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Semi-classical Electrodynamics

J. P. Lestone
Computational Physics Division, Los Alamos National Laboratory
Los Alamos, NM 87545, USA
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Quantum electrodynamics is complex and its associated mathematics can appear overwhelming for those not trained in this field. Here, semi-classical approaches are used to obtain a more intuitive feel for several QED processes including electrostatics, Compton scattering, and the anomalous magnetic moment of leptons. These intuitive arguments lead to a possible answer to the question of the nature of charge. By invoking a far-field L=0 photon-particle interaction cross section of $\pi\lambda\!\!\!^{-2}$, with the inclusion of near-field effects, the model calculated elementary charge is q=1.602177×10^{-19} C with a corresponding calculated inverse fine structure constant of α^{-1}=137.036.

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I. Introduction

Quantum electrodynamics (QED) is one of the most successful and tested theories ever developed. It can be viewed as one of the pinnacles of human thought. Its development was not easy and it took several decades of concerted effort by many authors to obtain a working theory capable of giving precise predictions. The first steps to a usable theory of the interaction of matter and light, consistent with both special relativity and quantum mechanics, were taken by Dirac in the late 1920s [1,2]. Significant contributions by others followed, culminating in a series of papers by Tomonaga [3], Schwinger [4-6], Feynman [7-9], and Dyson [10,11]. Those trained in QED have since been able to calculate many experimental observables with extraordinary precision. Even though doable, precise calculations can still require monumental effort. For example, the anomalous magnetic moment of the electron, $(g-2)/2$, was first obtained by Schwinger to 2nd order, $\alpha/(2\pi)$, in 1947 [4]. Calculations to higher order required considerable effort. Exact 4th and 6th order corrections were not obtained until 1957 [12] and 1996 [13], respectively. Numerical estimates to 8th [14,15] and 10th [16] order have since been obtained. The theoretical relationship between $(g-2)/2$ and the fine structure constant, α, is now considered to be so strong, and the modern $(g-2)/2$ measurements so precise [17], that the modern estimate of α^{-1}=137.0359991 [18] is inferred from $(g-2)/2$ measurements via QED theory.

In the present paper, we use semi-classical arguments without full quantum theory or detailed special relativity to enable those not trained in field theory to obtain a better intuitive feel for several QED processes, including electrostatics, Compton scattering, and the anomalous magnetic moment of leptons. The reader should be aware that the semi-classical calculations presented here are no substitute for full quantum field theory calculations which form the cornerstone of our understanding of elementary particles and fields.

If the semi-classical recipes presented here only gave values consistent with results already obtainable via QED then they would only be of interest as potential teaching tools to introduce some QED processes to an undergraduate audience. However, the concepts presented here lead to a possible answer to the question of the nature of charge. By invoking a photon-particle interaction cross section that does not pay respect to conservation of energy, and the inclusion of near-field effects with QED based corrections, a universal charge of 1.602177×10^{-19} C emerges from the model.

II. Electrostatics

The fine structure constant is defined relative to the elementary charge e via

$$e^2 = \alpha\hbar c\, 4\pi\varepsilon_o . \tag{1}$$

The electrostatic force between two leptons separated by a distance d can be expressed as

$$F = \frac{e^2}{4\pi\varepsilon_o d^2} = \frac{\alpha\hbar c}{d^2} . \tag{2}$$

Assuming this force is associated with the radiation pressure generated by the exchange of photons with energy ε, we rewrite Eq. (2) as

$$F = \left(\frac{2\varepsilon}{\hbar}\right)\times\left(\frac{2\varepsilon}{c}\right)\times\left(\frac{\alpha\pi(\hbar c/\varepsilon)^2}{4\pi d^2}\right). \tag{3}$$

The first term is the assumed rate of attempted emissions of virtual photons from each lepton obtained via the time-energy uncertainty principle. The second is the momentum change inflicted on the leptons via the two-way exchange of these photons. The third term is the probability that converts an attempted virtual emission into an exchange. The corresponding photon-lepton absorption cross section is

$$\sigma_a = \alpha\pi(\hbar c/\varepsilon)^2 = \alpha\pi ƛ^2 , \tag{4}$$

and was set based on the need for Eq. (3) to be the same as Eq. (2). This cross section does not respect conservation of energy and must always be associated with another process that reestablishes conservation of energy within a time period of $\sim\hbar/(2\varepsilon)$. To avoid complexities associated with near-field corrections, Eq. (3) was obtained assuming the reduced wavelength of the exchanging photons, ƛ, is much smaller than d. To avoid relativistic effects, ε is here assumed to be << mc^2. The real situation is more complex, with the uncertainty principle requiring ƛ to be near to or larger than d. This complexity is initially ignored, but discussed in more detail in sections V and VI.

The force associated with the semi-classical exchange of photons between two like-charged leptons represented by Eq. (3) can only generate repulsion. However, an attractive force between oppositely charged objects can be obtained by assuming the opposite charge is associated with a hole in a Fermi-sea of negative-energy particles [19].

