Intuition-based AI for solutions of NP-complete problems.

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Abstract.
The challenge of this paper is to relate artificial intuition-based intelligence, represented by self-supervised systems, to solutions of NP-complete problems. By self-supervised systems we understand systems that are capable to move from disorder to order without external effort, i.e. in violation of the second law of thermodynamics. It has been demonstrated, [1], that such systems exist in the ***mathematical*** world: they are presented by ODE coupled with their Liouville equation, but they belong neither to Newtonian nor to quantum physics since they are capable to violate the second law of thermodynamics. That suggests that machines could not ***simulate*** intuition-based intelligence if they are composed only of physical parts, but without digital components. Nevertheless it was found such quantum-classical ***hybrids***, [1], that simulates some of self-supervised systems. The main achievement of this work is a demonstration that self-supervised systems can solve NP-complete problems in polynomial time by replacing an enumeration of exponentially large number of possible choices with a short cut provided by a non-Newtonian and non-quantum nature of self-supervised systems.

1.Introduction.

A human intelligence, and in particular, its most mysterious kind – intuition - has always been an enigma for physicists, and an obstacle for artificial intelligence. It was well understood that human behavior, and in particular, the decision making process, is governed by feedbacks from the external world, and this part of the problem was successfully simulated in the most sophisticated way by control systems. However, in addition to that, when the external world does not provide sufficient information, a human turns for "advise" to his experience, and that is associated with intuition. In other words, intuition is a phenomenon of the mind that describes the ability to acquire knowledge without inference or the use of reason.

In this paper, intuition-based intelligence is implemented by a feedback from the self-image (a concept adapted from psychology), and we will illustrate its physical model in connection with the decision-making process.

A decision making process can be modeled by a time evolution of a vector π whose components $\pi_i\left(i=1,2\ldots N\right)$ present a probability distribution over *N* different choices. The evolution of this vector can be written in the form of a Markov chain:

$$\pi_i(t+\tau)=\sum_{j=1}^{n}\pi_j(t)p_{ij},\qquad \sum_{i=1}^{n}\pi_i=1,\qquad \pi_i\geq 0,\quad i=1,2,..n \tag{1}$$

where p_{ij} is the transition matrix representing a decision making policy. If p_{ij} =const., the process (1) approaches some final distribution π^{∞} regardless of the initial state π^{o}. In particular, in the case of doubly stochastic transition matrix, i.e., when

$$\sum_{j=1}^{N}p_{ij}=1\quad and\quad \sum_{i=1}^{N}p_{ij}=1 \tag{2}$$

all the final choices become equally probable

$$\pi_i=\pi_j=1/N \tag{1}$$

i.e., the system approaches its thermodynamics limit which is characterized by the maximum entropy. When the external world is changing, such a rigid behavior is unsatisfactory, and the matrix p_{ij} has to be changed accordingly, i.e., $p_{ij} = p_{ij}(t)$. Obviously this change can be implemented only if the external information is available, and there are certain sets of rules for correct responses. However, in real world situations, the number of rules grows exponentially with the dimensionalities of external factors, and therefore, any man-made device fails to implement such rules in full.

The main departure from this strategy can be observed in human approach to decision-making process. Indeed, faced with an uncertainty, a human uses an intuition-based approach relying upon his previous experience and knowledge in the form of certain invariants or patterns of behavior that are suitable for the whole class of similar situations. Such ability follows from the fact that a human possesses a self-image, and interacts with it. This concept which is widely exploited in psychology has been known as far back as to ancient philosophers, but so far its mathematical formalization has never been linked to the decision making model (1).

First we will start with an abstract mathematical question: can the system (1) change its evolution, and consequently, its limit distribution, without any external "forces"? The formal answer is definitely positive. Indeed, if the transition matrix depends upon the current probability distribution

$$p_{ij} = p_{ij}(\pi_k) \tag{4}$$

then the evolution (1) becomes nonlinear, and it may have many different scenarios depending upon the initial state π^0. In particular case (4), it could "overcome" the second law of thermodynamics decreasing its final entropy by using only the "internal" resources. Indeed let us assume that the objective of the system is to approach the deterministic state

$$\pi_1 = 1, \quad \pi_2 = 0 \tag{5}$$

Then as shown in [1], if the feedback is chosen as

$$p_{11} = \frac{\pi_1^{\,2}}{2\pi_1^{\,2} - 2\pi_1 + 1} \qquad p_{12} = \frac{(1-\pi_1)^2}{2\pi_1^{\,2} - 2\pi_1 + 1}$$
$$p_{21} = \frac{(1+\pi_1)^2}{2\pi_1^{\,2} + 2} \qquad p_{22} = \frac{(1-\pi_1)^2}{2\pi_1^{\,2} + 2} \tag{6}$$

the evolution of the probability π_1 can be presented as:

$$\pi_1^{(n+1)} = \pi_1^{(n)} p_{11} + (1-\pi^{(n)}) p_{21} \tag{7}$$

in which p_{11} and p_{22} are substituted from Eqs. (6).

It is easily verifiable that

$$\pi_1^{\infty} = 1, \ \pi_2^{\infty} = 0 \tag{8}$$

i.e., the objective is achieved due to the "internal" feedback (6)

The implementation of the stochastic process which probabilities are described by the Markov chains (1) with the feedback (6) has been described in [1]. This stochastic process can be simulated by quantum recurrent nets (QRN). (See Fig. 1)

This QRN is described by the following set of difference equations with constant time delay

$$a_i(t+1) = \sigma_1\{\sum U_{ij} a_j(t)\}, \qquad i = 1.2...n \tag{9}$$

The curly brackets are intended to emphasize that σ_1 is to be taken as a measurement operation with the effect similar to those of a sigmoid function in classical neural networks.

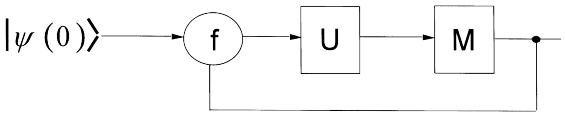


Fig. 1. A one-dimensional quantum recurrent network.

