Formula of the sieve of Eratosthenes

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Abstract

This article offers a solution in relation to the distribution of the primes, in this research we provide simple formulas and unpublished with a new approach that allow us to assimilate and conclude that the entities primales are sorted as regular as possible. We provide a new vision for addressing what that since ancient times has been a real challenge for the minds linked to the world mathematician, we deliver the reader a key to unravel the structure and behavior of the primes without open the door to the complexity.

Keywords: Prime Numbers, patterns, distribution, Number Theory, Mathematics, mystery resolved, Order, behavior, New Vision, Sieve, Eratosthenes.

"There are two facts about the distribution of prime numbers which I hope to convince so undeniable that shall be permanently recorded on their hearts. The first is that, despite its simple definition and role as the bricks with which are built the natural numbers, the prime numbers grow as weeds among the natural numbers and do not seem to obey any other law that the random, and nobody can predict where will spring up the next. The second fact is even more amazing, because it says the opposite: that the prime numbers show a regularity appalling, that there are laws that govern their behavior and who obey these laws precisely almost military."

[Fr Bernard *Zagier,* mathematician E.U.A]

1. Introduction

The world in which we move this surrounded by numbers, we find them everywhere, are part of our everyday life and even help us to communicate [1], with them we understand the world that surrounds us as stated Filolao, a Greek philosopher that affirms the important role of the numbers:

"The number resides in everything that is known. Without him it is impossible to think of anything or knowing anything."

The numbers are immersed in all the activities that the human being develops, what expressed leads to conceive of a dependency of the Mathematical forced, the numbers are essential in our daily living, particularly in mathematics are foundation and column as stated by the mathematician Gauss of German origin, one of the greatest mathematicians of history known as the prince of Mathematics:

Gauss
(1777-1855)

"Math is the queen of the sciences and the theory of numbers is the queen of Mathematics".

The most important numbers are the prime numbers, accordingly is brewing a question to the air:

What are the prime numbers and why we note their importance?

There are two basic characteristics about what is a prime number:

- *Is divisible by one*
- *Is divisible by itself*

Then a prime number is one that can only be divided by two numbers; the one and the same number, i.e. has two dividers.

Through the following table we liken the concept with an example:

Number	**What is the cousin?**	**Description**
7	Yes	Has two dividers: 1 and 7
9	No	Has three:1,3,9 Dividers

The list of the first prime numbers is the following:

$$\boldsymbol{P} = \{\mathbf{2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97}, \ldots\}$$

But there is more fabric of where cut about this subject, the prime numbers are the basis of Mathematics, yes math outside a castle prime numbers would be the bricks. All the numbers that are not cousins can be built by multiplying Prime Numbers, for example:

33 = 3*11, where 3 and 11 are prime numbers.
12= 2*2*3, where 2 and 3 are prime numbers.

We interpret that the prime numbers are the atoms of the Mathematics, are as hydrogen and oxygen in the universe of the numbers, here is its importance.

Since more than two thousand years ago, the mathematicians have been interested in these fascinating and important numbers, in ancient Greece various characters explored and deciphered their importance. The name of Euclid jumps to the light, Euclid showed that the prime numbers were infinite, specifically proved that his nature was inexhaustible and did so from the logical reasoning.

After the enormous contribution of Euclid, would emerge a new question:

Is there a pattern that allows to interpret the distribution of Prime Numbers on its way to the Infinite?

If we imagine the natural numbers in a row: 1, 2, 3, 4, 5, 6, 7,... , gives us the impression that the prime numbers are placed randomly, that their behavior is sympathetic with the random, look at the illustration:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**…**
***Prime Numbers in gray**																								

One of the missions of all mathematician is the search for patterns, to build models that predict and interpret the chaos of our around, necessarily the most brilliant minds of all times are penetrated in the conquest of patterns that determine the behavior of the Prime Numbers, we can highlight Gauss, Euler and many others: mathematical geniuses who wondered with insistence:

Is there an order in the prime numbers, what is the pattern that followed?

