Projektive Unified Field Theory today

Ernst Schmutzer
Friedrich Schiller University Jena
Jena, Germany

March 15, 2016

Abstract

The short reproduction of the Projective Unified Field Theory of the author (including empiric predictions) will be presented during the subsequent time:

- Fundamental 5-dimensional physical laws within the projective space and projection of these basic laws onto space-time.
- Transition from this 4-dimensional complex of the physical laws to the (better understandable) 3-dimensional version with new additional physical terms, being prepared for a physical interpretation (influence of cosmological expansion).
- Numerical presentation of the predicted astrophysical and cosmological effects: anomalous motion of bodies according to the pioneer effect, anomalous rotation curve of rotating bodies around a gravitational center. One should take note of my hypothetic result that the true origin of these both effects (pioneer effect and rotation curve effect) seems to have the same cause, namely the overwhelming dark matter existing in our Universe.

Email: eschmutzer@t-online.de

Introduction

After A. Einstein's great success in his geometrical 4-dimensional General Theory of Relativity and Gravitation (1915/16), particularly Th. Kaluza (1921) [1] and O. Klein (1926) [2] tried to generalize this theory to five dimensions, but without physical success. In the period between the two Great Wars only a few theorists followed this line, since the more interesting quantum field theory has been just detected. At the end of this time, inspired by P. A. M. Dirac's hypothesis on extreme large cosmological numbers, P. Jordan [3] (supported by his co-workers) and G. Ludwig [4] tested own separate attempts, mainly to find an approach to a gravitational theory with a variable time-dependent "gravitational constant". In this context one may not forget A. Lichnerowicz [5] who tried to overcome the difficulties to a generalized 5-dimensional geometrical field theory by a rather abstract geometrical input, not going into physical details.

Jordan's physical main goals: 1. Developing a theory with a variable gravitational parameter (instead of a constant); 2. Finding such generalized field equations which application to the cosmology lead to a production of normal matter (Ambarzumian's explosions), in contrast to the usually taught conservation of matter. Since suitable exact solutions of Jordan's program could not be found, parameter approximations were not avoidable. Roughly speaking, Jordan's generalization of the Einstein theory culminated in two important facts: variable gravitational parameter and using a new additional scalar field. This type of theory was at the same time apart from Jordan mainly by C. Brans and R. H. Dicke [6] extensively investigated. Nowadays for this theoretical approach often the name Jordan–Brans–Dicke-Theory is used.

It is historically important to remind in this context that the young W. Pauli already in 1921 published on the rather new relativity theory [7] and enriched Dirac's quantum theory of the electron by new insight into the spinor formalism. Of course one understands that, following the Kaluza–Klein five-dimensionality, he worked for some years on the inclusion of the electromagnetism in addition to gravitation within a unified physical field theory [8]. Final he failed in the attempt to explain the magnetic field of celestial bodies by their rotation. By his negative experience with the five-dimensionality he came to his well-known hard criticism of the Jordan theory [9]. Furthermore Pauli was supported in his argumentation by the majority of geologists who did not believe in Jordan's numerical prediction of the change of the Newtonian gravitational constant. In this dejected state Jordan finished in 1961/62 his research on his tried 5-dimensional field theory, particularly on his by a scalar field amplified 4-dimensional space-time [10]. Some of his co-workers remained in the large area of the Einstein theory concentrating on the search for exact solutions of rotating bodies (planets, stars, galaxies) and properties of black holes, etc.

My own interests after my doctor-degree at the Rostock University (1955) were determined by modern theoretical physics (relativity theory and quantum theory). In the following years I mainly studied the international attempts to the large field of unified field theories (above all Einstein's non-symmetric 4-dimensional theories and 5-dimensional symmetric approaches as the Kaluza-Klein scheme). I was strongly impressed by the 5-dimensionality. After my habilitation degree at the Jena University in 1958 my mostly research (aside from teaching duties) was devoted to my 5-dimensional Projective Unified Field Theory (PUFT). This is a unified field theory, possessing metric symmetry in the 5-dimensional projective space and in the 4-dimensional space-time. But here further additional terms with respect to gravitation, electromagnetism und scalarism, which I introduced as a new physical notion for a proper hypothetic physical entity of our world, at the 9th International Conference on General Relativity and Gravitation (GR9) [11] in Jena 1980.

