On the pattern of Standard Model fermions and charges

Cris A. Fitch
San Diego, CA
Email: cfitch@alum.mit.edu

Abstract

We observe that the Standard Model's fermions can be mapped onto a 7-bit pattern, and that these bits can be used to calculate the various charges (color, weak isospin, hypercharge, and electromagnetic) for these particles. A geometrical object, the trihepton, is proposed as means of understanding where the pattern of fermions and the simple formulas for the charges come from. Its relationship with the Fano plane from projective geometry is considered. Issues and implications of the model are discussed. A fourth generation of fermions with spin 3/2 and absolute charge (2, 5/3, 4/3, 1) is hypothesized, and it is also suggested that there may be bosons associated with neutrino oscillation.

Introduction

When inquiring as to the fundamental nature of charge, one can reasonably ask why there are the particles we detect with their given values. Why not something else? When we describe sets of particles in three generations, with matter and anti-matter, and with leptons of integer charge and quarks of charge in multiples of one third, where do these patterns come from? Why aren't there two or four generations or fractional charges of one fifth or one seventh?

The Standard Model (SM) has a particular set of fermions, with associated charges with regard to several forces (SU(3)C, SU(2)L, U(1)Y, U(1)Q). One might legitimately ask, why this particular set with these given charges? One example of this is the question, why are the electromagnetic charges of the proton and the electron equal and opposite, exactly, at least as far as we can tell.

Although composite theories of the fermions are out-of-vogue, the questions concerning the zoo of fermions and their charges remain a nagging aspect to the Standard Model. A positive aspect of a successor Beyond Standard Model (BSM) theory would be some explanation of this collection of particles and charges.

In this article we present a geometrical illustration, perhaps bordering on a theory, of natural way to conceive of the current set of fermionic particles and their charges. One aspect of this model is that it predicts a few additional particles that have not been observed - something of a weakness, but in the event of their observation, perhaps a strength.

Let us consider the concept of charge. For a given force there should be a valence on each particle, its charge, such that the force which multiplies the field indicates how the particle will be pushed or pulled (accelerated) by the field. More accurately it may be stated that a fermionic (spin=1/2) field should be affected, more or less with charge, by a vector bosonic field (spin=1) such that eigenstates (valid particle states) are selected with those characteristic valences.

The Standard Model fermions

At the energy of the weak scale we are aware of three vector fields, denoted by the symmetry groups SU(3)C x SU(2)L x U(1)Y, which use charges strong color, weak isospin, and weak hypercharge. At energies below this the electroweak symmetry is broken, yielding SU(3)C x U(1)EM, and the charges strong color and electromagnetic Q. It is useful to note that the weak eigenstates are slightly at odds with the strong and electromagnetic eigenstates.

The Standard Model presents the following zoo of fermionic particles: three generations of leptons and quarks, each with a corresponding anti-matter particle. Each generation has two leptons, an electromagnetic charge Q=-1 particle (e.g. the electron), and a corresponding Q=0 neutrino. Each generation also has two quarks, a charge Q=-1/3 quark (e.g. down) and a charge Q=+2/3 quark (e.g. up). Each quark also comes in three colors (red, green, blue), a property leptons do not possess and as a result they do are not affected by the SU(3)C/strong force. Each particle also comes in a left-handed and right-handed variety, with the exception of the neutrinos, for which we have not detected (and may not be able to detect) the right-handed variety. The right-handed fermions do not interact with the SU(2)L/weak force, and are said to have weak isospin charge t3 of 0. The left-handed fermions are found in pairs, so that (up, down) and (electron,neutrino) have isospin (+1/2, -1/2). A weak hypercharge Y can be assigned to each of the fermions such that Q=Y/2 + t3, and corresponds to a U(1) charge each fermion would have before the main SU(3)xSU(2)xU(1)Y symmetry of the Standard Model is broken by spontaneous symmetry breaking (SSB) into SU(3)xU(1)Q we observe in our everyday low-energy world.

Counting each of these fermionic cases separately, we have 90 different particles, each of which has 4 different charges (C, t3, Y, Q). This is what we seek to illustrate/explain with our model.

