedge \ldots \wedge e^{\tau_q}) \wedge (\Omega^1_{\upsilon_1 \ldots \upsilon_p} e^{\upsilon_1} \wedge \ldots \wedge e^{\upsilon_p}) \\ &= (-1)^{pq} \Omega^2 \wedge \Omega^1 \end{aligned} \qquad (10)$$

If a suitable coordinate is chosen, it will be seen that our electrogravity (EG) equations, will be appropriate for infinitely small five dimensional regions. Let x_1, x_2, x_3 and x_4 be the space coordinates and x_5 be the time coordinate in an appropriate unit. Here the appropriate unit is the unit in which the time unit is chosen such that the speed of light is unity (c=1) in the local coordinate. If a unit measure is chosen, the coordinates with a given orientation of the coordinates have direct physical meaning. In relativity theory, the following expression had a value which was independent of the orientation of the local system of coordinates

$$ds^2 = -dx_1^2 - dx_2^2 - dx_3^2 - dx_4^2 + dx_5^2 \qquad (11)$$

Let *ds* be the magnitude of linear element pertaining to points of the five-dimensional continuum in infinite proximity. To the mentioned linear element or to the two infinitely proximate point events, there are correspond definite differentials dx_1, dx_2, dx_3, dx_4 and dx_5. In this system the dx_υ is represented here by definite linear homogeneous expression of the dx_σ:

$$dx_\upsilon = \sum_\sigma \alpha_{\upsilon\sigma} dx_\sigma \qquad (12)$$

Inserting these expressions in above equation, it is obtained:

$$ds^2 = \sum_{\tau\sigma} g_{\sigma\tau} dx_\sigma dx_\tau \qquad (13)$$

where $g_{\sigma\tau}$ are functions of x_σ. These are independent from the orientation and the state of motion of the local system of the coordinates. *ds* is independent of any particular choice of coordinates.

If it is possible to choose the system of coordinate in the finite region in such a way that the $g_{\mu\nu}$ has constant values:

$$g_{\mu\upsilon}=\begin{bmatrix}-1 & 0 & 0 & 0 & 0\\ 0 & -1 & 0 & 0 & 0\\ 0 & 0 & -1 & 0 & 0\\ 0 & 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 0 & +1\end{bmatrix} \qquad (14)$$

It will be seen that a free material point moves, relatively to this system, with uniform motion on a straight line. But if a new space-time coordinates x_1, x_2, x_3, x_4, and x_5, is chosen, the $g_{\mu\nu}$ in the new system will not be constant, but functions of space and time, and the motion of free material point will be a curvilinear motion. This motion must be interpreted as a motion under the influence of the EG field. So we find the occurrence of an EG field connected with the space-time variables of $g_{\mu\upsilon}$ .So the $g_{\mu\upsilon}$ representing the EG field at the same time define the metrical properties of the space-time. Now a five vector can be defined as following, In a $(x_1, x_2, x_3, x_4, x_5)$ coordinate, let's use the following metric for this purpose,

$$ds^2 = A_1^2 dx_1^2 - A_2^2 dx_2^2 - A_3^2 dx_3^2 - A_4^2 dx_4^2 - A_5^2 dx_5^2 \qquad (15)$$

The five vector $h_\mu = (h_1, h_2, h_3, h_4, h_5)$ is defined as following:

$$h_1 = A_1, h_2 = A_2, h_3 = A_3,\ h_4 = A_4\ ,\ h_5 = A_5 \qquad (16)$$

As,

$$g^{\alpha\tau} R^{\mu}_{\upsilon\sigma\tau} = R^{\mu\alpha}_{\upsilon\sigma} \qquad (17)$$

where $R^{\mu}_{\upsilon\sigma\tau}$ is the Riemann tensor, one can write:

$$h_\alpha R^{\mu\alpha}_{\upsilon\sigma} = R^{\mu\alpha}_{\upsilon\sigma\alpha} \qquad (18)$$

