Divergence-Free Quantum Electrodynamics

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Abstract

We present results of applying our divergence-free effective action quantum field theory techniques to the theory of Maxwell-Dirac electrodynamics. This describes the interaction of the electromagnetic photon field with a charged fermion (the electron). This gives another example of the applicability of our divergence-free methods to a system with Abelian gauge invariance. Results of loop computations are given, implementing the principle of gauge-covariant momentum-space quantization.

1 Introduction

Besides evading loop divegences in all quantum field theories while preserving fundamental gauge and coordinate invariances, the divergence-free effective action approach to quantum field theory[1],...,[8] can be formulated in a much more powerful and simplified manner based on the principle of gauge-covariant as well as coordinate-covariant momentum-space quantization. Here we give an example of applying the latter formulation to the theory of a massless Maxwellian (photon) gauge field and massive matter represented by a charged fermionic spinor field (the electron).

After presenting the Lagrangian and the associated Feynman rules, we give the results of applying our divergence-free methods to two loop computations, again suppressing much of the detailed derivations (given together with applications to non-Abelian gauge and gravitational theories, in comprehensive reports elsewhere[9]), and conclude with a brief discussion.

2 The Lagrangian of Maxwell-Dirac Electrodynamics and Graphical Rules

The Lagrangian of Maxwell-Dirac electrodynamics is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \bar{\Psi}\left(i\gamma^{\mu}\nabla_{\mu} - m\right)\Psi \tag{1}$$

Here, $F_{\mu\nu}$ is the field tensor, given in terms of the 4-potential by

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} \tag{2}$$

We also have

$$\nabla_\mu \Psi = \partial_\mu \Psi - ieA_\mu \Psi \tag{3}$$

Here, Ψ is a Dirac spinor, $\bar{\Psi}$ is the corresponding Dirac adjoint, e is the (dimensionless) electromagnetic coupling constant. Notice that whereas the charged (complex) fermion $\Psi, \bar{\Psi}$ has a mass m, the photon field A_μ is massless, which fact is associated with Abelian gauge invariance with respect to the infinitesimal transformations (with real parameter ω):

$$\delta\Psi = ie\omega\Psi \qquad \delta\bar{\Psi} = -ie\omega\bar{\Psi} \qquad \delta A_\mu = \partial_\mu \omega \tag{4}$$

The above Lagrangian can be rewritten in the following form

$$\begin{cases} \frac{1}{2} A_\mu \left(\partial^2 \eta_{\mu\nu} - \partial_\mu \partial_\nu\right) A_\nu \\ +\bar{\Psi}\left(i\gamma^\mu \partial_\mu - m\right)\Psi \\ +e\bar{\Psi}\gamma^\mu \Psi A_\mu \end{cases} \tag{5}$$

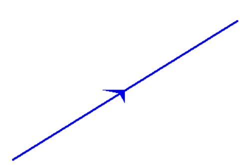
displaying the bilinear terms, and the trilinear coupling.

Now according to the scheme of the effective action, the fields would be split like $A_\mu \to A_\mu + \mathcal{A}_\mu$, $\Psi \to \Psi + \psi$, and $\bar{\Psi} \to \bar{\Psi} + \bar{\psi}$, where $\mathcal{A}, \psi, \bar{\psi}$ are *virtual* fields. Accordingly, the virtual vector $\mathcal{A}_\mu$ will be constrained with the (gauge-invariant) condition $\partial_\mu \mathcal{A}_\mu = 0$. Hence, we derive the following basic graphical rules (in Minkowskian momentum space):

- For every internal or bare propagator of the charged fermion (depicted with an arrow) with momentum p, we write

  $$-\frac{1}{-p^2 + m^2}\left(\gamma_\mu p^\mu + m\right)$$

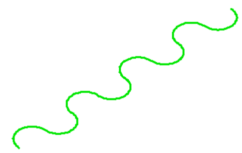
  Notice that this is just the bare propagator. But how is gauge invariance guaranteed in the perturbative development via appropriate insertions, and related matters, is explained in detail, in earlier letters, and other comprehensive reports[1],...,[9]. This is depicted like