It is here where springs one of the greatest mysteries of Mathematics:

<u>The pattern followed by the Prime Numbers</u>

This problem for many represents the holy grail of mathematics, it is a challenge that has plagued all those sages who defied conquering.

Mathematicians of all races have dared to resolve the mysterious and still unknown pattern with the Working Prime numbers, their intention is to rest that ghost primal that are harassed and implores its logical service to be able to rest in peace.

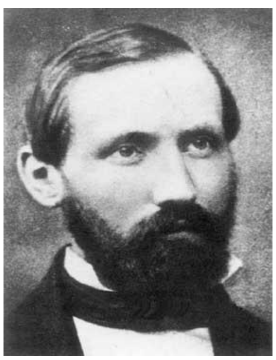
What progress there are today in relation to the pattern of the primes, of these mathematical entities that seem to mock the order?

It is necessary and fair to mention Gauss, who sculpted the most significant advance in terms of the pattern of the prime numbers as well as his pupil and disciple Bernard Riemann.

After failing in the attempt to find a model that is interpreted by the order of the primes, Gauss conducted a rotation of one hundred and eighty degrees to the problem and in alternative is asked:

How many prime numbers there are in a given interval? , for example how many prime numbers there are less than 10, less than 100 minors 1000 and so on?

Began to construct tables and tables of prime numbers and finally found a pattern with which develops a formula that gives approximate information in regard to the amount of prime numbers in a given range, though it is an approximation of what its formula of density material produces, Gauss opened a window of light to that mystery.



Another of the geniuses who opened more windows lighting around the already mentioned mystery was: Riemann, German mathematician (1826 - 1866) who was a disciple of Gauss. Riemann worked with a mathematical function call zeta function which the be carried to a three-dimensional mathematical scenario showed a connection with the distribution of the primes, this brought him to the conception of a new geometry, zeta function could decrypt the secrets of these numbers, this approach would improve accuracy the work of Gauss demonstrated that the correspondence of its formula with the prime numbers are fulfilled to infinity, to date it has not been demonstrated being for posterity the famous "Riemann Hypothesis", you will be given a million dollars for which the proof.

Euler (Swiss mathematician) some day affirm:

"Until the day of today, Mathematicians have tried in vain to find some order in the succession of the primes, and we have reasons to believe that it is a mystery in which the mind never will penetrate".

2. A NEW APPROACH TO THE INTERPRETATION OF THE ORDER OF THE PRIME NUMBERS

The last three years have been one of the most exciting trips of my life, I have done research varied in relation to the Prime Numbers, mainly in the mystery of his behavior, hence the issue has been of great interest not only for myself but also for many mathematicians throughout history, which can highlight to Gauss, Riemann, Euler, Euclid and many others.

I have entered the world of this sublime mystery which for many is known as the holy grail of mathematics, and I want to share through this article the final conclusion that I have drawn in relation to the distribution of these entities mathematics. My research on prime numbers and mathematics can see through the internet in a special blog entitled: ***Matemático Fresnillense*** [5].

I worked with many mathematical concepts linked to the world of prime numbers as they are:

The Fibonacci Sequence [6]*, Primoriales, Constant Kaprekar, assumptions china, etc.*

Until recently changed approach to the vision of the patterns associated with the Prime Numbers, with much respect I dare to say that this issue has been dealt with in the way more complex by a large percentage of the mathematical community, it seems to me to understand the behavior of the primes deals rotate the perspective, and so it was that I concluded that all the investigations that had made i had provided the response to this challenge from a long time ago, the solution was always there in front of my eyes, I refused to observe, but today things are different and then les I present my findings and conclusions. I also learned that there are many edges that lead us to the truth, I gave them a simple path through this document, many thanks in advance.

I would like to begin first explaining some mathematical concepts that will help us in the present work to understand the distribution of the Prime Numbers through this new perspective, to then show the results and conclusions.

