Continued radicals , Mathematical constants

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Abstract

In this note we show continued radicals for some mathematical constants

Resumen

En esta nota mostramos una colección de radicales continuos para algunas constantes clásicas:

$$\pi\ , e\ , \frac{10}{\pi} , \frac{8}{e}\ , \frac{2}{\gamma}\ , 2\varphi\ , \dots , etc.$$

I. Introducción : Master Functions

1. Definición1:

Sean $n \in \mathbb{N}\ y\ f_1, f_2, \dots , f_n \geq 0$, se define la función R como sigue:

$$R:\ \{f_1, f_1, \dots , f_n\} \to \mathbb{R}^+$$

$$R(f_1, f_2, \dots , f_n) = \sqrt{f_1 + \sqrt{f_2 + \cdots + \sqrt{f_n}}}$$

2. Definición 2:

Sean $x \in [2,4]$, $M = \{2,4,6,8,10,12\}$, se define la función $f(x,n) \equiv f_x(n) \equiv f_n$ como sigue:

$$f_x:\ \mathbb{N} \to M$$

$$n \to f_x(n) \equiv f_n$$

$$f_n = \begin{cases} f_1 = \begin{cases} 12 \quad si \quad \sqrt{14} \leq x \\ max\{m \in M:\ \sqrt{m+2} \leq x\} \end{cases} & ,\ \ n = 1 \\ max\{m \in M : R(f_1, f_2, \dots , f_{n-1}, m+2) \leq x\} & , n \geq 2 \end{cases}$$

Con las definiciones anteriores es válida la siguiente representación:

$$x = \sqrt{f_1 + \sqrt{f_2 + \sqrt{f_3 + \cdots}}} = \sqrt{f_1, f_2, f_3, \dots}$$

Para detalles del análisis ver referencia (1).

II. Algunas constantes clásicas

(1) *Constante Pi*:

$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.1415926535\ldots$$

$$\pi = \sqrt{6,12,6,6,2,2,10,4,4,2,8,\ldots}$$

$$\frac{7}{\pi} = \sqrt{2,6,4,12,2,8,6,6,4,10,2,\ldots}$$

(2) *Constante e de Euler*:

$$e = \sum_{n=0}^{\infty}\frac{1}{n!} = 2.7182818284\ldots$$

$$e = \sqrt{4,8,10,2,4,6,2,8,4,6,10,\ldots}$$

$$\frac{6}{e} = \sqrt{2,6,2,6,8,8,2,4,12,6,4,\ldots}$$

(3) *Golden Ratio* φ:

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.6180339887\ldots$$

$$2\varphi = \sqrt{8,4,2,4,2,2,4,8,1,2,2,2,\ldots}$$

$$\frac{4}{\varphi} = \sqrt{4,2,4,2,2,4,8,1,2,2,2,2,\ldots}$$

(4) *Constante gama* γ *de Euler*:

$$\gamma = \lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\ln n\right) = 0.5772156649\ldots$$

$$4\gamma = \sqrt{2,8,6,10,4,12,2,8,2,6,4,\ldots}$$

$$\frac{2}{\gamma} = \sqrt{10,2,2,2,2,10,6,8,4,6,6,\ldots}$$

(5) *Constante de Catalan* G:

$$G = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2} = 0.9159655941\ldots$$

$$3G = \sqrt{4,10,4,4,12,6,10,2,2,6,10,\ldots}$$

$$\frac{2}{G} = \sqrt{2,4,10,8,10,2,10,6,2,10,8,\ldots}$$

(6) *Constante ln*2:

$$\ln 2 = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 0.6931471805\ldots$$

$$4\ln 2 = \sqrt{4,10,10,6,4,4,6,12,10,6,12,\ldots}$$

$$\frac{2}{\ln 2} = \sqrt{6,2,8,10,6,8,8,2,2,2,8,\ldots}$$

(7) *Constante ln*3:

$$\ln 3 = 2\sum_{n=0}^{\infty} \frac{1}{2n+1}\left(\frac{2}{3}\right)^{2n+1} = 1.0986122886\ldots$$

$$3\ln 3 = \sqrt{8,6,2,4,12,8,8,6,10,6,6,\ldots}$$

$$\frac{3}{\ln 3} = \sqrt{4,8,12,10,6,4,10,2,2,4,2,\ldots}$$

(8) *Constante de Grothendieck* K_R:

$$K_R = \frac{\pi}{2\ln\left(1+\sqrt{2}\right)} = 1.7822139781\ldots$$

$$2K_R = \sqrt{10,4,8,6,6,8,4,12,,6,12,\ldots}$$

$$\frac{4}{K_R} = \sqrt{2,6,8,2,12,2,6,8,4,4,12,\ldots}$$

(9) *Constante de Khinchin* K:

$$K = \prod_{n=1}^{\infty}\left(1+\frac{1}{n(n+1)}\right)^{\frac{\ln n}{\ln 2}} = 2.6854520010\ldots$$

$$K = \sqrt{4,8,2,8,8,4,8,12,6,4,2,\ldots}$$

$$\frac{6}{K} = \sqrt{2,6,6,4,8,8,4,4,6,2,12,\ldots}$$

(10) *Constante de Glaisher − Kinkelin* A:

$$A = e^{\frac{1}{12}-\zeta'(-1)} = 1.2824271291\ldots$$

$$2A = \sqrt{4,4,4,6,6,10,4,4,2,4,2,\ldots}$$

$$\frac{3}{A} = \sqrt{2,10,2,2,6,6,4,4,4,4,2,\ldots}$$

(11) *Constante Omega* Ω:

$$\Omega\, e^{\Omega} = 1 \,,\, \Omega = 0.5671432904\ldots$$

$$4\Omega = \sqrt{2,6,12,8,2,8,6,4,4,4,4,\ldots}$$

$$\frac{2}{\Omega} = \sqrt{10,2,12,10,2,2,4,8,8,2,4,\ldots}$$

(12) *Constante de Ramanujan − Soldner* μ:

$$Li(\mu) = \int_0^{\mu} \frac{dx}{\ln x} = 0 \,, \qquad \mu = 1.4513692348\ldots$$

$$2\mu = \sqrt{6,2,12,6,10,4,8,8,4,6,\ldots}$$

$$\frac{4}{\mu} = \sqrt{4,10,6,4,4,6,4,12,4,8,\ldots}$$

(13) *Número de Dottie* d:

$$\cos d = d \;, \qquad d = 0.7390851332\ldots$$

$$3d = \sqrt{2,6,4,2,6,12,12,8,12,4,\ldots}$$

$$\frac{2}{d} = \sqrt{4,8,6,8,4,10,4,2,4,8,\ldots}$$

(14) *Constante de Apéry* $\zeta(3)$:

$$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3} = \int_0^1\int_0^1\int_0^1 \frac{dx\,dy\,dz}{1 - x\,y\,z} = 1.2020569031\ldots$$

$$2\,\zeta(3) = \sqrt{2,12,2,8,2,12,6,6,8,12,\ldots}$$

$$\frac{3}{\zeta(3)} = \sqrt{4,2,6,4,12,4,12,12,4,2,\ldots}$$

(15) *Constante de Smarandache* S_1:

$$S_1 = \sum_{n=2}^{\infty} \frac{1}{\mu(n)!} = 1.0931704591\ldots$$

$$2S_1 = \sqrt{2,4,10,12,8,4,2,12,6,2,\ldots}$$

$$\frac{3}{S_1} = \sqrt{4,10,4,2,2,12,2,12,2,6,\ldots}$$

$\mu(n)$: función de Kempner:es el número más pequeño por el que $\mu(n)!$ es divisible por n.

(16) $Dimensión\ fractal\ del\ conjunto\ de\ Cantor\ d_f(k)$:

$$d_f(k) = \frac{\ln 2}{\ln 3} = 0.6309297535 \dots$$

$$4d_f(k) = \sqrt{4,2,10,6,8,6,8,2,4,12, \dots}$$

$$\frac{2}{d_f(k)} = \sqrt{8,2,2,4,12,10,12,12,2,6, \dots}$$

(17) $Constante\ de\ Erdos - Borwein\ E_B$:

$$E_B = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{2^{mn}} = \sum_{n=1}^{\infty}\frac{1}{2^n - 1} = 1.6066951524 \dots$$

$$2E_B = \sqrt{8,2,8,10,6,12,8,6,6,2, \dots}$$

$$\frac{4}{E_B} = \sqrt{4,2,6,2,2,2,6,12,2,12, \dots}$$

(18) $Constante\ de\ Gelfond\ e^{\pi}$:

$$e^{\pi} = 23.1406926327 \dots$$

$$\frac{e^{\pi}}{10} = \sqrt{2,8,8,4,4,10,4,8,2,10, \dots}$$

$$\frac{50}{e^{\pi}} = \sqrt{2,4,6,12,2,2,2,6,4,12,2,2, \dots}$$

(19) $Constante\ de\ Gelfond - Schneider\ G_{GS}$:

$$G_{GS} = 2^{\sqrt{2}} = 2.6651441426 \dots$$

$$G_{GS} = \sqrt{4,6,10,8,2,2,2,12,10,12, \dots}$$

$$\frac{6}{G_{Gs}} = \sqrt{2,6,8,10,8,8,12,2,4,6, \dots}$$

(20) $Número\ de\ Kasner\ R$:

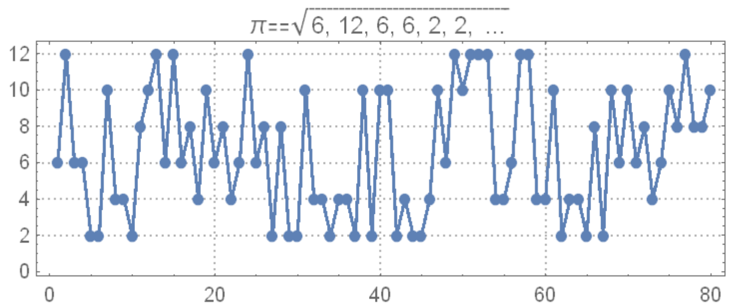
$$R = \sqrt{1 + \sqrt{2 + \sqrt{3 + \sqrt{4 + \cdots}}}} = 1.7579327566 \dots$$

$$2R = \sqrt{10,2,10,4,12,2,4,6,12,6, \dots}$$

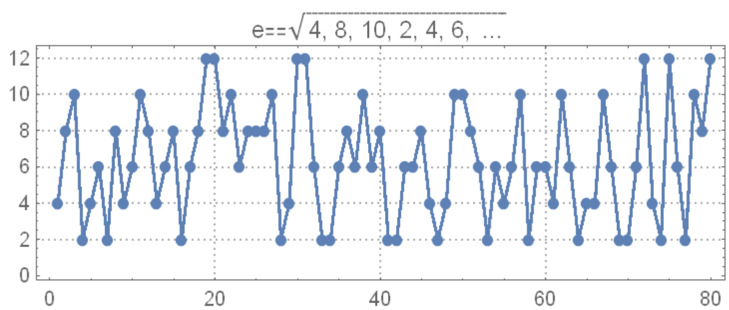
$$\frac{4}{R} = \sqrt{2,8,2,2,10,12,8,8,2,4, \dots}$$

III. Graficos

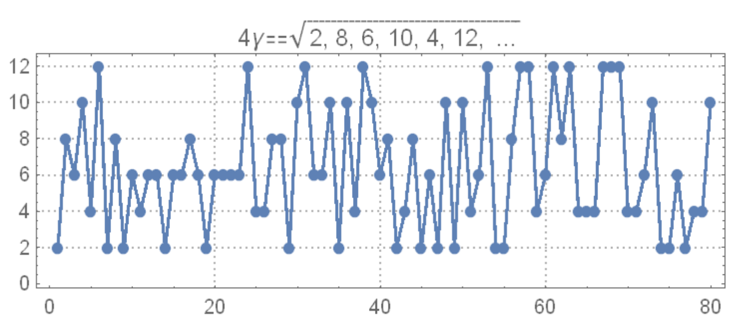
(18) $$\pi = \sqrt{6,12,6,6,2,2,10,4,4,2,8,10,12,6,12,6,8,4,10,6,\ldots}$$



(19) $$e = \sqrt{4,8,10,2,4,6,2,8,4,6,10,8,4,6,8,2,6,8,12,12,\ldots}$$



(20) $$\gamma = \frac{1}{4}\sqrt{2,8,6,10,4,12,2,8,2,6,4,6,6,2,6,6,8,6,2,6,\ldots}$$



IV. Fórmulas de Ramanujan

(21)
$$\frac{1}{1\ +}\frac{e^{-2\pi}}{1\ +}\frac{e^{-4\pi}}{1\ +}\frac{e^{-6\pi}}{1\ +\cdots}=\left(\sqrt{\frac{5+\sqrt{5}}{2}}-\frac{\sqrt{5}+1}{2}\right)e^{2\pi/5}$$
$$=\frac{1}{3}\sqrt{6,6,4,12,4,10,4,2,10,10,2,12,6,10,10,8,4,6,8,6,\ldots}$$