Fermion patterns

The late 1970's and early 1980's saw a variety composite fermion models (e.g. - Rashons) to attempt to explain the pattern of particles and charges. Figures 1 and 2 show the fermion patterns involved.

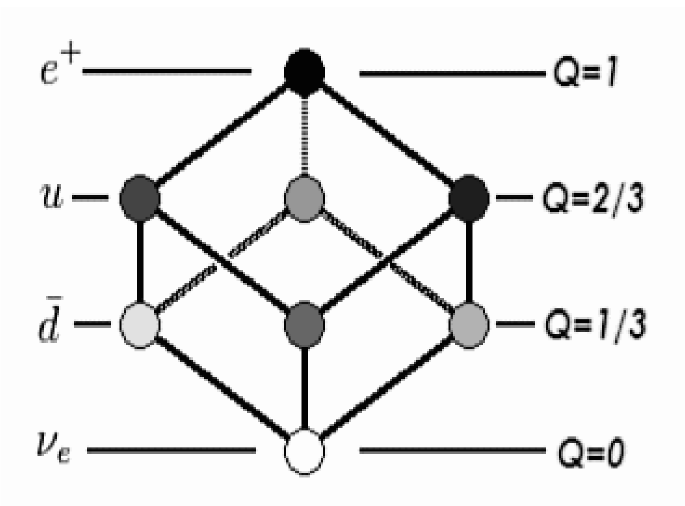


Figure 1: Basic fermion cube (Glashow 1979)

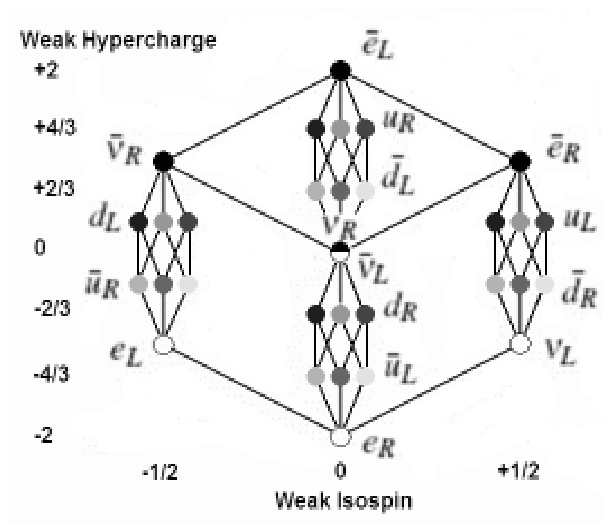


Figure 2: Hypercharge vs weak isospin (Brannen 2004)

The basic fermion cube hints at a bitwise approach to the problem, and a diagram of weak hypercharge vs. weak isospin further argues for it (5 bits). Attempts at incorporating the three generations of fermions lead one from needing 5 bits to needing 7 bits, as in an SO(14) GUT model.

Figures 3 and 4 show a proposed 7-bit mapping of the Standard Model's fermion set.

