Deciphering the fine structure constant

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Abstract

In order to explain the strongly differing forces of the fundamental interactions, we use optimal consecutive exponentiation (power towers) as a mathematical instrument for the optimization of spacetime volumes, and thus for information storage in the universe.
From this approach it is possible to derive and calculate the value of the fine structure constant as **137. 035 999 099 966**.

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Dimensions and interactions in the universe: What is it about?

Keywords: Information, information volume, information storage, information economy, spacetime volume, dimensions, optimization, power towers, fundamental interactions, wave-particle-dualism, relativistic corrections, Lamb shift, fine structure constant

- It is about the question how information is best stored in the universe.
  Space-time on the tiniest scale being two-dimensional
  we calculate with two- and four-dimensional space-time volumes.
  Information density means information per space-time volume.

- When we compare power towers as a mathematical instrument
  of optimization of information storage in space-time
  there are three transitions to ever higher power towers.

- The transition values correlate with the coupling parameters
  of the fundamental interactions: The comparison shows relationships
  between these strengths as well as with the dimensions in the universe.

- The transition values can be used for calculating the storage capacity
  and information density of a two- and four-dimensional world.

- The space-time volume formed by the transition values differs
  from the four-dimensional information volume shown by the CODATA-value
  of the fine structure constant by a significant amount $\alpha^{3/2}$
  which can be interpreted as vacuum energy fluctuations:
  $$2w_3 * e^{(\ln w_1 \cdot \ln w_2)} + \alpha^{3/2} = \alpha^4$$

- Using this difference we can calculate
  the exact value of the fine structure constant: **137. 035 999 099 966**.

- This α-value describes volume and density of the electromagnetic field
  as a system of unmoving two-dimensional electron
  interacting with vacuum energy fluctuations.

Deciphering the fine structure constant

I. The hierarchy of the fundamental forces

"The belief in the ultimate simplicity and unity
behind the rules that constrain the universe
leads us to expect that there exists
a single unchanging pattern behind the appearances.
Under different conditions this single pattern will crystallize
into superficially distinct patterns that show up
as the four separate forces governing the world around us."
(John Barrow, The Constants of Nature, 2002, p.55)

Two things catch one's eye in connection with the creation of the universe:

- that the number of dimensions is in agreement
  with the number of fundamental forces: four, and
- that the forces of fundamental interactions differ extremely.

Force	Coupling parameter
Strong Force	1 / 3.3 … 5
Weak Force	1/30
Electromagnetic	1/137.036
Gravity	10^{-39}

"QED-like at short distance r ≤ 0.1 fm, quarks are tightly bound: $\alpha_S \approx$ **0.2 … 0.3**"
(Franz Muheim, https://www2.ph.ed.ac.uk/~muheim/teaching/np3/lect-qcd.pdf)

"$\alpha_W = G^2/4\pi \sim$ **1/30** … The intrinsic strength of the weak interaction
is actually greater than that of the electromagnetic interaction. At low energies
it appears weak owing to the massive propagator." (Tina Potter, The Weak Force …,
https://www.hep.phy.cam.ac.uk/~chpotter/particleandnuclearphysics/mainpage.html)

„Why is nature so hierarchical? Why is the difference between the strength
of the strongest and the weakest force so huge?
The hierarchy problem contains two challenges. The first is to determine
what sets the constants, what makes ratios large. The second is how they stay there."
(Lee Smolin, The Trouble with Physics, 2006, p.70/71)

According to the Margolus-Levitin theorem and to Landauer's postulate that "*information is physical*" there is a relationship between information and energy.

„It seems that *information* is not an abstract concept invented by the human mind. It is a real, physical thing with real, physical consequences."

(Jim Baggott, Farewell to Reality, 2013, p.245)

" … what quantum gravity is all about: information and entropy, densely packed."

(Leonard Susskind, The Black Hole War, 2008)

We interpret the **dimension-less value of the fine structure constant**, as well as of the other interactions, to be an expression of the **density of information**.

The natural constants (e, h, c) constituting the fine structure constant being maximum (physical) values in their field we assume that α is a **result of (mathematical) optimization** itself.

The enormous differences in size of the forces of the four interactions suggest that the reason has to do with logarithmics.
So we use an exponential approach going beyond simple exponentiation to find out about the information economy of the universe.

II. Use of optimal power towers

We use the hyper-operator tetration
in the form of **optimal consecutive exponentiation (optimal power towers)**
as a mathematical instrument for the optimization
of volumes or densities of information storage in the universe.

An optimal power tower is the result
of dividing a substrate and exponentiating its fragments to an optimum:
For a_1+a_2+ …. we get a_1^a_2^ …. → maximum!
For a_1^a_2^ …. we get a_1+a_2+ …. → minimum!

The table shows the maximum values gained by exponentiating fragments of a substrate.
To be read as: an original value of 7 gives a maximum when separated into three fragments resp. levels, basis plus two exponents:

1,161.167 = 1.78098^2.64544^2.57358,
as the maxima gained by two:
83.119 = 3.2246^3.7754,
as well as by four fragments:
18.77 = 1.61^2.041^1.986^1.363,
are by far lower than the one gained by three fragments.