By contracting (18) two times, it will be obtained:

$$R^{\mu\alpha}_{\upsilon\mu\alpha} = R_\upsilon \qquad (19)$$

Now to find the geometrical form of the antisymmetric tensor, that was mentioned in the introduction, and its relationship with gravitational geometry, we define $S_{\mu\upsilon}$ using the following wedge product:

$$S_{\mu\upsilon} = R_\mu \wedge h_\upsilon \qquad (20)$$

where $S_{\mu\upsilon}$ is an antisymmetric tensor. Now the EG tensor can be defined as:

$$\frac{\partial S_{\mu\upsilon}}{\partial x_\sigma} - \Gamma^{\tau}_{\mu\sigma} S_{\tau\upsilon} - \Gamma^{\tau}_{\sigma\upsilon} S_{\mu\tau} + h_\mu R_{\upsilon\sigma} \qquad (21)$$

where $R_{\upsilon\sigma}$ is the Ricci tensor.

3. The Geodesic Equation:

Variations of a contravariant vector-field Y along a curve λ on N over time (t) can be formulated by,

$$(\delta Y)^{\upsilon} = Y^{\upsilon}_{,\tau} \frac{dx^{\tau}(\lambda(t))}{dt} \delta t \qquad (22)$$

where a δt increment in time results in displacement along (time dependent) λ and therefore change in Y, i.e. δY, in a local coordinate system.

When we use a Cartesian coordinate system in Euclidean geometry, parallel propagation of Y along λ is achieved by having $\delta Y = 0$. To generalize this notation for a general differentiable manifold with a connection, we would say that a vector Y is 'parallely propagated' along λ, if

$$(DY)^{\upsilon} = (\nabla_{\partial_\tau} Y)^{\upsilon} \frac{dx^{\tau}(\lambda(t))}{dt} \delta t = Y^{\upsilon}_{;\tau} \frac{dx^{\tau}(\lambda(t))}{dt} \delta t = 0 \qquad (23)$$

the above equation can be re-written as:

$$(Y^{\upsilon}_{,\tau} + Y^{\sigma} \Gamma^{\upsilon}_{\sigma\tau}) \frac{dx^{\tau}(\lambda(t))}{dt} \delta t = 0 \ . \qquad (24)$$

In other words, parallel propagation of Y along λ requires the following condition to be met:

$$(\delta Y)^{\upsilon} = -Y^{\sigma} \Gamma^{\upsilon}_{\sigma\tau} \frac{dx^{\tau}(\lambda(t))}{dt} \delta t \ . \qquad (25)$$

Now if Y is the tangent vector to the curve λ, then Y= $\frac{dx^{\upsilon}(\lambda(t))}{dt}$ and having it parallely propagated along λ, we have:

$$\delta\left(\frac{dx^{\upsilon}(\lambda(t))}{dt}\right) = -\Gamma^{\upsilon}_{\sigma\tau} \frac{dx^{\sigma}(\lambda(t))}{dt} \frac{dx^{\tau}(\lambda(t))}{dt} \delta t \ . \qquad (26)$$

When the tangent vector to λ, parallely propagated, remains a multiple of itself, we call geodesic to the curve λ on N . Therefore, for λ to be a geodesic, the following condition needs to be met in which $\phi(t)$ is some function of t:

$$\frac{dx^{\upsilon}(\lambda(t))}{dt} - \Gamma^{\upsilon}_{\sigma\tau} \frac{dx^{\sigma}(\lambda(t))}{dt} \frac{dx^{\tau}(\lambda(t))}{dt} \delta t = [1 - \phi(t)\delta t]\left[\frac{dx^{\upsilon}(\lambda(t))}{dt} + \frac{d^2 x^{\upsilon}(\lambda(t))}{dt^2} \delta t\right] \qquad (27)$$

