Dark matter cosmology based on the Projective Unified Field Theory

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Abstract

The Projective Unified Field Theory (PUFT) of the author is applied for a dark matter cosmos. Detailed results appeared recently in two volumes [1]. The set of formulas needed for cosmology (including hypothetic empiric predictions) is presented in [2]. The main essence of this paper lays in following assumptions: dark matter basis of the mechanical part of the cosmological gas component; taking into account the hypothetical scalarism phenomenon, introduced by the author at the GRG-congress 1980 in Jena [3]. One of our main results is the singularity-free beginning of the cosmos model investigated (Urstart instead of Big Bang). The occurring numerical result for the Hubble-parameter is physically full satisfactory.

1 Cosmology derived by specialization of the fundamentals of the Projective Unified Field Theory

1.1 Simplification of the cosmos model

In order to minimize the number of free parameters of the model studied we specialize our considerations to a spatially closed cosmological model with the properties: homogeneity and isotropy. For this reason we use the Robertson-Walker metric in the well-known form

$$\mathrm{d}s^2 = K^2(\xi)\big[\mathrm{d}\chi^2 + \sin^2\chi\big(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\big)\big] - \mathrm{d}\xi^2 \,, \tag{1}$$

where $K(\xi)$ with $\xi = x^4 = ct$ is the time-dependent world radius and χ, θ, φ are the polar angles ($x^1 \to \chi$, $x^2 \to \theta$, $x^3 \to \varphi$). Hence follows the metrical matrix

$$
(g_{ij}) = \begin{pmatrix} K^2 & 0 & 0 & 0 \\ 0 & K^2 \sin^2 \chi & 0 & 0 \\ 0 & 0 & K^2 \sin^2 \chi \sin^2 \theta & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{2}
$$

with the metrical determinants

$$
\det(g_{ij}) = -g = -K^6 \sin^4 \chi \sin^2 \theta \quad \text{(4-dimensional)} \tag{3}
$$

and

$$
\det(g_{ab}) = -\overset{3}{g} = g \quad \text{(3-dimensional).} \tag{4}
$$

Now we remember the gravitational field equation [2]:

$$
R^{mn} - \frac{1}{2} g^{mn} R - \frac{\lambda_{\mathrm{S}}}{S_0^2} \mathrm{e}^{-2\sigma} g^{mn} = \varkappa_0 (E^{mn} + S^{mn} + \Theta^{mn}), \tag{5}
$$

where E^{mn}, S^{mn}, and Θ^{mn} are the energy tensors of electromagnetism, scalarism and matter.

Since we take as a basis of our cosmological model its electrical neutrality ($B_{mk} = 0$), we can avoid the electromagnetic field equations. Nevertheless, we take into account an electromagnetic gas (photons) with the following structure of the electromagnetic quantities:

$$
B_{ij} H^{ij} = 0, \qquad (E^{ij}) = \left(\begin{array}{c|c} -p_{(r)} g^{ab} & 0 \\ \hline 0 & -3p_{(r)} \end{array} \right); \tag{6}
$$

$$
p_{(r)} = \frac{1}{3} w_{(r)} \geqslant 0, \qquad \mu_{(r)} = \frac{1}{c^2} w_{(r)} \tag{7}
$$

($p_{(r)}$ radiation pressure, $w_{(r)}$ radiation energy density, $\mu_{(r)}$ radiation mass density).

Still we have to grasp the scalarism as a hypothetically important phenomenon.

The scalaric field equation reads:

$$
\sigma^{,k}{}_{;k} - \frac{\lambda_{\mathrm{S}} \mathrm{e}^{-2\sigma}}{S_0^2} = -\frac{\varkappa_0}{2} \vartheta. \tag{8}
$$

With respect to the emotional aspect of the cosmological model investigated, we remind to the continuum-mechanical equation of motion [2]:

$$
\Theta^{mk}{}_{;k} = -\frac{1}{c} B^m{}_k j^k + \vartheta \sigma^{,m}. \tag{9}
$$

1.2 Modelling of the mechanical properties of the cosmos investigated

In the last equation (9) appear the two very basic quantities: (mechanical substrate energy tensor of the non-geometrized substrate called "matter") and the scalerg density (scalaric substrate energy density) [2]. The mathematical formulation of both quantities needs (at least approximately) concretization of the matter (substrate) of the cosmological model.

Historically up till now the cosmological research was theoretically mostly thinking of a homogeneous gas of normal matter, as used in the cosmological literature. Nowadays, with respect to the new empirical experience [3], we believe that our cosmos in good approximation consists of about 95 % dark matter and "dark energy", and 5 % normal matter. In order to simplify our theoretical image of the gas used, here for our numerical mathematical calculations we approximately investigate the behaviour of a fully dark matter model as a subject of study. This means that as a first step of experience, we have to look on the behaviour of such a provisional model.