Substrate	**2 fragments**	**3 fragments**	**4 fragments**
3	2.029		
3.93862	**3.93862**		
4	**4.135**		
6	**27.129**	17.768	
6.26217	**36.01442**	**36.01442**	
7	83.119	**1,161.167**	18.77
7.5	151.056	**117,286**	2,564.8
7.6783	187.974	**1,296,106**	**1,296,106**
8	280.905	$4.481 * 10^{8}$	$\mathbf{2.2974 * 10^{27}}$
8.3	411.875	$1.228 * 10^{12}$	$\mathbf{1.7432 * 10^{225}}$

The two tables above and below show the **transition points between rising power towers**:

S = 2.20144+1.73718 = W = 2.20144^1.73718 = **3.93862** **w_1**

S = 2.98913+3.27304 = 1.86504 +2.36373+ 2.03338 = 6.26217
W = 2.98913^3.27304 = 1.86504^(2.36373^2.03338) = **36.01442** **w_2**

S= 1.7265+ 2.8883+3.06352 = 1.2427+ 1.842+ 2.4321+ 2.1615 = 7.6783
W= 1.7265^(2.8883^3.06352) = 1.2427^(1.842^(2.4321^2.1615)) = **1,296,106** **w_3**

The values w_1, w_2 and w_3 show the transitions in consecutive exponentiation from one power tower to the next, from which in each respective case a resulting value can be reached more economically, i.e. with less exponentiation substrate for the same result than in the lower power tower.

The three transition values are two-fold optimal values:
First as optimal mathematical-numerical values within a power tower height (number of exponential levels of a power tower),
second as transition values where the next height comes into action.

These transition values can be calculated numerically. They are fix values, result of simple comparative calculation which is physically the basis of the optimization process in information storage.

They create a basic mathematical structure of multidimensional spacetime volumes based on information economy which fits exactly upon the wave-particle-dualism and allows a very exact calculation of α.

III. An exponential basis for the relative strengths of the fundamental interactions

Now we bring mathematics and physics together

and try to find out

if the mathematically optimized transition values

have any physical connection

with the coupling parameters of the fundamental interactions,

here of the fine structure constant.

> „The enormous usefulness of mathematics in the natural sciences
> is something bordering on the mysterious and there is no rational explanation for it."
> (Eugene Wigner: The unreasonable effectiveness of mathematics in the natural sciences,
> Communications in Pure and Applied Mathematics, Vol.13, No.1, 1960)

Comparing the coupling parameters of the fundamental interactions

by putting them on a common exponential basis,

the table below shows that they are connected

- first, with each other via the transition values
  between optimal power towers,

- second, with the dimensions in the universe
  (corresponding to the faculties and exponents, marked in red).

Interaction	**Coupling parameter** (inverse value)		exponential
Strong	**3.3 … 5**		
Weak	**30**		
W_1	3.93862	e^1.3708	e^(1!*1.3708^1)
W_2	36.01442	e^(1*1.3708*2*1.3072)	e^(2!*1.3386²)
Electromagn.	**137.036**	e^(1*1.3708*2*1.3072*1*1.3729)	e^(2!*1.3500³)
W_3	1296106	e^(1*1.3708*2*1.3072*3*1.3091)	e^(3!*1.3287³)
Gravity	**$1.7*10^{38}$**	e^(1*2*3*4*1.3839^4)	e^(4!*1.3839⁴)

IV. <u>The transition values as spacetime volumes</u>

> "Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."
> (1908; https://en.wikiquote.org/wiki/Hermann_Minkowski)

> "… Spacetime on the Tiniest Scale May Be Two-Dimensional:
> … recent work in loop quantum gravity, high temperature string theory,
> renormalization group analysis applied to general relativity and other areas
> of quantum gravity research,
> <u>all hints at a two dimensional spacetime on the smallest scale</u>.
> In most of these cases, the number of dimensions simply collapses
> in a process called spontaneous dimensional reduction as the scale reduces."
> (https://www.technologyreview.com/s/420717/why-spacetime-on-the-tiniest-scale-may-be-two-dimensional/)

Our approach to explain a possible connection
between the transition values and the strengths of fundamental interactions
is to use the **transition values** and their combinations **as spacetime volumes**.

Based on the assumptions that

- quantum gravity is about densely packed information,
- the particle-wave dualism can be seen
  as a phenomenon integrating two- and four-dimensionality,
- the figures for faculties and exponents (table on page 4)
  correspond with the number of dimensions,
- spontaneous dimensional transitions, like e.g. spontaneous dimensional reduction as the scale reduces, happen abruptly in no time, and
- the natural constants that form the fine structure constant are invariable,

we postulate that these spontaneous **dimensional transitions**
depend on the **density of information** stored in a space-time volume.

The question is - if there is any sense in creating such spacetime volumes -
which combinations by mathematical operators
(addition, multiplication, exponentiation, hyper-operator tetration)
correspond to the physical values.

V. The three transition values as integral parts of α

The three transition values are two-fold optimal values:

- First as optimal mathematical-numerical values within a power tower height (number of exponential levels of a power tower),
- second as transition values where the next height comes into action.