III. Compton Scattering

Using the absorption cross section given by Eq. (4), the magnitude of the Compton cross section, σ_C, can be understood via a three-step recipe. First, the lepton makes an attempt to absorb the incident photon with cross section σ_a. If there were no other steps, this attempted absorption must fail because photon absorption by a fixed-mass particle violates conservation of energy and momentum, and the incident photon must continue on as though there was no interaction. However, the time-energy uncertainty principle prevents any interaction from occurring at an instant. Therefore, we at first allow the attempted absorption. In the second step, the lepton is assumed to recoil with velocity $v=\varepsilon/(mc)$, which it obtains over a time scale $\tau = \hbar/(2\varepsilon)$. The recoil thus involves a lepton acceleration of

a=$2\varepsilon^2/(\hbar mc)$ over a time period of τ. The power of emission from an accelerating classical charge, in the non-relativistic limit, is given by [20]

$$P = \frac{2}{3}\frac{e^2}{4\pi\varepsilon_0}\frac{a^2}{c^3} = \frac{2\alpha\hbar c a^2}{3c^3}. \tag{5}$$

The electromagnetic energy emitted by the lepton during the acceleration phase of the attempted absorption is therefore

$$E = \frac{2\alpha\hbar c a^2\tau}{3c^3} = \frac{2\alpha\hbar c}{3c^3}\frac{4\varepsilon^4}{(\hbar mc)^2}\frac{\hbar}{2\varepsilon} = \frac{4\alpha}{3}\frac{\varepsilon^3}{(mc^2)^2}. \tag{6}$$

If the emitted energy associated with the recoil manifests itself as a photon of energy ε then conservation of energy and momentum can be reestablished. The probability of emitting such an energy-and-momentum conservation-reestablishing photon from an individual recoiling lepton can be expressed as

$$\mathrm{P} = \frac{4\alpha}{3}\frac{\varepsilon^2}{(mc^2)^2}. \tag{7}$$

If the initial absorption attempt is only successful if the energy-and-momentum conservation-reestablishing photon emission occurs, then the interaction cross section is given by

$$\sigma = \mathrm{P}\sigma_a = \frac{4\alpha}{3}\frac{\varepsilon^2}{(mc^2)^2}\frac{\alpha\pi(\hbar c)^2}{\varepsilon^2} = \frac{4\pi}{3}\left(\frac{\alpha\hbar c}{mc^2}\right)^2. \tag{8}$$

The third step is to realize that Eq. (8) is only half the real cross section because, in obtaining Eq. (8) the photon absorption is assumed to occur before the recoil-induced scattered photon. However, the order of these processes can be reversed and doubles Eq. (8) giving our final result for the Compton cross section, in the non-relativistic limit, of

$$\sigma_{\mathrm{C}} = \frac{8\pi}{3}\left(\frac{\alpha\hbar c}{mc^2}\right)^2 = \frac{8\pi\alpha^2 r_{\mathrm{C}}^2}{3} = \frac{8\pi r_e^2}{3}, \tag{9}$$

where r_{C} is the reduced Compton wavelength, and r_e is the classical radius.

IV. Intrinsic Particle Fuzziness

Here, we use semi-classical means to calculate some of the consequences of leptons constantly "trying" to emit virtual photons. We proceed by considering the possibility that the emission and absorption of virtual photons from leptons have similarities to the emission and absorption of unphysical L=0 photons from black holes with a temperature T_{bh}. One method for calculating the emission rate of massless particles from an object at temperature T is to use transition-state theory [21] and write the decay width for emission from a spherical object as

$$\Gamma = \frac{1}{2\pi}\int_0^\infty \sum_{\text{helicity}}\sum_{L=0}^{\infty}(2L+1)T_L(\varepsilon)\exp(-\varepsilon/T)d\varepsilon \;, \tag{10}$$

where the $T_L(\varepsilon)$ are the angular momentum and energy dependent transmission coefficients. In the limit of a large classical object, the maximum orbital angular momentum that can be taken away by a massless particle is $L_{\max}\hbar = r\varepsilon/c$, where r is the radius. Substituting in $T_L(\varepsilon)$=1 for $L \le L_{\max}$ and $T_L(\varepsilon)$=0 for $L > L_{\max}$ in the limit of a large object, assuming two states of helicity (polarization), and making the semi-classical replacement of $\exp(-\varepsilon/T)$ with $1/(\exp(\varepsilon/T) - 1)$ to include stimulated emission from a black body [22] gives the result

$$\Gamma_\gamma = \frac{4\pi r^2}{4\pi^2\hbar^2c^2}\int_0^\infty \frac{\varepsilon^2}{\exp(\varepsilon/T)-1}d\varepsilon\,. \tag{11}$$