Furthermore, we first try to test the use of an ideal gas (without friction etc.).

As we know via recent research, the substrate energy tensor reads:

$$\Theta^{ij} = -\Big(\mu + \frac{p}{c^2}\Big)\, u^i u^j - p\, g^{ij} \tag{10}$$

(μ mass density of dark matter, p pressure of dark matter, u^i four velocity of dark matter).

For simplification we consider the expanding motion of dark matter in a comoving frame of reference, called by us cosmological frame of reference:

$$\text{a)} \quad (u^m) = (0,0,0,c) \quad \text{with} \quad \text{b)} \quad \mathrm{d}\tau = \mathrm{d}t = \frac{1}{c}\,\mathrm{d}x^4\,. \tag{11}$$

Then the substrate energy tensor takes the form

$$(\Theta^{ij}) = \left(\begin{array}{c|c} -pg^{ab} & 0 \\ \hline 0 & -\mu c^2 \end{array}\right)\,. \tag{12}$$

1.3 Field equations, continuum-mechanical equation of motion and local conservation law

For further treatment we present once more the general equations for gravitation:

$$R^{mn} - \frac{1}{2}\, g^{mn} R - \frac{\lambda_{\mathrm{S}}}{S_0^2}\, \mathrm{e}^{-2\sigma}\, g^{mn} = \varkappa_0 (E^{mn} + S^{mn} + \Theta^{mn})\,, \tag{13}$$

and for scalarism:

$$\sigma^{,k}{}_{;k} - \frac{\lambda_{\mathrm{S}}\,\mathrm{e}^{-2\sigma}}{S_0^2} = -\frac{\varkappa_0}{2}\,\vartheta\,. \tag{14}$$

As usually derived, from the field equations result the continuum-mechanical equation of motion ($\nu = \mu + p/c^2$):

$$\nu\,\frac{\mathrm{D}u^m}{\mathrm{D}\tau} = -\left(p^{,m} + \frac{1}{c^2}\,\frac{\mathrm{d}p}{\mathrm{d}\tau}\,u^m\right) - \vartheta\left(\sigma^{,m} + \frac{1}{c^2}\,\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\,u^m\right) + \frac{\varrho_0}{c}\,B^m{}_k u^k \tag{15}$$

with

$$\frac{\mathrm{D}u^m}{\mathrm{D}\tau} = u^m{}_{;k}u^k\,, \tag{16}$$

and furthermore the local conservation law:

$$\left(\nu u^k\right)_{;k} = \frac{1}{c^2}\left(\frac{\mathrm{d}p}{\mathrm{d}\tau} + \vartheta\,\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\right). \tag{17}$$

1.4 Motion of a test body in the expanding universe

For the particular case of the motion of a test-body, by local integration the following equation of motion results:

$$M\frac{\mathrm{D}u^m}{\mathrm{D}\tau} = -D\left(\sigma^{,m} + \frac{1}{c^2}\,\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\,u^m\right) - \Pi^m + \frac{Q}{c}\,B^m{}_k u^k\,, \tag{18}$$

the occurring quantities having the physical meaning:

$$M_\mu = \int \mu\,\mathrm{d}^{(3)}V \qquad \text{(mechanical mass)}, \tag{19}$$

$$M_p = \frac{1}{c^2}\int p\,\mathrm{d}^{(3)}V \qquad \text{(pressure mass)}, \tag{20}$$

$$M = M_\mu + M_p \qquad \text{(inert mass:)}, \tag{21}$$

$$D = \int \vartheta\,\mathrm{d}^{(3)}V \qquad \text{(scalerg)}, \tag{22}$$

$$\Pi^m = \int \left(p^{,m} + \frac{1}{c^2}\,\frac{\mathrm{d}p}{\mathrm{d}\tau}\,u^m\right)\mathrm{d}^{(3)}V \quad \text{(relativistic pressure force)}, \tag{23}$$

$$Q = \int \varrho\,\mathrm{d}^{(3)}V \qquad \text{(electric charge (body))}. \tag{24}$$

Spatial (3-dimensional) motion of a body

Using α and β for the spacial indices, we arrive at the following equation of motion:

$$M\frac{\mathrm{D}u^a}{\mathrm{D}\tau} = -\frac{D}{c^2}\,\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\;u^a - \Pi^a + \frac{Q}{c}\left(B^a{}_b u^b + B^a{}_4 u^4\right) \tag{25}$$

with the abbreviations

$$\text{a)}\quad \frac{\mathrm{D}u^a}{\mathrm{D}\tau} = u^a{}_{;m}u^m\,, \quad \text{and} \quad \text{b)}\quad \Pi^a = \frac{1}{c^2}\int \frac{\mathrm{d}p}{\mathrm{d}\tau}\,u^a\,\mathrm{d}^{(3)}V\,. \tag{26}$$