- For every internal photon propagator (to be depicted by a wavy line), with momentum q, we write

  $$\frac{1}{-q^2 + m^2}\left\{\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right\}$$

  This is depicted like

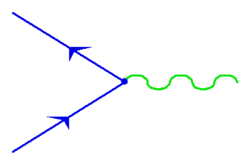


Notice that we have included a projection operator corresponding to the constraint applied to the virtual vector, and we have included a mass-regulating parameter as well. That the mass-regulating parameter taken here is equal to the mass of the charged fermion should not be of much concern. In general the two masses could be taken different, and the arbitrary mass of the virtual photon is *the subject of fundamental physical interpretation* in detailed and comprehensive works[9]. However, for simplicity in presenting the results of this letter, we proceed as indicated above. We stress that the above prescription is gauge invariant.

- For the bare trilinear vertex, we write

$$-e\gamma_\mu$$

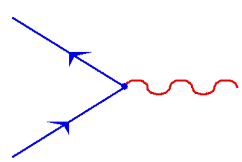
Here, p is the momentum of the incoming charged fermion, and r is the momentum of the incoming photon, according to the following depiction:



- Whereas the above coupling concerns the virtual photon field only, we also have an external field-strength insertion (in terms of $F^{\mu\nu}$ rather than the potential A^μ) on fermion propagators,

$$-e\frac{i}{2}\gamma_{\mu\nu}$$

corresponding to the following depiction



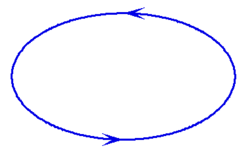
- We must associate a factor of i for each propagator, a factor of i for each vertex, and an overall factor of $-i$ for each graph.
- We must associate a factor of -1 for each fermion loop.
- We must supply the appropriate combinatoric factors for each graph.
- Most importantly, we must supply the appropriate *regularizing parameters* and the corresponding *pole-removing operators*, together with the gamma functions factors, and Feynman parameter combinations, all according to our divergence-free methods.[1]

In the following section, we shall display associated graphics and computational results suppressing all details.

3 Vacuum Contributions

3.1 One-Loop Contributions

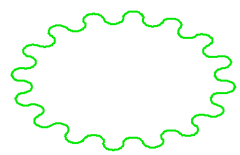
We have two 1-loop vacuum contributions. The 1st contribution corresponds the fermionic loop graph



This gives

$$\frac{m^4}{16\pi^2}\left\{-3+2\ln(m^2)\right\} \tag{6}$$

The 2nd contribution corresponds the photonic loop graph



This gives

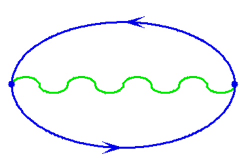
$$\frac{3m^4}{128\pi^2}\left\{3-2\ln(m^2)\right\} \tag{7}$$

The total one-loop vacuum contribution is

$$\frac{5m^4}{128\pi^2}\left\{-3+2\ln(m^2)\right\} \tag{8}$$

3.2 Two-Loop Contributions

There is only one 2-loop vacuum contribution. This corresponds to the graph



This gives (approx.)

$$\frac{e^2m^4}{806400\pi^4}\left\{35609+480\ln(m^2)-37800\ln^2(m^2)\right\} \tag{9}$$

3.3 Fixing the Vacuum

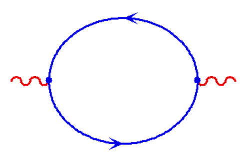
Consistency requires that we must determine the value of $ln(m^2)$ by setting the total vacuum contribution equal to zero, and inverting the perturbative series. The value of $ln(m^2)$ begins with $\frac{3}{2}$ and receives serial contributions in the coupling constant (here e^2). In more realistic models, when more than one mass parameter is present in the vacuum contributions, the mass parameter scale to be determined perturbatively would correspond to a central $ln(m^2)$. The physical interpretation of such a central infrared-regulating mass, such as associated with the virtual photon (the virtual gluons and the virtual graviton as well) would be very significant at the fundamental level[9].