My creation of PUFT during a period of about sixty years research had a clear scientific goal. Obviously some changes in the axiomatization of the 5-dimensional geometric apparatus were necessary in order to prepare a consistent procedure for the projection formalism of my 5-dimensional construction of my proposed fundamental 5-dimensional laws of Nature onto the 4-dimensional space-time with Riemannian geometry. The physical and astrophysical outcome of this theory including partly the comparison with new empirical results will be treated in three papers, the first being this contribution.

Concluding this introductory part, I would like to point to following ArXiv-publications, where detailed treatment of my Projective Unified Field Theory was presented: [12–20]. Further theoretic-physical calculations can be considered in my compendium on theoretical physics [21] and in both of my first representations of the Projective Unified Field Theory [22].

Because of the diversity of scientific literature on my hypothetic Projective Unified Field Theory (PUFT) it seemed to me reasonable to present the whole, rather voluminous part of field theory in its entirety in the form of two new volumes [23]:

Volume I: Compact presentation of the 5-dimensional PUFT comprising the following area of the proposed 5-dimensional physics: gravitation, electromagnetism, scalarism. Projecting this whole complex of basic physical laws from the 5-dimensional projective space onto the 4-dimensional space-time, receiving the pocket of fundamental 4-dimensional laws (Einstein's 4-dimensional physics, 4-dimensional electromagnetism, 4-dimensional laws of scalarism, new interacting terms of the fields just mentioned). Final interpretation of this 4-dimensional scheme of physics with view on the known 3-dimensional physics (Newton, Maxwell, Faraday, etc.).

Volume II: Application of PUFT to important (partly hypothetic) predictions: common physical origin of the anomalous motion of celestial bodies in astrophysics (pioneer effect) and anomalous spiral motion of celestial bodies around heavy central masses (rotation curve effect). Further prediction of emergence in moving bodies by the expansion of our Universe, heath production in moving bodies by the expansion of our Universe, etc.

Unfortunately I was not able to find a publisher for an edition in English language.

Axiomatic basis of the proposed 5-dimensional physics, projection of the physical laws onto the four-dimensional space-time, and physical interpretation

1 Mathematical preparation

As it is well known, in astrophysics and cosmology mostly the Gauss system (g, cm, s) for comparison with the empirical measuring process is used. We follow here this historical tradition, since for avoiding theoretical misleading we try to be near to the physical situation.

As we always emphasized in treating the start of PUFT in the 5-dimensional projective space, we remind that the basic mathematical quantities used are projectors, introduced and applied by O. Veblen [4]. The reason for this prominent position of these quantities results from their peculiar property that the postulated 5-dimensional bastion of fundamental physics (covariance for homogenous 5-dimensional coordinate transformations) by projection onto the 4-dimensional space-time yields the general-covariant concept of basic physics (covariance for general-relativistic 4-dimensional coordinate transformations).

For this whole article: Greek indices run form 1 to 5, Latin indices from 1 to 4. The signatures are: of the 5-dimensional metric $(+++-+)$, of the 4-dimensional space-time metric $(+++-)$. Comma means the partial derivative and semicolon the covariant derivative. As usually used, x^i are the 4-dimensioal coordinates and X^ν the 5-dimensional coordinates. The coordinates-connection function for these two types of coordinates reads:

$$\begin{aligned} &\text{a)}\quad x^i = x^i(X^\mu) \quad \text{with} \\ &\text{b)}\quad x^i{}_{,\mu} X^\mu = 0 \quad \text{(homogeneity relation of degree 0).} \end{aligned} \tag{1}$$

Using the abbreviation

$$g^k_\mu = x^k{}_{,\mu} \quad \text{(projection cosines),} \tag{2}$$

so the equation (1b) takes the form

$$g^i_\mu X^\mu = \boldsymbol{e}^i \boldsymbol{R} = 0\,. \tag{3}$$

For getting through the occurring large complex of calculations it is advantageous to perform a large part of this task by calculating means compact vectors. For this case we define by help of the 5-dimensional radius vector $\boldsymbol{R}$ and the invariant scalar S (amount of the radius vector with the physical dimension of length):

$$\text{a)}\quad \boldsymbol{R} = \boldsymbol{e}_\mu X^\mu \quad \text{with} \quad \text{b)}\quad S^2 = g_{\mu\nu} X^\mu X^\nu\,, \tag{4}$$

the radial unit vector:

$$\text{a)}\quad \boldsymbol{s} = \frac{\boldsymbol{R}}{S} = \boldsymbol{e}_\mu s^\mu \quad \text{with} \quad \text{b)} \quad s^\mu = \frac{X^\mu}{S}\,, \quad \text{and} \quad \text{c)} \quad |\boldsymbol{s}| = 1\,. \tag{5}$$