When $\delta t \to 0$, the geodesic equation becomes,

$$\frac{d^2 x^{\upsilon}}{dt^2} + \Gamma^{\upsilon}_{\sigma\tau} \frac{dx^{\sigma}}{dt} \frac{dx^{\tau}}{dt} = \phi(t) \frac{dx^{\upsilon}}{dt} \qquad (28)$$

By reparameterize the curve λ using the s variable

$$s = \int^{t} dt'' \exp\left\{\int^{t''} dt' \phi(t')\right\} \qquad (29)$$

The equation (28) turns into:

$$\frac{d^2x^{\upsilon}}{ds^2}+\Gamma^{\upsilon}_{\sigma\tau}\frac{dx^{\sigma}}{ds}\frac{dx^{\tau}}{ds}=0 \qquad (30)$$

which is the geodesic equation[6].

4. The Electrogravity Field Equations in the Absence of Matter:

The mathematical importance of the above-mentioned EG tensor is that, if there is a coordinate system with reference to which the $g_{\mu\nu}$ are constant, then all components of the EG tensor will vanish . If any new system of coordinates is chosen, instead of the original one, the $g_{\mu\nu}$ will not be constant, but in result of its tensor nature, the transformed components of the EG tensor will still vanish in the new system. Relative to this system, all components of the EG tensor vanish in any other system of coordinates. Thus, the required equations of the matter-free EG field must be satisfied if all components of the EG tensor vanish.

Therefore using 21, the equations of matter-free field are:

$$\frac{\partial S_{\mu\upsilon}}{\partial x_{\sigma}}-\Gamma^{\tau}_{\mu\sigma}S_{\tau\upsilon}-\Gamma^{\tau}_{\sigma\upsilon}S_{\mu\tau}+h_{\mu}R_{\upsilon\sigma}=0 \qquad (31)$$

In approximation for considering the electromagnetic behavior of this equation, it is better to focus on the microscopic scales. If some electric charges are placed in the media, it is clear that, in this scale the effects of gravitational field are so much smaller than electromagnetic field, so the gravitational field can be ignored in approximation, and because the existing mass (the mass of electric charges) is very small ($m'\to 0$), then $T_{\upsilon\sigma}\to 0$, where $T_{\upsilon\sigma}$ is the energy momentum tensor. Thus we have $R_{\upsilon\sigma}\to 0$ and expression (21) turns into:

$$\frac{\partial S_{\mu\upsilon}}{\partial x_{\sigma}}-\Gamma^{\tau}_{\mu\sigma}S_{\tau\upsilon}-\Gamma^{\tau}_{\sigma\upsilon}S_{\mu\tau} \qquad (32)$$

Using 32 , we can additionally write the following two tensors:

$$\frac{\partial S_{\upsilon\sigma}}{\partial x_{\mu}}-\Gamma^{\tau}{}_{\mu\upsilon}S_{\tau\sigma}-\Gamma^{\tau}{}_{\mu\sigma}S_{\upsilon\tau} \qquad (33)$$

$$\frac{\partial S_{\sigma\mu}}{\partial x_{\nu}}-\Gamma^{\tau}{}_{\upsilon\sigma}S_{\tau\mu}-\Gamma^{\tau}{}_{\upsilon\mu}S_{\sigma\tau} \qquad (34)$$

From eq.31 , it is seen that all components of the EG tensor vanished, so the sum of above three tensors (32,33 and 34) must be zero also. Now adding 32,33 and 34, it is obtained:

$$\begin{aligned}&\frac{\partial S_{\mu\upsilon}}{\partial x_{\sigma}}+\frac{\partial S_{\upsilon\sigma}}{\partial x_{\mu}}+\frac{\partial S_{\sigma\mu}}{\partial x_{\upsilon}}-\Gamma^{\tau}{}_{\mu\sigma}S_{\tau\upsilon}-\Gamma^{\tau}{}_{\sigma\upsilon}S_{\mu\tau}-\Gamma^{\tau}{}_{\mu\upsilon}S_{\tau\sigma}-\\&\Gamma^{\tau}{}_{\mu\sigma}S_{\upsilon\tau}-\Gamma^{\tau}{}_{\upsilon\sigma}S_{\tau\mu}-\Gamma^{\tau}{}_{\upsilon\mu}S_{\sigma\tau}=0\end{aligned} \qquad (35)$$