	000	001	010	100	Q	Q	011	101	110	111	k
000		${}_L e^{+}{}_R$	${}_L \mu^{+}{}_R$	${}_L \tau^{+}{}_R$	1	0	${}_R \bar{\nu}^{\tau}{}_L$	${}_R \bar{\nu}^{\mu}{}_L$	${}_R \bar{\nu}^{e}{}_L$		+3/2
001		${}_R u^{r}{}_L$	${}_R c^{r}{}_L$	${}_R t^{r}{}_L$	2/3	-1/3	${}_L b^{r}{}_R$	${}_L s^{r}{}_R$	${}_L d^{r}{}_R$		+1/2
010		${}_R u^{g}{}_L$	${}_R c^{g}{}_L$	${}_R t^{g}{}_L$	2/3	-1/3	${}_L b^{g}{}_R$	${}_L s^{g}{}_R$	${}_L d^{g}{}_R$		
100		${}_R u^{b}{}_L$	${}_R c^{b}{}_L$	${}_R t^{b}{}_L$	2/3	-1/3	${}_L b^{b}{}_R$	${}_L s^{b}{}_R$	${}_L d^{b}{}_R$		
011		${}_L \bar{d}^{b}{}_R$	${}_L \bar{s}^{b}{}_R$	${}_L \bar{b}^{b}{}_R$	1/3	-2/3	${}_R \bar{t}^{b}{}_L$	${}_R \bar{c}^{b}{}_L$	${}_R \bar{u}^{b}{}_L$		−1/2
101		${}_L \bar{d}^{g}{}_R$	${}_L \bar{s}^{g}{}_R$	${}_L \bar{b}^{g}{}_R$	1/3	-2/3	${}_R \bar{t}^{g}{}_L$	${}_R \bar{c}^{g}{}_L$	${}_R \bar{u}^{g}{}_L$		
110		${}_L \bar{d}^{r}{}_R$	${}_L \bar{s}^{r}{}_R$	${}_L \bar{b}^{r}{}_R$	1/3	-2/3	${}_R \bar{t}^{r}{}_L$	${}_R \bar{c}^{r}{}_L$	${}_R \bar{u}^{r}{}_L$		
111		${}_R \nu^{e}{}_L$	${}_R \nu^{\mu}{}_L$	${}_R \nu^{\tau}{}_L$	0	-1	${}_L \tau^{-}{}_R$	${}_L \mu^{-}{}_R$	${}_L e^{-}{}_R$		−3/2
		l=+1/2					l=−1/2				

Figure 3: 7-bit chart of the fermions

$SU(3)$	$SU(2)_L$	$U(1)_Y$	t_3	q	l	k	h_0
$\square$	$\square$	$+\frac{1}{6}$	$+\frac{1}{2}$	$+\frac{2}{3}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$
			$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$
$\bar{\square}$	$\mathbb{I}$	$-\frac{2}{3}$	0	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$\bar{\square}$	$\mathbb{I}$	$+\frac{1}{3}$	0	$+\frac{1}{3}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$
$\mathbb{I}$	$\square$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{1}{2}$
			$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$-\frac{3}{2}$	$+\frac{1}{2}$
$\mathbb{I}$	$\mathbb{I}$	0	0	0	$-\frac{1}{2}$	$+\frac{3}{2}$	$-\frac{1}{2}$
$\mathbb{I}$	$\mathbb{I}$	$+1$	0	$+1$	$+\frac{1}{2}$	$+\frac{3}{2}$	$+\frac{1}{2}$

Figure 4: Fermion groupings (l,k,h0 cols added to Toledo's table)

The Trihepton

Let us define an "essential eddy" to be a unit circle with two possible states: a flow of +1/2 (in a right-handed direction of circulation) [heads], and a flow of -1/2 (in a left-handed direction of circulation) [tails].