These results lead to following correspondence of two different five-legs spanning the 5-dimensional projective space:

$$\bigl\{\boldsymbol{e}_i\,,\,\boldsymbol{s}\bigr\} \longleftrightarrow \bigl\{\boldsymbol{e}_\mu\bigr\}\,. \tag{6}$$

Our next step is now the identification of the abbreviation (2) as scalar product:

$$\text{a)}\quad g_i^\mu = \boldsymbol{e}^\mu \boldsymbol{e}_i\,, \quad \text{b)} \quad g_\mu^i = \boldsymbol{e}_\mu \boldsymbol{e}^i\,. \tag{7}$$

Hence the following vectorial set of important formulas results which are very useful for further calculations:

$$\begin{aligned} &\text{a)}\quad \boldsymbol{e}_i = \boldsymbol{e}_\mu g_i^\mu\,, && \text{b)}\quad \boldsymbol{e}_\mu = \boldsymbol{e}_i g_\mu^i + \boldsymbol{s} s_\mu\,, \\ &\text{c)}\quad g_i^\mu\, g_\mu^j = g_i^j\,, && \text{d)}\quad g_\nu^i\, g_i^\mu = g_\nu^\mu - s_\nu\, s^\mu\,. \end{aligned} \tag{8}$$

2 Geodetic

First we define the square of the 5-dimensional line element by the equation

$$(\mathrm{d}\overset{5}{\boldsymbol{s}})^2 = (\mathrm{d}\overset{5}{s})^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu\,. \tag{9}$$

Hence the following equation of the geodetic results:

$$\left(\frac{\mathrm{d}X^\mu}{\mathrm{d}\overset{5}{s}}\right)_{;\alpha} \frac{\mathrm{d}X^\alpha}{\mathrm{d}\overset{5}{s}} = 0\,. \tag{10}$$

Further we find

$$\beta = X_\mu\, \frac{\mathrm{d}X^\mu}{\mathrm{d}\overset{5}{s}} = \boldsymbol{R}\overset{5}{\boldsymbol{t}} = \text{const}\,. \tag{11}$$

By projection of (10) onto space-time the 4-dimensional form of the geodetic reads:

$$\frac{\mathrm{d}^2 x^m}{\mathrm{d}\overset{5}{s}{}^2} + \begin{Bmatrix} m \\ ij \end{Bmatrix} \frac{\mathrm{d}x^i}{\mathrm{d}\overset{5}{s}}\, \frac{\mathrm{d}x^j}{\mathrm{d}\overset{5}{s}} - \frac{\beta}{S^2}\, X^m{}_k\, \frac{\mathrm{d}x^k}{\mathrm{d}\overset{5}{s}} - \frac{\beta^2}{S^3}\, S^{,m} = 0\,, \tag{12}$$

where X_{mn} is the 4-dimensional antisymmetric representative of the electromagnetic field strength tensor B_{mn}.

Finally we mention the connection between the 5-dimensional und 4-dimensional line element:

$$\mathrm{d}\overset{5}{s} = \underset{(-)}{+}\, \frac{ic\mathrm{d}\tau}{\sqrt{1 - \dfrac{\beta^2}{S^2}}}\,. \tag{13}$$

3 Hamilton-Lagrange formalism and field equations

3.1 Projective space:

The following formula for the 5-dimensional field theoretical action integral is immediately understandable:

$$\text{a)}\quad W = \frac{1}{c}\int\limits_{V_5} \overset{5}{L}\, \mathrm{d}^{(5)} f = \frac{1}{c}\int\limits_{V_5} \overset{5}{\mathcal{L}}\, \mathrm{d}^{(5)} X \qquad \text{mit} \qquad \text{b)}\ \overset{5}{\mathcal{L}} = \overset{5}{L}\sqrt{\overset{5}{g}}\,, \tag{14}$$

where well-known quantities occur: $\overset{5}{L}$ Lagrange density, $\overset{5}{\mathcal{L}}$ Lagrange function, $\overset{5}{g} = -\det(g_{\mu\nu})$ negative metrical determinant, $\mathrm{d}^{(5)} f$ 5-dimensional volume element.