An initial state, $|\psi(0)\rangle$, is fed into the network, transformed under the action of a unitary operator, *U*, subjected to a measurement indicated by the measurement operator M{ }, and the result of the measurement is used to control the new state fed back into the network at the next iteration. One is free to record, duplicate or even monitor the sequence of measurement outcomes, as they are all merely bits and hence constitute classical information. Moreover, one is free to choose the function used during the reset phase, including the possibility of adding no offset state whatsoever. Such flexibility makes the QRN architecture remarkably versatile. To simulate a Markov process, it is sufficient to return just the last output state to the next input at each iteration.

From physical viewpoint, the example described above can be associated with a particle that escapes from the Brownian motion using its own "internal effort" in violation of the second law of thermodynamics while the entropy decreases from infinity to zero. In other words, as a result of interaction with his own image and without any "external" enforcement, the decision maker can depart from the thermodynamics limit (3) of his performance "against the second law." Obviously, the enforcement in the form of the feedback (6) is an ***internal*** one since the image (1) is the uniquely defined product of the dynamical evolution (9), i.e., such a "free will" effort is in a disagreement with the second law of thermodynamics. The philosophical consequences of this result have been discussed in [1].

The objective of this work is to select an appropriate model from the class of ODE coupled with their Liouville equation to demonstrate its capability to solve the problem of search in unsorted database that is an NP-complete problem.

2. Self-supervised systems.

In order to illuminate specific features of intuition-based intelligence, we will start with control dynamics that described by a system of ODE:

$$\frac{d\mathbf{v}}{dt} = \mathbf{F}[\mathbf{v}, U] \qquad (10)$$

Here

$\mathbf{v} = v_1, v_2, \ldots v_n$ is the vector of state variables to be controlled,

$u = u_1, u_2, \ldots u_m$ is the control vector that represents ***external*** actuators.

Let us compare the control system Eq. (10) with the following system

$$\frac{d\mathbf{v}}{dt} = \mathbf{F}[\rho(\mathbf{v})] \qquad (11)$$

where the probability ρ is introduced via the Liouville equation corresponding to Eq. (11)

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{F}) = 0 \qquad (12)$$

It describes the continuity of the probability density flow originated by the error distribution

$$\rho_0 = \rho(t = 0) \qquad (13)$$

in the initial condition of ODE (12).

Comparison of Eqs.(10) and (11) shows that they have similar structure, and the role of the external actuator U in the control system (10) is played by the term $\rho(\mathbf{v})$ in the system (11). However the origins of these actuators are fundamentally different: the actuator U represents an external force, while the actuator $\rho(\mathbf{v})$ is an internal one. Indeed it is defined by Eq. (12) that, in turn, uniquely follows from Eq. (11). That is why the system (11),(12) can be called self- controlled, or self-supervised.

From the physical viewpoint, the feedback from the Liouville equation is a fundamental step in our approach: in Newtonian dynamics, the probability never explicitly enters the equation of motion. In addition to that, the Liouville equation generated by Eq. (11) is nonlinear with respect to the probability density ρ

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \{\rho \mathbf{F}[\rho(\mathbf{V})]\} = 0 \qquad (14)$$

and therefore, the system (11),(12) departs from Newtonian dynamics. However although it has the same topology as quantum mechanics (since now the equation of motion is coupled with the equation of continuity of probability density as it does in the Madelung version of the Schrödinger equation), it does not belong to it either. Indeed Eq. (11) is more general than the Hamilton-Jacoby equation: it is not necessarily conservative, and **F** is not necessarily the quantum potential although further we will impose some restriction upon it that links **F** to the concept of information. The relation of the system (11), (12) to Newtonian and quantum physics is illustrated in Fig.2.

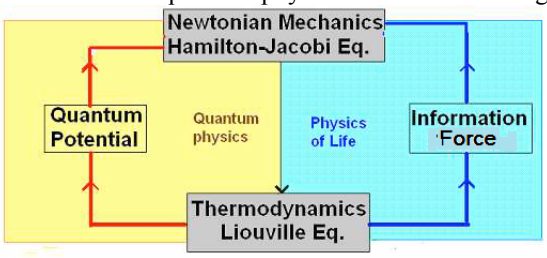


Figure 2. Classic Physics, Quantum Physics and Physics of Life

Remark. Here and below we make distinction between the random *variable v(t)* and its *values V* in probability space.

3. Selected self-supervised dynamical system.

In this section we will concentrate on a special type of the self-supervised system Eqs. (11),(12).

We will start with derivation of an auxiliary result that illuminates departure from Newtonian dynamics. For mathematical clarity, we will consider here a one-dimensional motion of a unit mass under action of a force *f* depending upon the *velocity v* and time *t* and present it in a dimensionless form

$$\dot{v} = f(v,t) \qquad (15)$$

referring all the variables to their representative values $v_0, t_0, etc.$

If initial conditions are not deterministic, and their probability density is given in the form

$$\rho_0 = \rho_0(V), \qquad where \quad \rho \geq 0, \quad and \quad \int_{-\infty}^{\infty} \rho dV = 1 \qquad (16)$$

while ρ is a *single- valued* function, then the evolution of this density is expressed by the corresponding Liouville equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial V}(\rho f) = 0 \qquad (17)$$

The solution of this equation subject to initial conditions and normalization constraints (16) determines probability density as a function of *V* and *t*:

$$\rho = \rho(V,t) \qquad (18)$$

In order to deal with the constraint (16) let us integrate Eq. (17) over the whole space assuming that $\rho \to 0$ at $|V| \to \infty$ and $|f| < \infty$. Then

$$\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\rho dV = 0, \int_{-\infty}^{\infty}\rho dV = const, \qquad (19)$$

Hence, the constraint (16) is satisfied for $t > 0$ if it is satisfied for $t = 0$.