For the subsequent calculations we use the well-known relations between the 4-velocity and the 3-velocity:

$$u^a = v^a\,\frac{\mathrm{d}t}{\mathrm{d}\tau}\;, \quad u^4 = c\,\frac{\mathrm{d}t}{\mathrm{d}\tau} \quad \text{and} \quad \frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{1}{\sqrt{1-(v/c)^2}}\,. \tag{27}$$

and the scalaric-dynamic mass:

$$M_d = M\,\frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{M}{\sqrt{1-(v/c)^2}}\,. \tag{28}$$

Now we apply this rather abstract theory to celestial bodies (similar to the planets of our solar system) and use the relation

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{D}{c^2}\,\frac{\mathrm{d}\sigma}{\mathrm{d}t} \tag{29}$$

between the mass und the scalerg of the moving body (V velocity of its motion around a big central mass). Finally we approximately arrive at the simplified differential equation

$$\frac{\mathrm{d}\ln\!\big(M_d V K\big)}{\mathrm{d}t} = 0\,, \tag{30}$$

whose integration leads to the amplified cosmological conservation relation

$$M_d V K = \frac{MVK}{\sqrt{1-(V/c)^2}} = p_d K = \bar{C} \tag{31}$$

between the momentum of the body and the cosmological radius K of the universe ($\bar{C}$ integration constant).

Temporal investigation of the moving body

Analogously to this spatial sight on the motion of the body considered, we now investigate the temporal sight of the equation (18) with the result:

$$\begin{aligned}\frac{\mathrm{d}M_d}{\mathrm{d}t} + \frac{M_d v^2}{c^2}\,\frac{1}{K}\,\frac{\mathrm{d}K}{\mathrm{d}t} &= \frac{\mathrm{d}M}{\mathrm{d}t}\,\frac{\mathrm{d}t}{\mathrm{d}\tau} - \frac{v^2}{c^2}\,\frac{\mathrm{d}t}{\mathrm{d}\tau}\,\frac{\mathrm{d}M_p}{\mathrm{d}t} \\ &\quad + \frac{D}{c^2}\,\frac{\mathrm{d}\sigma}{\mathrm{d}t}\,\frac{\mathrm{d}\tau}{\mathrm{d}t}\left[1-\left(\frac{\mathrm{d}t}{\mathrm{d}\tau}\right)^2\right] + \frac{Q}{c^2}\,E_a v^a\,. \end{aligned} \tag{32}$$

By the definition of the energy of the moving body

$$\mathcal{E}_d = M_d c^2 \,, \tag{33}$$

we arrive at the temporal change of this quantity:

$$\frac{\mathrm{d}\mathcal{E}_d}{\mathrm{d}t} = -M_d v^2 \frac{1}{K}\,\frac{\mathrm{d}K}{\mathrm{d}t} + \frac{\mathrm{d}M}{\mathrm{d}t}\,c^2\sqrt{1-(v/c)^2} + Q(\boldsymbol{E}\boldsymbol{v})\,. \tag{34}$$

In similarity to the usual mechanics we define the amplified kinetic energy of the body in the form

$$T_{\mathrm{kin}} = \mathcal{E}_d - Mc^2 \,, \tag{35}$$

which underlies the temporal balance equation

$$\begin{aligned}\frac{\mathrm{d}T_{\mathrm{kin}}}{\mathrm{d}t} &= -\frac{Mv^2}{\sqrt{1-(v/c)^2}}\,\frac{1}{K}\,\frac{\mathrm{d}K}{\mathrm{d}t} \\ &\quad + \frac{\mathrm{d}M}{\mathrm{d}t}\,c^2\big(\sqrt{1-(v/c)^2}-1\big) + Q(\boldsymbol{E}\boldsymbol{v})\,.\end{aligned} \tag{36}$$

2 Two-component cosmological gas-mixture (mechanical dark-matter-gas and electromagnetic photon-gas) as corners of the whole cosmos (homogeneity, isotropy, spatially closeness), approximately investigated as a first suggestion

As a first trial we start with a two-component ideal gas-mixture:

1. Mechanical dark-matter-gas of new strange particles (with properties only partly understood).

2. Electrically neutral photon-gas.

2.1 Mechanical dark-matter gas

Higgs-boson and wimp as candidates

Using the mostly accepted empirical astrophysical results that about 5% of the matter is usually normal and about 95% is dark matter and dark energy with up to now not fully understandable strange behaviour. According to the literature cited above one is thinking of particles with the physical properties:

– gravitation and inertia, according to the laws of Newton and Einstein;

– ground state of the particles (aside gravitation and inertia) only weak physical interaction;

– ground state spin of the particles (probably) zero.