The corresponding emitted power is

$$P = \frac{4\pi r^2}{4\pi^2\hbar^3c^2}\int_0^\infty \frac{\varepsilon^3}{\exp(\varepsilon/T)-1}d\varepsilon = 4\sigma_{\mathrm{a}}\sigma_{\mathrm{SB}}T^4 = A\sigma_{\mathrm{SB}}T^4\ , \tag{12}$$

where σ_{a} and A are the classical absorption cross section and the surface area of the black body, and $\sigma_{\mathrm{SB}}=\pi^2/(60\hbar^3c^2)$ is the Stefan-Boltzmann constant. In the limit of low gravity, Eq. (12) gives the power of Hawking radiation from a large black hole with temperature $T_{\mathrm{bh}}=\hbar c/(4\pi r_{\mathrm{s}})$ [23-25] where r_{S} is the Schwarzschild radius.

The emission and absorption of photons by a single Schwarzschild black hole are controlled by the energy-dependent cross section [26,27]

$$\sigma_{\mathrm{bh}} = \sum_{L=1}^{\infty}\frac{(2L+1)\pi r_S^2}{4(M\omega)^2}T_L(M\omega) = \sum_{L=1}^{\infty}(2L+1)\pi\lambda\!\!\!^{-}{}^2T_L(\varepsilon), \tag{13}$$

where M is the black-hole mass and ω is the angular frequency. The conversion between the $M\omega$ used by Crispino *et al*. [27] and the photon energy is $M\omega=\varepsilon/(8\pi T_{\mathrm{bh}})$. The L=0 photon emission from black holes is known to be unphysical, in part, because its forced inclusion into the absorption cross section would cause the emitted power to be infinite. These unphysical L=0 emissions are assumed to be analogous with virtual-photon emission from leptons. We use transition-state theory to write the semi-classical emission rate of unphysical L=0 photons as

$$R = \frac{1}{\pi\hbar}\int_0^\infty \exp(-\varepsilon/T_{\mathrm{bh}})\,d\varepsilon. \tag{14}$$

Of course, none of this emission can escape an isolated fixed-mass particle without violating conservation of energy. The virtual emission must therefore be in a constant emission and absorption dance with an isolated lepton. After recoiling, the maximum kinetic energy of the lepton is $K=\varepsilon^2/(2mc^2)$. Assuming harmonic-oscillator motion, the corresponding wave function for the lepton's location is

$$\psi \propto \exp\left(\frac{-x^2}{2}\left(\frac{mc^2}{\hbar c}\right)^2\right) = \exp\!\left(\frac{-x^2}{2r_{\mathrm{C}}^2}\right), \tag{15}$$

independent of $\lambda\!\!\!^{-}$. The corresponding photon wave function is obtained by replacing mc^2 with ε and is

$$\psi_\gamma \propto \exp\!\left(\frac{-x^2}{2\lambda\!\!\!^{-}{}^2}\right). \tag{16}$$

These calculations suggest fixed-mass black holes have an intrinsic fuzziness with a central matter core of radius ~r_{C}, and a cloud of virtual photons extending out a distance ~$\lambda\!\!\!^{-}$, with a distribution of wavelengths controlled by the temperature.

V. Anomalous Magnetic Moment

The anomalous magnetic moment of the electron has been measured to extraordinary precision and is known to be $(g-2)/2=0.001159652181$ [28]. Precise measurements of both $(g-2)/2$ and the Lamb shift [29] have been used to test QED. For example, the electron-only QED calculation of $(g-2)/2$ can be written as [16]

$$\frac{g-2}{2} = \sum_{n=1}^{\infty}A_1^{(2n)}\left(\frac{\alpha}{\pi}\right)^n. \tag{17}$$

The first three $A_1^{(2n)}$ are known precisely and are $A_1^{(2)}$=0.5, $A_1^{(4)}$= −0.328478965579…, and $A_1^{(6)}$=1.1812456587… Substituting α^{-1}=137.036 into Eq. (17) and using only terms with $n \leq 3$ gives $(g-2)/2$=0.00115965222. The small difference from the measured value is due to a combination of even higher-order corrections, and non-electron and hadronic effects. Here, we are not looking for this level of accuracy but instead attempt to understand $(g-2)/2$ using semi-classical arguments.

To obtain a semi-classical recipe of $(g-2)/2$ we assume leptons are constantly emitting photons and self-absorbing them on a time scale of $\sim\hbar/(2\varepsilon)$. After each emission, the lepton is assumed to recoil with a velocity $v=\varepsilon/(mc)$ and curl in the external magnetic field with an orbital angular moment of ħ, in a direction such that the recoil-induced magnetic dipole of one Bohr-magneton is added to the corresponding spin-induced magnetic dipole of one Bohr-magneton. If this interpretation is correct then the known 2nd order QED correction to the magnetic moment of leptons would imply leptons spend $\alpha/(2\pi)$ of their time in a state where a photon has been emitted but has not yet been self-absorbed, i.e. "dressed" by a single virtual photon.