The 5-dimensional Lagrange equation reads:

$$\frac{\delta \overset{5}{\mathcal{L}}}{\delta g_{\mu\nu}} = \frac{\partial \overset{5}{\mathcal{L}}}{\partial g_{\mu\nu}} - \left(\frac{\partial \overset{5}{\mathcal{L}}}{\partial g_{\mu\nu,\tau}}\right)_{,\tau} + \left(\frac{\partial \overset{5}{\mathcal{L}}}{\partial g_{\mu\nu,\tau,\sigma}}\right)_{,\tau,\sigma} = 0\,. \tag{15}$$

For further treatment, the Lagrange density is split into a geometrical and a substrate part:

$$\overset{5}{L} = \overset{5}{L}{}^{(\mathrm{geom})} + \overset{5}{L}{}^{(\Theta)}\,, \tag{16}$$

where the substrate part belongs to the non-geometized basic matter (e.g. quantum matter) which is not treated in detail in our considerations, but abstractly investigated.

In physical investigations we arrived at the Lagrange density for the geometrical part:

$$\overset{5}{L}{}^{(\mathrm{geom})} = -\frac{1}{2\varkappa_0 S}\overset{5}{R} + \frac{2}{\varkappa_0 S^3} S_{,\alpha}S^{,\alpha} - \frac{\lambda_{\mathrm{S}}}{\varkappa_0 S^3} \tag{17}$$

($\overset{5}{R}$ 5-dimensional curvature invariant, $\varkappa_0$ Einstein's gravitational constant, λ_{S} cosmological constant).

Further the definition of the 5-dimensional energy projector

$$\Theta^{\mu\nu} = -\frac{2S}{\sqrt{\overset{5}{g}}}\,\frac{\delta \overset{5}{\mathcal{L}}{}^{(\Theta)}}{\delta g_{\mu\nu}} \tag{18}$$

should be mentioned.

Under these conditions the 5-dimensional field equation for the 5-dimensional metric tensor $g_{\mu\nu}$ reads ($\overset{5}{R}{}^{\alpha}{}_{\varkappa\mu\nu}$ 5-dimensional curvature tensor):

$$R^{\mu\nu} - \tfrac{1}{2}\, g^{\mu\nu}\overset{5}{R} - \frac{1}{S}\, S^{,\mu;\nu} - \frac{2}{S^2}\, S^{,\mu}S^{,\nu} - \frac{4s^{\mu}s^{\nu}}{S}\Big(S^{,\tau}{}_{;\tau} - \frac{3}{2S}\, S_{,\tau}S^{,\tau}$$
$$- \frac{S}{8}\overset{5}{R} - \frac{3\lambda_{\mathrm{S}}}{4S}\Big) + \frac{1}{S}\, g^{\mu\nu}\Big(S^{,\tau}{}_{;\tau} - \frac{\lambda_{\mathrm{S}}}{S}\Big) = \varkappa_0\Theta^{\mu\nu}\,. \tag{19}$$

3.2 Space-time:

The projection formalism (transition from the projective space onto space-time) means: radial projection (by $\boldsymbol{s}$) and 4-leg projection (by g^i_μ). Applying this procedure to the 5-dimensional field equation (19) leads to three sets of 4-dimensional field equations:

Gravitation:

$$R^{mn} - \tfrac{1}{2}\,g^{mn}R - \frac{\lambda_{\mathrm{S}}}{S^2}\,g^{mn} = \frac{1}{2S^2}\left(X^m{}_i X^{in} + \frac{1}{4}\,g^{mn}X_{ij}X^{ij}\right) + \frac{2}{S^2}\left(S^{,m}S^{,n} - \frac{1}{2}\,g^{mn}S_{,k}S^{,k}\right) + \varkappa_0\Theta^{mn}\,; \tag{20}$$

Electromagnetism (basic equation):

$$X^{lm}{}_{;m} = -2\varkappa_0\overset{5}{\Theta}{}^{m\nu}X_\nu\,. \tag{21}$$

With respect to the right hand side of this equation we expect a connection to the electric current density j^i.