The Constant Kaprekar* [5]*, is the number 6174 that was discovered by the Indian mathematician D. R. Kaprekar. The number of Kaprekar has a mysterious and interesting property:

To get it we select any number of 4 digits in the that the digits are not equal, for example the 8253. Now we will simply reorder the digits to get the largest number (8532) and the lowest possible number (2358), we are now the difference:

8532-2358 = 6174

Always arrives in 6174, in particular with this number to the first. The maximum number of iterations is seven, although the most frequent is that they are only three iterations.

Yes we select the number 1562:

6521 - 1256 = 5265

6552 - 2556 = 3996

9963 - 3699 = 6264

6642 - 2466 = 4176

7641 - 1467 = 6174

There are more constant Kaprekar for numbers with a different amount of figures in this research we focus only on the ***6174.***

Sieve of Eratosthenes, is an algorithm that allows you to find all the prime numbers less than a given natural number n.

Module function or residue (MOD), is a mathematical operation that returns the remainder of a division, for example: 16 Mod 3 to carry out the division 16/3 the result is 5, but the excess or residue is 1 therefore 16 Mod 3 =1.

Greatest common divisor function (G.C.D), defines the greatest common divisor (abbreviated "G.C.D" or "M.C.D") of two or more integers the largest number that divides them without leaving a remainder. For example, the g.c.d of 42 and 56 is 14.

2.1 **Pattern of prime numbers**

In an investigation of March 2013 [5] an intuition made that tried to relate the constant Kaprekar with Prime Numbers, I wondered if the famous algorithm of the sieve of Eratosthenes could materialize in a formula. I started to work in the Wolfram Matemathica software with a formula, my idea, which included the Kaprekar constant , the functions greatest common divisor and module and the prime numbers 2,3,5,7, for me surprise a pattern began to manifest itself, I came to an interesting expression that I produced sequences of zeros and prime numbers, which I published in the blog of my research [5] and forgot for some time until recently, where I went back to the study and found pure gold, which along with other research and other mathematical concepts already had pushed me and open doors and windows of light in the face of this mystery.

The formula with which he worked at that time was the following:

We proceed from the expression:

n*((GCD [5, n (((GCD [6174,n] mod n) mod 3) mod 2)] mod 5) mod n)

We generated for the first three hundred natural values of 'n' the sequence:

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11, 0, 13, 0, 0, 0, 17, 0, 19, 0, 0, 0, 23, 0, 0, 0, 0, 0, 29, 0, 31, 0, 0, 0, 0, 0, 37, 0, 0, 0, 41, 0, 43, 0, 0, 0, 47, 0, 0, 0, 0, 0, 53, 0, 0, 0, 0,
 0, 59, 0, 61, 0, 0, 0, 0, 0, 67, 0, 0, 0, 71, 0, 73, 0, 0, 0, 77, 0, 79, 0, 0, 0, 83, 0, 0, 0, 0, 0, 89, 0, 91, 0, 0, 0, 0, 0, 97, 0, 0, 0, 101, 0, 103, 0, 0, 0, 107, 0,
 109, 0, 0, 0, 113, 0, 0, 0, 0, 0, 119, 0, 121, 0, 0, 0, 0, 0, 127, 0, 0, 0, 131, 0, 133, 0, 0, 0, 137, 0, 139, 0, 0, 0, 143, 0, 0, 0, 0, 0, 149, 0, 151, 0, 0, 0, 0, 0,
 157, 0, 0, 0, 161, 0, 163, 0, 0, 0, 167, 0, 169, 0, 0, 0, 173, 0, 0, 0, 0, 0, 179, 0, 181, 0, 0, 0, 0, 0, 187, 0, 0, 0, 191, 0, 193, 0, 0, 0, 197, 0, 199, 0, 0, 0, 203,
 0, 0, 0, 0, 0, 209, 0, 211, 0, 0, 0, 0, 0, 217, 0, 0, 0, 221, 0, 223, 0, 0, 0, 227, 0, 229, 0, 0, 0, 233, 0, 0, 0, 0, 0, 239, 0, 241, 0, 0, 0, 0, 0, 247, 0, 0, 0, 251, 0,
 253, 0, 0, 0, 257, 0, 259, 0, 0, 0, 263, 0, 0, 0, 0, 0, 269, 0, 271, 0, 0, 0, 0, 0, 277, 0, 0, 0, 281, 0, 283, 0, 0, 0, 287, 0, 289, 0, 0, 0, 293, 0, 0, 0, 0, 0, 299, 0}
```

We realize that occur zeros, prime numbers and some numbers compounds multiples of seven, eleven, thirteen and seventeen, the windows of light begin its opening.

You change the formula in order to make disappear the parasites of multiples of the compounds numbers seven, eleven, thirteen and seventeen we now have the expression:

n (GCD [7, n ((GCD[5,n (((GCD[6174,n] mod n) mod 3) mod 2)] mod 5) mod n)] mod 7)

Now the formula we produced for the first three hundred natural values of 'n':

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11, 0, 13, 0, 0, 0, 17, 0, 19, 0, 0, 0, 23, 0, 0, 0, 0, 0, 29, 0, 31, 0, 0, 0, 0, 0, 37, 0, 0, 0, 41, 0, 43, 0, 0, 0, 47, 0, 0, 0, 0, 0, 53, 0, 0, 0, 0, 0, 59, 0, 61, 0, 0, 0, 0, 0, 67, 0, 0, 0, 71, 0, 73, 0, 0, 0, 0, 0, 79, 0, 0, 0, 83, 0, 0, 0, 0, 0, 89, 0, 0, 0, 0, 0, 0, 0, 97, 0, 0, 0, 101, 0, 103, 0, 0, 0, 107, 0, 109, 0, 0, 0, 113, 0, 0, 0, 0, 0, 0, 0, 121, 0, 0, 0, 0, 0, 127, 0, 0, 0, 131, 0, 0, 0, 0, 0, 137, 0, 139, 0, 0, 0, 143, 0, 0, 0, 0, 0, 149, 0, 151, 0, 0, 0, 0, 0, 157, 0, 0, 0, 0, 0, 163, 0, 0, 0, 167, 0, 169, 0, 0, 0, 173, 0, 0, 0, 0, 0, 179, 0, 181, 0, 0, 0, 0, 0, 187, 0, 0, 0, 191, 0, 193, 0, 0, 0, 197, 0, 199, 0, 0, 0, 0, 0, 0, 0, 0, 0, 209, 0, 211, 0, 0, 0, 0, 0, 0, 0, 0, 0, 221, 0, 223, 0, 0, 0, 227, 0, 229, 0, 0, 0, 233, 0, 0, 0, 0, 0, 239, 0, 241, 0, 0, 0, 0, 0, 247, 0, 0, 0, 251, 0, 253, 0, 0, 0, 257, 0, 0, 0, 0, 0, 263, 0, 0, 0, 0, 0, 269, 0, 271, 0, 0, 0, 0, 0, 277, 0, 0, 0, 281, 0, 283, 0, 0, 0, 0, 0, 289, 0, 0, 0, 293, 0, 0, 0, 0, 0, 299, 0}

We note after the calculation that we make disappear to compound numbers multiples of seven, but now part of produce zeros and prime numbers occur parasites compounds multiples of the number eleven, thirteen and seventeen that are the following prime numbers after the seven.