V.A No Near-field Corrections

Given the above discussed assumptions, after an attempted virtual emission of a photon, the recoiling lepton travels along an arc with radius $r=ƛ$. We assume the attempted emission does not produce real consequences unless it is self-absorbed within a time $\tau\sim\hbar/(2\varepsilon)$. In this time the lepton can travel a path length $\sim r_C/2$ around the assumed arc. We write the self-absorption probability as

$$P_{\mathrm{sa}} = \frac{2}{r_{\mathrm{C}} T_{\mathrm{sa}}} \int_0^\infty \int_0^\infty \exp\left(\frac{-2x}{r_{\mathrm{C}}}\right) \exp\left(\frac{-\varepsilon}{T_{\mathrm{sa}}}\right) \frac{\sigma_{\mathrm{a}}}{4\pi d^2}\, dx\, d\varepsilon \,, \tag{18}$$

where x is the path length traveled around the arc, and d is the distance between the attempted emission and self-absorption locations. We have introduced a self-absorption temperature, T_{sa}, to correct for relativistic effects that limit the self-absorption of high-energy virtual photons. This is similar to the high-energy cutoff used to obtain simple estimates of the Lamb shift [19,30,31]. However, sharp cutoffs are not intuitively pleasing, and we instead assume an exponential cutoff. The square of the distance between emission and self-absorption locations is given by $d^2=2ƛ^2\{1-\cos(x/ƛ)\}$. We immediately see a problem if the far-field absorption cross section from Eq. (4) is blindly substituted into Eq. (18) because of the divergence generated by the $1/d^2$ term.

V.B Near-field Corrections

Ignoring the unphysical nature of L=0 black-hole photons, their absorption cross section is here assumed to be $\piƛ^2$ in the far-field limit where the photons approach from infinity as plane waves. This means that, in a semi-classical sense, black holes absorb L=0 photons that approach from a distance many times larger than ƛ, as though they are classical spherical objects with a radius $r=ƛ$. This is the expected length scale for near-field effects. For the $(g-2)/2$ calculation considered in the present section, the probability of self-absorption of $\sigma_{\mathrm{a}}/(4\pi d^2)$ can only be used when the separation between emitter and absorber locations is greater than ƛ. For shorter separations, the semi-classical absorption sphere will overlap with the emission location and lead to a reduction in the self-absorption probability. Therefore, when $d < ƛ$, the near-field corrections should be significant.

We assume that around each lepton there is a distribution of photon interaction sites, and that the effective density of these sites can be represented by Eq. (16). When d is comparable to or smaller than ƛ, the emission location lies within the distribution of the absorption sites of the translated recoiling lepton. We assume this overlap controls the increase of the near-field corrections as d decreases through ƛ. Guided by the wave function presented in Eq. (16) we assume the L=0 effective interaction radius is

$$r_{\mathrm{n}}(d,ƛ) = \frac{2}{\sqrt{2\pi}}\int_0^d \exp(\frac{-z^2}{2ƛ^2})dz. \tag{19}$$

Notice that as $d\rightarrow\infty$, r_n approaches the far-field value of ƛ. The corresponding near-field modified transmission coefficients are

$$T_{L=0}(ƛ,d) = \left(\frac{r_n}{ƛ}\right)^2 = \mathrm{erf}^2(\frac{d}{ƛ\sqrt{2}}). \tag{20}$$

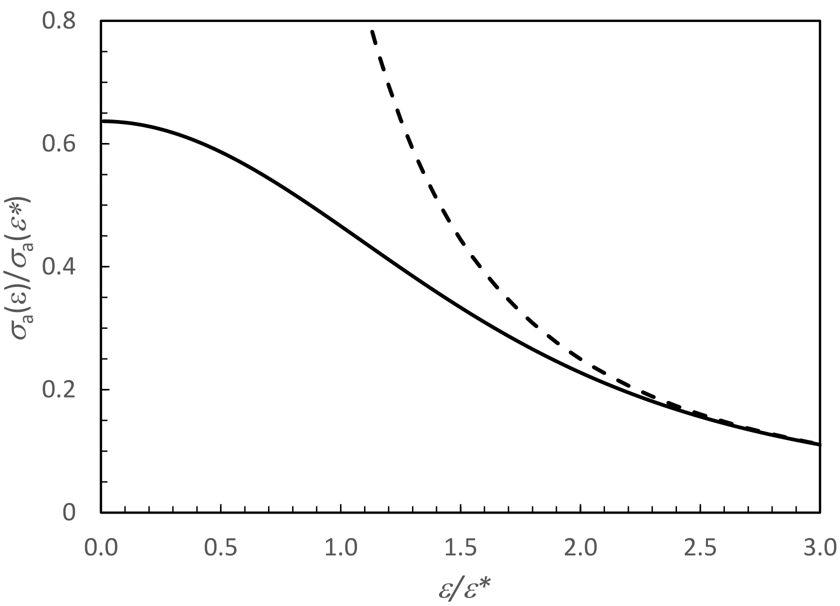

Fig. 1. Effective interaction cross sections obtained using the near-field corrected transmission coefficients represented by Eq. (20) (solid curve). The energy axis is in units of $\varepsilon^*=\hbar c/d$. At $\varepsilon/\varepsilon^*=1$ and 2, the separations between emitter and absorber are ƛ and 2ƛ, respectively. The dashed curve displays the far-field result.