Electromagnetism (cyclic equation):

Different from the basic equation, this equivalently important cyclic equation

$$\left[\left(\frac{X_{\varrho\sigma}}{S^2}\right)_{\shortparallel\tau}\right]_{<\varrho\sigma\tau>} = 0\,, \tag{22}$$

is not a result of the above projection procedure, but is a consequence of the former axiomatized mathematics by postulating the Killing equation

$$X_{\beta\,;\mu\,;\tau} + X_{\mu\,;\beta\,;\tau} = 0\,. \tag{23}$$

which follows from symmetry considerations:

$$\left[\left(\frac{X_{ij}}{S^2}\right)_{;\,k}\right]_{<ijk>} = 0\,. \tag{24}$$

Scalarism:

$$S^{,k}{}_{;k} - \frac{1}{S}\,S_{,k}S^{,k} - \frac{\lambda_{\mathrm{S}}}{S} + \frac{1}{8S}\,X_{ij}X^{ij} = \frac{\varkappa_0 S}{2}\left(\Theta_m{}^m - \overset{5}{\Theta}\right), \tag{25}$$

This above part of calculations is rather tedious. The interested reader may look into my two volumes edition [23]. Aside from this remark, one should immediately take notice of following definitions:

$$\text{a)}\quad X_{mn} = X_{\mu\nu}g^\mu_m g^\nu_n \quad\text{with}\quad \text{b)}\quad X_{\alpha\varrho} = X_{\varrho\,,\alpha} - X_{\alpha\,,\varrho} = -X_{\varrho\alpha}\,; \tag{26}$$

$$\text{a)} \quad \overset{5}{R}{}^{mn} = R^{\mu\nu} g^m_\mu \, g^n_\nu \,, \quad \text{b)} \quad \Theta^{mn} = \Theta^{\mu\nu} g^m_\mu \, g^n_\nu \,. \tag{27}$$

4 Local conservation law

Similar to the procedure in the General Relativity Theory, where by covariant differentiation the local conservation law follows, here in PUFT from (19) results the different local conservation law:

$$\left(\frac{\Theta^{\mu\nu}}{S}\right)_{;\nu} = 0 \,. \tag{28}$$

By radial projection we arrive at:

$$\left(S \overset{5}{\Theta}{}^{k\mu} s_\mu\right)_{;k} = 0 \,, \tag{29}$$

whereas we find by 4-leg projection:

$$\Theta^{mk}{}_{;k} = \overset{5}{\Theta}{}^{\beta k} s_\beta s^m{}_k + \frac{1}{S} \, \Theta^{\beta\nu} s_\beta s_\nu S^{,m} \,. \tag{30}$$

5 Physical interpretation of the 4-dimensional geometrical structures

According to its definition (4b) the scalar field S with the physical dimension of length dominated in the above derived physical laws. For physical reasons we tended to prefer the dimensionless quantity σ as the basis field, describing our hypothetical new phenomenon of Nature “scalarism”, on an equivalent level like electromagnetism, etc. Since the notion ”scalaron” was long ago introduced in physics by solid state physics, we chose the word “scalaric”. The definition of σ can be taken from the equation

$$\text{a)} \quad S = S_0 e^\sigma \quad \text{or} \quad \text{b)} \quad \sigma = \ln \frac{S}{S_0} \,. \tag{31}$$

We name the here introduced constant parameter S_0 “scalaric length constant”. The detailed calculations lead to following result for its numerical value:

$$S_0 = e_0 \sqrt{\frac{\varkappa_0}{2\pi}} = \frac{2e_0}{c^2} \sqrt{\gamma_{\mathrm{N}}} = 2,763 \cdot 10^{-34} \, \mathrm{cm} \tag{32}$$

In this relation we meet the electric elementary charge:

$$e_0 = 4.8066 \cdot 10^{-10} \, \mathrm{g}^{1/2} \mathrm{cm}^{3/2} \mathrm{s}^{-1} \,. \tag{33}$$