We grow a little more to the formula modifying it for now disappear the compounds multiples of the eleven and thirteen arrived at:

n *(GCD[13,n (GCD[11,n (GCD[7,n ((GCD[5,n (((GCD[6174,n] mod n) mod 3) mod 2)] mod 5) mod n)] mod 7)] mod 11)] mod 13)

For the first five hundred natural values of 'n' is generated the sequence:

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 17, 0, 19, 0, 0, 0, 23, 0, 0, 0, 0, 0, 29, 0, 31, 0, 0, 0, 0, 0, 37, 0, 0, 0, 41, 0, 43, 0, 0, 0, 47, 0, 0, 0, 0, 0, 53, 0, 0, 0, 0, 0, 0, 59, 0, 61, 0, 0, 0, 0, 0, 67, 0, 0, 0, 71, 0, 73, 0, 0, 0, 0, 0, 79, 0, 0, 0, 83, 0, 0, 0, 0, 0, 89, 0, 0, 0, 0, 0, 0, 0, 97, 0, 0, 0, 101, 0, 103, 0, 0, 0, 107, 0, 109, 0, 0, 0, 113, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 127, 0, 0, 0, 131, 0, 0, 0, 0, 0, 137, 0, 139, 0, 0, 0, 0, 0, 0, 0, 0, 0, 149, 0, 151, 0, 0, 0, 0, 0, 157, 0, 0, 0, 0, 0, 163, 0, 0, 0, 167, 0, 0, 0, 0, 0, 173, 0, 0, 0, 0, 0, 179, 0, 181, 0, 0, 0, 0, 0, 0, 0, 0, 0, 191, 0, 193, 0, 0, 0, 197, 0, 199, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 211, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 223, 0, 0, 0, 227, 0, 229, 0, 0, 0, 233, 0, 0, 0, 0, 0, 239, 0, 241, 0, 0, 0, 0, 0, 0, 0, 0, 0, 251, 0, 0, 0, 0, 0, 257, 0, 0, 0, 0, 0, 263, 0, 0, 0, 0, 0, 269, 0, 271, 0, 0, 0, 0, 0, 277, 0, 0, 0, 281, 0, 283, 0, 0, 0, 0, 0, 289, 0, 0, 0, 293, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 307, 0, 0, 0, 311, 0, 313, 0, 0, 0, 317, 0, 0, 0, 0, 0, 323, 0, 0, 0, 0, 0, 0, 0, 331, 0, 0, 0, 0, 0, 337, 0, 0, 0, 0, 0, 0, 0, 0, 0, 347, 0, 349, 0, 0, 0, 353, 0, 0, 0, 0, 0, 359, 0, 361, 0, 0, 0, 0, 0, 367, 0, 0, 0, 0, 0, 373, 0, 0, 0, 0, 0, 379, 0, 0, 0, 383, 0, 0, 0, 0, 0, 389, 0, 391, 0, 0, 0, 0, 0, 397, 0, 0, 0, 401, 0, 0, 0, 0, 0, 0, 0, 409, 0, 0, 0, 0, 0, 0, 0, 0, 0, 419, 0, 421, 0, 0, 0, 0, 0, 0, 0, 0, 0, 431, 0, 433, 0, 0, 0, 437, 0, 439, 0, 0, 0, 443, 0, 0, 0, 0, 0, 449, 0, 0, 0, 0, 0, 0, 0, 457, 0, 0, 0, 461, 0, 463, 0, 0, 0, 467, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 479, 0, 0, 0, 0, 0, 0, 0, 487, 0, 0, 0, 491, 0, 493, 0, 0, 0, 0, 0, 499, 0}

Where we generate zeros, prime numbers and parasites compounds multiples of the following prime numbers after 13, 17 and 19.

Until this time if they have been the analytical final pattern prime numbers begins be latent.

Once more we return to modify the formula to make disappear parasites compounds in multiples of 17, 19 and 23, we now have the expression :

n*(GCD[23,n (GCD[19,n (GCD[17,n (GCD[13,n (GCD[11,n (GCD[7,n ((GCD[5,n (((GCD[6174,n] mod n) mod 3) mod 2)] mod 5) mod n)] mod 7)] mod 11)] mod 13)] mod 17)] mod 19)] mod 23)

For the first eight hundred forty natural values of 'n' we have:

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 29, 0, 31, 0, 0, 0, 0, 0, 37, 0, 0, 0, 41, 0, 43, 0, 0, 0, 47, 0, 0, 0, 0, 0, 53, 0,