Both sorts of particles mentioned above seem to be physically related:

Higgs-boson [3] with the suggested rest mass:

$$m_{\text{Higgs}} = 126\,\frac{\text{GeV}}{c^2} = 2,246 \cdot 10^{-22}\,\text{g} \tag{37}$$

and

wimp (weakly interacting massive particle) [4] with the eventual rest mass:

$$m_{\text{wimp}} = 1,783 \cdot 10^{-22}\,\text{g}\,. \tag{38}$$

Both particles being in talk are loans from the standard model of the elementary particle theory, combined with empirical insights in the motion of galactic particles.

In contrast to this quantum field theoretical approach, we frequently tried to get some knowledge from our 5-dimensional PUFT which belongs (analoguesly to the Einstein theory) to the class of classical field theories. We hoped to receive some information on elementary particles in exploiting the richness of our field-theoretical five-dimensionality [5]. In the following part we will speak about some ideas in this direction.

Scalon and mechanical scalon-gas

Of course, at the begin of our research on scalarity (scalarism) we thought about the particle (assigned to the scalaric field) which we named "scalon", since the notion "scalaron" was already used in solid state physics. First it seemed rather obvious that this particle should be understood according to the parton conception of R. P. Feynman (1969). Nowadays it seems to be clear that the existence of overwhelming dark matter with the basis particle wimp dominates the scene. As pointed out above, the basis of our approach to the mechanical dark-matter gas is the motion of a single dark-matter particle. The list of physical relations reads [1]:

Scalaric-dynamic mass of a scalon:

$$m_d = \frac{m}{\sqrt{1-(v/c)^2}} \tag{39}$$

($m = m(\sigma)$ scalaric rest mass of a scalon);

Particle number density of the scalon-gas:

$$n = \frac{\mu}{m_d} = \frac{\mu}{m}\sqrt{1-\left(\frac{v}{c}\right)^2}\,; \tag{40}$$

Scalaric-cosmological press parameter (with the integration constant (31)):

$$H_{\mathrm{S}} = \left(\frac{mcK}{\bar{C}}\right)^2 . \tag{41}$$

By means of these connections for later physical calculations the following relationships are useful:

$$\frac{v}{c} = \frac{1}{\sqrt{1+H_{\mathrm{S}}}} ; \tag{42}$$

$$\sqrt{1-(v/c)^2} = \frac{\mathrm{d}\tau}{\mathrm{d}t} = \sqrt{\frac{H_{\mathrm{S}}}{1+H_{\mathrm{S}}}} ; \tag{43}$$

$$m = \frac{\bar{C}\sqrt{H_{\mathrm{S}}}}{cK} \quad \text{and} \quad m_d = \frac{\bar{C}\sqrt{1+H_{\mathrm{S}}}}{cK} ; \tag{44}$$

$$n = \frac{\mu}{m_d} = \frac{\mu}{m\sqrt{1+H_{\mathrm{S}}}} . \tag{45}$$

Mechanical pressure of the scalon-gas and other basic notions of the gas

In the subsequent text we reset the above notion "dark-matter gas" by "scalon-gas", suspecting that the dark matter of our world could eventually be connected with its five-dimensionality investigated by our search .

A very important notion for the mechanical pressure of the scalon-gas is its pressure which we define as

$$p = \frac{n\varepsilon_d}{3}\Big(1 - H_{\mathrm{S}}\Big) , \tag{46}$$

where the quantity

$$\varepsilon_d = m_d c^2 \tag{47}$$

means the scalaric-dynamic energy of the scalon.

These lengthy calculations just shown lead to following useful different formula for the pressure of the scalon-gas:

$$p = \frac{\mu c^2}{3\big(1+H_{\mathrm{S}}\big)} = \frac{n m_d c^2}{3\big(1+H_{\mathrm{S}}\big)} . \tag{48}$$

Kinetic temperature of the scalon-gas

In order to concentrate to typical main effects between pressure and temperature, we specialize our considerations to an ideal gas. In this case the well-known ideal gas equation (k Boltzmann constant)

$$T = \frac{p}{n\mathrm{k}} \tag{49}$$

is used.

Let us finish this subjection with the following remark on the above new definitions of basic notations of the theory of the mechanical gas of our world model: The choose applied is just so determined that for the non-relativistic case consistence with usual thermodynamics holds.