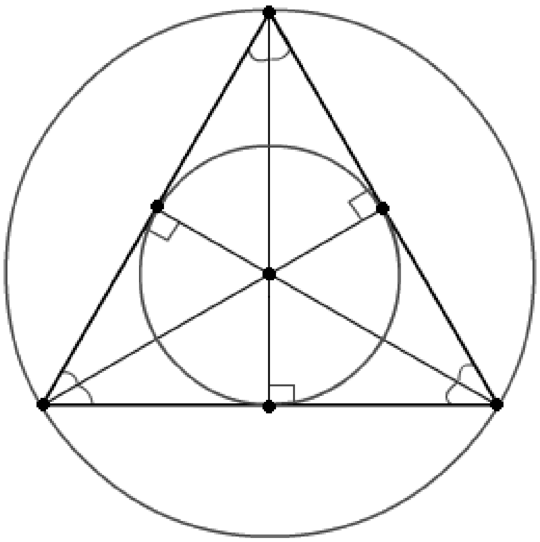


Figure 5: Bisected equilateral triangle

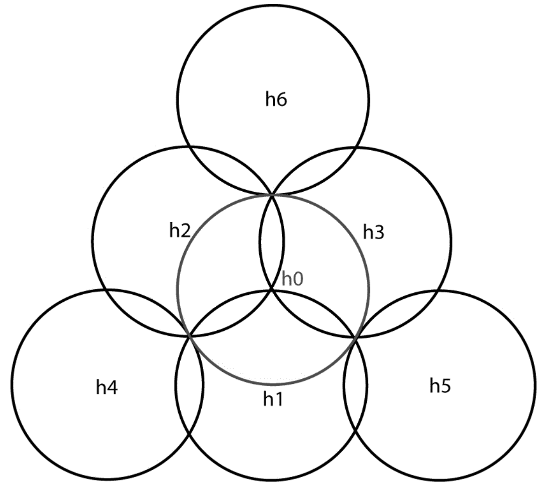


Figure 6: The Trihepton

Consider a bisected equilateral triangle. It has seven points of note: the three vertices, the three halfway points along the sides, and the intersection of the three bisecting lines. We consider the last of these to be the center point (p0), the three halfway points to be the inner points (p1, p2, p3), and the three vertices to be the outer points (p4, p5, p6). It is easily observed that the ratio of the distance from the center point to the outer points is twice the distance from the center point to the inner points. Let us assign the distance from the center point to the inner points to be one.

The "trihepton" is an object consisting of seven essential eddies, each centered on unique points of the bisected equilateral triangle. We label these values h0, h1, h2, h3, h4, h5, and h6. The object is called the trihepton as it is formed of a triangle (tri-) and there are seven eddies (hept-). Note that essential eddy of the central point overlaps with the circle which joins the inner three points. Let us call this circle the unit boundary.

Before moving on, it is worth noting a related basic concept from Projective Geometry discovered over a hundred years ago, the Fano plane. The Fano plane can be imagined as a bisected equilateral triangle with an additional circle (also considered a line) connecting the three inner points. This object then has the properties that each point is in exactly three lines, and that each line goes through only three of these points. It has a total of seven points and seven lines, and there is a fundamental symmetry between the points and lines.

Inside and Outside

The trihepton can have a total of 128 (2^7) possible modes. For each mode we will want to calculate a set of directional flows. If we imagine a scenario in which the unit boundary does not permit transport across it, one can see that there might be observable values describing the directional flow inside and outside of the unit boundary. The exterior total directional flow, including the flow at the boundary itself, is

$$Y = h0 + 2/3(h1+h2+h3) + h4+h5+h6$$

This corresponds with weak hypercharge in the Standard Model.

The interior directional flows depend only on the inner three eddies, and can be expressed as:

R = (red, anti-red) depending on h1
G = (green, anti-green) depending on h2
B = (blue, anti-blue) depending on h3

These correspond with the strong color charges.

There is a third charge of note, the "transterior" flow consisting of the outer three eddy values minus the unit boundary eddy.

2 t3 = h4+h5+h6 - h0

This is twice the third (of Z) component of weak isospin.

An average of the transterior and exterior flows, important as the electroweak symmetry is broken, gives us

Q = h4+h5+h6 + 1/3(h1+h2+h3)

which is just the electromagnetic charge. Note that the central eddy plays no part in the electromagnetic charge.

Spin is given by absolute value of the outer flows.