And using $S_{\mu\upsilon}=-S_{\upsilon\mu}$ in equation (35) ,

$$\frac{\partial S_{\mu\upsilon}}{\partial x_{\sigma}}+\frac{\partial S_{\upsilon\sigma}}{\partial x_{\mu}}+\frac{\partial S_{\sigma\mu}}{\partial x_{\upsilon}}=0 \qquad (36)$$

To find a complete solution for equation (31) which contains the effects of both gravity and electromagnetism in a five dimensional coordinate, one could try to combine two spherical and cylindrical coordinates to make a new five dimensional coordinate, then write the metric in this coordinate and find the metric functions. However one can try to solve the EG field equations in any other arbitrary coordinate.

5. Equation of Motion of a Material Point in the Electrogravity Field:

According to the special theory of relativity a freely movable body, in the absence of the external forces, moves uniformly on a straight line. If a system of coordinate, K_0, in five dimensional space-time is chosen so that the $g_{\mu v}$ s are constant, then the material point will be moved on a straight line. If we consider this movement from any other system of coordinates, K_1, the law of motion with respect to K_1 results from the following consideration. The body movement in K_0 coordinate corresponds to a five dimensional straight line which is a geodetic line [5]. Since the geodetic line is defined independently of the system of reference, its equations will also be the equation of motion of the material point with respect to K_1. So the equation of motion of the point with respect to K_1 becomes:

$$\ddot{x}^{\upsilon} + \Gamma^{\upsilon}_{\sigma\tau}\dot{x}^{\sigma}\dot{x}^{\tau} = 0 \qquad (37)$$

Now we say, this system of equations also defines the motion of the point in the EG field.

6. The General Form of the Electrogravity Field Equations:

The field equations (eq.31), which are obtained for matter free space-time, are to be compared with the field equation $\nabla^2\varphi = 0$ of Newton's theory or $R_{\mu\upsilon} = 0$ of Einstein gravity field equations in vacuum. We require the equation corresponding to Poisson's equation: $\nabla^2\varphi = 4k\pi\rho$ or $R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}R = -kT_{\mu\upsilon}$ of Einstein general form of the gravitational field equation. For this purpose we define $T_{\mu v\sigma}$ as following:

$$T_{\mu\upsilon\sigma} = g_{\alpha\upsilon}(kT^{\alpha}_{\sigma} + k'T'^{\alpha}_{\sigma})h_{\mu} \qquad (38)$$

where k and k' are two constants related to the gravity and electromagnetism respectively, and $T_{\upsilon\sigma} = g_{\alpha\upsilon}T^{\alpha}_{\sigma}$ is the energy-momentum tensor and T'^{α}_{σ} is the electromagnetic energy tensor where:

$$g_{\alpha\upsilon}T'^{\alpha}_{\sigma}\ h_{\mu} = h_{\mu}T'_{\upsilon\sigma} = T'_{\mu\upsilon\sigma} \qquad (39)$$

Thus instead of eq.31 we write:

$$\frac{\partial S_{\mu\upsilon}}{\partial x_{\sigma}} - \Gamma^{\tau}_{\mu\sigma}S_{\tau\upsilon} - \Gamma^{\tau}_{\sigma\upsilon}S_{\mu\tau} + h_{\mu}R_{\sigma\upsilon} - \frac{1}{2}g_{\sigma\upsilon}h_{\mu}R = -T_{\mu\upsilon\sigma} \qquad (40)$$

Therefore equation.40 is the required general form of the EG field equations.

