Bare Charge and Bare Mass in Quantum Electrodynamics

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Abstract

The existence of the bare mass and the bare charge in Quantum Electrodynamics is analyzed in terms of the Standard Model of particle physics. QED arises as a renormalized theory as a consequense of spontaneous symmetry breaking by Englert-Brout-Higgs mechanism as $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{em}$.

Keywords: bare charge, bare mass, Old Standard Model, New Standard Model, Renormalization

The physical mass m of the electron to order α is $m = m_0 + \delta m$ where m_0 is the bare mass of the electron. This is the mass of the electron in the absence of the electromagnetic interaction, that is mass of an uncharged electron [1]. This is an imporatnt point. The idea of renormalizability is that m_0 which is physically not measurable is made to depend upon a cutoff Λ i.e., $m_0(\Lambda)$. We choose m_0 to depend upon Λ in such a manner that m is independent of Λ.

The same for the renormalized charge in QED [1] $e = (Z_3)^{\frac{1}{2}} e_0$ where e is the physically measured charge and e_0 is the bare charge of the electron. Note that this bare charge is independent of any mass parameter of the electron. Therefore the bare mass and the bare charge are independent of each other but of course, defining the same electron of which they are the primitive bare quantities.

QED is renormalizable in terms of only two parameters, the bare mass m_0 and the bare charge e_0. When we calculate the true mass m and the true charge e (the actually measured quantities) we find that these expressions diverge. Next we regularize the theory by introducing an unphysical parameter Λ such that the resulting mass and charge are finite. Note the parallel and complementary roles of bare mass and bare charge in the following assumed structures. We express this as,

$$
\begin{aligned}
m &= m(m_0, e_0, \Lambda) \\
e &= e(m_0, e_0, \Lambda)
\end{aligned}
\tag{1}
$$

To make physical sense out of the regularizd theory we make m and e independent of Λ. Thus we make sure that m_0 and e_0 are functions of Λ such that m and e do not depend upon Λ. That is

$$
\begin{aligned}
\frac{dm}{d\Lambda} &= 0 = \frac{\partial m}{\partial m_0}\frac{\partial m_0}{\partial \Lambda} + \frac{\partial m}{\partial e_0}\frac{\partial e_0}{\partial \Lambda} + \frac{\partial m}{\partial \Lambda} \\
\frac{de}{d\Lambda} &= 0 = \frac{\partial e}{\partial m_0}\frac{\partial m_0}{\partial \Lambda} + \frac{\partial e}{\partial e_0}\frac{\partial e_0}{\partial \Lambda} + \frac{\partial e}{\partial \Lambda}
\end{aligned}
\tag{2}
$$

One has to solve these equations. These coupled equations are first order and their initial conditions are provided by the experimentally measured or renormalized mass and charge. Thereafter one uses the standard renormalization techniques [1].

What we have stated above can be summarized in terms of the following three basic points:

Point 1: Bare mass m_0 is mass of an uncharged electron.

Point 2: Bare charge e_0 is charge of a massless electron.

Point 3: These two should arise jointly and simultaneously in any consistent theory of QED.

We look at the Standard Model and see how much consistent it is with the above structure of Quantum Electrodynamics. Let us look at the first generation of the particles in the Standard Model built around the group structure $SU(3)_C \supset SU(2)_L \otimes U(1)_Y$. It is well known that in this Standard Model the electric charge is not quantized and that its definition is conventional. There are two popular conventional definitions of the electric charges existing in the Standard Model literature. One is [2]

$$Q = (I_3)_L + \frac{Y_W}{2} \tag{3}$$

Here the charges of all the members of the first generation are obtained by picking up values of Y_W so as to obtain the correct charges for the doublet, e.g., $Y_W = -1$, which produces the correct charges of $(\nu_e, e^-)_L$ and $Y_W = +\frac{1}{3}$ to yield the correct charges for the $(u, d)_L$ pair.

Next, the other definition is [3]

$$Q = (T_3)_L + Y'_W \tag{4}$$

We distinguish Y'_w as a generator of the group $U(1)_{Y_W}$ in Equation 4 from Y_W used in Equation 3. We may do so and still obtain the correct electric charges for the first generation pair with $Y'_W = -\frac{1}{2}$ and $Y'_W = \frac{1}{6}$ for the lepton and the quark pairs, respectively.

So as per the above picture the electric charge existed well before the Electroweak phase transition in the early universe. Hence we take the above charge as definig the bare charge e_o. Then the bare mass may be associated with the Yukawa mass generated after the Spoantanoeus Symmetry Breaking $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{em}$. Now these bare quantities do not fulfill the Point 3 above. Point 2 may hold but Point 1 is not valid anymore. Thus the Standard Model as shown above does not satisfy the structure of the bare mass and the bare charge in a consistent manner. This is so in spite of the fact that QED arises from the SSB mechanism therein.

The above Standard Model charges had other difficulties too. Witten et al. [4] had used the above charges to study QCD in the limit of large number of colours. The above charges are independent of colour and thus static as one changes colours in QCD for arbitrary number of colours. This leads to fundamental theoretical difficulites and inconsistencies [5]. Thus Witten et al. [4] colour independent charges of the Standard Model have to be discarded, adding to the woes of the Standard Model. The author had obtained new colour dependent charges which arise on fundamental grounds in the Standard Model and showed that those were the correct charges to use for a proper understanding of the Standard Model [5]. These are for the Standard Model group $SU(N_c) \otimes SU(2)_2 \otimes U1_Y$ given as [5]

$$Q_u = \frac{1}{2}\left(1 + \frac{1}{N_c}\right) \tag{5}$$

$$Q_d = \frac{1}{2}\left(-1 + \frac{1}{N_c}\right) \tag{6}$$

To distinguish this new understanding of the Standard Model as being so radically different from the above (Old) Standand Model, that we may call this new understanding and structure as the New Standard Model (NSM).