s = | h4+h5+h6 |

Figure 7: Charges derived from the trihepton

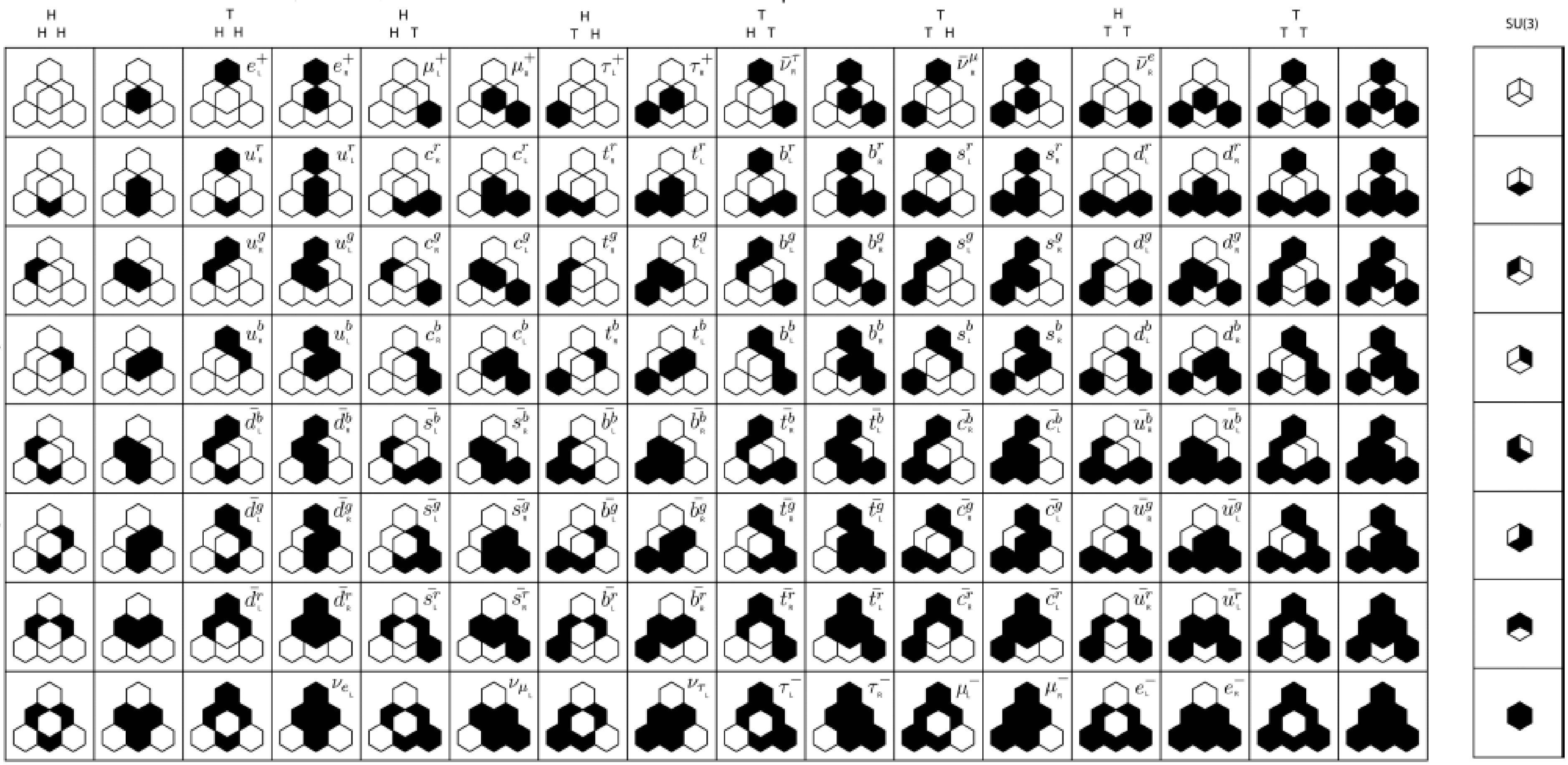
The background field that breaks the SU(2)L x U(1)Y electroweak symmetry is the (spin 0) Higgs field. As it interacts with the trihepton, it causes the central eddy, h0, to flip back and forth. It can do so without changing the electromagnetic charge, Q. This is the mechanism by which these particles acquire mass, and the coupling coefficients which moderate this process are the Yukawa couplings.

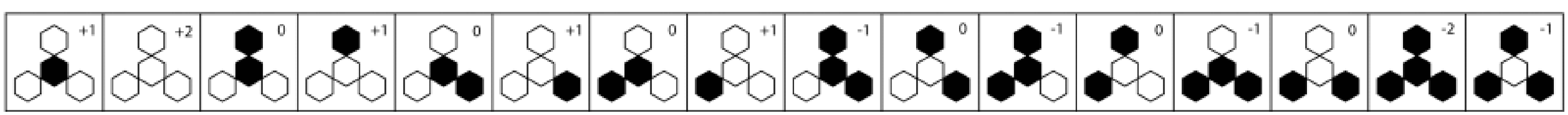
A Fourth Generation of Particles

Certain experimental evidence argues strongly that there should be no fourth generation of fermions within the normal framework. This section will argue that there might be a fourth generation of fermions, but with distinctly different properties such as spin=3/2. Figure 8 provides a look at the full set of modes of the trihepton, including this abnormal fourth generation.