0, 0, 0, 0, 59, 0, 61, 0, 0, 0, 0, 0, 67, 0, 0, 0, 71, 0, 73, 0, 0, 0, 0, 0, 79, 0, 0, 0, 83, 0, 0, 0, 0, 0, 89, 0, 0, 0, 0, 0, 0, 0, 97, 0, 0, 0, 101, 0, 103, 0,
0, 0, 107, 0, 109, 0, 0, 0, 113, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 127, 0, 0, 0, 131, 0, 0, 0, 0, 0, 137, 0, 139, 0, 0, 0, 0, 0, 0, 0, 0, 0, 149, 0, 151, 0,
0, 0, 0, 0, 157, 0, 0, 0, 0, 0, 163, 0, 0, 0, 167, 0, 0, 0, 0, 0, 173, 0, 0, 0, 0, 0, 179, 0, 181, 0, 0, 0, 0, 0, 0, 0, 0, 0, 191, 0, 193, 0, 0, 0, 197, 0, 199,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 211, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 223, 0, 0, 0, 227, 0, 229, 0, 0, 0, 233, 0, 0, 0, 0, 0, 239, 0, 241, 0, 0, 0, 0, 0, 0, 0, 0,
0, 251, 0, 0, 0, 0, 0, 257, 0, 0, 0, 0, 0, 263, 0, 0, 0, 0, 0, 269, 0, 271, 0, 0, 0, 0, 0, 277, 0, 0, 0, 281, 0, 283, 0, 0, 0, 0, 0, 0, 0, 0, 0, 293, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 307, 0, 0, 0, 311, 0, 313, 0, 0, 0, 317, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 331, 0, 0, 0, 0, 0, 337, 0, 0, 0, 0, 0, 0, 0, 0, 0, 347,
0, 349, 0, 0, 0, 353, 0, 0, 0, 0, 0, 359, 0, 0, 0, 0, 0, 0, 0, 367, 0, 0, 0, 0, 0, 373, 0, 0, 0, 0, 0, 379, 0, 0, 0, 383, 0, 0, 0, 0, 0, 389, 0, 0, 0, 0, 0, 0, 0,
397, 0, 0, 0, 401, 0, 0, 0, 0, 0, 0, 0, 409, 0, 0, 0, 0, 0, 0, 0, 0, 0, 419, 0, 421, 0, 0, 0, 0, 0, 0, 0, 0, 0, 431, 0, 433, 0, 0, 0, 0, 0, 439, 0, 0, 0, 443, 0,
0, 0, 0, 0, 449, 0, 0, 0, 0, 0, 0, 0, 457, 0, 0, 0, 461, 0, 463, 0, 0, 0, 467, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 479, 0, 0, 0, 0, 0, 0, 0, 487, 0, 0, 0, 491, 0, 0,
0, 0, 0, 0, 0, 499, 0, 0, 0, 503, 0, 0, 0, 0, 0, 509, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 521, 0, 523, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 541, 0, 0, 0,
0, 0, 547, 0, 0, 0, 0, 0, 0, 0, 0, 0, 557, 0, 0, 0, 0, 0, 563, 0, 0, 0, 0, 0, 569, 0, 571, 0, 0, 0, 0, 0, 577, 0, 0, 0, 0, 0, 0, 0, 0, 0, 587, 0, 0, 0, 0, 0, 593,
0, 0, 0, 0, 0, 599, 0, 601, 0, 0, 0, 0, 0, 607, 0, 0, 0, 0, 0, 613, 0, 0, 0, 617, 0, 619, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 631, 0, 0, 0, 0, 0, 0, 0, 0, 0, 641, 0,
643, 0, 0, 0, 647, 0, 0, 0, 0, 0, 653, 0, 0, 0, 0, 0, 659, 0, 661, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 673, 0, 0, 0, 677, 0, 0, 0, 0, 0, 683, 0, 0, 0, 0, 0, 0, 0, 691,
0, 0, 0, 0, 0, 0, 0, 0, 0, 701, 0, 0, 0, 0, 0, 0, 0, 709, 0, 0, 0, 0, 0, 0, 0, 0, 0, 719, 0, 0, 0, 0, 0, 0, 0, 727, 0, 0, 0, 0, 0, 733, 0, 0, 0, 0, 0, 739, 0, 0,
0, 743, 0, 0, 0, 0, 0, 0, 0, 751, 0, 0, 0, 0, 0, 757, 0, 0, 0, 761, 0, 0, 0, 0, 0, 0, 0, 769, 0, 0, 0, 773, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 787, 0, 0, 0, 0,
0, 0, 0, 0, 0, 797, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 809, 0, 811, 0, 0, 0, 0, 0, 0, 0, 0, 0, 821, 0, 823, 0, 0, 0, 827, 0, 829, 0, 0, 0, 0, 0, 0, 0, 0, 0, 839, 0}

Observe that beautiful sequence occurs, pure poetry, zeros and pure prime numbers, if we exclude the zeros we have 137 cousins effective:

{29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211,
223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409,
419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599, 601, 607, 613, 617,
619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691, 701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809, 811, 821, 823, 827, 829, 839}

After all these tests, analysis of tables and calculations, the ultimate key to understand the distribution of the prime numbers is more than obvious from the following conclusion and vision:

The Prime numbers depend on themselves to propagate through the infinite, are generated in blocks/ litters, being its behavior and distribution as regular as possible, definitely order exist in them.

2.2 Pattern Analysis END OF THE PRIME NUMBERS FROM AN EXAMPLE

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 29, 0, 31, 0, 0, 0, 0, 0, 37, 0, 0, 0, 41, 0, 43, 0, 0, 0, 47, 0, 0, 0, 0, 0, 53, 0,
0, 0, 0, 0, 59, 0, 61, 0, 0, 0, 0, 0, 67, 0, 0, 0, 71, 0, 73, 0, 0, 0, 0, 0, 79, 0, 0, 0, 83, 0, 0, 0, 0, 0, 89, 0, 0, 0, 0, 0, 0, 0, 97, 0, 0, 0, 101, 0, 103, 0,
0, 0, 107, 0, 109, 0, 0, 0, 113, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 127, 0, 0, 0, 131, 0, 0, 0, 0, 0, 137, 0, 139, 0, 0, 0, 0, 0, 0, 0, 0, 0, 149, 0, 151, 0,
0, 0, 0, 0, 157, 0, 0, 0, 0, 0, 163, 0, 0, 0, 167, 0, 0, 0, 0, 0, 173, 0, 0, 0, 0, 0, 179, 0, 181, 0, 0, 0, 0, 0, 0, 0, 0, 0, 191, 0, 193, 0, 0, 0, 197, 0, 199,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 211, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 223, 0, 0, 0, 227, 0, 229, 0, 0, 0, 233, 0, 0, 0, 0, 0, 239, 0, 241, 0, 0, 0, 0, 0, 0, 0, 0,
0, 251, 0, 0, 0, 0, 0, 257, 0, 0, 0, 0, 0, 263, 0, 0, 0, 0, 0, 269, 0, 271, 0, 0, 0, 0, 0, 277, 0, 0, 0, 281, 0, 283, 0, 0, 0, 0, 0, 0, 0, 0, 0, 293, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 307, 0, 0, 0, 311, 0, 313, 0, 0, 0, 317, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 331, 0, 0, 0, 0, 0, 337, 0, 0, 0, 0, 0, 0, 0, 0, 0, 347,
0, 349, 0, 0, 0, 353, 0, 0, 0, 0, 0, 359, 0, 0, 0, 0, 0, 0, 0, 367, 0, 0, 0, 0, 0, 373, 0, 0, 0, 0, 0, 379, 0, 0, 0, 383, 0, 0, 0, 0, 0, 389, 0, 0, 0, 0, 0, 0, 0,
397, 0, 0, 0, 401, 0, 0, 0, 0, 0, 0, 0, 409, 0, 0, 0, 0, 0, 0, 0, 0, 0, 419, 0, 421, 0, 0, 0, 0, 0, 0, 0, 0, 0, 431, 0, 433, 0, 0, 0, 0, 0, 439, 0, 0, 0, 443, 0,
0, 0, 0, 0, 449, 0, 0, 0, 0, 0, 0, 0, 457, 0, 0, 0, 461, 0, 463, 0, 0, 0, 467, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 479, 0, 0, 0, 0, 0, 0, 0, 487, 0, 0, 0, 491, 0, 0,
0, 0, 0, 0, 0, 499, 0, 0, 0, 503, 0, 0, 0, 0, 0, 509, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 521, 0, 523, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 541, 0, 0, 0,
0, 0, 547, 0, 0, 0, 0, 0, 0, 0, 0, 0, 557, 0, 0, 0, 0, 0, 563, 0, 0, 0, 0, 0, 569, 0, 571, 0, 0, 0, 0, 0, 577, 0, 0, 0, 0, 0, 0, 0, 0, 0, 587, 0, 0, 0, 0, 0, 593,