Let us now specify the force f as a feedback from the Liouville equation

$$f(v,t) = \varphi[\rho(v,t)] \qquad (20)$$

and analyze the motion after substituting the force (20) into Eq.(16)

$$\dot{v} = \varphi[\rho(v,t)], \qquad (21)$$

Although the theory of ODE does not impose any restrictions upon the force as a function of space coordinates, the Newtonian physics does: equations of motion are never coupled with the corresponding Liouville equation. Moreover, it can be shown that such a coupling leads to non-Newtonian properties of the underlying model. Indeed, substituting the force f from Eq. (20) into Eq. (17), one arrives at the *nonlinear* equation of evolution of the probability density

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial V}\{\rho\varphi[\rho(V,t)]\} = 0 \qquad (22)$$

Let us now demonstrate the destabilizing effect of the feedback (20). For that purpose, it should be noticed that the derivative $\partial\rho/\partial v$ must change its sign at least once, within the interval $-\infty < v < \infty$, in order to satisfy the normalization constraint (16).

But since

$$Sign\frac{\partial \dot{v}}{\partial v} = Sign\frac{d\varphi}{d\rho}Sign\frac{\partial \rho}{\partial v} \qquad (23)$$

there will be regions of *v* where the motion is unstable, and this instability generates randomness with the probability distribution guided by the Liouville equation (22). It should be noticed that the condition (23) may lead to exponential or polynomial growth of *v* (in the last case the motion is called neutrally stable, however, as will be shown below, it causes the emergence of randomness as well if prior to the polynomial growth, the Lipchitz condition is violated).

3.1. Emergene of self-generated stochasticity. In order to illustrate mathematical aspects of the concepts of Liouville feedback in systems under consideration as well as associated with it instability and randomness, let us take the feedback (20) in the form

$$f = -\sigma^2\frac{\partial}{\partial v}\ln\rho, \qquad (24)$$

to obtain the following equation of motion

$$\dot{v} = -\sigma^2\frac{\partial}{\partial v}\ln\rho, \qquad (25)$$

This equation should be complemented by the corresponding Liouville equation (in this particular case, the Liouville equation takes the form of the Fokker-Planck equation)

$$\frac{\partial\rho}{\partial t} = \sigma^2\frac{\partial^2\rho}{\partial V^2} \qquad (26)$$

Here *v* stands for a particle velocity, and σ^2 is the constant diffusion coefficient.

The solution of Eq. (26) subject to the sharp initial condition

$$\rho = \frac{1}{2\sigma\sqrt{\pi t}}\exp(-\frac{V^2}{4\sigma^2 t}) \qquad (27)$$

describes diffusion of the probability density, and that is why the feedback (24) will be called a diffusion feedback.

Substituting this solution into Eq. (25) at ***V=v*** one arrives at the differential equation with respect to *v (t)*

$$\dot{v} = \frac{v}{2t} \qquad (28)$$

and therefore,

$$v = C\sqrt{t} \qquad (29)$$

where C is an arbitrary constant. Since $v=0$ at $t=0$ for any value of C, the solution (29) is consistent with the sharp initial condition for the solution (27) of the corresponding Liouvile equation (26). The solution (29) describes the simplest irreversible motion: it is characterized by the "beginning of time" where all the trajectories intersect (that results from the violation of Lipcsitz condition at $t=0$, Fig3), while the backward motion obtained by replacement of t with $(-t)$ leads to imaginary values of velocities. One can notice that the probability density (27) possesses the same properties.

It is easily verifiable that the solution (29) has the same structure as the solution of the Madelung equation [1], although the dynamical system (25), (26) is not quantum! The explanation of such a "coincidence" is very simple: the system (25), (26) has the same dynamical topology as that of the Madelung equation where the equation of conservation of the probability is coupled with the equation of conservation of the momentum (see Fig.2). As will be shown below, the system (25), (26) neither quantum nor Newtonian, and we will call such systems quantum-inspired, or self-supervised.

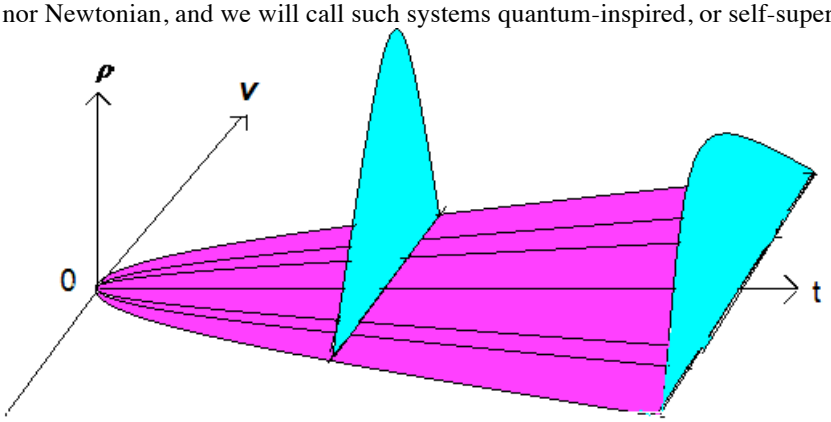


Figure 3. Stochastic process and probability density.

Further analysis of the solution (29) demonstrates that it is *unstable* since

$$\frac{d\dot{v}}{dv} = \frac{1}{2t} > 0 \qquad (30)$$

and therefore, an initial error always grows generating *randomness*. Initially, at $t=0$, this growth is of infinite rate since the Lipchitz condition at this point is violated

$$\frac{d\dot{v}}{dv} \to \infty \ at \qquad t \to 0 \qquad (31)$$

This type of instability has been introduced and analyzed in [2]. The unstable equilibrium point ($v = 0$) has been called a terminal repeller, and the instability triggered by the violation of the Lipchitz condition – a non-Lipchitz instability. The basic property of the non- Lipchitz instability is the following: if the initial condition is infinitely close to the repeller, the transient solution will escape the repeller during a *bounded* time while for a regular repeller the time would be *unbounded*. Indeed, an escape from the simplest regular repeller can be described by the exponent $v = v_0 e^t$. Obviously $v \to 0$ if $v_0 \to 0$, unless the time period is unbounded. On the contrary, the period of escape from the terminal repeller (29) is bounded (and even infinitesimal) if the initial condition is infinitely small, (see Eq. (31)).