First significant finding

If we write, as indicated in the table on page 4,

$w_1 = 3.93862 = e$^(1*1.3708) and $w_2 = 36.01442 = e$^(1*1.3708*2*1.3072),

we can rewrite

137.036 = e^(1*1.3708*2*1.3072*1*1.3729) = **e**^(**1*1.3708*2*1.3072*1*1.3708***1.001485)

= $\mathbf{e}^{\ln w1 * \ln w2 * 1.001485}$ = $\mathbf{e}^{\ln w1 * \ln w2}$ *(1 + 1/**137.036**),

with $e^{\ln w1 * \ln w2} = w_1^{\ln w2} = w_2^{\ln w1} = 136.03985$. This result

- follows the mathematical dimensional structure based on figure **e**,
- makes the transition values w_1 and w_2 alternately exchangeable, and
- differs from α by its reciprocal, that is **1/α**.

Second significant finding

As relativistic corrections occur via even-numbered powers of α

we assume that there is another 1/α-difference

hidden somewhere as a binomial complement, and indeed:

- When we double the third transition value w_3 (2*1296106 = 2592212)

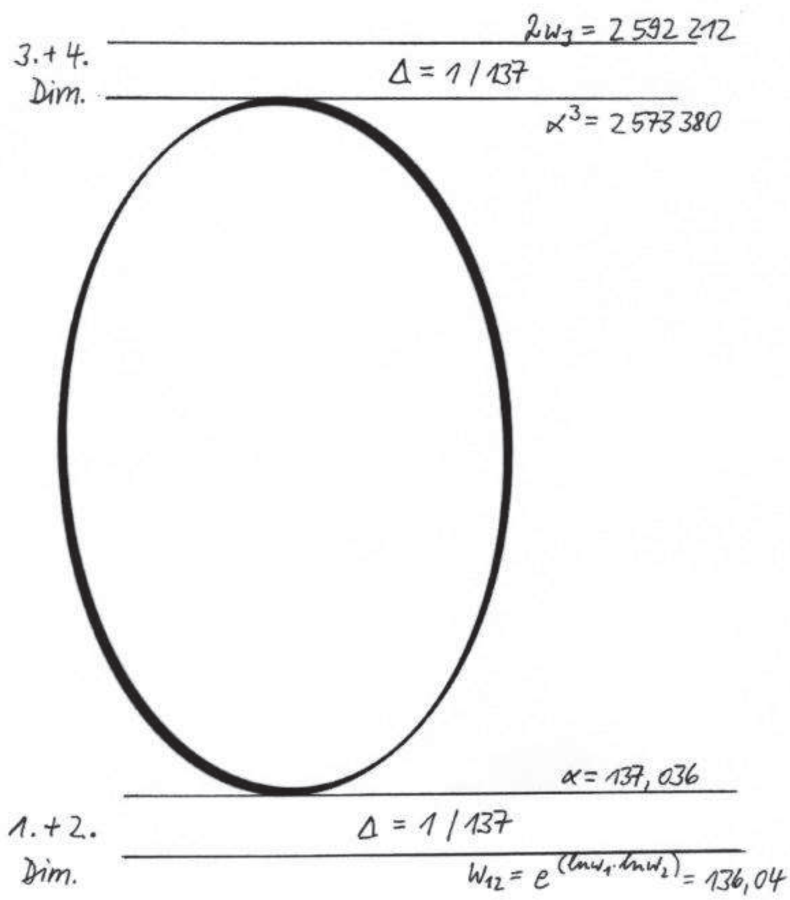
  the product differs from the third power of α (α^3 = 2573380)

  again by the reciprocal **1/α**.

The significant differences of the modified transition values

$$\alpha / e^{(\ln w_1 \cdot \ln w_2)} = 137.036 / 136.04 = \left[1 + \frac{1}{\alpha}\right]$$

$$\alpha^3 / 2w_3 = 2{,}573{,}380 / 2{,}592{,}212 = \left[1 - \frac{1}{\alpha}\right]$$



The space-time volumes (resp. multiplication factors) of single dimensions correspond with transition values.

In their exponential and multiplicative combination they stand for basic storage volumes of information.

Given the relative differences (of each $1/\alpha$)
of the modified transition values $e^{(\ln w_1 \cdot \ln w_2)}$ and $2w_3$
from the measured values of α und α^3 we would assume
that the four-dimensional volume has a binomial difference of $1/\alpha^2$
from the measured value of α^4 but ...

(continued on page 10)

Multiplying the two expressions we get the following formula:

$$\alpha^4 \sim 2 \cdot w_3 \cdot e^{(\ln w_1 \cdot \ln w_2)} \quad \text{or} \quad \alpha \sim \sqrt[4]{2 \cdot w_3 \cdot e^{(\ln w_1 \cdot \ln w_2)}}$$

which contains the three transition values as integral parts.

What we can read from the formula is the way the dimensions interact when it comes to their volumes, resp. how these act as multiplication factors if we interpret our result as a phenomenon integrating two- und four-dimensionality, particle and wave (dualism).