5.1 Field equations:

Now it is convenient, first to transcribe the space-time laws of physics (20), (21), (25) into a new form and then to explain the new occurring quantities:

$$R^{mn} - \tfrac{1}{2}\, g^{mn} R - \frac{\lambda_{\mathrm{S}}}{S_0^2}\, e^{-2\sigma} g^{mn} = \varkappa_0 \Bigg[\frac{1}{4\pi} \Big(B^{mk} H_k{}^n + \tfrac{1}{4}\, g^{mn} B_{kl} H^{kl} \Big) + \frac{2}{\varkappa_0} \Big(\sigma^{,m} \sigma^{,n} - \tfrac{1}{2}\, g^{mn} \sigma_{,k} \sigma^{,k} \Big) + \Theta^{mn} \Bigg] \tag{34}$$

with

$$R = -\frac{4\lambda_{\mathrm{S}}}{S_0^2}\, e^{-2\sigma} + 2\sigma_{,k}\sigma^{,k} - \varkappa_0 \Theta\,; \tag{35}$$

$$\text{a)} \quad H^{ij}{}_{;j} = \frac{4\pi}{c}\, j^i \quad \text{with} \quad \text{b)} \quad j^i = c\sqrt{\frac{\varkappa_0}{2\pi}}\, \mathrm{e}^{\sigma} \overset{5}{\Theta}{}^{i\nu} s_\nu\,, \tag{36}$$

$$B_{<ij,k>} = 0\,. \tag{37}$$

$$\sigma^{,k}{}_{;k} - \frac{\lambda_{\mathrm{S}}}{S^2} = -\frac{\varkappa_0}{2} \Big(\vartheta + \frac{1}{8\pi}\, B_{ij} H^{ij} \Big)\,. \tag{38}$$

Aside from S_0 and e_0 further new quantities occurring in the above field equations are:

$$\gamma_{\mathrm{N}} = 6,6742 \cdot 10^{-8}\, \mathrm{g}^{-1}\mathrm{cm}^3\mathrm{s}^{-2} \tag{39}$$

(Newton's gravitational constant),

$$\varkappa_0 = \frac{8\pi\gamma_{\mathrm{N}}}{c^4} = 2,0767 \cdot 10^{-48}\, \mathrm{g}^{-1}\mathrm{cm}^{-1}\mathrm{s}^2 \tag{40}$$

(Einstein's gravitational constant);

$$B_{ij} = -e_0 \frac{s_{ij}}{S} = -e_0 \frac{X_{ij}}{S^2} \tag{41a}$$

(electromagnetic field strength tensor),

$$H^{ij} = \varepsilon B^{ij} = -\frac{e_0}{S_0^2}\, X^{ij} \tag{41b}$$

(electromagnetic induction tensor).,

$$\varepsilon = e^{2\sigma}\,, \tag{41c}$$

(scalaric electromagnetic vacuum dielectricity);

$$\overset{5}{\Theta} = \Theta^{\mu}{}_{\mu} \tag{42a}$$

(5-dimensional substrate trace),

$$\Theta^{mn} = \Theta^{\mu\nu} g^m_\mu g^n_\nu \tag{42b}$$

(substrate energy tensor),

$$\Theta = \Theta_k{}^k \tag{42c}$$

(4-dimensional substrate trace),

$$\vartheta = \Theta_\mu{}^\mu - \Theta_k{}^k = \overset{5}{\Theta} - \Theta \tag{42d}$$

(scalaric substrate energy density = scalerg density);

$$j^i = c\sqrt{\frac{\varkappa_0}{2\pi}}\, \mathrm{e}^\sigma \overset{5}{\Theta}{}^{i\nu} s_\nu \tag{42e}$$

(electric current density). Here it is convenient to simplify the derived field equations (35), (36), (37), (38) the partly well-known abbreviations:

$$\text{a)} \quad E^{mn} = \frac{1}{4\pi}\left(B^{mk} H_k^n + \tfrac{1}{4}\, g^{mn} B_{kl} H^{kl}\right) \quad \text{with} \quad \text{b)} \quad E_i^i = 0 \tag{43}$$

(electromagnetic energy tensor),

$$\Lambda_\mathrm{L} = B_{ij} H^{ij} \tag{44}$$

(Larmor invariant),

$$S^{mn} = \frac{2}{\varkappa_0}\left(\sigma^{,m}\sigma^{,n} - \tfrac{1}{2}\, g^{mn} \sigma_{,k}\sigma^{,k}\right) \tag{45}$$

(scalar energy tensor). Hence the field equations (35) and (38) receive the much simple shape:

$$R^{mn} - \tfrac{1}{2}\, g^{mn} R - \frac{\lambda_\mathrm{S}}{S_0^2}\, e^{-2\sigma} g^{mn} = \varkappa_0 \Big[E^{mn} + S^{mn} + \Theta^{mn}\Big], \tag{46}$$

$$\sigma^{,k}{}_{;k} - \frac{\lambda_\mathrm{S}}{S_0^2} e^{-2\sigma} = -\frac{\varkappa_0}{2}\left(\vartheta + \frac{1}{8\pi}\, \Lambda_\mathrm{L}\right). \tag{47}$$