Considering first Eq. (29) at fixed C as a sample of the underlying stochastic process (26), and then varying C, one arrives at the whole ensemble characterizing that process, (see Fig. 3). One can verify that, as follows from Eq. (27), [3], the expectation and the variance of this process are, respectively

$$\bar{v} = 0, \quad \tilde{v} = 2\sigma^2 t \qquad (32)$$

The same results follow from the ensemble (29) at $-\infty \le C \le \infty$. Indeed, the first equality in (32) results from symmetry of the ensemble with respect to *v=0*; the second one follows from the fact that

$$\widetilde{v} \propto v^2 \propto t \qquad (33)$$

It is interesting to notice that the stochastic process (29) is an alternative to the following Langevin equation, [3]

$$\dot{v} = \Gamma(t),\ \overline{\Gamma} = 0, \quad \widetilde{\Gamma} = \sigma \qquad (34)$$

that corresponds to the *same* Fokker-Planck equation (26). Here $\Gamma(t)$ is the Langevin (random) force with zero mean and constant variance σ.

Thus, the emergence of self-generated stochasticity is the first basic non-Newtonian property of the dynamics with the Liouville feedback.

3.2. Second law of thermodynamics. In order to demonstrate another non-Newtonian property of the systems considered above, let us start with the dimensionless form of the Langevin equation for a one-dimensional Brownian motion of a particle subjected to a random force, [3]

$$\dot{v} = -kv + \Gamma(t), \quad <\Gamma(t)>=0, \quad <\Gamma(t)\Gamma(t')>=2\sigma\delta(t-t'),\ [\Gamma] = 1/s \qquad (35)$$

Here v is the dimensionless velocity of the particle (referred to a representative velocity v_0), k is the coefficient of a linear damping force, $\Gamma(t)$ is the Langevin (random) force per unit mass, $\sigma > 0$ is the noise strength. The representative velocity v_0 can be chosen, for instance, as the initial velocity of the motion under consideration.

The corresponding continuity equation for the probability density ρ is the following Fokker-Planck equation

$$\frac{\partial \rho}{\partial t} = k\frac{\partial (V\rho)}{\partial V} + \sigma\frac{\partial^2 \rho}{\partial V^2}, \quad \int_{-\infty}^{\infty} \rho\, dV = 1 \qquad (36)$$

Obviously without external control, the particle cannot escape the Brownian motion.

Let us now introduce a new force (referred to unit mass and divided by v_0) as a Liouville feedback

$$f = \sigma \exp\sqrt{D}\,\frac{\partial}{\partial v}\ln\rho, \qquad [f] = 1/s \qquad (37)$$

Here D is the dimensionless variance of the stochastic process $D(t) = \int_{-\infty}^{\infty} \rho V^2\, dV$,

Then the new equation of motion takes the form

$$\dot{v} = -kv + \Gamma(t) + \sigma \exp\sqrt{D}\,\frac{\partial}{\partial v}\ln\rho, \qquad (38)$$

and the corresponding Fokker-Planck equation becomes ***nonlinear***

$$\frac{\partial \rho}{\partial t} = k\frac{\partial (V\rho)}{\partial V} + \sigma(1-\exp\sqrt{D})\frac{\partial^2 \rho}{\partial V^2}, \qquad \int_{-\infty}^{\infty} \rho\, dV = 1 \qquad (39)$$

Obviously the diffusion coefficient in Eq. (39) is *negative.* Multiplying Eq. (39) by V^2, then integrating it with respect to V over the whole space, one arrives at ODE for the variance D

$$\dot{D} = 2[\sigma(1-\exp\sqrt{D}) - kD] \qquad (40)$$

Thus, as a result of *negative* diffusion, the variance D monotonously vanishes regardless of the initial value $D(0)$. It is interesting to note that the time T of approaching the point $D=0$ is finite

$$T = \frac{1}{2}\int_{D(0)}^{0} \frac{dD}{\sigma(1-\exp\sqrt{D}) - kD} \le \frac{1}{2\sigma}\int_{0}^{\infty} \frac{dD}{\exp\sqrt{D} - 1} = \frac{\pi}{6\sigma} \qquad (41)$$

This terminal effect is due to violation of the Lipchitz condition, at $D = 0$, [6].

Let us review the structure of the force (37): it is composed only out of the probability density and its variance, i.e. out of the components of the conservation equation (39); at the same time, Eq. (39) itself is generated by the equation of motion (38). Consequently, the force (37) is *not* an external force. Nevertheless, it allows the particle to escape from the Brownian motion using its own "internal effort". It would be reasonable to call the force (37) an *information force* since it links to information rather than to energy.

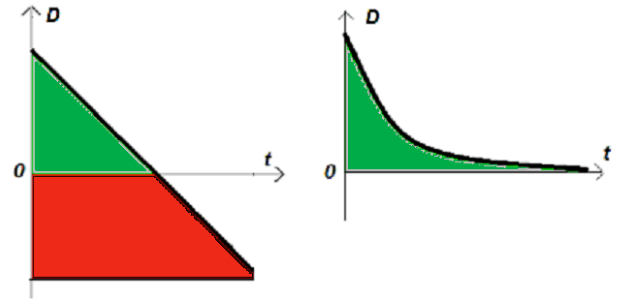
One may ask why the negative diffusion was chosen to be nonlinear. Let us turn to the linear version of Eq. (40)

$$\frac{\partial \rho}{\partial t} = -\sigma^2 \frac{\partial^2 \rho}{\partial V^2}, \qquad \int_{-\infty}^{\infty} \rho\, dV = 1 \qquad (42)$$

and discuss the negative diffusion in more details. As follows from the linear equivalent of Eq. (40)

$$\dot{D} = -2\sigma, i.e.\ D = D_0 - 2\sigma t < 0 \quad at \quad t > D_0/(2\sigma) \qquad (43)$$

Thus, eventually the variance becomes negative, and that disqualifies Eq. (43) from being meaningful. As shown in [4], the initial value problem for this equation is ill-posed: its solution is not differentiable at any point. Therefore, a *negative diffusion must be nonlinear* in order to protect the variance from becoming negative, Fig.5. The proof of the ill-possednes of linear negative diffusion can be found in [4].