The corresponding modification to the absorption cross section is displayed in Fig. 1. The near-field correction to the transmission coefficients provides an effective smooth low-energy cutoff, similar to the artificial sharp cutoffs employed elsewhere to obtain simple estimates of the Lamb shift [19,30,31]. The corresponding update to Eq. (18) is

$$P_{\mathrm{sa}} = \frac{2}{r_{\mathrm{C}}T_{\mathrm{sa}}}\int_0^\infty\int_0^\infty \frac{\exp(\frac{-2x}{r_{\mathrm{C}}})\exp(\frac{-\varepsilon}{T_{\mathrm{sa}}})\mathrm{erf}^2(\sqrt{1-\cos(\theta)})\alpha\pi ƛ^2}{4\pi ƛ^2\, 2(1-\cos(\theta))}dx\,d\varepsilon\ , \tag{21}$$

where $\theta=x/ƛ$ is the angle the recoiling lepton traverses around the assumed arc. This appears to be a promising step forward because, in the limit that the energy of all the virtual photons are $<<mc^2$, i.e. with $T_{\mathrm{sa}}<<mc^2$, Eq. (21) gives the fraction of attempted emissions that are successfully self-absorbed as $\alpha/(2\pi)$. However, this is still not intuitively satisfying because it is unreasonable to assume that all virtual photons have energies $<<mc^2$ [30]. This situation can be rectified by the inclusion of speculative QED corrections to the near-field effects.

V.C QED Corrections

Standard QED corrections to the magnetic moment of leptons are related to the relative path lengths in space-time where the lepton location is fuzzy due to a surrounding cloud of virtual photons, and the corresponding length scale for when the lepton is naked. The transfer problem in the semi-classical recipe for (g–2)/2 presented here is related to effective absorption cross sections modified by near-field effects, and thus has a change in symmetry relative to that associated with the traditional QED corrections to the electron's magnetic moment. Perhaps, for the semi-classical transfer problem, we need a ratio of a QED-corrected effective fuzzy surface area to the corresponding sharp surface area. Inspired by this suggestion, we speculate that the $A_1^{(2n)}$ in Eq. (17) are related to dimensionless length scales. For our semi-classical near-field correction problem, we convert these length scales into effective surface areas of spheres and re-express Eq. (21) as

$$P_{\mathrm{sa}} = \frac{2\alpha}{r_{\mathrm{C}} T_{\mathrm{sa}}} \int_0^\infty \int_0^\infty \frac{\exp(\frac{-2x}{r_{\mathrm{C}}}) \exp(\frac{-\varepsilon}{T_{\mathrm{sa}}}) \mathrm{erf}^{\,2/(1+\eta(\alpha))}(\sqrt{1-\cos(\theta)})}{8(1-\cos(\theta))} dx\, d\varepsilon \,, \tag{22}$$

with a QED near-field correction term

$$\eta(\alpha) = \sum_{n=1}^{\infty} 4\pi (A_1^{(2n)})^2 (\frac{\alpha}{\pi})^n . \tag{23}$$

A symmetry between length and energy is obtained if we choose $T_{\mathrm{sa}}=mc^2/2$. Switching to length and energy units of r_{C} and mc^2, respectively, gives the self-absorption probability

$$P_{\mathrm{sa}} = \frac{\alpha}{2} \int_0^\infty \int_0^\infty \frac{\exp(-2x)\exp(-2\varepsilon)\mathrm{erf}^{\,2/(1+\eta(\alpha))}(\sqrt{1-\cos(x\varepsilon)})}{1-\cos(x\varepsilon)}\, dx d\varepsilon \,. \tag{24}$$

Solving numerically gives $(g–2)/2=0.001161$ in agreement with 2nd order QED. This partially justifies the choice of the self-absorption temperature, $T_{\mathrm{sa}}=mc^2/2$, and the assumed QED correction to the near-field effects.

VI. Revisiting Electrostatic Repulsion

Despite many attempts, there is still no accepted theory to explain the elementary charge $e=1.602177\times10^{-19}$ C [32]. In this section, based on the apparent success of the previous sections, we consider the possibility that electromagnetism and its strength are generated by L=0 virtual-photon exchanges between particles with some properties resembling black holes. It is known that L=0 photons cannot be emitted to infinity by a single Schwarzschild black hole [27]. However, we consider the possibility that low-energy L=0 photons can be exchanged between two quantum micro black holes via quantum means.