5.2 Local conservation law:

Using this new form of designation for the above introduced physical quantities, hence we receive from (30) the 4-dimensional continuum-mechanical equation of motion:

$$\Theta^{mk}{}_{;k} = -\frac{1}{c}\, B^m{}_k j^k + \vartheta\sigma^{,m} \tag{48}$$

and from (29) the electrical continuity equation

$$j^k{}_{;k} = 0\,. \tag{49}$$

References

[1] Th. Kaluza. Sitzungsber. d. preuss. Akad. d. Wiss., Phys.-math. Klasse 966 (1921).

[2] O. Klein. Z. Phys. 37, 895 (1926).

[3] P. Jordan. Schwerkraft und Weltall, Vieweg Braunschweig 1955

[4] G. Ludwig. Fortschritte der projektiven Relativitätstheorie, Vieweg Braunschweig 1951; O. Veblen. Projektive Relativitaetstheorie, Springer-Verlag Berlin 1933; D. van Dantzig. Math. Ann. 106, 400 (1932).

[5] A. Lichnerowicz. Théories relativistes de la Gravitation et de l´Electromagnetisme, Masson Paris 1955.

[6] C. Brans and R.H. Dicke. Phys. Rev. **124**, 925 (1961).

[7] W. Pauli. Relativitätstheeorie. Enzykl. Math. Phys. **V** 2 (1921) 539.

[8] W. Pauli. Ann. Physik **18** (1933) 305.

[9] P. Jordan. Die Wissenschaft **124**, Vieweg Braunschweig 1966.

[10] E. L. Schuecking. Jordan, Pauli, Politics and the variable gravitational constant, Physics today **52** (10) 26; doi: 10.10. 6311.882858.

[11] E. Schmutzer. Proceedings of the 9th International Conference on General Relativity and Gravitation, Deutscher Verlag der Wissenschaften Berlin 1983; E. Schmutzer. Relativistische Physik, Teubner Verlagsgesellschaft, Leipzig 1968.

[12] E. Schmutzer. arXiv: astro- ph/0502538v1 25 Feb 2005.

[13] E. Schmutzer. arXiv: astro- ph/0505222v1 11 May 2005.

[14] E. Schmutzer. arXiv: astro- ph/0201007v1 2 Jan 2002.

[15] E. Schmutzer. arXiv: gr-qc/0106050v1 14 Jun 2001.

[16] E. Schmutzer. arXiv: gr-qc/0106049v1 14 Jun 2001.

[17] A. K. Gorbatsievich. arXiv: gr-qc/0003053v1 14 Mar 2000.

[18] A. Gorbatsievich and E. Schmutzer. arXiv: 1203.5212v2 [gr-qc] 17 May 2012.

[19] A.A. Blinkouski and A. K. Gorbatsievich. arXiv: gr-qc/0312092v2 22 Dec 2003.

[20] A. J. Sanders, G. T. Gillies and E. Schmutzer: 1005.3226v2 [gr-qc].

[21] E. Schmutzer. Grundlagen der Theoretischen Physik, Vol. 1 and Vol. 2, 3th edition, Wiley-VCH Weinheim 2005.

[22] E. Schmutzer. Projektive Einheitliche Feldtheorie mit Anwendungen in Kosmologie und Astrophysik (Neues Weltbild ohne Urknall?), Harri Deutsch Frankfurt/Main 2004; E. Schmutzer. Fünfdimensionale Physik, Wissenschaftsverlag Thueringen, Langewiesen 2009.

[23] E. Schmutzer. Fünfdimensionale Physik (Projektive Einheitliche Feldtheorie), Vol.1, 2nd edition, Wissenschaftsverlag Thueringen, Langewiesen 2013; E. Schmutzer. Fünfdimensionale Physik (Projektive Einheitliche Feldtheorie), Vol.2, 2nd edition, Wissenschaftsverlag Thueringen, Langewiesen 2015.

The author would like to thank Professor A. Gorbatsievich/Minsk for many helpful discussions on PUFT and for technical help.