Linear negative diffusion. *Nonlinear negative diffusion.*

Figure 4. Negative diffusion.

In the next sub-section we will demonstrate again that formally the dynamics introduced above does not belong to the Newtonian world; nevertheless its self-supervising capability may associate such a dynamics with a potential model for intelligent behavior. For that purpose we will turn to even simpler version of this dynamics by removing the external Langevin force and simplifying the information force.

In 1945 Schrödinger wrote in his book "What is life": *"Life is to create order in the disordered environment against the second law of thermodynamics"*. The self-supervised dynamical system introduced above is fully consistent with this statement. Indeed, consider a simplified version of Eqs. (35) and (36)

$$\dot{v} = \sigma\sqrt{D}\frac{\partial}{\partial v}\ln\rho, \qquad (44)$$

$$\frac{\partial \rho}{\partial t} = -\sigma\sqrt{D}\,\frac{\partial^2 \rho}{\partial V^2}, \quad \int_{-\infty}^{\infty} \rho\, dV = 1 \qquad (45)$$

Removal of the Langevin and damping forces makes the particle ***isolated***. Nevertheless the particle has a capability of moving from disorder to order. For demonstration of this property we will assume that the Langevin force was suddenly removed at $t = 0$ so that the initial variance $D_0 > 0.$ Then

$$\dot{D} = -2\sigma\sqrt{D} \qquad (46)$$

whence $$D = (\sqrt{D_0} - \sigma t)^2 \qquad (47)$$

As follows from Eq. (47), as a result of *internal, self-generated* force

$$F = \sigma\sqrt{D}\,\frac{\partial}{\partial v}\ln \rho, \qquad (48)$$

the Brownian motion gradually disappears and then vanishes abruptly:

$$D \to 0, \qquad \dot{D} \to 0, \qquad \frac{d\dot{D}}{dD} \to \infty \qquad at \qquad t \to \frac{\sqrt{D_0}}{\sigma} \qquad (49)$$

(3.36)

Thus the probability density shrinks to a delta-function at. Consequently, the entropy $H(t) = -\int_V \rho \ln \rho dV$ decreases down to zero, and that violates the second law of thermodynamics.

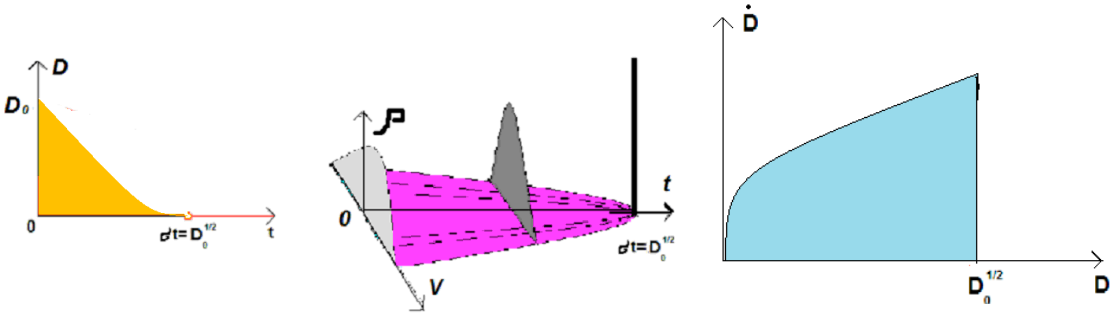


Figure 5. Vanishing Brownian motion.

3.3. Self-supervised system for solving NP problem. Let us introduce the following inhomogeneous version of Eq. (44)

$$\dot{v} = -\frac{1}{\rho}[a^2\,\frac{\partial \rho}{\partial v} - e^{-\omega t}\sum_{k=1}^{k=m}\frac{l}{2\pi k}\sin\frac{2\pi k}{l}v], 0 \le v \le l, t > 0, \qquad (50)$$

Then the corresponding Liouville equation takes the form of an inhomogeneous parabolic equation subject to an aperiodic force

$$\frac{\partial \rho}{\partial t} - a^2 \frac{\partial^2 \rho}{\partial V^2} = e^{-\omega t} \sum_{k=1}^{k=m} \cos \frac{2\pi k}{l} v \qquad (51)$$

It should be noticed that the sums in Eqs. (50) and (51) are finite, and they do not represent even truncated Fourier expansions, while all the harmonic terms are equally powerful. Obviously this system is still self-supervising, but ***not*** isolated any more.

We will solve this equation subject to the following initial and boundary conditions

$$\rho(v,0) = \delta(v = 0.5l), \qquad \frac{\partial \rho}{\partial V}(0,t) = 0, \qquad \frac{\partial \rho}{\partial V}(l,t) = 0 \qquad (52)$$

and the normalization constraint

$$\int_0^l \rho(\zeta,t) d\zeta = 1 \qquad (53)$$

Before writing down the solution, we will verify satisfaction of the constraint (53). For that purpose, let us integrate Eq. (2) with respect to v

$$\frac{\partial}{\partial t} \int_0^l \rho d\zeta - a^2 \int_0^l \frac{\partial^2 \rho}{\partial V^2} = e^{-\omega t} \sum_{k=1}^{k=m} \int_0^l \cos \frac{2\pi k}{l} \zeta d\zeta \qquad (54)$$

As follows from the boundary conditions in (52),

$\frac{\partial \rho}{\partial V}|_{V=0} = \frac{\partial \rho}{\partial V}|_{V=l} = 0$, and therefore, $\int_0^l \frac{\partial^2 \rho}{\partial V^2} d\zeta = 0$; obviously, $\int_0^l \cos \frac{2\pi k}{l} \zeta d\zeta = 0$ as well.