So the NSM predicts correct colour dependent quantized electric charge. What does this NSM has to say about the above problem of the bare charge and the bare mass of the renormalized QED?

Let us define our model [5,6] as broken down into three distinct steps:

STEP 1:

Let us start with the first generation of particles. In the SM this is represented in the group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ as

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (3, 2, Y_q); u_R \sim (3, 1, Y_u); d_R \sim (3, 1, Y_d)$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \sim (1, 2, Y_l); e_R \sim (1, 1, Y_e) \tag{7}$$

Let us now define the electric charge in the most general way in terms of the diagonal generators of $SU(2)_L \otimes U(1)_Y$ as

$$Q = I_3 + bY \tag{8}$$

where b is an arbitrary parameter.

In the SM, $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is spontaneously broken through a Englert-Brout-Higgs (EBH) mechanism to the group $SU(3)_c \otimes U(1)_{em}$. Here it is assumed to be a doublet ϕ with arbitrary hypercharge Y_ϕ. The isospin $I_3 = -\frac{1}{2}$ component of the EBH field develops a nonzero vacuum expectation value $< \phi >_o$. Since we want the $U(1)_{em}$ generator Q to be unbroken we require $Q < \phi >_o = 0$. This right away fixes b in Equation 8 and we get

$$Q = I_3 + (\frac{1}{2Y_\phi})Y \tag{9}$$

To proceed further one imposes the anomaly cancellation conditions to establish constraints on the various hypercharges above. First $[SU(3)_c]^2 U(1)_Y$ gives $2Y_q = Y_u + Y_d$ and $[SU(2)_L]^2 U(1)_Y$ gives $3Y_q = -Y_l$. Next $[U(1)_Y]^3$ does not provide any new constraints. So the anomaly conditions themselves are not sufficient to tell us anything about the masses of matter particles or provide quantization of electric charge in the SM.

STEP 2:

Note that thus far the electron (for example) is charge less as well as masssless. One has to provide new physical inputs to proceed further. There are two independent ways to do so.

Method A:

Here one demands that fermions acquire masses through Yukawa coupling in the SM. Note that there is no consistent and non-chiral charge existing so far. Hence electron shall develop a Yukawa mass but has no charge within this structre. This is like demanding Point 1 above. This brings about the following constraints:

$$Y_u = Y_q + Y_\phi ; Y_d = Y_q - Y_\phi ; Y_e = Y_l - Y_\phi \tag{10}$$

Method B:

Simultanoeus to Method A, we have no electron mass (for example) but now in conformity to Point 2 above we obtain the bare charge. So ignoring Yukawa coupling (and hence no mass exists) we impose the vector nature of the electric charge [4,5] which means that photon couples identically to the left handed and the right handed charges. That is $Q_L = Q_R$

$$\frac{1}{2}(1+\frac{Y_q}{Y_\phi}) = \frac{1}{2}\frac{Y_u}{Y_\phi}; giving : Y_u = Y_q + Y_\phi$$

$$Q(d) = \frac{1}{2}(-1+\frac{Y_q}{Y_\phi}) = \frac{1}{2}\frac{Y_d}{Y_\phi}; giving : Y_d = Y_q - Y_\phi$$

$$\frac{1}{2}(-1+\frac{Y_l}{Y_\phi}) = \frac{1}{2}\frac{Y_e}{Y_\phi}; giving : Y_e = Y_l - Y_\phi \tag{11}$$

Hence the new constraints on hyeprcharges in Equation 11 are exactly the same as in Equation 10. So this is like Point 2 above, that is bare charge with no mass terms. Note that Point 1 and 2 arise as simultaneosuly existing mathematical reality bringing in the same constraints on the hypercharges and which are exactly indentical. So all the conditions Point 1 , Point 2, and Point 3 are consistently satisfied in the New Standard Model.

STEP 3:

Note that $2Y_q = Y_u + Y_d$ from the anomaly cancellation condition for $[SU(3)_c]^2 U(1)_Y$ is automatically satisfied here from the Yukawa conditions or the vector charge conditions above. Now using $3Y_q = -Y_l$ from anomaly cancellation, along with simultaneous and independent Yukawa term as well and the non-chiral character of the electric charge, when put in above $[U(1)_Y]^3$, does provide a new constraint of $Y_l = -Y_\phi$. Putting all these together one immediately gets charge quantization in the Standard Model [5,6] as below:

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, Y_q = \frac{Y_\phi}{3}; Q(u) = \frac{1}{2}(1+\frac{1}{3}); Q(d) = \frac{1}{2}(-1+\frac{1}{3})$$

$$u_R, Y_u = Y_\phi(1+\frac{1}{3}); Q(u_R) = \frac{1}{2}(1+\frac{1}{3}); d_R, Y_d = Y_\phi(-1+\frac{1}{3}); Q(d_R) = \frac{1}{2}(-1+\frac{1}{3})$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L; Y_l = -Y_\phi; Q(\nu) = 0, Q(e) = -1; e_R, Y_e = -2Y_\phi; Q(e_R) = -1 \tag{12}$$

On may add a chargeless right-handed neutrino and the the above analysis is not affected. A repetitive structure for the other generations of particles gives charges for the other fermions as well.

Thus we see that the New Standard Model, in contrast to the (Old) Standard Model, is completely consistent with the structure of the bare charge and the bare mass in QED.

Also note that the colour group sits outside the electrowek group in the (old) Standard Model. However, as colour sits inside the expression of the electric charge in the New Standard Model, we should consider it as actually already unified. Here the group structure $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ sits as one complete and unified whole. Take away the colour part and one does not even have an electric charge existing in the New Standard Model anymore!

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