Teleportation of information without classical communication channel

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Abstract: The spin projection expectation value of electrons in magnetic field in direction perpendicular to the magnetic induction B depends on the magnitude of B and the time t, spent by the electron in the field. Consequently choosing the value of the product B.t one can have spin statistics biased to +1/2 or -1/2. The spin statistics of electrons from a quantity of EPR pairs is manipulated this way. The spin statistics of the partner electrons will show the opposite statistics, thus realizing teleportation of information without the use of a classical channel in contrast to the Bennett teleportation protocol [1].

The proposed protocol starts with a quantity (necessary for statistics) of entangled by spin electron pairs (or entangled electron – photon pairs), where the first partners - electrons are at disposal of one laboratory (Alice), while the second partners - electrons are at disposal of another distant laboratory (Brian). The pairs are prepared in one of the four Bell's states. By example:

$$\Phi_i = \left|\uparrow\right\rangle_i \left|\downarrow\right\rangle_i + \left|\downarrow\right\rangle_i \left|\uparrow\right\rangle_i \quad \text{where } i = 1,\ldots N \text{ is the number of the pair}$$

Alice lets her electrons pass an area of constant magnetic field along the magnetic induction B (for simplicity). The spin of an electron in constant magnetic field can not become projected on any axis including the axis of the magnetic field before a measurement is done. In fact the spin starts to rotate (precession known in Nuclear Magnetic Resonance). Therefore any electron from the entangled pairs in Alice disposal positioned in an area of constant magnetic field does not become disentangled with its partner in Brian's disposal. There must be a measurement which will tell apart electrons with spin up or down (such apparatus is the well-known Stern-Gerlach setup). Only after this measurement the pairs become disentangled.

Let's look what happens to the spin of an electron in constant magnetic field. Because the interaction between the magnetic induction B and the spin does not depend on the velocity of the electron the case with v=0 or more generally v parallel to B is considered.

The **Schrodinger equation for the spin** is:

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi \qquad \text{where } \Psi = \begin{pmatrix}\chi \\ \eta\end{pmatrix}$$

The **Hamiltonian** in the case of an electron positioned in magnetic field of magnetic induction B with velocity v = 0 (or with v along the z axis where is also B_z = B=const; B_x=0; B_y=0) must be:

$$H\Psi = -\frac{e\hbar}{2m}B\sigma_3 = \hbar k\sigma_3$$

where k is positive constant, as 'e' is negative. So:

$$i\frac{\partial\Psi}{\partial t} = k\sigma_3\Psi$$

$$\text{where } \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{is the Dirac matrix on z axis}$$

$$\text{Then } i\frac{\partial\chi}{\partial t} = k\chi \quad \text{with solution } \chi = \mathrm{e}^{-ikt}$$

$$\text{Analogously for } \eta = \mathrm{e}^{ikt}$$

Then the **wavefunction** after normalization is:

$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{e}^{-ikt} \\ \mathrm{e}^{ikt} \end{pmatrix}$$

What are the **spin projection and the spin projection expectation value of this wavefunction**?

$$\text{It is not eigenstate of } \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\text{Also is it is not eigenstate of } \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ or } \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Spin projection on Z:

$$Z:\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\rightarrow\frac{1}{\sqrt{2}}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}e^{-ikt}\\e^{ikt}\end{pmatrix}=\frac{i}{\sqrt{2}}\begin{pmatrix}e^{-ikt}\\-e^{ikt}\end{pmatrix}$$

not eigenstate of σ_3

The **expectation value on Z:**

$$\langle\Psi|\sigma_3|\Psi\rangle=\frac{\hbar}{4}\begin{pmatrix}e^{-ikt}\\e^{ikt}\end{pmatrix}^+\begin{pmatrix}e^{-ikt}\\-e^{ikt}\end{pmatrix}=\frac{\hbar}{4}\begin{pmatrix}e^{ikt}&e^{-ikt}\end{pmatrix}\begin{pmatrix}e^{-ikt}\\-e^{ikt}\end{pmatrix}=\frac{\hbar i}{4}\left(e^0-e^0\right)=0$$

