# Serie slowly convergent

**Edgar Valdebenito**

**abstract**

We exhibit a serie slowly convergent for pi:

$$\pi = 3.14159265358979\ ...$$

$$\frac{1}{\pi} = 0.31830988618379\ ...$$


## I. Serie de convergencia lenta

$$\frac{1}{\pi} = \frac{1}{32} \sum_{n=0}^{\infty} \frac{(-1)^n (1/2)_n^3}{((n+1)!)^3} \left(12 n^2 + 18 n + 7\right) (4 n + 3 - (-1)^n) \tag{1}$$

la serie (1) se puede escribir como:

$$\frac{1}{\pi} = \sum_{n=0}^{\infty} (-1)^n \binom{2n}{n}^3 2^{-6n-5} \left( \frac{(12 n^2 + 18 n + 7)(4 n + 3 - (-1)^n)}{(n+1)^3} \right) \tag{2}$$

la serie anterior se puede obtener por distintas teorías alternativas ( Ej. series de ramanujan).

Sea

$$f(n) = \binom{2n}{n}^3 2^{-6n-5} \left( \frac{(12 n^2 + 18 n + 7)(4 n + 3 - (-1)^n)}{(n+1)^3} \right) \tag{3}$$

se tiene

$$f(n) > 0\ ,\ \ n = 0,\ 1,\ 2,\ 3,\ ... \tag{4}$$

$$0 < f(n+1) < f(n)\ ,\ \ n = 0,\ 1,\ 2,\ 3,\ ... \tag{5}$$

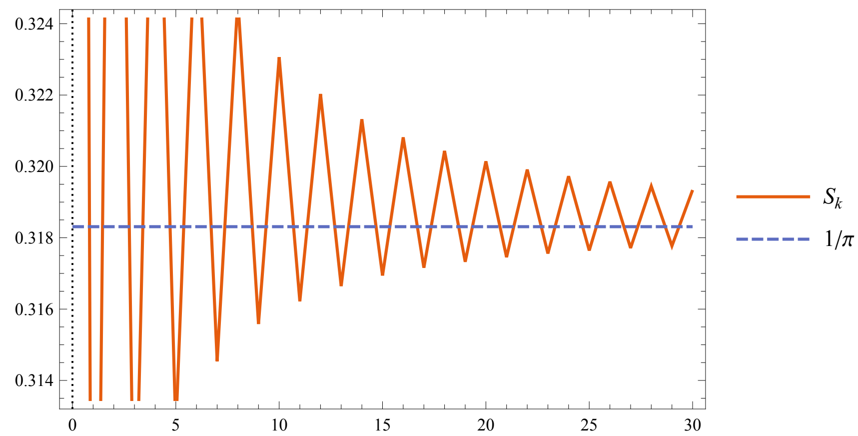
$$\lim_{n \to \infty} f(n) = 0 \tag{6}$$

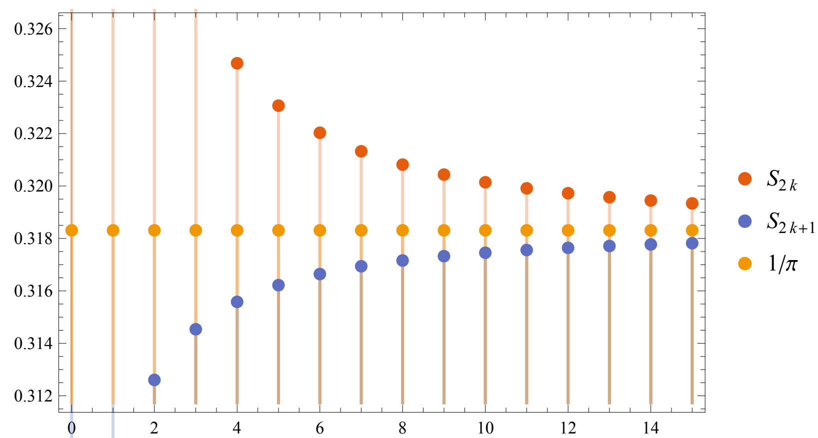
$$f(n) = O\left(n^{-3/2}\right) \tag{7}$$

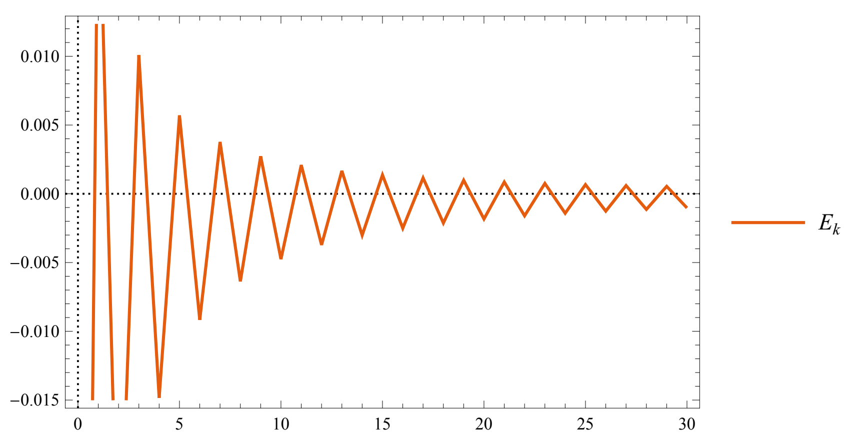
Sea

$$S_k = \sum_{n=0}^{k} (-1)^n f(n)\ ,\ \ k = 0,\ 1,\ 2,\ 3,\ ... \tag{8}$$

$$E_k = \frac{1}{\pi} - S_k\ ,\ \ k = 0,\ 1,\ 2,\ 3,\ ... \tag{9}$$


0.324
0.322
0.320
0.318
0.316
0.314
0
5
10
15
20
25
30
$S_k$
$1/\pi$


0.326
0.324
0.322
0.320
0.318
0.316
0.314
0.312
0
2
4
6
8
10
12
14
$S_{2k}$
$S_{2k+1}$
$1/\pi$