2.2 Further treatment of the cosmological system of differential equations

In order to receive knowledge on the electromagnetic pressure of the cosmological photon-gas, we need some important insight in the above derived system of differential equations. Via lengthy calculation from (19) and (20) the two differential equations of second order result for the world radius $K(\xi)$ and the scalaric cosmological world function $\sigma(\xi)$:

$$\begin{gathered}\frac{K''}{K} - \frac{2}{3}\,\sigma'^2 - \frac{1}{3}\,\Lambda_{\mathrm{S}}\,\mathrm{e}^{-2\sigma} + \frac{1}{6}\,\varkappa_0(\mu c^2 + 3p) + \varkappa_0 p_{(r)} = 0 \\ \Big(K' = \frac{\mathrm{d}K}{\mathrm{d}\xi}\Big)\end{gathered} \tag{50}$$

and

$$\sigma'' + \frac{3}{K}\,K'\sigma' + \Lambda_{\mathrm{S}}\,\mathrm{e}^{-2\sigma} - \frac{1}{2}\,\varkappa_0\vartheta = 0 \tag{51}$$

with the scalaric-cosmological constant (physical dimension: reciproce square of length)

$$\Lambda_{\mathrm{S}} = \frac{\lambda_{\mathrm{S}}}{S_0^2}\,. \tag{52}$$

Further we mention the result

$$\frac{K'}{K} + \frac{\mu' - \dfrac{\vartheta}{c^2}\,\sigma'}{3\Big(\mu + \dfrac{p}{c^2}\Big)} = 0\,, \tag{53}$$

derived from the mechanical balance equation. By means of partial integration of (50) and (51) the resulting intermediate additional condition (intermediate equation) has to be noted:

$$\frac{1}{K^2}\big(K'^2 + 1\big) - \frac{1}{3}\,\Lambda_{\mathrm{S}}\,\mathrm{e}^{-2\sigma} + \frac{1}{3}\,\sigma'^2 - \varkappa_0\Big(\frac{1}{3}\,\mu c^2 + p_{(r)}\Big) = 0\,. \tag{54}$$

2.3 Photon-gas

Electromagnetic radiation pressure

By differentiation of (53) and further reshapings one is led to the differential equation for the radiation pressure:

$$p'_{(r)} + 4\frac{K'}{K}\,p_{(r)} = 0\,. \tag{55}$$

Resolution gives (Λ_0 electromagnetic radiation constant as an integration constant)

$$p_{(r)} = \frac{\Lambda_0}{K^4}\,. \tag{56}$$

By means of these new results the basic cosmological equations (50), (51) and (52) take the new mathematical form:

$$\frac{K''}{K} - \frac{2}{3}\,\sigma'^2 - \frac{1}{3}\,\Lambda_\mathrm{S}\,\mathrm{e}^{-2\sigma} + \frac{1}{6}\,\varkappa_0(\mu c^2 + 3p) + \frac{\varkappa_0\Lambda_0}{K^4} = 0\,, \tag{57a}$$

$$\sigma'' + \frac{3}{K}\,K'\sigma' + \Lambda_\mathrm{S}\,\mathrm{e}^{-2\sigma} - \frac{1}{2}\,\varkappa_0\vartheta = 0\,, \tag{57b}$$

$$K'\Big(\mu + \frac{p}{c^2}\Big) + \frac{1}{3}\,K\Big(\mu' - \frac{\vartheta}{c^2}\,\sigma'\Big) = 0\,. \tag{57c}$$

Furthermore, we may not forget the intermediate equation (54):

$$\frac{1}{K^2}\,(K'^2 + 1) - \frac{1}{3}\Lambda_\mathrm{S}\,\mathrm{e}^{-2\sigma} + \frac{1}{3}\,\sigma'^2 - \varkappa_0\Big(\frac{1}{3}\,\mu c^2 + \frac{\Lambda_0}{K^4}\Big) = 0\,. \tag{58}$$

Electromagnetic radiation energy and photon number density of the cosmos

Since our world model treated here was taken as closed, we know the volume of the cosmos

$$V_\mathrm{cosm} = 2\pi^2 K^3\,. \tag{59}$$

Let us adopt the often used relation between electromagnetic energy density and radiation pressure:

$$w_{(r)} = 3p_{(r)} = \frac{3\Lambda_0}{K^4}\,. \tag{60}$$

Hence for the electromagnetic radiation energy of the cosmos, not being a conserved quantity, results

$$E_{(r)} = w_{(r)} V_\mathrm{cosm} = \frac{6\pi^2\Lambda_0}{K} > 0\,. \tag{61}$$

For cosmological expansion it finally vanishes.