Multidimensional space-time volumes of electromagnetic radiation

$e^{(\ln w_1 \cdot \ln w_2)}$	**= 136.03985**	**$\sim\alpha$**	two-dimensional	**(particle)**
$2w_3 * e^{(\ln w_1 \cdot \ln w_2)}$	**= 352,644,168**	**$\sim\alpha^4$**	four-dimensional	**(wave)**

The exact relative volume of the elementary wave comprising all four dimensions is $2w_3 * e^{(\ln w_1 \cdot \ln w_2)}$ = 2*1,296,106.0912992190*136.0398545813999 = 352,644,168.364825.

But with a 2018-CODATA-value of α^4 = 137.035999084^4 = 352,645,772.376447 there is an obvious surplus of 1604.01162 which equals significantly $\alpha^{3/2}$ or expressed as a ratio of the wave volume (α^4) it is $1/\alpha^{5/2}$ which is the average density of two- and four-dimensionality, of particle and wave.

Significant difference between the four-dimensional space-time volumes based on transition values and based on the fourth power of α:

α^4	**= 352,645,772**		**space-time volume**
$2w_3 * e^{(\ln w_1 \cdot \ln w_2)}$	**= 352,644,168**		**space-time volume**
difference	**1,604.01162**	= **$\alpha^{3/2}$**	

Now as for mathematical space-time volumes we have to justify physically

why we first work with exponentiation, then with multiplication.

Therefore we make the following **two assumptions**:

- **Identity of space volume and time volume: Time depicts space!**
- **Density maximization: Operator must see to low space-time volume!**

In two-dimensionality time depicts space by the interaction of the first two dimensions, the total volume of which - created by alternately exchangeable transition values - follows the formula $e^{(\ln w_1 \cdot \ln w_2)} = w_1^{\ln w2} = w_2^{\ln w1}$.

From the comparison of $\underline{2}w_3$ and α^3 resp. of $2w_3 * e^{(\ln w_1 \cdot \ln w_2)}$ and α^4 we conclude that the leap from two to four dimensions is implemented mathematically by inserting and doubling the next multiplier w_3.

Time depicts space at maximum density is assured as first $w_1^{\ln w2} = w_2^{\ln w1} < w_1 * w_2$

and second $2w_3 * e^{(\ln w_1 \cdot \ln w_2)} < e^{(\ln w_1 \cdot \ln w_2 \cdot \ln w_3)}$,

that is why we first work with exponentiation, then with multiplication.

VI. Relativistic and QED-corrections

Looking at the table of significant differences on page 6 we would have assumed

that the total binomial difference between the fourth power of α

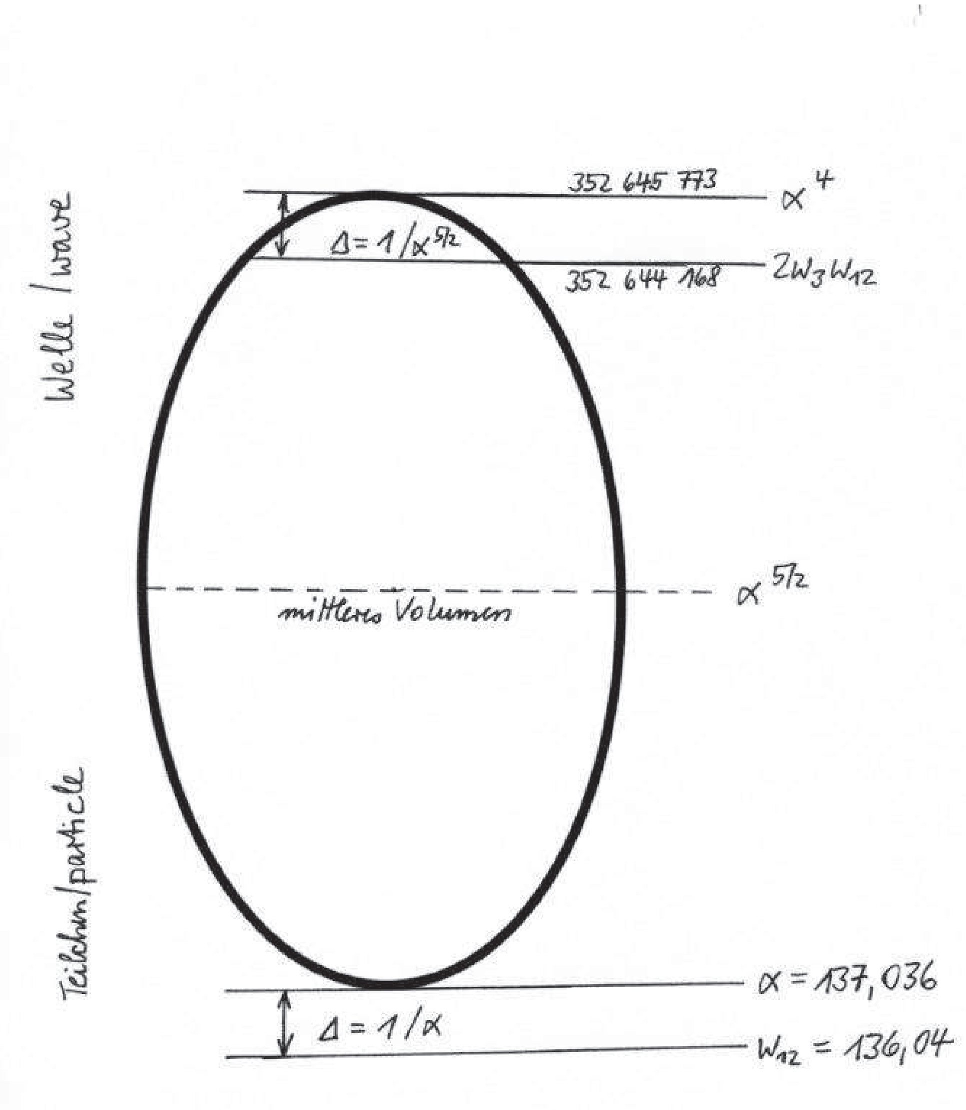
and the product of the transition values covering four dimensions was $1/\alpha^2$.

