

NEW FORM OF THE LIOUVILLE FUNCTION

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ABSTRACT

This short paper presents a new form of the Liouville function. To the author's best knowledge, this relationship is previously unknown.

The Liouville function $\lambda(n)$ is defined as,

$$\lambda(n) = (-1)^{\Omega(n)}$$

Where $\Omega(n)$ is the number of prime factors of n counted with multiplicity.

Its summatory function $L(n)$ is defined as,

$$L(n) = \sum_{j=1}^n (-1)^{\Omega(j)}$$

From this one can see that its calculation requires calculation of prime factors. I present a new form here,

$$\lambda(n) = i^{\tau(n^2)-1}$$

Where $\tau(n)$ is the divisor function, the number of divisors of n , and i is the imaginary unit.

Its summatory function now becomes,

$$L(n) = \sum_{j=1}^n i^{\tau(j^2)-1}$$

This new form now only requires the calculation of the number of divisors of n^2 . The author is not making any deterministic claim of increased computing complexity. However, this new form may be easier to compute, because it doesn't require the computation of the number of prime factors, or even that n is prime.