Let $//a, b, c, d, e \in \mathbb{Z}$
satisfying $1 < a < b < c < d < e$.
Prove that $\frac{1}{\text{lcm}(a,b)} + \frac{1}{\text{lcm}(b,c)} + \frac{1}{\text{lcm}(c,d)} + \frac{1}{\text{lcm}(d,e)} < 1$

PROOF $u = gcd(a, b)$
$v = gcd(b, c)$
$w = gcd(c, d)$
$z = gcd(d, e)$
Then , we need to prove
$u/(ab) + v/(bc) + w/(cd) + z/(de) < 1$
Suppose not,
Then,
$u/(ab) + v/(bc) + w/(cd) + z/(de) >= 1$
Now,
$u <= a$
$v <= b$
$w <= c$
$z <= d$
So,
$u/(ab) + v/(bc) + w/(cd) + z/(de) <= a/(ab) + b/(bc) + c/(cd) + d/(de) = 1/b + 1/c + 1/d + 1/e$
Now
$a, b, c, d, e \in \mathbb{Z}$
and,
$1 < a < b < c < d < e$
=¿
$a >= 2$
$b >= 3$
$c >= 4$
$d >= 5$
$e >= 6$
Thus,
$1/b + 1/c + 1/d + 1/e <= (1/3) + (1/4) + (1/5) + (1/6) = ((1/3) + (1/6)) + ((1/4) + (1/5)) = 1/2 + 1/4 + 1/5 < 1$
Thus,
$\frac{1}{\text{lcm}(a,b)} + \frac{1}{\text{lcm}(b,c)} + \frac{1}{\text{lcm}(c,d)} + \frac{1}{\text{lcm}(d,e)} < 1$
Hence Proved

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