For a single quantum black hole with a fixed mass, the unphysical L=0 emission would be completely suppressed by energy conservation. We temporarily ignore this important fact and use transition-state theory to write the semi-classical power of unphysical L=0 photons emitted to infinity from a quantum micro black hole (A) as (see Eq. (14))

$$P_{A\to\infty} = \frac{1}{\pi\hbar} \int_0^\infty \varepsilon \exp(-\varepsilon / T_{\mathrm{bh}}) d\varepsilon. \tag{25}$$

The violation of conservation of energy by the emission from a single quantum black hole can be rectified by the absorption of the photon by a neighboring quantum black hole on a time scale of $\sim\hbar/(2\varepsilon)$. The photon energies involved in the exchange are therefore

$$T_{\rm ex} \sim \frac{\hbar c}{2d}. \tag{26}$$

For widely spaced quantum black holes, the constraints on the exchanging photon energies imposed by the time-energy uncertainty principle rule out all but low-energy unphysical L=0 exchanges. Assuming that Eq. (26) defines an effective exchange temperature, we write the photon power being exchanged from quantum micro black hole A to black hole B as

$$P_{A\to B} = \frac{1}{\pi\hbar}\int_0^\infty \varepsilon\, T_{L=0}(\varepsilon)\exp(-\varepsilon/T_{\rm ex})\ \frac{\sigma_{L=0}(\varepsilon)}{4\pi d^2}\, d\varepsilon. \tag{27}$$

Here we assume $T_{\rm bh}>>T_{\rm ex}$. The $\exp(-\varepsilon/T_{\rm ex})$ term is the probability that a virtual photon can jump the distance between the particle pair. The $\sigma/(4\pi d^2)$ term is the probability of finding the partner assuming the jump across the distance d has been made. Despite the fact that for emission to infinity $T_{L=0}(\varepsilon)=1$, we leave it in Eq. (27) [see Eq. (10)] because, as previously discussed in section V, the transmission coefficients need to be modified for the case of exchanges between objects where ƛ is comparable to or larger than d. Given Eq. (27), the force generated by the two-way exchange of unphysical photons is not dependent on the size and/or temperature of either black hole and is

$$F = \frac{2}{\pi\hbar c}\int_0^\infty \varepsilon\, T_{L=0}(\varepsilon)\exp(-\varepsilon/T_{\rm ex})\, \frac{\sigma_{L=0}(\varepsilon)}{4\pi d^2}\, d\varepsilon. \tag{28}$$

Substituting $\sigma_{L=0}(\varepsilon) = \pi(\hbar c/\varepsilon)^2\cdot T_{L=0}(\varepsilon)$ [see Eq. (13)] into Eq. (28) gives

$$F = \frac{\hbar c}{2\pi d^2}\int_0^\infty \frac{(T_{L=0}(\varepsilon))^2\exp(-\varepsilon/T_{\rm ex})}{\varepsilon}\, d\varepsilon. \tag{29}$$

If this force is assumed to be the origin of electromagnetism, then the fine structure constant can be expressed as

$$\alpha = \frac{1}{2\pi}\int_0^\infty \frac{(T_{L=0}(\varepsilon))^2\exp(-\varepsilon/T_{\rm ex})}{\varepsilon}\, d\varepsilon. \tag{30}$$

Using the far-field value of $T_{L=0}(\varepsilon)=1$ gives an infinite fine structure constant. However, the origin of the divergence is the lowest-energy photons where ƛ>d and the transmission coefficients need to be modified to lower values to correct for near-field effects.

Guided by our apparent success in calculating the $(g\!-\!2)/2$ in section V.C, we use the same near-field correction to the transmission coefficients [see Eqs. (20), (22), and (23)] and rewrite Eq. (30) as

$$\alpha = \frac{1}{2\pi}\int_0^\infty \frac{\mathrm{erf}^{\,4/(1+\eta(\alpha))}(\varepsilon/(2^{3/2}))\exp(-\varepsilon)}{\varepsilon}\, d\varepsilon. \tag{31}$$

For convenience, here we have switched to energy in units of $T_{\rm ex}$. Eq. (31) can be solved iteratively, and rapidly converges to a calculated inverse fine structure constant of α^{-1}=166.1. However, Eq. (31) only allows for the spontaneous emission of virtual photons. It is known that photons falling into a black hole cause a simulated time-reversed emission with a probability $\exp(-\varepsilon/T_{\rm bh})$ [33]. In the limit of $T_{\rm bh}>>T_{\rm ex}$ the probability that the original spontaneous virtual exchange generates a simulated virtual photon, which makes the exchange back to the original spontaneous emission site, is $\exp(-\varepsilon)$ [energy is units of $T_{\rm ex}$]. We assume the time-reversed photon automatically finds the original emitter (assuming a static system). Including only single and double exchanges gives a ratio of the number of exchanges per original spontaneous exchange as

$$\frac{n_2}{n_1} = 1-\exp(-\varepsilon)+2\exp(-\varepsilon) = 1+\exp(-\varepsilon). \tag{32}$$