Hence, $\frac{\partial}{\partial t} \int_0^l \rho d\zeta = 0$. But, according to the initial condition in (52) $\int_o^l \rho d\zeta = \int_o^l \delta(\zeta = 0.5l) d\zeta = 1$ at *t=0.*

Therefore, the normalization constraint will be satisfied for all $t \geq 0$

Exploiting the superposition principle for the linear equation (2), we will represent the solution as a sum of free and forced components. These components are, respectfully

$$\rho_1 = \frac{2}{l} \sum_{j=1}^{\infty} e^{-(\frac{\pi j}{l})^2 a^2 t} \cos \frac{\pi j}{l} V \cdot \cos \frac{\pi j}{2l} + \frac{1}{l} \qquad (55)$$

$$\rho_2 = \sum_{k=1}^{m} [\int_0^t e^{-(\frac{\pi k}{l})^2 a^2 (t-\tau) - \omega t} d\tau] \cos \frac{2\pi k}{l} V \quad if \quad \omega \neq (\frac{\pi k}{l})^2 a^2 \qquad (56)$$

$$\rho_2^* = t e^{-\omega t} \cos \frac{2\pi k}{l} V \quad if \quad \omega = (\frac{\pi k}{l})^2 a^2 \qquad (57)$$

Here we will be interested only in the case (57) that represents a resonance between two aperiodic terms, namely: exponentially decaying force and exponentially decaying free motion. Indeed, the solution (57) has a well-pronounced maximum at

$$t^* = 1/\omega \quad if \quad \omega = (\frac{\pi k}{i})^2 a^2 \qquad (58)$$

while the solutions (55) and (56) are monotonously decay.

Let us now reaffirm the scenario of transition from deterministic to random state described by Eqs. (25),(26). For that purpose, rewrite Eq. (55) in a different, but an equivalent form (based upon reflections from the boundaries)

$$\rho_1 = \frac{1}{2a\sqrt{\pi t}} \sum_{n=-\infty}^{\infty} \{ e^{-\frac{(V-\zeta+2nl)^2}{4a^2 t}} + e^{-\frac{(V+\zeta-2nl)^2}{4a^2 t}} \} \qquad (59)$$

It can be verified that for vanishingly small times

$$\rho_1 \to \frac{1}{2a\sqrt{\pi t}} \exp(-\frac{V^2}{4a^2 t}), \qquad and \qquad \rho_2 * \to 0 \qquad at \qquad t \to 0 \quad (60)$$

and therefore, the transition scenario remains the same.

It should be noticed that prior to running Eq. (50), the analytical solution of Eq. (51) in the form of the sum of Eqs. (55), (56), and (59) is to be substituted for ρ.

Before moving to *n*-dimensional case, we will discuss the basic properties of the solution to Eqs. (25), (26). Although there are many similarities to quantum systems, we will concentrate again on superposition since it will be essential for the described approach.

In quantum mechanics, any observable quantity corresponds to an eigenstate of a Hermitian linear operator. The linear combination of two or more eigenstates results in quantum superposition of two or more values of the quantity. If the quantity is measured, the projection postulate states that the state will be randomly collapsed onto one of the values in the superposition (with a probability proportional to the square of the amplitude of that eigenstate in the linear combination). Let us compare the behavior of the model of self-supervised systems from that viewpoint. As follows from Eq. (29), all the particular solutions intersect at the same point *v=0* at *t=0,* and that leads to non-uniqueness of the solution due to violation of the Lipcshitz condition. Therefore, the same initial condition *v=0* at *t=0* yields infinite number of different solutions forming a family (29); each solution of this family appears with a certain probability guided by the corresponding Fokker-Planck equation, Fig. 6 .

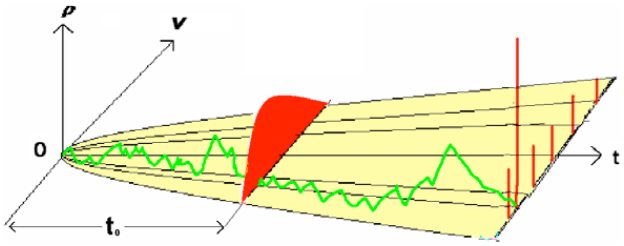


Figure 6. Resonance in the probability space

Turning to *n*-dimensional case we have

$$\dot{v}_i = -\frac{1}{\rho}[a^2 \frac{\partial \rho}{\partial v_i} - e^{-\omega t} \sum_{k=1}^{k=m} \frac{l}{2\pi k} \sin \frac{2\pi k}{l_i} v_i], \qquad 0 \le v_i \le l_i, \qquad (61)$$

$$\frac{\partial \rho}{\partial t} - a^2 \sum_{i=1}^{n} \frac{\partial^2 \rho}{\partial V_i^2} = e^{-\omega t} \sum_{i=1}^{n} \sum_{k=1}^{m} \cos \frac{2\pi k}{l_i} V_i, \qquad (62)$$

$$i = 1,2,...n, \quad k = 1,2...m.$$

Eq. (62) has the following eigen-values of decay

$$\omega_{j_{k_1} \ldots j_{k_n}} = (a^2 \pi^2 \sum_{i=1}^{i=n} \frac{j_{k_i}^2}{l_i^2}), j = 1,2,..n, i = 1,2,..n, k = 1,2...m \qquad (63)$$