(22)
$$\frac{1}{1\ -}\frac{e^{-\pi}}{1\ +}\frac{e^{-2\pi}}{1\ -}\frac{e^{-3\pi}}{1\ +\cdots}=\left(\sqrt{\frac{5-\sqrt{5}}{2}}-\frac{\sqrt{5}-1}{2}\right)e^{\pi/5}$$
$$=\frac{1}{3}\sqrt{6,12,4,6,10,10,2,6,2,12,6,12,6,12,6,2,2,4,6,4,\ldots}$$

V. Otras representaciones

(23)
$$\pi=2\sqrt{1,1,0,0,2,0,0,1,1,1,2,1,1,0,2,\ldots}=2\sqrt{1+\sqrt{1+\sqrt[8]{2+\sqrt[8]{1+\sqrt{1+\sqrt{1+\cdots}}}}}}$$

donde

$$f_n=f(n)=\begin{cases}1 & si\ n=1\\ max\{m\in\{0,1,2\}:R(1,f_2,\ldots,f_{n-1},m+1)\le\pi/2\} & ,n\ge 2\end{cases}$$

(24)
$$\pi=3\sqrt{0,0,0,1,0,0,1,0,0,0,0,0,1,1,0,2,\ldots}=3\sqrt[16]{1+\sqrt[8]{1+\sqrt[64]{1+\sqrt{1+\sqrt[4]{2+\cdots}}}}}$$

donde

$$f_n = f(n) = \begin{cases} 0 & si\ n = 1 \\ max\{m \in \{0,1,2\} : R(0, f_2, \dots, f_{n-1}, m+1) \le \pi/3\} & ,n \ge 2 \end{cases}$$

(25)

$$\frac{1}{\gamma} = \sqrt{2,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,1,1,\dots} = \sqrt{2 + \sqrt[512]{1 + \sqrt[16]{1 + \sqrt[16]{1 + \sqrt{1 + \sqrt[8]{2 + \cdots}}}}}}$$

donde

$$f_n = f(n) = \begin{cases} 2 & si\ n = 1 \\ max\{m \in \{0,1,2\} : R(2, f_2, \dots, f_{n-1}, m+1) \le 1/\gamma\} & ,n \ge 2 \end{cases}$$

(26)

$$e = 2\sqrt{0,2,0,2,2,2,1,2,0,1,2,0,2,\dots} = 2\sqrt[4]{2 + \sqrt[4]{2 + \sqrt{2 + \sqrt{2 + \sqrt{1 + \sqrt{2 + \sqrt[4]{1 + \sqrt{2 + \cdots}}}}}}}}$$

donde

$$f_n = f(n) = \begin{cases} 0 & si\ n = 1 \\ max\{m \in \{0,1,2\} : R(0, f_2, \dots, f_{n-1}, m+1) \le e/2\} & ,n \ge 2 \end{cases}$$

VI. Número Pi

(27) $$\pi = \sqrt{7,6,2,6,6,5,5,2,4,6,3,4,\dots}$$

(28) $$\pi = \sqrt{8,2,1,0,1,0,2,0,0,2,2,1,\dots}$$

(29) $$\pi = \sqrt{8,1,4,2,6,5,4,6,4,2,4,6,\dots}$$

(30) $$\pi = \sqrt{2^3, 2^1, 2^1, 2^{-5}, 2^{-11}, 2^{-27}, 2^{-55}, 2^{-114}, \dots}$$

(31) $$\pi = \sqrt{9 + \frac{1}{\sqrt{1 + \frac{1}{\sqrt{9 + \frac{1}{\sqrt{2 + \cdots}}}}}}} = \sqrt{f_1 + \frac{1}{\sqrt{f_2 + \frac{1}{\sqrt{f_3 + \cdots}}}}}$$

$$f_n = \{9,1,9,2,2,1,28,5,1,9,469,3,\ldots\}$$

(32) $$\pi = \sqrt{6,10,19,27,29,32,42,45,56,67,75,94,109,122,138,144,151,152,172,\ldots}$$

(33) $$\pi = \sqrt{8+\sqrt[3]{5+\sqrt[4]{4+\sqrt[5]{7+\sqrt[6]{6+\sqrt[7]{1+\cdots}}}}}}$$

VII. Referencia

1. Jamie Johnson and Tom Richmond:"Continued Radicals".Department of Mathematics,Western Kentucky University.January 14,2005.To appear in the Ramanujan Journal.