4 Photonic Bilinear Contributions

According to our scheme, the effective Lagrangian terms would contain the photon field either through the covariant derivative of fermionic matter, or through curvature (field strength) forms. Hence, the following results for photonic bilinears would need to be multiplied by $F^{\mu\nu}F^{\lambda\rho}$ in order to construct the effective Lagrangian terms. The reader should be able to do that easily. Much more details are given elsewhere[9].

4.1 One-Loop Contributions

There is only one, 1-loop contribution for the photonic bilinears. This corresponds to the graph

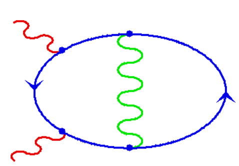


and gives

$$\frac{e^2}{32\pi^2}\left\{\eta_{\mu\lambda}\eta_{\nu\rho} - \eta_{\nu\lambda}\eta_{\mu\rho}\right\}\ln(m^2) \tag{10}$$

4.2 Two-Loop Contributions

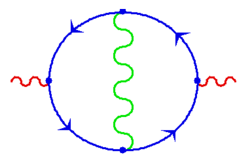
There are two, 2-loop contributions for the photonic bilinears. The 1st corresponds to the graph



and gives (approx.)

$$\frac{e^4}{6451200\pi^4}\left\{\eta_{\mu\lambda}\eta_{\nu\rho}-\eta_{\nu\lambda}\eta_{\mu\rho}\right\}\left\{-39803+27960\ln(m^2)\right\} \tag{11}$$

The 2nd corresponds to the graph



and gives (approx.)

$$-\frac{e^4}{645120\pi^4}\left\{\eta_{\mu\lambda}\eta_{\nu\rho}-\eta_{\nu\lambda}\eta_{\mu\rho}\right\}\left\{-7657+5778\ln(m^2)\right\} \tag{12}$$

The total 2-loop contribution to the photonic bilinears becomes (approx.)

$$-\frac{e^4}{6451200\pi^4}\left\{\eta_{\mu\lambda}\eta_{\nu\rho}-\eta_{\nu\lambda}\eta_{\mu\rho}\right\}\left\{-36767+29820\ln(m^2)\right\} \tag{13}$$

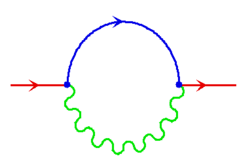
5 Fermionic Bilinear Contributions

The following results for 1- and 2-loop contributions to the fermionic bilinears should be interpreted according to the followings. Whereas the momentum of the effective fermion is denoted by r, the constant of momentum-independent terms correspond to corrections to the mass term $m\bar{\Psi}\Psi$. Contributions that have the factor $(\gamma\cdot r)$ would correspond to corrections to the kinetic term $\bar{\Psi}(i\gamma\cdot\partial)\Psi$, and according to our scheme would actually correspond to the gauge-invariant form $\bar{\Psi}(i\gamma\cdot\nabla)\Psi$. Contributions with a quadratic momentum factor (r^2) would correspond to the Lagrangian term $\partial^\mu\bar{\Psi}\partial_\mu\Psi$, better in fact to the gauge-invariant form $\nabla^\mu\bar{\Psi}\nabla_\mu\Psi$.

In the followings, we give the 1-loop contributions to second order in the external momentum, and give the 2-loop contributions to first order.