This equation in its special forms easily gives us the general form of the Einstein field equations in general relativity and all Maxwell's equations.

As Einstein has mentioned in his paper, the Christoffel symbols are used as the field components of gravitation [5], So in the absence of matter, by approximation, when the gravitational field is very small: $\Gamma^{\tau}_{\sigma\upsilon} \to 0$ then $R_{\sigma\upsilon}$ will be very small. For example it can be reached to this approximation in a region without any matter unless some electrical charges, like electron, that have a very small mass. So in the mentioned approximation, eq.40 turns into:

$$\frac{\partial S_{\mu\upsilon}}{\partial x_{\sigma}} = -T_{\mu\upsilon\sigma} \qquad (41)$$

Let's suppose that, we are working in a region that the total electric charge density of the area is ρ_q and the current density is j_i. Now in one special case, we suppose that all components of $T_{\mu\nu\sigma}$ are zero unless five components of $T_{5\upsilon 5}$. Now a 5_vector $T_{5\upsilon 5}$ can be defined as:

$$T_{5\upsilon 5} = j_{\upsilon} = k\,(\rho_q, j_i) \text{ and } \quad i = 1,2,3,4 \qquad (42)$$

In above approximation, by ignoring the gravitation (because, here we are working in a microscopic scale and in this scale the effects of gravitational field is so much smaller than electromagnetic field), in the absence of gravity the space time can be considered as the four dimensional part of R_5, and in this space-time, j_{υ} turns into:

$$j_{\upsilon} = k\,(\rho_q, j_1, j_2, j_3)\,. \qquad (43)$$

Now from eqs.41 and 42:

$$\frac{\partial S_{\mu\upsilon}}{\partial x_{\mu}} = -k\,(\rho_q, j_1, j_2, j_3) = j_{\upsilon} \Rightarrow \frac{\partial S_{\mu\upsilon}}{\partial x_{\mu}} = j_{\upsilon} \qquad (44)$$

where $S_{\mu\nu}$ is an antisymmetric tensor.

Therefore, from eqs.36 and 44, on identifying $S_{\mu\upsilon}$ with the electromagnetic tensor $F_{\mu\upsilon}$, one will recognize eqs.36 and 44 as the Maxwell's equations. Therefore, $S_{\mu\upsilon}$ can be identified as the electromagnetic tensor ($F_{\mu\upsilon}$).

Moreover, equation 40 in its special form easily gives us the general form of the Einstein field equations for gravitation.

At the end, for completing the equations, it is better to add a new constant (Λ) to the EG field equations, which is different from gravitational cosmological constant Λ, and it is considered as the sum of two electromagnetic and gravitational cosmological constants

$$\frac{\partial S_{\mu\upsilon}}{\partial x_{\sigma}} - \Gamma^{\tau}_{\mu\sigma} S_{\tau\upsilon} - \Gamma^{\tau}_{\sigma\upsilon} S_{\mu\tau} + h_{\mu} R_{\sigma\upsilon} - \frac{1}{2} g_{\sigma\upsilon} h_{\mu} R + g_{\sigma\upsilon} h_{\mu} \Lambda = -T_{\mu\upsilon\sigma} \qquad (45)$$

7. Conclusion:

In this research, the unified field equation for both electromagnetism and gravitation (Electrogravity field equation) is presented. Doing more considerations on the EG equations can solve an important problem in physics, which is the geometrical nature of the electromagnetic field.

In contrast to gravity field, which it makes a curvature in the space time, the presence of the electromagnetic field can be visualized to make another kind of curvature in the space time with a very small radius of curvature (in contrast to gravity curvature radius), and from geometrical view, for example, this can be the actual reason of the fact that, a charged particle goes on a cylindrical curved path (helical path) in the presence of the electromagnetic field. This motion was justified by Lorentz force, in no geometrical thinking.

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