Let us now in this context remind the Stefan-Boltzmann constant

$$a_\mathrm{SB} = \frac{8\pi^5\mathrm{k}^4}{15c^3h^3} = 7.5659\cdot 10^{-15}\,\mathrm{g\,cm^{-1}s^{-2}K^{-4}} \tag{62}$$

which is a very important quantity in electromagnetic radiation theory:

$$\text{a)}\quad w_{(r)} = a_\mathrm{SB}T_{ph}^4\,,\quad \text{b)}\quad T_{ph} = \Big(\frac{w_{(r)}}{a_\mathrm{SB}}\Big)^{1/4} \quad\text{and}\quad \text{c)}\quad \varepsilon_{ph} = 3\,\mathrm{k}T_{ph}\,. \tag{63}$$

If we apply this theory mentioned to the electromagnetic energy density of the present state of our cosmos, using the measuring value of the present background radiation:

$$T_{ph} = 2,728\,\mathrm{K}\,. \tag{64}$$

for the present radiation energy density we find the result

$$w_{(r)\mathrm{p}} = a_{\mathrm{SB}} T^4_{ph|p} = 4.1902 \cdot 10^{-13}\,\mathrm{g\,cm^{-1}s^{-2}}\,. \tag{65}$$

Closing this subsection, we apply the formula (63c) and find for the photon number density of the cosmos:

$$n_{(r)} = \frac{\hat{C}}{\mathrm{k}K^3}\,, \tag{66}$$

where the abbreviation

$$\hat{C} = \left(\frac{3\Lambda_0}{a_{\mathrm{SB}}}\right)^{1/4} \tag{67}$$

is used.

3 Mathematical treatment of our cosmological system of differential equations

3.1 4-dimensional ansatz for the 5-dimensional scalerg density

In retrospect at our efforts in deriving a cosmological system of differential equations we arrived our aim as follows:

The task desired is concentrated in the basic cosmological system of differential equations (57a), (57b), (57c) including the intermediate consistence equation (58). The field quantities to be determined are: world radius $K(\xi)$, scalaric world function $\sigma(\xi)$ and mechanic mass density $\vartheta(\xi)$.

In this 4-dimensional cosmological system we are confronted with a typical 5-dimensional mechanical quantity, namely the scalerg density:

$$\vartheta = \Theta^{\mu\nu} s_\mu s_\nu\,. \tag{68}$$

It seemed to us obvious that in the case of an ideal gas there should be a connection between scalerg density ϑ with mass density μ and mechanical pressure p. But how in detail? I tried several versions. My last attempt was:

$$\vartheta = -\frac{\mu c^2 + p}{1 + \mathrm{e}^{2\sigma}} = -\frac{\nu c^2}{1 + \mathrm{e}^{2\sigma}}\,. \tag{69}$$

Hence from this step chosen we arrive at following relationships:

$$\text{a)}\quad M = \int \Big(\mu + \frac{p}{c^2}\Big) \mathrm{d}^{(3)}V\,, \quad \text{b)} \qquad D = \int \vartheta \mathrm{d}^{(3)}V\,; \tag{70}$$

$$\begin{aligned} &\text{a)}\quad D = \frac{\mathrm{d}M}{\mathrm{d}\sigma}\, c^2\,, \quad \text{b)} \quad D = -\frac{Mc^2}{1+\mathrm{e}^{2\sigma}}\,, \\ &\text{c)}\quad \frac{\mathrm{d}\ln M}{\mathrm{d}\sigma} = -\frac{1}{1+\mathrm{e}^{2\sigma}}\,. \end{aligned} \tag{71}$$

Let us further in this context mention the formula for the scalaric mass and hence for the relative scalaric mass:

$$\begin{aligned} &\text{a)}\quad m(\sigma) = m_0\sqrt{1+\mathrm{e}^{-2\sigma}} \quad \text{and} \\ &\text{b)}\quad m_{\mathrm{rel}} = \frac{m}{m_0} = \sqrt{1+\mathrm{e}^{-2\sigma}}\,. \end{aligned} \tag{72}$$

3.2 Rescaling of the 4-dimensional cosmological system of differential equations and use of new physical quantities for simplifying the subsequent calculations

Up to know both cosmological quantities (with physical dimension of length):K (world radius) and ξ (quasi-time parameter) plaid a fundamental role. The cosmological system of equations becomes more transparent by change to dimensionless quantities L and η:

$$\text{a)}\quad K = A_0 L\,, \quad \text{b)}\quad \xi = ct = A_0\eta\,, \tag{73}$$

where the rescaling constant reads:

$$A_0 = 10^{27}\,\mathrm{cm}\,. \tag{74}$$

Using the new Parameters:

$$\bar{A} = \frac{1}{3}\varkappa_0 c^2 A_0^2 = 6,2197\cdot 10^{26}\,\mathrm{g}^{-1}\mathrm{cm}^3\,, \tag{75a}$$

$$\Delta = \frac{\varkappa_0\Lambda_0}{A_0^2}\,, \tag{75b}$$

$$\Lambda_0 = p_{(r)\mathrm{p}}K_{\mathrm{p}}^4 = \frac{1}{3}\,w_{(r)\mathrm{p}}A_0^4L_{\mathrm{p}}^4 = 1,3967\cdot 10^{95}\,L_{\mathrm{p}}^4\,\mathrm{g\,cm}^3\mathrm{s}^{-2}\,, \tag{75c}$$