5.1 One-Loop Contributions

There is only one, 1-loop contribution to the fermionic bilinears. This corresponds to the graph

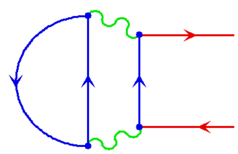


and gives

$$\frac{3e^2m}{16\pi^2}\left\{1+\ln(m^2)\right\}+\frac{3e^2}{64\pi^2}\gamma\cdot r-\frac{e^2}{32\pi^2}\frac{r^2}{m} \tag{14}$$

5.2 2-Loop Contributions

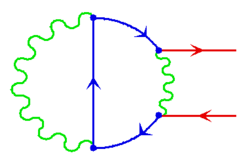
There are three, 2-loop contributions to the fermionic bilinears. The 1st corresponds to the graph



and gives (approx.)

$$\frac{e^4 m}{322560\pi^4}\left\{3281 - 2940\ln(m^2)\right\} + \frac{e^4}{6451200\pi^4}\left\{-18211 + 15780\ln(m^2)\right\}\gamma\cdot r \quad (15)$$

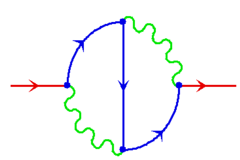
The 2nd corresponds to the graph



and gives (approx.)

$$\frac{3e^4 m}{10240\pi^4}\left\{-41 + 34\ln(m^2)\right\} + \frac{e^4}{20480\pi^4}\left\{-109 + 90\ln(m^2)\right\}\gamma\cdot r \quad (16)$$

The 3rd corresponds to the graph



and gives (approx.)

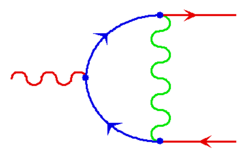
$$\frac{e^4 m}{860160\pi^4}\left\{4929 + 16012\ln(m^2)\right\} + \frac{37e^4}{1720320\pi^4}\left\{359 - 676\ln(m^2)\right\}\gamma\cdot r \quad (17)$$

The total 2-loop contribution to the fermionic bilinear is

$$\frac{e^4 m}{2580480\pi^4}\left\{10039 + 50220\ln(m^2)\right\} - \frac{e^4}{25804800\pi^4}\left\{10939 + 198660\ln(m^2)\right\}\gamma\cdot r \quad (18)$$

6 Trilinear Vertex

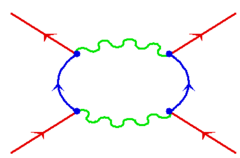
For the photon-fermion vertex in one-loop, that is the contribution to the effective Lagrangian term of the form $\frac{i}{2}\bar{\Psi}\gamma_{\mu\nu}\Psi F^{\mu\nu}$, there is one contribution corresponding to the graph



However, this contribution gives a vanishing result.

7 Fermionic Quartilinear Contributions

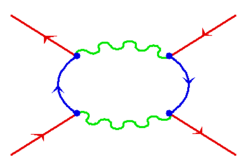
Here we give the results for computing the one-loop contributions to the effective vertex that is quartic (and neutral) in the charged fermionic fields. We have two contributions. The 1st contribution corresponds to the graph



and gives

$$\frac{3e^4}{64\pi^2 m^2} + \frac{e^4}{384\pi^2 m^2}\gamma_{\mu\nu} \otimes \gamma^{\mu\nu} + \frac{e^4}{384\pi^2 m^2}\gamma_{\mu\nu\lambda} \otimes \gamma^{\mu\nu\lambda} \tag{19}$$

The 2nd corresponds to the graph



and gives

$$-\frac{e^4}{32\pi^2 m^2} - \frac{e^4}{192\pi^2 m^2}\gamma_{\mu\nu} \otimes \gamma^{\mu\nu} + \frac{e^4}{192\pi^2 m^2}\gamma_{\mu\nu\lambda} \otimes \gamma^{\mu\nu\lambda} \tag{20}$$

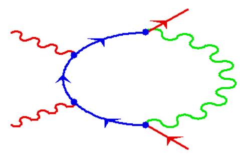
For the total 1-loop contribution we have

$$-\frac{e^4}{16\pi^2 m^2} - \frac{e^4}{96\pi^2 m^2}\gamma_{\mu\nu} \otimes \gamma^{\mu\nu} + \frac{e^4}{96\pi^2 m^2}\gamma_{\mu\nu\lambda} \otimes \gamma^{\mu\nu\lambda} \tag{21}$$