As in this paper we calculate with space-time volumes rather than densities,
we take $\alpha = 137$ and $1/\alpha^2 = 1/137^2 = (v/c)^2$.

In fact the difference is not a simple relativistic $1/\alpha^2$ but is $1/\alpha^{5/2}$.

The significant differences of the modified transition values (used as multipliers)

$\alpha / e^{(\ln w_1 \cdot \ln w_2)}$	= 137.036 / 136.04	=	$\left[1+\frac{1}{\alpha}\right]$
$\alpha^3 / 2w_3$	= 2,573,380 / 2,592,212	=	$\left[1-\frac{1}{\alpha}\right]$
$\alpha^4 / 2w_3 * e^{(\ln w_1 \cdot \ln w_2)}$	= 352,645,772 / 352,644,168	=	$\left[1+\frac{1}{\alpha^{5/2}}\right]$



(Continued from page 7)

… this is not the case, but

the (relative) difference is **$1/\alpha^{5/2}$**,

which corresponds to the **average density of particle and wave**

which itself is the reciprocal of the square root of the product of volume values α und α^4.

$2w_3{*}\, e^{(\ln w_1 \cdot \ln w_2)}$ is the space-time volume in four-dimensionality, created by the transition values.

α^4 is the total space-time volume - including information - of the wave.

$1/\alpha^{5/2}$ as part of the four-dimensional space-time volume **α^4** equals **$\alpha^{3/2}$**.

The ellipse on page 7 is contained in the above ellipse,
meaning that four-dimensional space-time can be regarded as a sequence of three-dimensional spaces.

$$2w_3 * e^{(\ln w_1 \cdot \ln w_2)} * \left[1+\frac{1}{\alpha^{5/2}}\right] \quad \sim \quad 2w_3 * e^{(\ln w_1 \cdot \ln w_2)} + \alpha^{3/2} \quad = \alpha^4$$

Having to explain why the two differences of 1/α

do not result in a binomial difference of $1/\alpha^2$

we give the expression the following form which contains the binomial difference:

$$\left[1+\frac{1}{\alpha^{5/2}}\right] = \left[1-\frac{1}{\alpha^2}\right] * \left[1+\sum\frac{1}{\alpha^{2n}}+\frac{1}{\alpha^{2n+1/2}}\right]$$

Now we have multiples of **$1/\alpha^2$** which can be interpreted as

relativistic corrections due to relativistic velocity and spin-orbit-coupling:

the relativistic dependence of the electron mass on its velocity,

and the spin-orbit term as interaction of the magnetic moment of the electron,

due to electron spin, with the effective magnetic field the electrons see

due to orbital motion around the nucleus.

But there is also another term, a "**substructure**", (for n=1) of $1/\alpha^{2n+1/2}$ = **$1/\alpha^{5/2}$**

as additional effect of only a fraction of the relativistic effects: $\alpha^{-1/2}$.

If we regroup the equation and take into account that there are

two relativistic (=velocity-dependent) effects which come out as multiples of $1/\alpha^2$

we can dissect the relative difference in space-time volumes as follows:

$$\left[1+\frac{1}{\alpha^{5/2}}\right] = \left[(1+\frac{1}{\alpha^2}) * (1-\frac{1}{\alpha^2})\right] * \left[1+\frac{1}{\alpha^{5/2}}+\frac{1}{\alpha^4}\ldots\right]$$

Spin-orbit rel.veloc. Lamb Hyp

As in this paper we calculate with space-time volumes rather than densities,

we take α =137 and $1/\alpha^2 = 1/137^2 = (v/c)^2$.

If the electron has no velocity

and thus there is no velocity-dependent, even-numbered exponent of α

a rest of $1/\alpha^{5/2}$ remains, the relative size of which

- compared to the relativistic effects in the hydrogen atom - is

$(1/\alpha^{5/2})/(1/\alpha^2) = \alpha^{-1/2}$ = 1/11.71 = 8.54%.

> "The dominant effect is the fine structure. The Lamb shift is about 10% of the fine structure."
> (https://indico.inp.nsk.su/event/1/session/12/contribution/.../1.pdf)

As the effects grow with increasing atomic number Z, an overall statistical result of 10%

is in accordance with the mathematical result of $\alpha^{-1/2}$ = 8.54%.

The relative sizes of relativistic and QED effects
are due to differing dimensional states
of the electron.
What they represent physically
can be described mathematically
by using the transition values.