$$\dot{L} = \frac{\mathrm{d}L}{\mathrm{d}\eta}\,, \quad \text{etc.} \tag{75d}$$

then the changed system of differential equations takes the form:

$$\ddot{L} - \frac{2}{3}\,\dot{\sigma}^2 L - \frac{1}{3}\,\Lambda_{\mathrm{S}}A_0^2 L\,\mathrm{e}^{-2\sigma} + \mu\bar{A}\frac{1+\dfrac{1}{2}\,H_{\mathrm{S}}}{1+H_{\mathrm{S}}} + \frac{\Delta}{L^3} = 0\,, \tag{76a}$$

$$\ddot{\sigma} + \frac{3\dot{L}\,\dot{\sigma}}{L} + \Lambda_{\mathrm{S}} A_0^2\,\mathrm{e}^{-2\sigma} + \frac{2\bar{A}\mu}{1+\,\mathrm{e}^{2\sigma}}\,\frac{1+\dfrac{3}{4}\,H_{\mathrm{S}}}{1+H_{\mathrm{S}}} = 0\,, \tag{76b}$$

$$\dot{\mu} + 4\mu\Big(\frac{\dot{L}}{L} + \frac{\dot{\sigma}}{3(1+\,\mathrm{e}^{2\sigma})}\Big)\frac{1+\dfrac{3}{4}\,H_{\mathrm{S}}}{1+H_{\mathrm{S}}} = 0\,, \tag{76c}$$

while the intermediate consistence equation holds:

$$\dot{L}^2 + 1 + \frac{1}{3}\,L^2\dot{\sigma}^2 - \frac{1}{3}\,\Lambda_{\mathrm{S}} A_0^2 L^2\,\mathrm{e}^{-2\sigma} - \bar{A}\mu L^2 - \frac{\Delta}{L^2} = 0\,. \tag{77}$$

Furthermore, the scalaric-cosmological pressure parameter (41) gets the simple fashion

$$H_{\mathrm{S}} = \Big(\frac{mcA_0L}{\bar{C}}\Big)^2\,. \tag{78}$$

The interested reader may find various context formulas in vol. 2 [1], where hints for the concrete performance of the numerical detail work are described. Most of this challenge was fulfilled with Wolfram Mathematica 10. The author is very grateful to the Computer Centre of Jena University for this helpful assistance in cosmological research.

3.3 Short review to the numerical integration of the cosmological system of differential equations

In contrast to the Einstein theory with its initial singularity (Big bang = Urknall) at the beginning of the expansion procedure of the cosmos ($\eta = 0$), in PUFT the beginning starts without singularity behaviour, called by me (1980): Urstart [2].

According to nowadays calculations the initial values are (slightly changed values in comparison to [1]) :

$$\text{a)}\quad L_0 = 4.88 \cdot 10^{-5}\,, \qquad \text{b)}\quad \dot{L}_0 = 4039\,; \tag{79}$$

$$\text{a)}\quad \sigma_0 = 0\,, \qquad \text{b)}\quad \dot{\sigma}_0 = 5.75 \cdot 10^{8}\,; \tag{80}$$

$$\mu_0 = 6.3 \cdot 10^{-11} \tag{81}$$

Because of continuity reasons one should note the value

$$L(13) \sim 35\,. \tag{82}$$

Furthermore, a series of up to now open parameters can be determined by satisfying well fixed empirical experience (in different contexts):

Cosmological constants:

$$\text{a)}\quad \Lambda_{\mathrm{S}} = 5,7639 \cdot 10^{-55}\,\mathrm{cm}^{-2} \quad \text{with} \quad \text{b)}\quad \lambda_{\mathrm{S}} = 4,4 \cdot 10^{-122}\,; \tag{83}$$

Velocity V_0 of the scalon at the urstart (nearby vacuum velocity of light):

$$\frac{V_0}{c} = 0.999\,. \tag{84}$$

Presence

Let us finally report on some physically interesting values of our cosmos model at cosmological presence (index p). For reasons of physical interpretation we remember the scale of physical time:

$$t = \frac{A_0}{c}\,\eta = 3,3357 \cdot 10^{16}\eta\,\mathrm{s} = 1,057 \cdot 10^{9}\eta\,\mathrm{y} \quad (\,\mathrm{y}\ \text{years}). \tag{85}$$

Many empiric arguments from physics, astrophysics, geophysics, chemistry estimate the age of our world (universe) as follows:

$$\text{a)} \quad t_{AW} = 13.7 \cdot 10^{9}\,\mathrm{y} \quad \text{i.e.} \quad \text{b)} \quad \eta_p = 13\,. \tag{86}$$

Our computer results read:

$$\text{a)} \quad L(13) = 35.5 \quad \text{(already mentioned)}, \quad \text{b)} \quad \dot{L}(13) = 2.7\,; \tag{87}$$

$$\text{a)} \quad \sigma(13) = 1.8\,, \quad \text{b)} \quad \dot{\sigma}(13) = -0.05\,; \tag{88}$$

$$\mu(13) = 3.34 \cdot 10^{-30}\,\mathrm{g\,cm^{-3}}\,. \tag{89}$$

$$H(13) = 70.26\,\frac{\mathrm{km}}{\mathrm{s\,Mpc}} = 2.28 \cdot 10^{-18}\,\mathrm{s^{-1}} = 7.2 \cdot 10^{-11}\,\mathrm{y^{-1}} \tag{90}$$

(Hubble-Parameter). Slightly different detailed numerical cosmological and astrophysical material can be found in our literature [1].