LEP experiments measuring the precise Z0 cross-section were able to determine that only three neutrinos exist with a mass smaller than 45 GeV (half the Z0 mass)[1]. The Particle Data Group database indicates that any additional quark or lepton doublet is ruled out if the difference in mass between up- and down-type fermions is larger than 85 GeV. Recent results with the Higgs boson also argue that there are only three generations of fermions.[2]

Modes of the Trihepton





The missing columns have been filled in with a distinctly different fourth generation, much as leptons are different from the quarks. There are four new particles and their corresponding anti-particles. It is believed that these will be spin 3/2 particles. Most obvious will be a charge +2 particle, the allon, reflective of the fact that all of the bits are on (well, h1-h6 at least). There will be two quark-like particles that are SU(3) triplets, the on and off quark, with charge magnitude +5/3 and +4/3 respectively. Finally there will be a neutrino equivalent, but with charge magnitude +1, the raton.

There are no clues as to what their Yukawa couplings (and thus masses) are, except they are probably larger than what we've yet reached.

Effect of the Bosons

The nature of the transitions between trihepton modes is a result of the absorption or emission of a boson. Interaction with the spin-0 Higgs field will be a change in the center eddy (h0), changing left-handed particles to right-handed particles, or vis-versa. Interaction with photons does not change the mode, nor does an interaction with the weak Z0 boson. The effect of the SU(3)C strong force is to rotate the states of (h1,h2,h3). An interaction with a weak W boson results in a complement/inversion of the states of (h4,h5,h6).

Oscillation of neutrinos can be seen as something similar to the effects of the strong force, except applied to (h4,h5,h6). It may be worth considering the possibility of a vector boson associated with neutrino oscillation, although it's bound to be difficult to detect. One should also note the correspondence here between the CKM matrix associated with quark transitions and the PMNS matrix associated with neutrino oscillation. These describe the shifting of particles from one generation to another through a shift between weak eigenstates and mass eigenstates. In essence rather than the quark pair (u, d) we have (u, d') and there is a slight rotation needed to get to the pure d state.

Unaddressed Questions

This model attempts to illustrate where the pattern of fermion particles and charges come from, but does not address several equally important questions. Where do the Yukawa couplings come from? Where does the SM potential / Higgs field come from? And one can also ponder the source of the CKM and PMNS matrices, which provide details of quark and neutrino oscillation characteristics. And the symmetries of the model are clearly broken in a number of ways, yielding but a few ways for modes to change - at least at our low energies.

Currently most of the modes of the trihepton have been observed. The two exceptions are the modes where the outer flows (h4,h5,h6) have all the same value, and the modes where all charges are zero (right handed-neutrinos). It is understandable that the second category should be difficult to observe, as these particles would not respond to any known detector. The first category should be observable, and should result in particles with electromagnetic charges of magnitudes 1, 4/3, 5/3, and 2 (spin 3/2).

Conclusion

A simple model of fermion substructure has been presented which may help students to better understand the matter particles of our universe. It may be useful for further exploring how the symmetries, forces, matter, and background potential of our universe interact.

References

T. Dorigo, [http://dorigo.wordpress.com/2008/03/25/thou-shalt-have-three-generations/].

O. Eberhardt et al, “Impact of a Higgs Boson at a Mass of 126 GeV on the Standard Model with Three and Four Fermion Generations”, Phys. Rev. Lett. 109, 241802, 2012.

S. L. Glashow, The Future of Elementary Particle Physics, p. 29, Lyman Laboratory of Physics, Harvard University, HUTP-79/A059, 1979.

P. Tanedo, [http://www.quantumdiaries.org/2012/04/17/name-these-brandsplants-name-these-particles/]

H. Harari, “A schematic model of quarks and leptons”, Phys.Lett. B, vol. 86, 83, 1979.

M. A. Shupe, “A composite model of leptons and quarks”,Phys. Lett. B, vol. 86, 87, 1979.

I. D'Souza, C.S. Kalman, Preons: Models of Leptons, Quarks and Gauge Bosons as Composite Objects, World Scientific 1992.

H. Saller, "The central correlations of hypercharge, isospin, colour and chirality in the Standard Model", [arXiv:hep-th/9802112v1].