Extending this to consider an infinite number of exchanges back and forth between the interacting particles gives

$$\frac{n_\infty}{n_1} = 1 + \sum_{n=1}^{\infty} \exp(-n\varepsilon) = \frac{1}{1-\exp(-\varepsilon)}. \tag{33}$$

Including these considerations modifies the result expressed in Eq. (31) to

$$\alpha = \frac{1}{2\pi}\int_0^\infty \frac{\mathrm{erf}^{\,4/(1+\eta(\alpha))}(\varepsilon/(2^{3/2}))}{\varepsilon(\exp(\varepsilon)-1)}\, d\varepsilon. \tag{34}$$

The corresponding calculated α^{-1} is 137.035891, which differs by 1 in the 7^{th} significant digit from the known value. The corresponding calculated fundamental unit of charge is q=1.602177×10^{-19} C. The photon-exchange power spectrum is displayed in Fig. 2. The photons responsible for the exchange force have a mean energy of $\sim 2T_{\text{ex}}=\varepsilon^*$ and $\lambda\!\!\!^{-}\sim d$.

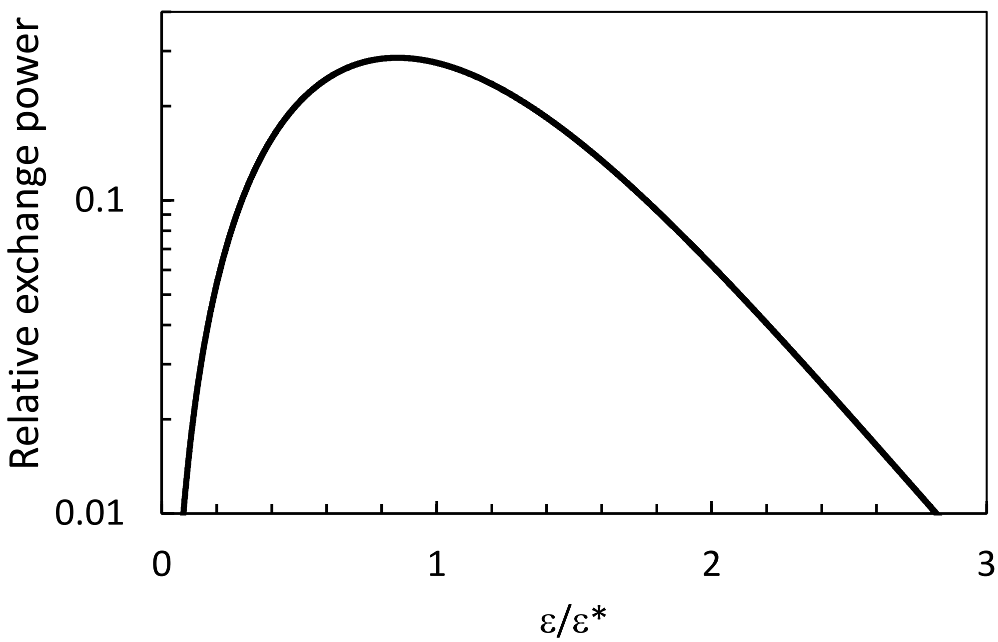

Fig. 2. Power spectrum for the exchanging photons displayed in units of $\varepsilon^*=2T_{\text{ex}}$ (solid curve).

VII. Summary

Using semi-classical approaches, several key QED processes can be understood using intuitive steps that could be explained to an undergraduate physics audience. These steps involve a combination of virtual-photon emission or absorption, associated recoil effects, and absorption or emission to re-establish conservation of energy and momentum on a time scale consistent with the time-energy uncertainty principle. Using these steps, semi-classical recipes have been developed which give the Compton cross section $\sigma_C=8\pi\alpha^2 r_C^2/3$, and $g/2$=1.001161.

Using Eq. (34), a semi-classical estimate of the magnitude of the repulsive force generated by the exchange of unphysical photons between a pair of widely spaced quantum micro black holes can be obtained. If only the far-field estimate of the photon black-hole interaction cross section is used, then the calculated force is infinite. Estimates of near-field corrections obtained by intuitive overlap arguments, removes the divergence, leads to a force that is inversely proportional to the square of the separation distance, and independent of the properties of the black holes (mass, size and temperature). The insensitivity to several black-hole properties suggests that fundamental particles do not necessarily have to be quantum micro black holes. However, they appear to have a propensity to emit L=0 photon power via Eq. (25) that is completely suppressed for emission to infinity, while the exchange of low-energy photons between like objects is allowed via the time-energy uncertainty principle.