If the excitation ω in Eqs. (61) and (62) are selected as following

$$\omega = \omega_{j_{k_1},\dots j_{k_n}} = (a^2\pi^2 \sum_{i=1}^{i=n} \frac{j_{k_i}^2}{l_i^2}), j = 1,2,\dots n, i = 1,2,\dots n, k = 1,2\dots m \quad (64)$$

it will generate resonance with the eigen-value (63), and the corresponding "decay" will dominate over the rest of decays; in terms of Eq. (62) this means that the probability density ϱ will tend to its maximum at

$$t^* = 1/\omega_{j_{k_1},\dots j_{k_n}} = (a^2\pi^2 \sum_{i=1}^{i=n} \frac{j_{k_i}^2}{l_i^2})^{-1}, \quad (65)$$

along that trajectory which is the "winning" solution of the system (61), Fig. 6. The value of this maximum is irrelevant, but its location is important: it is given by the following values of the coordinates

$$v_i^* = v_i(t^*), i = 1,2,..n. \quad (66)$$

4. Search in unsorted database.

In this section we will apply the self-supervise dynamics introduced above to solve the problem of search in unsorted database by improving the Grover's algorithm. Grover's algorithm is a quantum algorithm that finds with high probability the unique input to a black box function that produces a particular output value, using just O($N^{1/2}$) evaluations of the function, where N is the size of the function's domain. It was originated by Lov Grover in 1996. The analogous problem in classical computation cannot be solved in fewer than O(N) evaluations (because, in the worst case, the Nth member of the domain might be the correct member). Unlike other quantum algorithms, which may provide exponential speedup over their classical counterparts, Grover's algorithm provides only a quadratic speedup. However, even quadratic speedup is considerable when N is large. At roughly the same time that Grover published his algorithm, it was proved that no quantum solution to the problem can evaluate the function fewer than O($N^{1/2}$) times, so Grover's algorithm is asymptotically optimal [5]. However that proof does not contradict our claim to improve the Grover algorithm since the self-supervised dynamics is neither quantum nor Newtonian: it represents a quantum-classical hybrid capable to violate the second law of thermodynamics.

The algorithm formulated as the following. Consider an unsorted data-base consisting of n^n items labeled with a string of numbers $j_1, j_2,\dots j_n$ as shown in Eq.(63) for $n=m$. Obviously a label includes permutations of these numbers. Turning to Eqs. (61), notice that each solution to this system can be labeled similarly if the winning solution in Eqs. (62) has its maximum at a point with the coordinates $v_1^*, v_2^*,\dots v^*_n$ defined by Eqs (65). Then one can introduce the forced excitation defined by Eq. (64) that provides the resonant solution of Eq. (62), and as a result, the coordinates $v_1^*, v_2^*,\dots v^*_n$ of this maximum will represent the address of the item in question. According to Eq. (64), the number of possible values of forced excitations ω providing required resonances is equal to n^n, and that is exactly the number of the items to be retrieved. Therefore, each item can be retrieved by the corresponding resonance with the forced excitation (having the values from the set (64)) with the probability that dominates over the probabilities for wrong addresses to occur. Strictly speaking, a non-resonance solution has a smaller, but non-zero probability to occur; then by a few number of Bernoulli trials, the most probable solution can be found. Indeed, the probability of success ρ_s and failure ρ_f after the first trial is, respectively

$$\rho_s = \bar{\psi}_1, \qquad \rho_f = 1 - \bar{\psi}_1 \quad (67)$$

Then the probability of success after M trials is

$$\rho_{sM} = 1 - (1 - \bar{\psi})^M \to 1 \qquad at \qquad M \to \infty \quad (68)$$

Therefore, after *polynomial* number of trials, one arrived at the solution to the problem (unless the function ψ is flat).

Let us now briefly review the procedure of the retrieval. Assume that the label of the item to be found is $\omega_{j_{k_1}\ldots j_{k_n}}$. The first step is to write down the analytical solution to Eq. (62) that consists of free and forced motions as in the one-dimensional case:

$$\rho = \rho_1 + \rho_2 * \tag{69}$$

Here

$$\rho_1 = \sum_{j=1}^{\infty} C_j e^{-(\frac{\pi j}{l})^2 a^2 t}, \tag{70}$$

$$\rho_2 * = te^{-\omega * t} \sum_{i=1}^{n} \sum_{k=1}^{m} \cos\frac{2\pi k}{l_i} V_i \tag{71}$$

where C_j are constants to be found from the initial conditions, and

$$\omega * = \omega_{j_{k_1},\ldots j_{k_n}} = (a^2\pi^2 \sum_{i=1}^{i=n} \frac{j_{k_i}^2}{l_i^2}), j = 1,2,\ldots n, i = 1,2,\ldots n, k = 1,2\ldots m \tag{73}$$

The second step is to substitute the solution (69) into Eq. (61). The third step is to run the system (61), measure the values of v_i at $t = 1/\omega *$ and obtain the address of the item in the form of a string of coordinates $v_1 *, v_2 *, \ldots v *_n$, Fig. 7.

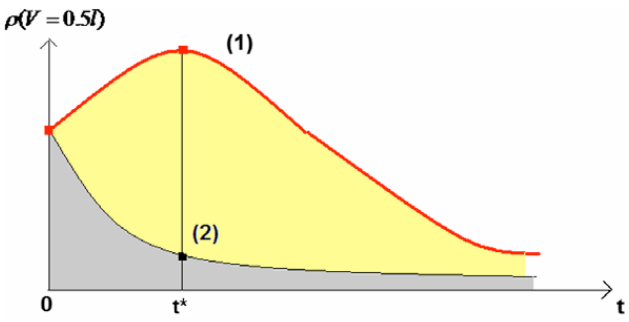


Figure 7. Maximum probability, selected (1) and not selected (2) items.

It should be noticed that the capacity of the unsorted database is of order $O(n^n)$ i.e. exponential with respect to its dimensionality *n*, while all the resources providing its implementation are of order *O(n)*, i.e. polynomial since the number of equations in the system (61) is *n*, and the number of terms in the analytical solution to Eq.(62) (to be substituted into Eqs. (61)) are of the order *O(n)* as well. Indeed, the infinite sum in Eq. (70) converges very fast to equal distribution of the probability density, and practically, only the forced component of the solution represented by Eq. (71) is important, and this component contains $O(n^2)$ number of terms.