Spin projection on X:

$$X:\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix}\rightarrow\frac{1}{\sqrt{2}}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}e^{-ikt}\\e^{ikt}\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}e^{ikt}\\e^{-ikt}\end{pmatrix}$$

not eigenstate of σ_1

The **expectation value on X**

$$\langle\Psi|\sigma_1|\Psi\rangle=\frac{\hbar}{2}\frac{1}{2}\begin{pmatrix}e^{-ikt}\\e^{ikt}\end{pmatrix}^+\begin{pmatrix}e^{ikt}\\e^{-ikt}\end{pmatrix}=\frac{\hbar}{4}\begin{pmatrix}e^{ikt}&e^{-ikt}\end{pmatrix}\begin{pmatrix}e^{ikt}\\e^{-ikt}\end{pmatrix}=$$

$$\frac{\hbar}{4}\left(e^{i2kt}+e^{-i2kt}\right)=\frac{\hbar}{2}\cos 2kt$$

Spin projection on Y:

$$\mathrm{Y}:\sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\rightarrow\frac{1}{\sqrt{2}}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}\mathrm{e}^{-\mathrm{i}kt}\\ \mathrm{e}^{\mathrm{i}kt}\end{pmatrix}=\frac{\mathrm{i}}{\sqrt{2}}\begin{pmatrix}-\mathrm{e}^{\mathrm{i}kt}\\ \mathrm{e}^{-\mathrm{i}kt}\end{pmatrix}$$

$$\text{not eigenstate of } \sigma_2$$

The **expectation value on Y:**

$$\langle\Psi|\sigma_2|\Psi\rangle=\frac{\hbar}{2}\frac{1}{2}\begin{pmatrix}\mathrm{e}^{-\mathrm{i}kt}\\ \mathrm{e}^{\mathrm{i}kt}\end{pmatrix}^{+}\begin{pmatrix}-\mathrm{e}^{\mathrm{i}kt}\\ \mathrm{e}^{-\mathrm{i}kt}\end{pmatrix}=\frac{\hbar}{4}\begin{pmatrix}\mathrm{e}^{\mathrm{i}kt} & \mathrm{e}^{-\mathrm{i}kt}\end{pmatrix}\begin{pmatrix}-\mathrm{e}^{\mathrm{i}kt}\\ \mathrm{e}^{-\mathrm{i}kt}\end{pmatrix}=$$

$$\frac{\hbar i}{4}\left(-\mathrm{e}^{\mathrm{i}2kt}+\mathrm{e}^{-\mathrm{i}2kt}\right)=\frac{\hbar}{2}\sin 2kt$$

While the expectation value along the direction of the magnetic induction Z is zero, which means that spin up or down along Z are equally possible, the expectations value of the spin both on X and Y depends on the product B.t.

This means that by choosing appropriately B.t, one can become a statistics biased to spin projection along X or Y either to be +1/2 or to be -1/2. It is obvious that when k.t is near *l*.π (l= 0,1,2,…) it is much more possible to receive +1/2 for spin along X. When k.t is close to (l+1/2) π the spin projection would be more likely -1/2. In order to accomplished this projection a measurement of spin is necessary by example by a subsequent Stern-Gerlach measurement.

After the magnetic field Alice makes a Stern-Gerlach measurement on her electrons. She ought to see a statistics of spin projection predominantly s_x =+1/2 if she has chosen k.t is near $l.\pi$ (l=0,1,2,…) or a statistics of spin projection predominantly s_x =-1/2 if she has chosen k.t is near (l+1/2).π .

As now these electrons (Alice) were previously entangled to their partner electrons (Brian) in Bell state:

$\Phi_i = \left|\uparrow\right\rangle_i \left|\downarrow\right\rangle_i + \left|\downarrow\right\rangle_i \left|\uparrow\right\rangle_i$ it is expected that Brian's electrons will show predominantly spin projection s_x =-1/2 if Alice has chosen k.t is near $l.\pi$ or a statistics of spin projection predominantly s_x =+1/2 if she has chosen k.t is near (l+1/2).π.

This is a realization of a communication channel between Alice and Brian which doesn't need a second classical communication channel (as it is used for the teleportation of unknown state in the Bennett's protocol [1]).

The entanglement of the Alice-Brian electron EPR pairs can be made routinely by swapping using entangled photon pairs. The Alice's electrons can be entangled in the area of the magnetic field in order to avoid an influence of the field at the entrance in it, leading to projection of the spin before the electron has passed the area with magnetic field. The Stern-Gerlach setup can also be in this area at the exit for the abovementioned reason.

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Reference:

[1] Charles H. Bennett, Gilles Brassard, Claude Crépeau, Richard Jozsa, Asher Peres, and William K. Wootters, Phys. Rev. Lett. 70, 1895 – Published 29 March 1993