0.010
0.005
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−0.005
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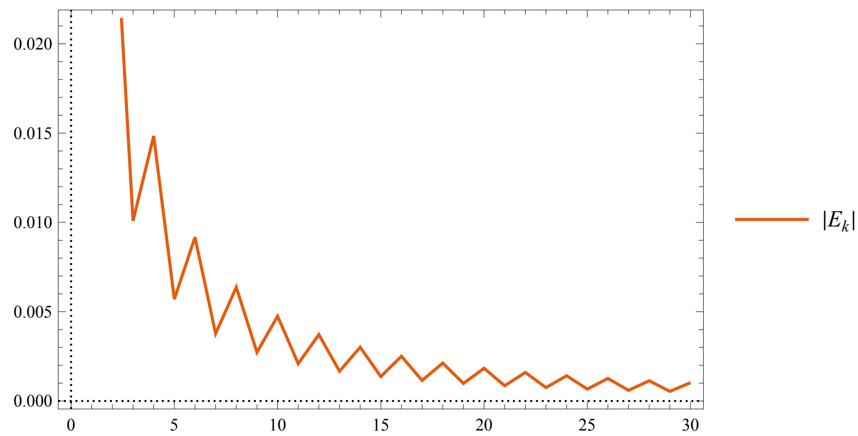
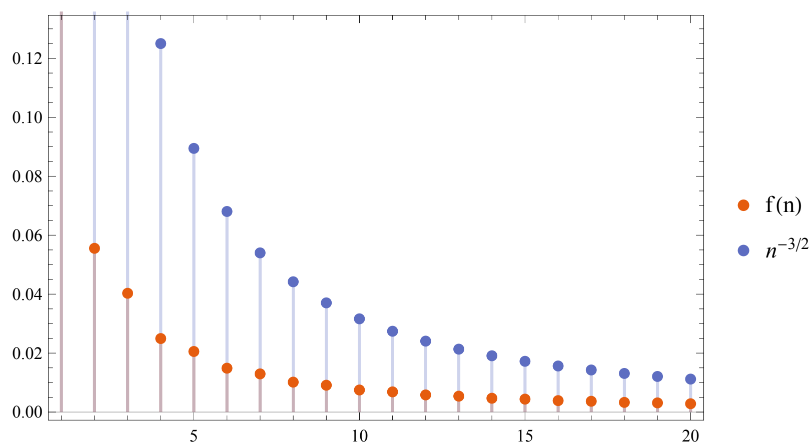
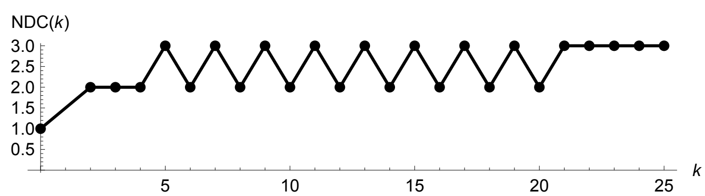
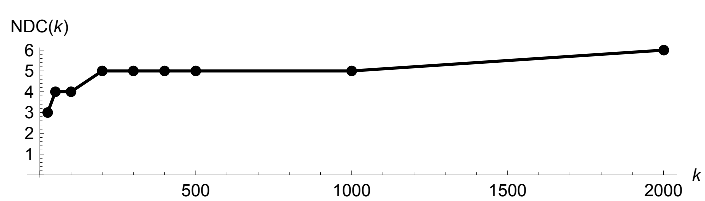
−0.015
0
5
10
15
20
25
30
$E_k$




NDC($k$) = número de digitos correctos en la $k$ − ésima suma parcial $S_k$, $0 \leq k \leq 25$




NDC($k$) = número de digitos correctos en la $k$ − ésima suma parcial $S_k$, $25 \leq k \leq 2001$

una fórmula relacionada con la serie (1):

$$\frac{1}{\pi} = \frac{(\Gamma(1/4))^2}{256\,((\Gamma(1/4))^2 - 8\,\Gamma(3/4)\,\Gamma(7/4))}\left(128\,F\left(\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}, \{1, 1\}, -1\right) - 64\,F\left(\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \{2, 2\}, -1\right) - 48\,F\left(\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \{2, 2\}, 1\right) - 3\,F\left(\left\{\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right\}, \{3, 3\}, 1\right)\right) \tag{10}$$

donde

$$F(\{a_1, a_2, a_3\}, \{b_1, b_2\}, z) = \sum_{n=0}^{\infty} \frac{(a_1)_n\,(a_2)_n\,(a_3)_n}{(b_1)_n\,(b_2)_n\,n!} z^n \tag{11}$$

$$\Gamma(x) = \int_0^{\infty} t^{x-1}\, e^{-t}\, dt \ \ ,\ \ x > 0 \tag{12}$$

En (11) se tiene:

$$(a)_0 = 1\ \ ,\ \ (a)_n = a\,(a+1)\,(a+2)\,...\,(a+n-1)\ ,\ \ n = 1, 2, 3, \ ... \tag{13}$$

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