A roadmap to the quark and lepton mass ratios

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The last six years have seen great strides in measuring the neutrino squared-mass splittings and heavy quark masses. It is therefore timely to reconsider the mass formulas introduced by the author in 2010, which then disagreed with the ratio of the neutrino squared-mass splittings.

I. MASS RATIOS

Let

$$\left.\begin{array}{l} n = 3 \\ f = 4.1 \end{array}\right\} \text{for the four leptonic mass ratios below,}$$

and

$$\left.\begin{array}{l} n = 1/3 \\ f = 10 \end{array}\right\} \text{for the four quark mass ratios below,}$$

where:

$$\frac{m_{\nu_3}^2}{m_{\nu_1}^2} f^{-0.5} = \frac{m_\tau}{m_e} = 4.1 n f^4 \qquad (1a)$$

$$\frac{m_{\nu_2}^2}{m_{\nu_1}^2} = \frac{m_\mu}{m_e} = n f^3 \qquad (1b)$$

$$\frac{m_s^2}{m_u^2} f^{-1.0} = \frac{m_t}{m_c} = 4.1 n f^2 \qquad (1c)$$

$$\frac{m_d^2}{m_u^2} = \frac{m_b}{m_c} = n f^1 \qquad (1d)$$

Table I shows that Eqs. (1a) and (1b) fit the 2014 CODATA [1] values for the tau- and muon-electron mass to one part in ∼2400 and one part in ∼40 000, respectively. And Tables II and III show how Eqs. (1c) and (1d) reproduce the quark masses and ratios reported by the Particle Data Group [2] within their limits of error.

TABLE I: The charged lepton mass ratios calculated using Eqs. (1a) and (1b) are compared against experiment.

Ratio	Calc.	Exp.
m_τ/m_e	$4.1^5 \times 3 \approx 3475.69$	3477.15 ± 0.31[a]
m_μ/m_e	$4.1^3 \times 3 \approx 206.763$	$206.768\,2826$[b]

[a] Ref. [1]. Fit to one part in ∼2400.
[b] Ref. [1]. Fit to one part in ∼40 000.

II. NEUTRINO SQUARED-MASS SPLITTINGS

Equations (1a) and (1b) determine the following ratio between neutrino squared-mass splittings:

$$\frac{m_{\nu_3}^2 - m_{\nu_1}^2}{m_{\nu_2}^2 - m_{\nu_1}^2} = \frac{4.1^{5.5} \times 3 - 1}{4.1^{3.0} \times 3 - 1} \approx 34.2 \quad . \qquad (2)$$

And a recent global analysis of solar, atmospheric, reactor, and accelerator neutrino data [3] gives

$$|\Delta m_{31}^2| = 2.457 \pm 0.047 \times 10^{-3}\ \Delta\text{eV}^2 \qquad (3a)$$

and

$$|\Delta m_{21}^2| = 7.5^{+0.19}_{-0.17} \times 10^{-5}\ \Delta\text{eV}^2 \quad . \qquad (3b)$$

These values form this ratio of neutrino squared-mass splittings

$$\frac{|\Delta m_{31}^2|}{|\Delta m_{21}^2|} = \frac{2.457 \times 10^{-3}\ \Delta\text{eV}^2}{7.5 \times 10^{-5}\ \Delta\text{eV}^2} \approx 32.8 \quad , \qquad (3c)$$

which is fit to within five per cent by Eq. (2).

III. NEW VERSUS OLD EQUATIONS

Equations (1a)–(1d) incorporate changes to the following set of similar equations, introduced in 2010 [4]:

$$\left(\frac{m_{\nu_3}}{m_{\nu_1}}\right)^2 = \frac{m_\tau}{m_e} = 4.1^5 \times 3 \qquad (4a)$$

$$\left(\frac{m_{\nu_2}}{m_{\nu_1}}\right)^2 = \frac{m_\mu}{m_e} = 4.1^3 \times 3 \qquad (4b)$$

$$\left(\frac{m_s}{m_u} \times 0.1\right)^2 = \frac{m_t}{m_c} \times 0.1 = 4.1^1 \times 3 \qquad (4c)$$

$$\left(\frac{m_d}{m_u}\right)^2 = \frac{m_b}{m_c} = 4.1^0 \times 3 \qquad (4d)$$

The above equations predicted $m_b/m_c = 3$, whereas the new equations predict $m_b/m_c = 10/3$. The new equations also resolve the "major discrepancy" — noted in 2010 [4]— between the above equations and the ratio of the observed neutrino squared-mass splittings.

Despite the new equations' simplicity they manage to fit today's improved measurements much better than the above equations fit 2010 data. Whether the new equations retain the simplicity of the above older equations the reader can best judge for himself. Note that the unusual constant 4.1, upon which Eqs. (1a)–(1c) all depend, occurs naturally in analogues of the Koide formula [5].

TABLE II: With the aid of Eqs. (1c) and (1d), the c- and b-quark masses are calculated from the top quark's mass of 173 210 ± 510 ± 710 MeV (from direct measurements) [2]. Below, these calculated masses are compared against their corresponding experimental values, which they fit to within one or two per cent.

Mass	Calc.	Exp.[a]
m_c	1267 MeV[b]	1275 ± 25 MeV
m_b	4225 MeV[c]	4180 ± 30 MeV

[a] Ref. [2]. It is important to recognize, however, that the experimental values for m_c and m_b are the "running" masses in the $\overline{\mathrm{MS}}$ scheme at $\mu = m_c$ and $\mu = m_b$, respectively.
[b] 173 210 MeV/$(4.1^1 \times 10 \times (10/3)) \approx 1267$ MeV.
[c] 173 210 MeV/$(4.1^1 \times 10) \approx 4225$ MeV.

TABLE III: With the aid of Eqs. (1c) and (1d), three light quark mass ratios are calculated. Below, these ratios are compared against their corresponding experimental values, which they fit within their limits of error.

Ratio	Calc.	Exp.[a]
m_u/m_d	$\sqrt{3/10} \approx 0.55$	0.38–0.58
m_s/m_d	$10\sqrt{4.1} \approx 20.2$	17–22
$\dfrac{m_s}{(m_u+m_d)/2}$	$\dfrac{10\sqrt{4.1}\sqrt{10/3}}{(1+\sqrt{10/3})/2} \approx 26.2$	27.5 ± 1.0

[a] Ref. [2].

[1] P. J. Mohr, D. B. Newell and B. N. Taylor, "CODATA Recommended Values of the Fundamental Physical Constants: 2014," arXiv:1507.07956.

[2] K. A. Olive *et al.* [Particle Data Group], "Review of particle physics," Chin. Phys. C **38**, 0900011 (2014) and 2015 update.

[3] M. C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, "Global Analyses of Neutrino Oscillation Experiments," arXiv:1512.06856 [hep-ph].

[4] J. S. Markovitch, "Symmetrically reproducing quark and lepton mass, charge, and generation" (2010) `http://www.vixra.org/abs/1010.0018`.

[5] J. S. Markovitch, "The Koide formula and its analogues" (2012) `http://www.vixra.org/abs/1207.0052`.