The power series resulting from the division

$$\left[1+\frac{1}{\alpha^{5/2}}\right] / \left[1+\frac{1}{\alpha}\right]$$

show in their first four components
the following quotients

$$= 1-\frac{1}{\alpha}+\frac{1}{\alpha^2}+\frac{1}{\alpha^{5/2}}$$

These quotients can be assigned to physical effects (in reality there are deviations, dependent on atomic and quantum numbers).

As the electron in the hydrogen atom has a **relativistic velocity** of c/137 it has a higher mass, therefore its spacetime-volume in two dimensions is higher,
so we conclude that this volume is:

$$e^{(\ln w_1 \cdot \ln w_2)} * \left[1+\frac{1}{\alpha}\right] = \alpha .$$

Due to the two-dimensional particle movement the two components $2w_3$ and $e^{(\ln w_1 \cdot \ln w_2)}$, the mathematical factors of the wave volume, compete according to relativity theory (as well as space and time themselves) for their share of space-time volume.

So we have to reduce the space-time volume of the following two dimensions by the same relative factor of 1/α:
We reduce the multiplication factor $2w_3$, which stands for the space-time volume of the two additional dimensions,
by a relative value of 1/α, and we get as result:

$$2w_3 * \left[1-\frac{1}{\alpha}\right] = \alpha^3.$$

The increase in space-time of the two-dimensional particle due to relativistic velocity is nearly compensated by an according decrease of the space-time volume of the two additional dimensions so that the total space-time volume remains unchanged apart from the binomial rest which is $\frac{1}{\alpha^2}$.

The second relativistic correction covers **spin-orbit coupling** due to electromagnetic interaction between the electron's magnetic dipole, its orbital motion, and the electrostatic field of the positively charged nucleus,
its order of size is also $\frac{1}{\alpha^2}$.

The third correction is a **QED correction**.

After we have created mathematically, by multiplication of both two-dimensional components w_{12} und $2w_3$,
the “empty” four-dimensional wave volume $2w_3 * e^{(\ln w_1 \cdot \ln w_2)}$
the electron in the empty space-time
Interacts with vacuum energy fluctuations, creating a difference in energy between two energy levels of the hydrogen atom.
The effect of this interaction increases the space-time volume by a relative factor of $\frac{1}{\alpha^{5/2}}$
which is the average space-time density of particle and wave.

Relativistic corrections
are due to the competitive relationship between the first two and the last two dimensions.

QED corrections result from the interrelation of two- and four-dimensionality.

The Lamb shift is - as an overall statistical result covering a multitude of cases - about 10% of the fine structure. As the effects grow with increasing atomic number Z, this is in accordance with the mathematical result of $\alpha^{-1/2}$ = 8.54%.

> "The Lamb shift, named after Willis Lamb, is a difference in energy between two energy levels $2S_{1/2}$ and $2P_{1/2}$ of the hydrogen atom … .
> Interaction between vacuum energy fluctuations and the hydrogen electron in these different orbitals is the cause of the Lamb shift …"
> (https://en.wikipedia.org/wiki/Lamb_shift)

The **Lamb shift** as an interaction between vacuum energy fluctuations and the electron corresponds, according to the formula, to an **increase in space-time volume**: an increase on the "empty" space-time volume of the vacuum, for which the product $2w_3 * e^{(\ln w_1 \cdot \ln w_2)}$ - created by optimal transitional values - stands.

The reason why the final increase in **space-time volume** of $\mathbf{1/\alpha^{5/2}}$ is identical with the **Lamb shift**'s share of $\mathbf{1/\alpha^{5/2}}$ is that there is no velocity, there are no relativistic effects in the final value of α, but only vacuum energy fluctuations (always) at work.
Any velocity of the electron has to be taken into account via perturbation theory.

fundamental interactions	coupling parameters	transition values	formulae	relation to α
strong	3.3 – 5	3.93862	w_1	
weak	29.5	36.01442	w_2	
(particle volume)		(136.04)	$e^{(\ln w_1 \cdot \ln w_2)} = w_1^{\ln w2} = w_2^{\ln w1}$	$\alpha*(1-1/\alpha)$
electromagn.	**137.036**		$[2w_3 * e^{(\ln w_1 \cdot \ln w_2)} + \alpha^{3/2}]^{1/4}$	α
		1296106	w_3	
		(2592212)	$2w_3$	$\alpha^3*(1+1/\alpha)$
(wave volume)		(352.65 mio)	$2w_3 * e^{(\ln w_1 \cdot \ln w_2)}$	$\alpha^4*(1-1/\alpha^{5/2})$
gravity	$2*10^{38}$		$w_1*w_2*w_3* e^{(\ln w_1 \cdot \ln w_2 \cdot \ln w_3)}$	

VII. How to calculate the value of the fine structure constant

> "Notice that α combines the basic constants from electrodynamics, special relativity
> and quantum mechanics, together with the fundamental unit of electric charge.
> It carries almost mystical significance to physicists,
> for whom **an ab initio calculation of α stands as the ultimate holy grail**."
> (David Griffiths, Revolutions in Twentieth-Century Physics, 2012, page 132, note 33)

Formulated in space-time volumes we have the **extended fine structure constant formula**:

$$2w_3 * e^{(\ln w_1 \cdot \ln w_2)} + \alpha^{3/2} = \alpha^4$$

The equation yields an α-value of **137. 035 999 099 966**

as **prognosis** for future high-precision measurements.