0, 0, 0, 0, 0, 599, 0, 601, 0, 0, 0, 0, 0, 607, 0, 0, 0, 0, 0, 613, 0, 0, 0, 617, 0, 619, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 631, 0, 0, 0, 0, 0, 0, 0, 0, 0, 641, 0,
643, 0, 0, 0, 647, 0, 0, 0, 0, 0, 653, 0, 0, 0, 0, 0, 659, 0, 661, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 673, 0, 0, 0, 677, 0, 0, 0, 0, 0, 683, 0, 0, 0, 0, 0, 0, 0, 691,
0, 0, 0, 0, 0, 0, 0, 0, 0, 701, 0, 0, 0, 0, 0, 0, 0, 709, 0, 0, 0, 0, 0, 0, 0, 0, 0, 719, 0, 0, 0, 0, 0, 0, 0, 727, 0, 0, 0, 0, 0, 733, 0, 0, 0, 0, 0, 739, 0, 0,
0, 743, 0, 0, 0, 0, 0, 0, 0, 751, 0, 0, 0, 0, 0, 757, 0, 0, 0, 761, 0, 0, 0, 0, 0, 0, 0, 769, 0, 0, 0, 773, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 787, 0, 0, 0, 0,
0, 0, 0, 0, 0, 797, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 809, 0, 811, 0, 0, 0, 0, 0, 0, 0, 0, 0, 821, 0, 823, 0, 0, 0, 827, 0, 829, 0, 0, 0, 0, 0, 0, 0, 0, 0, 839, 0}

This sequence is generated by the formula :

n*(GCD[23,n (GCD[19,n (GCD[17,n (GCD[13,n (GCD[11,n (GCD[7,n ((GCD[5,n (((GCD[6174,n] mod n) mod 3) mod 2)] mod 5) mod n)] mod 7)] mod 11)] mod 13)] mod 17)] mod 19)] mod 23)

Excluding the zeros we have the prime numbers from 29 up to 839, missing the first elements primales that are: 2, 3, 5, 7, 11, 13, 17, 19, 23, to see that these elements are immersed in our formula, i.e. are required to generate the sequence shown, that is why I have

come to the conclusion that the numbers depend on themselves to propagate through the infinite, and also occur in blocks.

{29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599, 601, 607, 613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691, 701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809, 811, 821, 823, 827, 829, 839}

If we look forward to continue producing Prime Numbers and avoid Numbers compounds, the formula must be modified under the same pattern and the number which then should be inserted in it we provides the sequence generated by the abovementioned formula:

{29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599, 601, 607, 613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691, 701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809, 811, 821, 823, 827, 829, 839}

The first number in the sequence is 29, this would be the next item to be inserted in the parameters of the formula and produce a block more than prime numbers.

It is concluded that the expression:

$f(n) = \mathrm{n} * (\mathrm{GCD}[\mathrm{P}(\mathrm{n}), \ldots$ n*(GCD[23,n (GCD[19,n (GCD[17,n (GCD[13,n (GCD[11,n (GCD[7,n ((GCD[5,n (((GCD[6174,n] mod n) mod 3) mod 2)] mod 5) mod n)] mod 7)] mod 11)] mod 13)] mod 17)] mod 19)] mod 23)… mod P(n)), where P(n) is the first element of the sequence produced according to the status of the formula.

It is our guide to produce all the prime numbers that we want, is one of many keys surely to decrypt the mystery of prime numbers.

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