The nature of the photon exchange calculated here has a QED feel, with the photons involved being unphysical (virtual) and with the dominant exchange photons having $\lambda\!\!\!^{-}\sim d$. In a semi-classical sense this means the energy, path, and direction of an individual exchange are not definable. It would thus not be surprising in a more detailed quantum mechanical calculation that the details of the semi-classical exchange suggested here are lost, and the only surviving property is a single photon-particle coupling constant. Possible reasons for the fractional charges of quarks are not discussed. Only a static configuration of a pair of particles is considered here. It would be interesting to consider the possibility of the generation of "real" photons in an extension of the presented scenario to a dynamical particle pair and/or triplet.

The calculated effective charge is q=1.6021773×10^{-19} C, which differs from the known elementary charge e=1.6021766×10^{-19} C [18] by 7 in the 8$^{\text{th}}$ significant digit. The list of assumptions needed to obtain this result is: an effective exchange temperature of $T_{\text{ex}}=\hbar c/(2d)$; near-field corrections controlled by a harmonic oscillator wave function with a length scale related to the wavelength of the exchanging photons; QED corrections to the near-field effects that are analogous with the QED calculation of the electron's $(g-2)/2$ but with the $A_1^{(2n)}$ terms replaced by $4\pi\cdot(A_1^{(2n)})^2$; and the existence of time-reversed simulated virtual emission with a probability of $\exp(-\varepsilon/T_{\text{ex}})$ of making the exchange back to the partner particle. The match to experiment may be fortuitous, and quantum field theory calculations are needed to confirm or negate the speculations presented here. In particular, the nature of near-field corrections and the possibility of QED corrections to near-field effects should be studied.

If the presented speculations are confirmed by quantum field theory calculations, the implications are too numerous to be discussed here. However, an important one is that the strength of electromagnetism would be controlled by simple geometrical factors and QED corrections, and α would be a mathematical constant like π and e, and not a physical one (at least in flat space-time and in the non-relativistic limit). This would have significant consequences for ideas related to possible time and/or spatial dependencies of α, the anthropic principle, string theories, and multiverse scenarios.

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[1] P.A.M Dirac, Proc. Roy. Soc. (London) **A114**, 243 (1927).
[2] P.A.M Dirac, Proc. Roy. Soc. (London) **A117**, 610 (1928).
[3] S. Tomonaga, Prog. Theor. Phys. **1**, 27 (1946).
[4] J. Schwinger, Phys. Rev. **73**, 416 (1948).
[5] J. Schwinger, Phys. Rev. **74**, 1439 (1949).
[6] J. Schwinger, Phys. Rev. **76**, 790 (1949).
[7] R. P. Feynman, Phys. Rev. **76**, 769 (1949).
[8] R. P. Feynman, Phys. Rev. **76**, 749 (1949).
[9] R. P. Feynman, Phys. Rev. **80**, 440 (1950).
[10] F. Dyson, Phys. Rev. **75**, 486 (1949).
[11] F. Dyson, Phys. Rev. **75**, 1736 (1949).
[12] A. Petermann, Helv. Phys. Acta, **30**, 407 (1957).
[13] S. Laporta and E. Remiddi, Phys. Lett. B, **379**, 283 (1996).
[14] T. Kinoshita and M. Nio, Phys. Rev. D **73**, 013003 (2006).

[15] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. Lett. **99**, 110406 (2007).
[16] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. D **85**, 033007 (2012).
[17] G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, Phys. Rev. Lett. **97**, 030802 (2006) and **99**, 039902 (2007).
[18] P. J. Mohr *et al*., http://physics.nist.gov/constants (2015), National Institute of Standards and Technology, Gaithersburg, MD 20899, USA.
[19] J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics, McGraw-Hill ISBN 07-005493-2 (1964).
[20] J. D. Jackson, Classical Electrodynamics, John Wiley & Sons, Inc. (1975).
[21] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).
[22] J. P. Lestone, Mod. Phys. Lett. A, **23**, 1067 (2008).
[23] S. W. Hawking, Nature **248**, 30 (1974).
[24] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[25] S. W. Hawking, Phys. Rev. D **13**, 191 (1976).
[26] R. Fabbri, Phys. Rev. **12**, 933 (1975).
[27] L. C. B. Crispino, E. S. Oliveira, A. Higuchi, and G. E. A. Matsas, Phys. Rev. D **75**, 104012 (2007).
[28] D. Hanneke, S. Fogwell Hoogerheide, and G. Gabrielse, Phys. Rev. A **83**, 052122 (2011).
[29] W. E. Lamb and R. C. Retherford, Phys. Rev. **72**, 241 (1947).
[30] H. A. Bethe, Phys. Rev. **72**, 339 (1947)
[31] T. A. Welton, Phys. Rev. **74**, 1157 (1948).
[32] U. D. Jentschura and I. Nandori, Eur. Phys. J. H, **39**, 591 (2014).
[33] J. D. Bekenstein and A. Meisels, Phys. Rev. D, **15**, 2775 (1977).