P. Zenczykowski, “The Harari-Shupe preon model and nonrelativistic quantum phase space”, [arXiv:0803.0223v1 [hep-th]].

M. Everaldo de Souza, “The Higgs-like Bosons and Quark Compositeness”,[arXiv:1301.0130v3 [hep-ph]].

A. G. Lisi, “An Exceptionally Simple Theory of Everything”, [arXiv:0711.0770 [hep-th]].

C. A. Brannen, “The Geometry of Fermions”, [http://www.brannenworks.com/], 2004.

G. Feldman, R. Holman, “SO(14) as a grand unification group and the need for intermediate symmetries”, J. Phys. G: Nucl. Phys. 9 (1983) 7-56.

W.J. Stirling, E. Vryonidou, “Effect of spin-3/2 top quark excitation on t¯t production at the LHC”, [arXiv:1110.1565v1 [hep-ph]].

C. J. C. Burges, H. J. Schnitzer, Nucl. Phys. B228, 464, 1983.

J. H. Kuhn, P. M. Zerwas, Phys. Lett. B147, 189, 1984.

J. H. Kuhn, H. D. Tholl, P. M. Zerwas, Phys. Lett. B158, 270, 1985.

B. Moussallam, V. Soni, Phys. Rev. D39, 1883-1891, 1989.

F. M. L. Almeida, Jr., J. H. Lopes, J. A. Martins Simoes, A. J. Ramalho, Phys. Rev. D53, 3555-3558, 1996. [arXiv:hep-ph/9509364].

D. A. Dicus, S. Gibbons, S. Nandi, [arXiv:hep-ph/9806312].

R. Walsh, A. J. Ramalho, Phys. Rev. D60, 077302, 1999. [arXiv:hep-ph/9907364].

O. Cakir, A. Ozansoy, Phys. Rev. D77, 035002, 2008. [arXiv:0709.2134 [hep-ph]].

L. Randall and R. Sundrum, “A Large Mass Hierarchy from a Small Extra Dimension”, Phys. Rev. Lett. 83, 3370, 1999.

P. B. Pal, "Dirac, Majorana and Weyl fermions", [arXiv:1006.1718v2 [hep-ph]].

Textbooks

P. Binétruy, Supersymmetry: Theory, Experiment, and Cosmology, Oxford University Press, New York, 2006.

D. Griffiths, Introduction to Elementary Particles, Wiley-VCH, New York, 2008.

G. Kane, Modern Elementary Particle Physics, Addison-Wesley Publishing, New York, 1987.

P. Labelle, Supersymmetry Demystified , McGraw Hill, New York, 2010.

M. Herrero, The Standard Model, [arXiv:9812242v1 [hep-ph]].

P. Langacker, Introduction to the Standard Model and Electroweak physics, [arXiv:0901.0241v1 [hep-ph]].

S. F. Novaes, Standard Model: An Introduction, [arXiv:0001283v1 [hep-ph]].

Standard Model

P. A. M. Dirac, “The Quantum theory of electron,” Proc. Roy. Soc. Lond. A 117 (1928) 610–624.
S. L. Glashow, Nucl. Phys. 22 (1961) 579.
S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
A. Salam in Elementary Particle Theory, ed. N. Svartholm (Almquist and Wiksells, Stockholm 1969), 367-377.
C.-N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).
M.Gell-Mann, Phys.Lett.8(1964) 214.
G.Zweig, CERN Report No. 8182/TH 401, (1964).
P. W. Anderson, Phys. Rev. 130, 439 (1963).
P. W. Higgs, Phys. Lett. 12, 132 (1964).
P. W. Higgs, Phys. Rev. 145, 1156 (1966).
F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964).
G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, Phys. Rev. Lett. 13, 585 (1964).
G. 't Hooft, Nucl. Phys. B35, 167 (1971).
G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B50, 318 (1972).
D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973).
H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).
H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. B47, 365 (1973).
N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).
M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
L. Wolfenstein, Phys. Rev. Lett. 51, p. 1945 (1983).