We finally mention that the number of cosmological photons at all during the process of expansion is a numerical constant:

$$N_{ph|\mathrm{cosm}} \sim 3 \cdot 10^{89}\,. \tag{91}$$

3.4 Basic graphic representations

An enlargement of the list of the above numerical values of other physical quantities can be found in our voluminous graphic insight in the literature [1] cited. As examples we choose the temporal course of some elementary quantities numerically calculated: L (rescaled world radius), σ (scalaric world function) and μ (cosmologic mass density) within the time interval: from urstart up to the present cosmological era. In order to learn more on the detailed cosmological situation immediately after the urstart, we show the behaviour of these quantities according to the initial conditions chosen in (79), (80), (81).

Rescaled curvature radius of the cosmos (dimensionless)

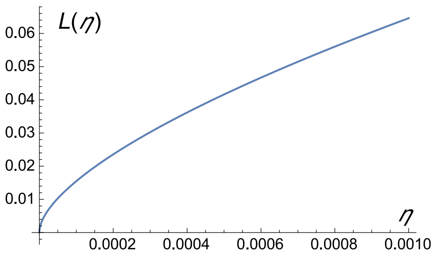

figure 1:
Temporal course immediately after the urstart

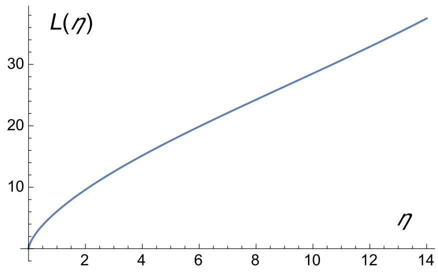

figure 2:
Temporal course between urstart and presence

Scalaric world function

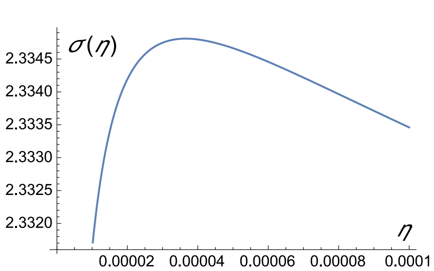

figure 3:
Temporal course immediately after the urstart

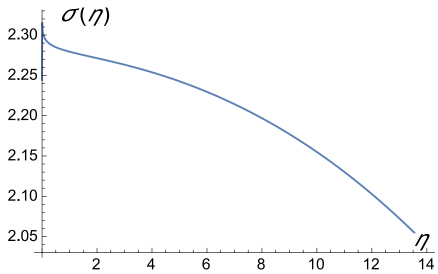

figure 4:
Temporal course between urstart and presence

Cosmologic mass density $[\mathrm{g\,cm^{-3}}]$

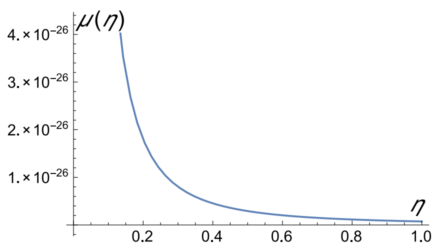

figure 5:
Temporal course immediately after the urstart

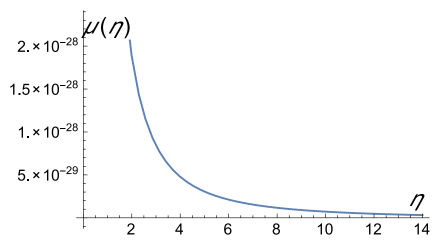

figure 6:
Temporal course between urstart and presence

3.5 Temporal finish of the world model

From the full temporal course of the scalar world function (see partially Fig. 4) we learn that this function continuously slows down and reaches the value $\sigma(\eta \sim 27) \sim 0$. Further on it changes to negative values not being in the realm of real physical consideration. Hence a rather free cosmological interpretation could believe in the finish (end state of existence of this world model). This above value $\eta \sim 27$ according to the recalculation of real time leads to the final existing time of this world model (85):

$$t_{\text{world existence}} \sim 28.9 \cdot 10^9 \,\text{years}\,. \tag{92}$$

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