5. **Discussion and conclusion**

The basic idea of this paper is to create a new kind of dynamical systems that would preserve superposition of random solutions, while allowing one to measure its state variables using classical methods. In other words, such a hybrid system would reinforce the advantages and minimize limitations of both quantum and classical aspects. These systems have been analyzed in [1] and [6]. It has been shown there that along with preservation of superposition, such an important property of quantum systems as direct-product-

decomposability in hybrids is lost. Let us recall that the main advantage of this property in terms of quantum information is in blowing up an input of a polynomial complexity into an output of exponential complexity, with no additional resources required, Fig. 8.

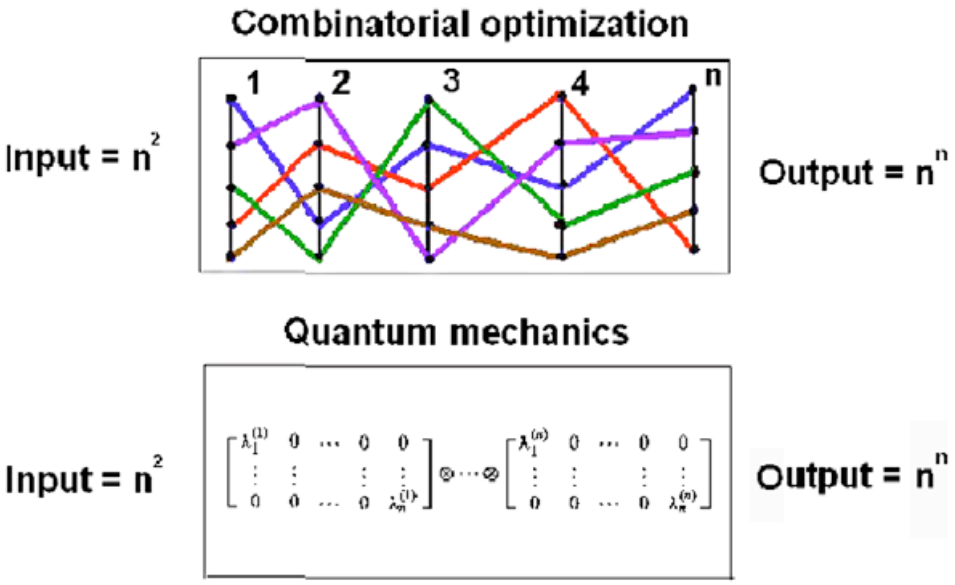


Figure 8. Mapping combinatorial optimization to quantum mechanics.

The greatest challenge of our approach in this paper was in finding a ‘‘replacement” for the fundamental property of the Schrödinger equation in quantum-classical hybrids. It turns out that eigen-values of linear parabolic PDE possess similar property. Indeed, consider a linear *n*-dimensional parabolic PDE subject to boundary conditions. Then the eigen-values corresponding to each variable form a sequence of monotonously increasing positive numbers $\lambda_i^{(1)} \ldots \lambda_i^{(n)}$. However, each linear combination of these eigen-values represents another eigen-value of the solution, and that is the same ‘‘combinatorial explosion” that is illustrated in Fig. 8. Due to that property, for each *n*-string-number label, one can find an excitation force that activates the corresponding eigen-value. The second challenge was to satisfy a global (normalization) constraint imposed upon the probability density (in addition to boundary conditions). That was achieved via a special form of the excitation force. Finally these work ads a positive comment to a question posed in [7]: Can **NP-complete problems** be solved efficiently in the physical universe? The answer given by the author, Scott Aaronson, is negative. To our opinion, it could be positive if we complement the “physical world” with self-supervised systems capable to violate the second law of thermodynamics in order to find short cuts to solutions of combinatorial problems. Another evidence of a “mysterious” power of self – supervise systems has been demonstrated in [6] where a different type of the ODE that belongs to the same class of self-supervising systems finds the global maximum of an integrable, but not necessarily differentiable function. The idea of the algorithm is very simple: introduce a positive function to be maximized as the probability density to which the solution is attracted. Then the larger value of this function will have the higher probability to appear. Special attention is paid to simulation of integer programming, NP-complete problems and information retrieval.

Thus the challenge of this paper is to relate artificial intuition-based intelligence, represented by self-supervised systems, to solutions of NP-complete problems. By self-supervised systems we understand systems that are capable to move from disorder to order without external effort, i.e. in violation of the second law of thermodynamics. It has been demonstrated, [1], that such systems exist in the ***mathematical*** world: they are presented by ODE coupled with their Liouville equation, but they belong neither to Newtonian nor to quantum physics since they are capable to violate the second law of thermodynamics. That suggests that machines could not ***simulate*** intuition-based intelligence if they are composed only of

physical parts, but without digital components. Nevertheless it was found such quantum-classical ***hybrids***, [1], that simulates some of self-supervised systems. The main achievement of this work is a demonstration that self-supervised systems can solve NP-complete problems in polynomial time by replacing an enumeration of exponentially large number of possible choices with a short cut provided by a non-Newtonian and non-quantum nature of self-supervised systems.

References

1. Zak,M., Non-Newtonian aspects of artificial intelligence. Foundations of Physics , Vol.46, May 2016
2. Zak, M., "Terminal Model of Newtonian Dynamics," Int. J. of Theoretical Physics, No.32, 159-190, 1992
3. Risken,H., 1989, The Fokker-Planck Equation, Springer, NY.
4. Zak, M., Introduction to quantum-inspired intelligence, PHYSICS ESSAYS **23,** 1.201
5. Bennett C.H., Bernstein E., Brassard G., Vazirani U. (1997). "The strengths and weaknesses of quantum computation". *SIAM Journal on Computing*. 26(5): 1510–1523
6. Zak,M.,Quantum-inspired maximizer, JOURNAL OF MATHEMATICAL PHYSICS **49,** 042702 ,2008
7. Aaronson, S., NP-complete Problems and Physical Reality, Institute for Advanced Study, Princeton,NJ