Notice that the above results are given to 0th order in external momenta. The meaning of the above results, in terms of effective Lagrangian terms, should be clear. The first term corresponds to an effective $(\bar{\Psi}\Psi)^2$ contribution, the second like $(\bar{\Psi}\gamma_{\mu\nu}\Psi)^2$, and the third gives $(\bar{\Psi}\gamma_\mu\gamma_5\Psi)^2$ (using $\gamma_{\mu\nu\lambda} \sim \epsilon_{\mu\nu\lambda\rho}\gamma^\rho\gamma_5$).

8 Photon-Fermion Quartilinear Contributions

Here we give the results of computing the 1-loop contributions to the effective photon-fermion quartic vertex. There is only one contribution corresponding to the graph



and gives

$$\frac{e^4}{128\pi^2 m^3}\epsilon_{\mu\nu\lambda\rho}\gamma_5 - \frac{e^4}{128\pi^2 m^3}\left\{\eta_{\mu\lambda}\eta_{\nu\rho} - \eta_{\nu\lambda}\eta_{\mu\rho}\right\} \tag{22}$$

Again, the above result is given to 0th order in the external momenta. The first term corresponds to an effective Lagrangian vertex of the form

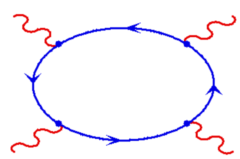
$$\epsilon_{\mu\nu\lambda\rho}F^{\mu\nu}F^{\lambda\rho}\bar{\Psi}\gamma_5\Psi$$

The second term corresponds to

$$(F_{\mu\nu})^2\bar{\Psi}\Psi$$

9 Photon-Photon Quartilinear Contributions

There is just one contribution, in 1-loop, to the effective photon-photon quartilinear vertex. This corresponds to the graph



This gives the effective Lagrangian terms

$$\frac{3e^4}{3200\pi^2 m^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - \frac{e^4}{800\pi^2 m^4}F_{\mu\nu}F_{\lambda\rho}F^{\mu\lambda}F^{\nu\rho} \tag{23}$$

10 Discussion

The foregoing work should have made it clear that our perfectly consistent divergence-free effective action scheme for quantum field theory, combined with the principle of momentum-space gauge-covariance, is a powerful and a very economical approach to the development of Feynman-like perturbative development.

Whereas we have given, in the present letter, results that pertain to Maxwell-Dirac quantum electrodynamics, a theory with Abelian gauge invariance, very much more impressive demonstrations can also be given for non-Abelian gauge theories as well as Einstein-like gravidynamic theories.[9]

References

[1] N.S. Baaklini, "Effective Action Framework for Divergence-Free Quantum Field Theory", *N.S.B. Letters*, **NSBL-QF-010**; http://www.vixra.org/abs/1312.0056

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[3] N.S. Baaklini, "The Divergence-Free Effective Action for Quantum Electrodynamics", *N.S.B. Letters*, **NSBL-QF-015**, http://www.vixra.org/abs/1401.0013

[4] N.S. Baaklini, "Framework for the Effective Action of Quantum Gauge and Gravitational Fields", *N.S.B. Letters*, **NSBL-QF-011**, http://www.vixra.org/abs/1401.0121

[5] N.S. Baaklini, "Graphs and Expressions for Higher-Loop Effective Quantum Action", *N.S.B. Computing*, **NSBC-QF-005**, http://www.vixra.org/abs/1402.0085

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[8] N.S. Baaklini, "Divergence-Free Scalar Electrodynamics", *N.S.B. Letters*, **NSBL-QF-045**, http://www.vixra.org/abs/1604.0341

[9] *Divergence-Free Quantum Field Theory* by N.S. Baaklini (Complete Updated Development)

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