As this value lies between the two most exact measurements of the fine structure constant,
those from Harvard (137.035999149) and from Berkeley (137.035999046),
it can also from a numerical point of view be regarded as a sensational result.

α^4 =	$2w_3w_{12}$		+ $\alpha^{3/2}$
	four-dimensional space-time volume		**QED-corrections**
	2*1296106.0913*136.03985		+ 1604.176
	352644168.3648		+ 1604.176
α^4 =	352645772.54	= **137.035999100^4**	

Why does the equation used here differ from those used by physicists?
Because physicists do not explain how to derive α by an ab initio calculation.
They make approximative calculations based on the dependency
on ordinal and quantum numbers and use Feynman diagrams.
That allows an approximation of the CODATA-value to the <u>real</u> α-value
which we have derived and calculated here.

VIII. The most precise experimental measurements of α

"Both the Guellati-Khélifa group at Laboratoire Kastler-Brossel (LKB, France) and Müller's group at Berkeley are measuring the fine-structure constant by measuring h/m, the ratio of the Planck constant and the mass of an atom."
(https://www.nist.gov/file/410611 about NIST Precision Measurement Grants 2017)

„ … the three best available determinations of α:

- the measurement of **LKB** …,
- the measurement using a_e from **Harvard** combined with the last calculation from Riken, and
- the recent measurement of h/m_{cs} from **Berkeley**."

(Cladé, Nez, Biraben, Guellati-Khelifa [all LKB Paris], State of the art in the determination of the fine structure constant and the ratio h/m_u, arXiv: 1901.01990v1, Jan 2019, p.9)

These values are:

LKB (Guellati-Khélifa): „This leads to the following value of α: α^{-1} = 137.035998996(85)"
(Cladé, Nez, Biraben, Guellati-Khelifa, loc cit)

Harvard/Riken (Gabrielse/Aoyama):
"If we assume that the theory of a_e is correct … we obtain an α which is more precise than that of (19): [Hint: (19) is the above LKB-value]
α^{-1} (a_e: 2017) = 137.035999**149**1 (15)(14)(330)"
(Aoyama, Kinoshita, Nio: Revised and improved value of the QED tenth-order electron anomalous magnetic moment, Physical Review D 97, 036001, Feb 2018)

Berkeley (Müller, Parker):
"Combining with precise measurements of the cesium and electron mass, we found α^{-1} = 137.035999**046**(27) with a statistical uncertainty of 0.16 ppb and a systematic uncertainty of 0.12 ppb (0.20 ppb total). Our result is a more than threefold improvement over previous direct measurements of α."
(Holger Müller, Richard Parker e.a., Measurement of the fine-structure constant as a test of the Standard Model, Science, 13 Apr 2018, Vol. 360, Issue 6385, pp. 191-195)

So the two best values at present are:

Harvard/Riken:	137.035999**149** (0.24 ppb)
Berkeley:	137.035999**046** (0.2 ppb)

Müller's result, weighted with the value of Gabrielse/Aoyama, leads to a value of (137.035999**046***0.24 + 137.035999**149***0.2) / (0.24 + 0.2) = 137.035999**093**.

But NIST in 2019 not only took the two best values when it fixed the new CODATA-2018-value of 137.035999**084** (https://physics.nist.gov/cgi-bin/cuu/Value?alphinv):

> "the most important new input data for alpha is from Mueller/Berkeley.
> There is some new theory, but I don't recall the impact being significant.
> The next significant impact is spectroscopy input data from muonic systems
> and from Hessel that impact the Rydberg constant, the proton radius, and alpha."
> (Mail from David Newell, NIST, 17 June 2019)

As for the exact experimental value there is wide agreement about the first six figures after the point:

137.035998 **996**	(LKB Paris, 2018)
137.035999 **046**	(Müller, Berkeley, 2018)
137.035999 **074**	(CODATA 2010)
137.035999 **084**	(Gabrielse, Hanneke, 2008)
137.035999 **084**	(CODATA 2018)
137.035999 **093**	(weighted: Müller, Gabrielse, Aoyama, 2018)
137.035999 **100**	(Ganter-prognosis, 2015)
137.035999 **139**	(CODATA 2014)
137.035999 **149**	(Gabrielse, Aoyama, 2018)

„NIST support will … allow the [Müller] group … to reduce the error bar further to 0.1 ppb and to develop a future apparatus that will even reach 0.01 ppb.“ (https://www.nist.gov/file/410611)

The difference between CODATA-2018 and Ganter's prognosis is 0.112 ppb.

IX. Some considerations on our result´s probability

*" … statement that **spacetime** itself*
is not a fundamental notion in physics,
but instead may arise as
an approximation to
a more basic mathematical structure
in quantum mechanics."
(Steve Giddings: The Event Horizon Telescope, the Hawking Effect and the Foundations of Physics; Scientific American; April 11, 2019)

What we have developed in our paper about the fine structure constant is a **basic mathematical structure of multidimensional spacetime volumes based on information economy**.

Regarding some numerical probabilities …

First

How probable is it that the mathematical construction $e^{(\ln w_1 \cdot \ln w_2)}$ differs from the CODATA-value by its inverse value 1/137.036 – and not by 1/200 or 1/137.034?

Second

How probable is it that the product of integer 2 and w_3 differs from the third power of the CODATA-value by its inverse value 1/137.036 – and not by 1/200 or 1/150 or 1/137.034 ?

Third

How probable is that the product $2 \cdot w_3 \cdot e^{(\ln w_1 \cdot \ln w_2)}$ differs from the fourth power of the CODATA-value by the (defined by our scheme) average of particle and wave density?

Fourth

How probable is it that the product $2 \cdot w_3 \cdot e^{(\ln w_1 \cdot \ln w_2)}$, completed by the difference defined as Lamb Shift, lies between the two most exact measurements and differs from the CODATA-value by just 1 against 9 billion?

Fifth

How probable is it that the substructure we found (Lamb shift: $1/\alpha^{5/2}$) fits so fine, as it is only about 10% of the relativistic structures, as measured by experiments?

Sixth …

And how probable is it that the number of exponential power tower heights, number of dimensions in the universe and number of fundamental interactions is the same (four!)?

And look (table page 4) at how close the first two transition values are to the parameters of Strong and Weak Force.

… the conclusion is:

It may be possible that one of these results happens by chance but the probability of chance goes (by simple multiplication of the probabilities of these mathematical findings) very fast and close to zero.

This **basic mathematical structure of multidimensional spacetime volumes based on information economy** fits exactly upon the wave-particle-dualism of physics and allows a very exact calculation of α.

X. Survey: Two sides of the same coin

Physics meets mathematics, information meets spacetime

Physics: Information	Mathematics: Spacetime

Constants of nature (e, ħ, c):	Adaptation of the constants of nature
extreme, unchanging,	and of the four interactions
result of optimization	to fit into optimized spacetime volumes

Information density	Spacetime volume:
determines coupling parameter	Reciprocal of information density
and strength of interaction	

4 dimensions	4 exponential levels

Spontaneous dimensional reduction:	transitions between optimal power towers,
Density-dependent, spontaneous,	values unchanging: w_1, w_2, w_3
from four to two dimensions	

The information value of electromagnetic radiation
can be stored as well in two (as particle) as in four (as wave) dimensions.
Quantum information remains preserved, does not get lost:
The four-dimensional wave function of a quantum system
contains its full and complete information.

Information: constant	Spacetime volumes
in wave-particle-dualism	simultaneously
particle	two-dimensional: $e^{(\ln w_1 \cdot \ln w_2)}$
wave	four-dimensional: $2w_3 * e^{(\ln w_1 \cdot \ln w_2)}$

4 exponential levels, 4 dimensions, 4 interactions

Dimensions pop up at and hook into the transition values
gained from the optimization of power towers
and create connections between optimized values: Volumes, densities, strengths.

XI. The precise transition values

Transition from first to second exponential level

$x+x*\ln(x) = x^{x*\ln(x)}$

= 2,20144017842653590880525^1,73718236179526542632168

= **3,93862**254022180133512693 w_1

Transition from second to third exponential level

x^(6,262168359366175062485985-x)

= 2,98912796195770^3,27304039740617506

= **36,0144165**040712926642873

$(6{,}26216835936617506-z-z*\ln(z))^{(z\text{^}(z*\ln(z)))}$

= 1,8650498849967043769869161^
2,3637339505421^2,0333845238273706855

= **36,0144165**040712926642873 w_2

Volume value of the first two exponential levels: $e^{\ln w1*\ln w2}$

e^(ln(3,93862254022180133512693)*ln(36,0144165040712926642873))

= 136,03985458139989976495839

Transition from third to fourth exponential level

$(7{,}67829495344594035082858 - x - x*\ln(x))^{(x\text{^}(x*\ln(x)))}$

= 1,72647933879796835^2,8882960660424^3,063519548605572

= **1296106**,0912992189723178

1,24242486627932858062^((6.43587008716661177021
-z-z*ln(z))^(z^(z*ln(z))))

= 1,24242486627932858062^(1,84195727165086147021
^(2,432194012658751^2,1617188028569993))

= **1296106**,0912992189723178 w_3

Volume value of the last two exponential levels: $2w_3$

2***1.296.106**,0912992189723178 = **2.592.212**,182598437944634

Survey: Dimensions, volumes, densities and corrections

	2 Dim.	+ 2 Dim.	= 4 Dim.	4 Dim. + Information
Spacetime volume	**136,04**	**2,592 Mio**	**352,644 Mio**	**352,644 Mio**
Formula	w_{12}	$2w_3$	$2w_3 * w_{12}$	$+ \alpha^{3/2}$ (1604 = **Information**)
Raw approximation	α	α^3	α^4	$= \alpha^4$
Exact approximation	$\alpha(1-1/\alpha)$	$\alpha^3(1+1/\alpha)$	$\alpha^4(1-1/\alpha^{5/2})$	
Information density	$1/\alpha$	$1/\alpha^3$	$1/\alpha^4$	$[1/\alpha^{5/2}$, from 2D+4D]
Average density (rel. corr.)	$1/\alpha^2$			
Average density (QED-corr.)		$1/\alpha^{5/2}$		